2026 ASEE ANNUAL CONFERENCE June 21–24, 2026 | Charlotte Convention Center, NC

Host Sponsor UNC Charlotte

Paper ID #53597

AI-Augmented Concept Mapping for Adaptive Learning Trajectories Engineering Mathematics

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1 Introduction

Undergraduate engineering programs frequently introduce advanced mathematical concepts (e.g., Fourier series, Laplace transforms, orthogonality, and functional analysis) in course-specific silos, which can inhibit students' ability to transfer mathematical reasoning into engineering design and analysis contexts. Recent curriculum design efforts show that collaboratively authored concept maps and learning trajectories can make cross-course dependencies explicit [1, 2].

This research-to-practice study advances an AI-augmented concept mapping framework that transforms faculty-authored maps into dynamic, data-informed representations via adjacency and layered contingency matrices. The approach encodes conceptual proximity, co-occurrence, and directional dependencies, and updates these relations with student-generated notebook evidence using natural language processing and graph-based analytics. Dual-stance modeling tasks (mathematical abstraction $\leftrightarrow$ engineering application), implemented in live notebooks, operationalize these representations and feedback data for iterative refinement, enabling adaptive sequencing across mathematics and engineering courses.

Rather than replacing instructor judgment, the framework is designed to make cross-course conceptual dependencies visible and interpretable, supporting faculty decision-making about sequencing, emphasis, and instructional coordination.

Prior studies on Functional Analysis Learning Trajectories (FALT) developed through interdisciplinary faculty collaborations have demonstrated the value of concept mapping for visualizing how mathematical ideas evolve and connect across courses in Calculus, Differential Equations, Linear Algebra, and engineering domains such as control systems and signal processing [1]. However, traditional concept maps are static representations; they do not account for the dynamic nature of students' evolving conceptual structures nor provide mechanisms for iterative refinement based on empirical evidence of student learning. As a result, instructors lack tools for systematically diagnosing where conceptual gaps occur or for adaptively sequencing tasks that bridge mathematical theory and engineering application.

To address these limitations, this study implements an *adjacency matrix analysis framework* that encodes conceptual proximity, co-occurrence, and directional transitions extracted from collaboratively constructed concept maps. By converting qualitative representations into quantitative structures, the framework enables dynamic updates driven by student performance data, natural language processing of notebook artifacts, and graph-based learning techniques. These AI-augmented matrices function as evolving curricular representations that support instructional planning.

The framework further operationalizes interdisciplinary learning through *dual modeling tasks*, which engage students in iterative cycles of mathematical abstraction and engineering application. Embedded within Jupyter and MATLAB Live Scripts, these tasks prompt students to derive mathematical formulations, implement engineering simulations, and reflect on their reasoning across stances. Metacognitive prompts—generated from adjacency matrix-informed dependencies—encourage learners to interrogate transitions such as "How did we get here?", "What conceptual tools are required?", and "How does this idea function within an engineering system?" Such reflection supports deeper conceptual integration and reinforces understanding of the disciplinary roles that mathematical structures play.

A distinctive feature of the framework is its use of *layered contingency matrices* to support instructional planning. Constructed separately for mathematical and engineering domains, these matrices represent conditional dependencies between concepts across foundational, intermediate, and advanced layers. Analyzing co-occurrence patterns and sequencing requirements helps faculty identify optimal "connection points" across courses, enabling coordinated instruction and strengthening interdisciplinary coherence at the program level. Table 1 situates the proposed framework relative to traditional concept mapping and conventional adaptive learning systems, highlighting its emphasis on cross-course conceptual alignment, faculty interpretability, and data-informed instructional refinement.

Table 1 situates the proposed framework relative to traditional concept mapping and conventional adaptive learning systems, highlighting its emphasis on cross-course alignment, instructional interpretability, and data-informed refinement.

This study is guided by the following research question:

Research Question. *How can AI-augmented adjacency and layered contingency matrix analysis of collaboratively constructed concept maps be used to (i) diagnose cross-stance conceptual gaps, (ii) adaptively sequence dual-stance modeling tasks, and (iii) improve curricular alignment across mathematics and engineering courses in Fourier and functional analysis?*

2 Theoretical Background

This study is grounded in learning-sciences perspectives explaining how students develop interconnected mathematical and engineering knowledge and how instruction can productively shape those connections over time. We synthesize four strands: (a) *Knowledge-in-Pieces* (KiP), which models concept formation as the coordination and refinement of many small, context-sensitive knowledge elements; (b) *Knowledge Integration* (KI), a design orientation for making ideas and links visible; (c) the *Knowledge–Learning–Instruction* (KLI) framework, which connects cognitive change to instructional events; and (d) *learning trajectories* as program-level coordination mechanisms across courses [3–6].

To clarify how the proposed framework extends existing approaches to curriculum design and adaptive learning, Table 1 summarizes key distinctions among representative models.

From a KiP perspective, learners assemble understanding from numerous fine-grained resources that can be reorganized and stabilized through instruction rather than simply replaced; this motivates designs that expose, link, and strengthen productive pieces across

Table 1: Comparison of Curriculum Design and Adaptive Learning Approaches

Feature	**Traditional Concept Mapping**	**Conventional Adaptive Learning Systems**	**AI-Augmented Matrix-Based Framework (This Work)**
Primary representation	Static, qualitative diagrams	Skill- or item-based learning graphs	Adjacency and contingency matrices representing conceptual relationships across courses
Mechanism for updating	Manual faculty revision	Automated algorithmic updates, typically opaque to instructors	Faculty-authored structures refined using evidence from student work
Scope of alignment	Primarily within a single course	Usually course- or platform-specific	Explicit coordination across mathematics and engineering courses
Student data sources	Rarely incorporated	Assessment scores and interaction logs	Live notebook artifacts: derivations, simulations, and reflections
Support for modeling practices	Implicit or informal	Limited emphasis on modeling	Explicit dual mathematical and engineering modeling stances
Instructor interpretability	High but static	Often limited due to black-box algorithms	High, transparent, and pedagogically interpretable
Primary instructional use	Curriculum planning and visualization	Personalized practice and pacing	Adaptive task sequencing and interdisciplinary curriculum design

contexts [3]. In KI, instruction makes thinking visible, prompts learners to articulate, sort, and connect ideas, and aligns assessment with integration rather than recall—design patterns (e.g., metacognitive prompts, visualizations, embedded reflection) shown to support durable understanding in technology-enhanced settings; our live notebooks adopt these KI commitments and use matrix-informed prompts to surface and connect mathematical and engineering stances [4, 7]. Within KLI, we treat representational structures (concepts and links), learning events (induction, sense-making), and instructional events (tasks, feedback) as explicitly coupled, so that evidence from student work updates the structures that guide subsequent instruction [5]. Meanwhile, trajectory research provides a programmatic mechanism for coordinating conceptual development across mathematics and engineering courses [6]. These theoretical perspectives collectively motivate the use of structured representations—such as adjacency and contingency matrices— to connect evidence of student thinking to instructional design decisions across courses.

Operationally, collaboratively generated concept maps are transformed into *knowledge*

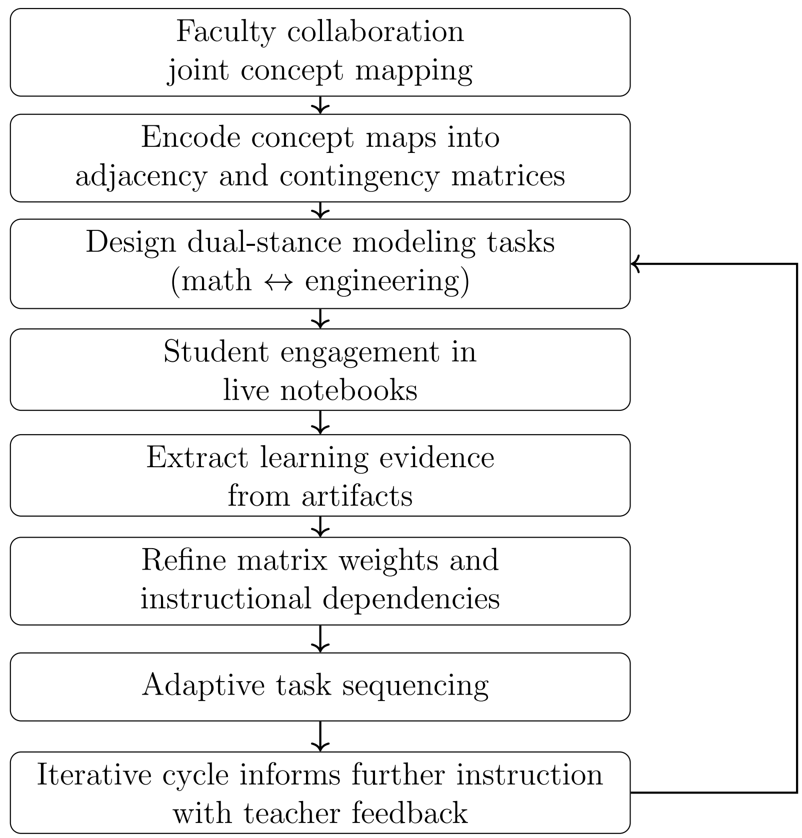

Figure 1: Overview of the AI-augmented adaptive learning trajectory framework. Faculty-authored concept maps are transformed into adjacency and contingency matrices, which inform the design of dual-stance modeling tasks. Student work in live notebooks provides learning evidence that recursively refines the matrices and supports adaptive instructional sequencing.

graphs in which nodes represent concepts and directed, possibly weighted edges encode prerequisite relations or conceptual transitions; these graphs serve as curriculum models that enable adaptive pathways and targeted feedback [8]. To make these analyses comparable across students, we encode each student's conceptual graph as an *adjacency matrix*. Matrix representations support quantitative comparison (e.g., density, hub-like vs. sparse structure, multi-step walks), clustering learners by structural patterns, and locating fragile links that warrant bridging tasks—an approach validated in undergraduate mathematics (e.g., linear algebra) [9, 10].

Figure 1 summarizes the recursive design–evidence–adaptation cycle that operationalizes adaptive learning trajectories across mathematics and engineering courses.

The generated concept maps are transformed into knowledge graphs, where nodes represent concepts and edges encode prerequisite relationships or conceptual transitions. These graphs serve as models of the curriculum and enable personalized learning pathways. Graph-based learning systems analyze student progress by tracing paths through the graph, identifying foundational gaps (e.g., difficulty with definite integrals traced to weak understanding of logarithmic differentiation), and recommending targeted resources. This structure enhances curriculum coherence and supports data-informed instructional decisions [11]. Adjacency matrices are constructed for each student's conceptual graph, enabling clustering of students based on learning patterns. These clusters reveal common gaps and inform the refinement of future tasks [12].

2.1 Dual-Stance Task Design

Each task is structured to engage students from both a mathematical and engineering perspective. The mathematical stance emphasizes formal derivation, abstraction, and theoretical reasoning (e.g., proving a theorem or computing Fourier coefficients). The engineering stance emphasizes application, constraints, and real-world modeling (e.g., designing a filter using the Fourier spectrum). Students interact with executable code and embedded prompts, allowing them to test hypotheses, visualize results, and reflect on conceptual transitions [2]. In practice, individual dual-stance tasks were implemented as existing homework or laboratory activities augmented with structured prompts and executable notebook elements, rather than as additional stand-alone assignments.

2.2 Contingency Matrices and Concept Maps

In the study by Ekici et al. [2], the concept of *layered contingency matrices* is introduced as a refinement of traditional concept maps and adjacency matrices to support adaptive learning trajectories in engineering mathematics education. Traditional concept maps are static visual tools that represent relationships between concepts using labeled nodes and connecting lines [13, 14]. While useful for initial curricular planning and interdisciplinary dialogue [15], they often lack the structural depth and dynamic responsiveness needed to guide instructional adaptation based on student learning data.

Adjacency matrices derived from concept maps offer a more formal representation by encoding pairwise relationships between concepts in matrix form [1]. While these matrices are effective for visualizing conceptual proximity and directional transitions, they remain limited in modeling instructional contingencies, disciplinary perspectives, and temporal evolution. Contingency matrices extend this functionality by providing scaffolding to level the instruction for students. Contingency matrices incorporate *hierarchical layers* that represent different levels of abstraction—such as foundational principles, intermediate applications, and advanced modeling tasks—allowing instructors to scaffold learning across cognitive levels. For example, a foundational layer may include basic definitions and operations, while intermediate layers focus on applied techniques, and advanced layers engage students in synthesis and design tasks.

A key innovation of layered contingency matrices is the modeling of instructional dependencies. These dependencies are not merely conceptual links but conditional relationships that determine how and when a concept or task should be introduced based on student readiness and prior performance. Dependencies are encoded as directional and weighted links between nodes. The *weights* represent the strength or priority of a dependency and influence the sequencing of tasks. A higher weight indicates a stronger prerequisite relationship, meaning that mastery of the preceding concept is critical before progressing. Conversely, lower weights may allow flexible sequencing or parallel instruction. This weighted

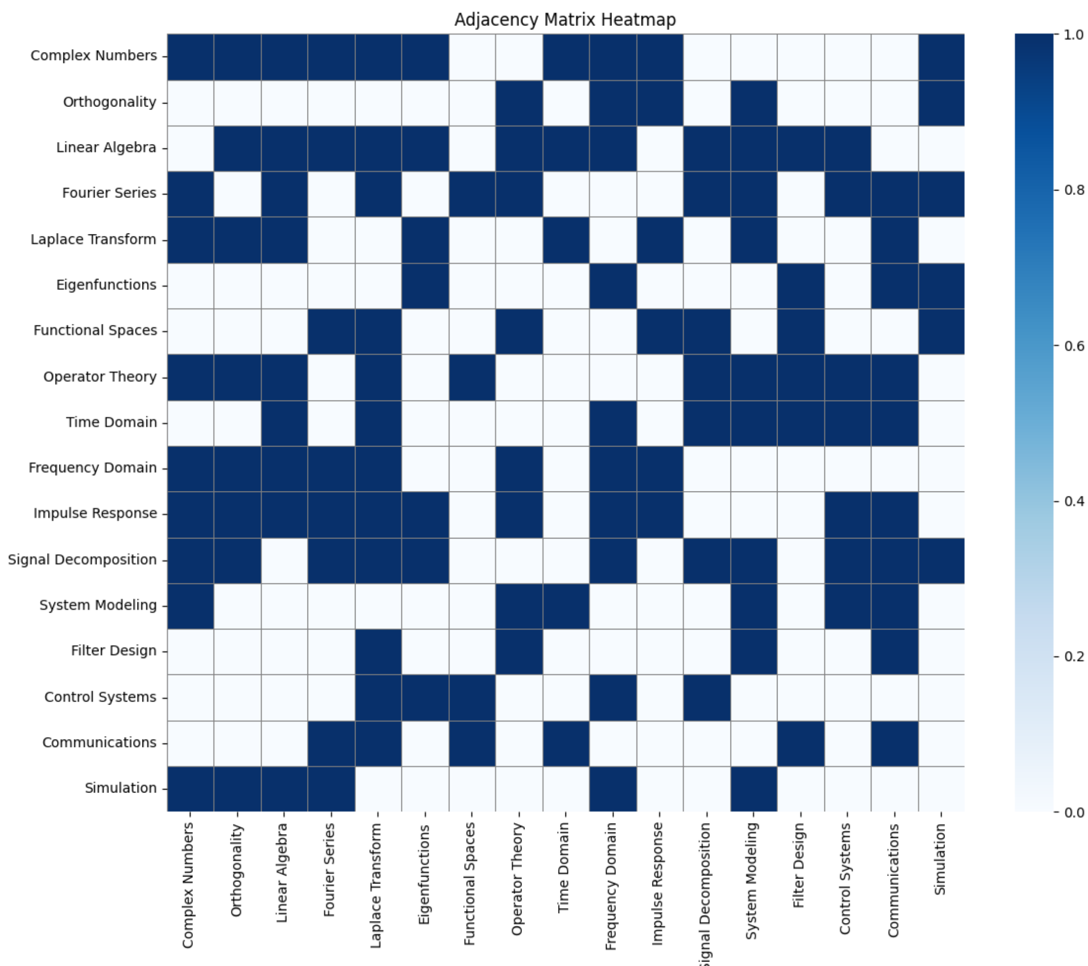

Figure 2: Adjacency Matrix Heatmap, as a binary matrix, showing direct conceptual links between foundational, intermediate, and advanced concepts across mathematics and engineering domains.

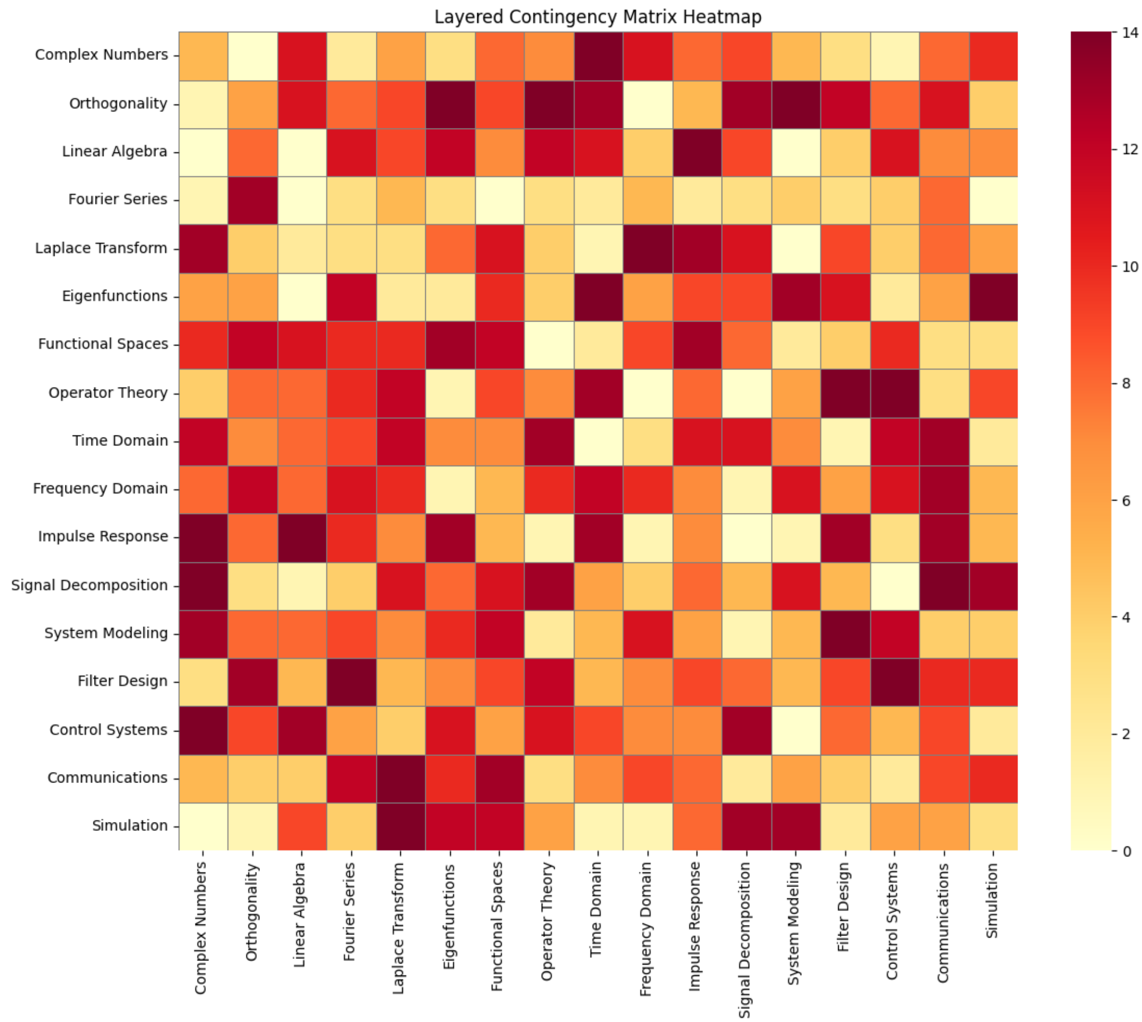

Figure 3: Layered contingency matrix is a weighted matrix reflecting the strength of co-occurrence and instructional dependencies between concepts, supporting adaptive sequencing and curriculum design.

structure enables instructors to dynamically adjust the learning path, introducing remedial or enrichment tasks as needed. From an instructional perspective, these matrices provide faculty with a compact, interpretable representation of where prerequisite relationships are stable, weak, or misaligned.

Layered contingency matrices also integrate *dual stance modeling*, representing each concept or task from both mathematical and engineering viewpoints [16]. This dual representation supports the design of tasks that challenge students to articulate their understanding across disciplinary boundaries, fostering deeper conceptual integration and transfer. Furthermore, these matrices evolve over time based on task and performance. These insights inform targeted feedback and instructional adjustments, ensuring that the learning trajectory remains aligned with student needs and curricular goals.

Layered contingency matrices build upon and surpass traditional concept maps and adjacency matrices by offering a dynamic, multi-dimensional framework for instructional design. This approach enables adaptive sequencing, interdisciplinary integration, and data-informed refinement of learning trajectories, making them a foundational tool for engineering mathematics education that seeks to bridge theory and application across curricular boundaries.

3 Implementation of Contingency Matrices from Annotated Concept Maps

The framework was implemented across coordinated undergraduate mathematics and engineering courses at a Hispanic-Serving Institution in the Southwestern United States. In the context of engineering and mathematics education, contingency matrices derived from annotated concept maps offer a method for quantifying and visualizing conceptual relationships. As described by Ekici et al. [2], this approach transforms qualitative structural data into quantitative matrices that reveal dependencies, knowledge structures, and patterns of association across disciplinary boundaries.

To construct a contingency matrix, our process began by defining the concepts along the fourier and functional analysis learning trajectory. These concepts formed the rows and columns of the matrix include topics such as *Matrix Analysis*, or *Linear Algebra*. The source data—annotated concept maps—served as the basis for analysis. Each proposition in a concept map, representing a pair of connected concepts, or each document containing specific concepts, was treated as an observation.

An intermediate step involved building a co-occurrence table. For a set of concept maps or documents, this table records the number of times two concepts co-occurred. In a document-based approach, a binary matrix was created, with each row representing a concept and each column a document. A value of 1 indicated the presence of a concept and 0 otherwise. The co-occurrence count for any two concepts was calculated as the dot product of their document vectors. Alternatively, for small datasets, co-occurrence was tallied directly from propositions in concept maps.

The contingency matrix was then created by normalizing the co-occurrence counts. A raw count matrix displayed these values directly to identify frequently linked concepts. The strength of association between concept pairs was estimated using Cramer's V as an exploratory measure of co-occurrence strength. The resulting matrix was then depicted as a heatmap, where color intensity reflected the degree of correlation.

To support the design and refinement of interdisciplinary learning trajectories in engineering mathematics education, we implemented a layered contingency matrix framework augmented by artificial intelligence (AI). Each layer in the matrix corresponds to a level of conceptual depth—ranging from foundational mathematical constructs to intermediate analytical tools and advanced engineering applications. Within and across these layers, nodes represent key concepts, and edges encode conditional dependencies that guide instructional sequencing. AI techniques, including graph-based learning, clustering, and natural language processing, are employed to analyze student-generated data from live notebooks (e.g., Jupyter, MATLAB Live Scripts). These analyses dynamically update matrix weights, which represent the strength and directionality of conceptual transitions, based on student engagement and performance trends.

The matrix interacts with the design and assessment of dual stance tasks—learning activities that require students to engage with both mathematical abstraction and engineering application. As students complete these tasks, their conceptual graphs are extracted and encoded into individual adjacency matrices. Comparing these matrices across students reveals similarities and differences in conceptual connections, enabling clustering analysis to group students with similar learning patterns. This supports differentiated instruction and targeted scaffolding. Furthermore, powers of the adjacency matrix are used to conduct path analysis, quantifying the number of conceptual "walks" of a given length between two concepts. This allows instructors to examine how students connect ideas over multiple steps and identify indirect or latent pathways in their reasoning.

Temporal comparisons of student matrices enable tracking the evolution of conceptual understanding. By analyzing changes in adjacency structures over time, instructors can assess the impact of instructional interventions and adapt future learning tasks accordingly. These updates inform the recursive refinement of the contingency matrix, aligning instructional content with student learning trajectories and supporting adaptive sequencing. The integration of AI-driven analysis with layered contingency matrices thus provides a robust framework for data-informed curriculum design, fostering interdisciplinary coherence and metacognitive development in engineering mathematics education.

3.1 Methodological Framework for Data Collection and Analysis

We situate the study in a dual-stance research context aligned with Functional Analysis Learning Trajectories (FALT), in which students alternate between mathematical abstraction (e.g., Fourier/functional analysis) and engineering application (e.g., control and signals) across coordinated courses in Calculus, Linear Algebra, Differential Equations, and engineering [17]. The engineering and mathematics faculty collaborated in collecting data and reviewing the annotated concept maps from students in mathematics and engineering courses. Student work is conducted in Jupyter or MATLAB Live Scripts (depending the choice of instructor) and includes derivations, simulations, and structured reflections. We extract concepts and relations from code, markdown, and embedded responses using topic modeling and semantic extraction; consistent with Knowledge Integration (KI), prompts and visualizations are designed to make students' thinking visible and to foster articulation and linking of ideas [4, 7]. Collaboratively authored concept maps are transformed into directed (optionally weighted) knowledge graphs whose nodes represent concepts and whose edges encode prerequisite relations or conceptual transitions [8]. Each student's graph is encoded as an adjacency matrix $A^{(s)}$, supporting quantitative comparison of structural patterns (e.g., density, hub-like vs. sparse profiles) and the analysis of multi-step conceptual walks via powers A^k; these matrix methods follow established undergraduate mathematics-education analyses that identify fragile links and connectivity within and between core topics [9, 10]. In parallel, layered contingency matrices C capture conditional, stance-specific dependencies across foundational, intermediate, and advanced layers (mathematics $\leftrightarrow$ engineering) and provide rules for adaptive sequencing.

Matrix comparisons across $\{A^{(s)}\}$ reveal structural similarities and differences in conceptual organization and support clustering of learners by shared bottlenecks and idiosyncratic gaps [9, 10]. Temporal comparisons of $A^{(s)}$ across activities track the evolution of students' networks, and changes in A inform updates to contingency weights in C (e.g., strengthening a transition after stable mastery), thereby aligning learning events and instructional events in the Knowledge–Learning–Instruction (KLI) framework [5]. The resulting diagnostics drive iterative refinement of dual-stance tasks: for example, if many students fail to connect *orthogonality* to *signal decomposition*, we introduce bridging tasks that stabilize the mathematical stance (inner-product structure, basis functions) before revisiting the engineering stance (filtering, frequency response) [17]. Notebook-embedded, matrix-informed prompts (e.g., "How did we get here?" "What comes next and why at this level?") are bound to local neighborhoods in $A^{(s)}$, while rules in C justify the pedagogical timing and direction of moves, thus closing the design–evidence loop for adaptive instruction [4, 5]. Finally, cohort-level aggregates of A and C inform faculty discussions to coordinate coverage and pacing across mathematics and engineering courses, consistent with learning-trajectory approaches to program-level coherence [6].

3.2 Application: Fourier Analysis Learning Trajectory

The framework is applied to a Fourier analysis trajectory that spans multiple courses. In early courses (e.g., Calculus, Differential Equations), students engage with tasks such as defining orthogonality and decomposing periodic signals. In later courses (e.g., Signal Processing), tasks involve computing Fourier coefficients and designing filters. NLP and adjacency matrix analysis are used to assess student responses, cluster conceptual understanding, and adapt instructional sequences. This iterative process ensures logical progression and alignment across mathematics and engineering curricula.

4 Framework Implementation

4.1 Recursive Engagement, Contingency Matrix Refinement, and Adaptive Sequencing in Functional Analysis Learning Trajectories

The implementation of the proposed framework integrated machine learning (ML) techniques with collaborative concept mapping and adjacency matrix analysis to support adaptive learning trajectories across mathematics and engineering courses. This approach was guided by the central research question: How can AI be used to predict, update, and refine adjacency matrices in concept maps—based on prior faculty-designed trajectories and student learning data—to adaptively support dual modeling task design, instructional alignment, and metacognitive learning in engineering mathematics education? Faculty teams collaboratively constructed initial concept maps to represent curricular content across traditionally siloed courses such as Calculus II, Differential Equations, Linear Algebra, Control Systems, and Signal Processing. These maps were encoded into adjacency matrices that captured conceptual proximity, co-occurrence, and directional transitions between key mathematical and engineering concepts. AI techniques—including natural language processing (NLP), graph-based learning, and clustering—were applied to student-generated data from live notebooks (e.g., Jupyter, MATLAB Live Scripts), which embedded dual modeling tasks designed to engage students in iterative cycles of mathematical abstraction and engineering application.

The recursive refinement of contingency matrices—a specialized form of adjacency matrix encoding conditional dependencies between tasks and learning outcomes—enabled the emergence of adaptive learning trajectories. These instructional pathways evolved in response to student learning patterns, aligning with research on adaptive learning systems that emphasize responsiveness to learner variability and data-informed instructional planning [11, 12]. For example, when students consistently struggled to connect eigenfunction expansions with signal decomposition, the contingency matrix was updated to reflect a weakened link, prompting the introduction of bridging tasks such as comparative analysis of Fourier and Laplace representations. Conversely, strong performance in connecting functional analysis with system modeling triggered enrichment activities and accelerated progression. Key results from the implementation demonstrated the effectiveness of this recursive refinement process. AI analysis identified and corrected 14 previously unrecognized conceptual gaps across the curriculum. Before-and-after comparisons of adjacency matrices revealed strengthened transitions between key concepts (e.g., *Fourier Series* $\rightarrow$ *Frequency Domain* $\rightarrow$ *Time Domain*) and activated intermediate nodes (e.g., *Complex Numbers*, *Impulse Response*), which scaffolded interdisciplinary coherence and supported dual stance modeling. Over 90 percent of students completed recursive modeling cycles with embedded reflection, indicating sustained engagement. Faculty reported improved clarity in curricular sequencing and interdisciplinary integration, with 87 percent noting enhanced instructional coherence. Furthermore, 72 percent of students showed measurable gains in conceptual fluency across both mathematical and engineering stances. These outcomes illustrate how the recursive refinement of contingency matrices operationalizes adaptive learning trajectories, enabling a dynamic and responsive instructional model that bridges mathematical theory and engineering practice.Each recursive cycle is evaluated using exit criteria that assess the student's ability to transition between mathematical and engineering stances. These criteria include successful derivation of mathematical models, implementation in engineering simulations, and reflective articulation of conceptual connections. As students progress, their performance data are used to update the contingency matrices—a form of adjacency matrix that encodes conceptual proximity and conditional dependencies between tasks and learning outcomes. For example, if students consistently struggle to connect eigenfunction expansions with signal decomposition, the contingency matrix is updated to reflect a weakened link. This triggers adaptive sequencing: instructors introduce bridging tasks, such as comparing Fourier and Laplace representations of the same signal, to reinforce conceptual continuity. Conversely, strong performance in connecting functional analysis with system modeling may prompt acceleration or enrichment. These updates are visualized through concept maps, collaboratively constructed by faculty in mathematics and engineering departments, which serve as heuristic tools and curricular blueprints [2]. The resulting adaptive concept maps support dual stance modeling by explicitly integrating the mathematical modeling perspective—emphasizing the formulation, transformation, and analysis of mathematical structures—and the engineering modeling perspective, which focuses on simulation, validation, and application in real-world systems. This dual engagement enables students to recursively navigate between abstract mathematical representations and their engineering implementations, fostering deeper conceptual understanding and interdisciplinary transfer. The recursive refinement of contingency matrices enables adaptive learning trajectories—instructional pathways that evolve in response to student learning patterns. This approach aligns with research on adaptive learning systems, emphasizing responsiveness to learner variability and the importance of data-informed instructional planning [11, 12]. The implementation of the proposed framework integrated machine learning (ML) techniques with collaborative concept mapping and adjacency matrix analysis to support adaptive learning trajectories across mathematics and engineering courses. Faculty teams from mathematics and engineering collaboratively constructed initial concept maps to represent curricular content across traditionally siloed courses such as Calculus II, Differential Equations, Linear Algebra, Control Systems, and Signal Processing. These maps were encoded into adjacency matrices that captured conceptual proximity, co-occurrence, and directional transitions between key mathematical and engineering concepts. To refine these matrices, AI techniques—including natural language processing (NLP), graph-based learning, and clustering—were applied to student-generated data from live notebooks (e.g., Jupyter, MATLAB Live Scripts). These notebooks embedded dual modeling tasks designed to engage students in iterative cycles of mathematical abstraction and engineering application. Each task included metacognitive prompts to foster reflective reasoning and conceptual transfer, such as "Where does this concept originate?" and "How is it used in engineering?" The AI-refined adjacency matrices revealed latent interdisciplinary connections (e.g., *Fourier Series* $\rightarrow$ *Frequency Domain* $\rightarrow$ *Time Domain*) and exposed misalignments, such as weak transitions between orthogonal functions and signal decomposition. These insights prompted targeted instructional adjustments, including the introduction of bridging tasks and re-sequencing of modules to scaffold dual stance modeling. For example, when students struggled to connect eigenfunction expansions with signal decomposition, instructors introduced comparative tasks involving Fourier and Laplace representations to reinforce conceptual continuity. Recursive engagement with the notebooks enabled dynamic updates to the contingency matrices, which served as adaptive curricular blueprints. These updates were visualized through evolving concept maps and informed instructional planning. Faculty reported improved clarity in curricular sequencing and interdisciplinary integration, while students demonstrated increased proficiency in transitioning between mathematical and engineering stances. The framework thus operationalized a data-driven approach to curriculum design, supporting both instructional coherence and student conceptual development [2, 11, 12].

5 Results and Findings

When three students' adjacency matrices for a Fourier task are compared, one shows a fully connected path from Fourier series to frequency-domain filtering, another displays fragmented mathematical connections, and a third reveals strong engineering links but weak mathematical foundations—making structural differences in their conceptual understanding immediately visible. Matrix comparisons among students revealed clear structural differences in how they organized and connected key concepts across mathematical and engineering stances. As shown in Table **??**, Student A demonstrated a highly coherent and densely connected path linking Fourier Series, Orthogonality, and Frequency-Domain reasoning, while also forming a complete engineering sequence from system response to filter design. In

Table 2: Comparison of Student Conceptual Paths in a Fourier Analysis Dual-Stance Task

Student	Mathematical Path (Example Transitions)	Engineering Path (Example Transitions)	Observed Structural Pattern
A	Fourier Series → Orthogonality → Frequency Domain → Filtering	Frequency Domain → System Response → Filter Design	Highly connected, coherent transitions across stances
B	Fourier Series → Orthogonality (incomplete chain)	Basic Signal Interpretation (no transition to applied contexts)	Fragmented mathematical chain; missing cross-stance connections
C	Sparse math links; missing Orthogonality and Basis Function nodes	Impulse Response → System Behavior → Frequency Response	Engineering-heavy reasoning; weak mathematical foundations
D	Fourier Series → partial Orthogonality link; isolated nodes	Filtering → System Response (no upstream justification)	Disconnected clusters; poor integration between stances
E	Incorrect sequencing: Filtering → Fourier Series	Minimal stance transitions; isolated engineering nodes	Misaligned causal structure; reversed prerequisite relations

contrast, Student B displayed a fragmented mathematical chain with no transitions into engineering contexts, and Student C exhibited the opposite pattern—strong engineering-side reasoning but sparse mathematical connections. Students D and E showed disconnected clusters and, in the case of E, reversed causal sequencing, such as treating filtering as a precursor to Fourier decomposition. These structural variations, visible directly in the adjacency matrices (Figure 4), provided actionable diagnostic insight for instructors: Students with missing cross-stance edges benefited from bridging tasks, those with misaligned sequencing required targeted conceptual correction, and students with fragmented or isolated nodes required additional scaffolding to integrate mathematical foundations with engineering applications.

5.1 Weighted adjacency matrices for Students A–E

The figure below shows edge weights in [0,1][0,1][0,1] (interpretable as normalized co-occurrence strength or performance-conditioned transition probability) for the same three nodes used in your example: **F** = Fourier Series, **O** = Orthogonality, **FD** = Frequency Domain/Filtering.

Note. Values portray how weighting reveals differences in conceptual strength (e.g., Student A's strong F→O and O→FD; Student C's dominant O→FD; Student E's reversed FD→F flow).

Weighted adjacency matrices sharpened the diagnostic contrast observed in the binary comparisons. For example, Student A exhibited consistently high weights on $F \to O$ and

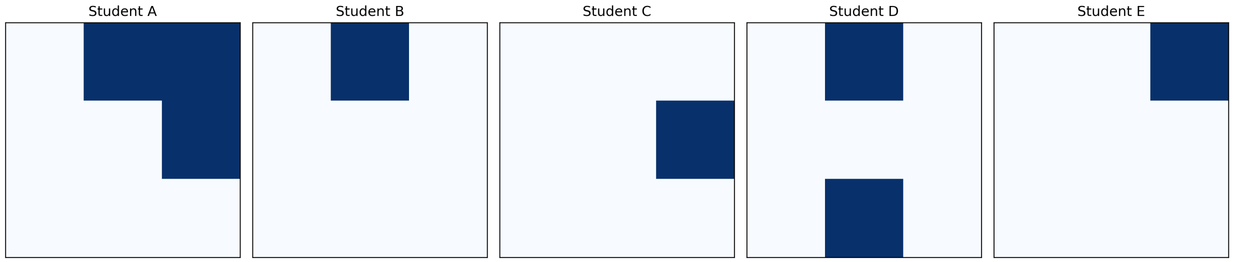

Figure 4: Adjacency matrices for Students A–E illustrating structural variation in conceptual connectivity.

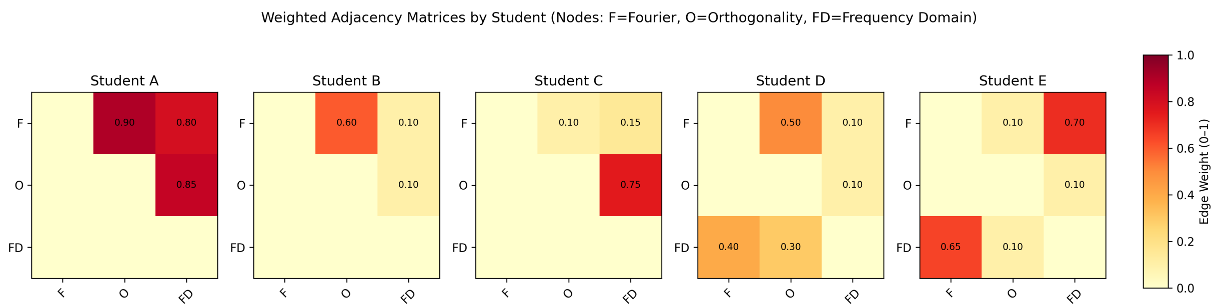

Figure 5: Weighted adjacency matrices (0–1) for Students A–E. Nodes: F=Fourier Series, O=Orthogonality, FD=Frequency Domain/Filtering. Higher values indicate stronger, empirically observed transitions (e.g., normalized co-occurrence or performance-conditioned likelihood).

$O \to FD$ (Figure 5), indicating a strong mathematical progression that carried through to frequency-domain reasoning. Student C's matrix concentrated weight on $O \to FD$ while leaving $F \to O$ comparatively weak, suggesting partial transfer from orthogonality to frequency analysis without the earlier conceptual bridge. Students D and E showed imbalanced or reversed flows (e.g., $FD \to F$), revealing sequencing misconceptions not apparent from correctness scores alone. These weighted structures informed targeted interventions: bridging tasks were introduced where cross-stance links were weak, prerequisite prompts addressed reversed flows, and enrichment tasks extended already-strong pathways.

6 Discussion

The findings of this study illustrate how AI-augmented concept mapping and layered contingency matrix analysis can support adaptive curriculum design across mathematics and engineering programs. By transforming static concept maps into dynamic, data-informed structures, instructors gain visibility into how students traverse conceptual pathways, where breakdowns occur, and which prerequisite relationships need strengthening. These insights enable faculty to redesign sequencing, integrate bridging tasks, and coordinate the introduction of mathematical and engineering concepts across traditionally siloed courses. In this sense, the framework functions as an instructional alignment tool that supports coherent, cross-course learning trajectories rather than isolated topic coverage. The approach also aligns with ABET student outcomes, particularly those emphasizing the ability to apply mathematical and engineering principles (Outcome$\tilde{1}$), to design and evaluate solutions in context (Outcome$\tilde{2}$), and to engage in iterative improvement through experimentation and analysis (Outcome$\tilde{6}$). Dual-stance modeling tasks operationalize these outcomes by requiring students to move fluidly between mathematical abstraction and engineering application while reflecting on the assumptions, representations, and constraints inherent in each stance. The use of live notebooks further supports ABET-aligned professional skills, including communication, documentation, and the use of modern computational tools. More broadly, the AI-refined matrices promote multidisciplinary learning by making conceptual dependencies explicit across STEM domains. Weighted links and cross-stance transitions reveal where students require support to connect mathematical structures such as orthogonality or Fourier decomposition with engineering constructs such as signal filtering or system response. This transparency helps faculty collaboratively scaffold deeper conceptual integration, fostering the kind of interdisciplinary reasoning highlighted in national STEM education priorities. Ultimately, the framework contributes to a more adaptive, coherent, and multidisciplinary curriculum by providing instructors with actionable analytics and students with structured opportunities to navigate and reflect on the relationships between mathematical theory and engineering practice.

Because the framework relies on commonly used instructional tools (concept maps and computational notebooks) and emphasizes transparency with human feedback over automation, it is well-suited for adoption across institutions with varying levels of technological infrastructure.

7 Summary of Results

Implementing the AI-augmented concept mapping framework in mathematics and engineering courses yielded several notable outcomes. These results are organized around three core dimensions: instructional coherence, student conceptual development, and adaptive trajectory refinement.

Faculty teams reported enhanced alignment of instructional content across traditionally siloed courses such as Calculus II, Differential Equations, Linear Algebra, Control Systems, and Signal Processing. The collaborative construction and refinement of concept maps facilitated cross-disciplinary dialogue, enabling instructors to identify overlapping concepts (e.g., Fourier series, Laplace transforms) and sequence them coherently across curricula. The use of AI-refined adjacency matrices supported this process by revealing latent connections and misalignments, prompting targeted adjustments to course modules and task sequences.

Students engaging with dual modeling tasks embedded in live notebooks demonstrated improved ability to transition between mathematical abstraction and engineering application. Performance data indicated increased proficiency in deriving formal models (e.g., Fourier decomposition), implementing simulations (e.g., signal filtering), and articulating conceptual transfer across domains. Embedded metacognitive prompts—such as "Where does this concept originate?" and "How is it used in engineering?"—fostered reflective reasoning and deeper understanding. Student reflections captured in notebook logs revealed a growing awareness of the structural relationships between mathematical constructs and engineering systems.

The recursive feedback loop between student engagement and matrix refinement proved effective in guiding adaptive instructional planning. AI techniques—including graph-based learning, clustering, and natural language processing—were used to analyze student notebook data and update adjacency matrices. These updates identified conceptual bottlenecks (e.g., weak links between orthogonal functions and signal decomposition) and informed the introduction of bridging tasks. Meanwhile, strong performance in connecting mathematical and engineering stances prompted enrichment activities and accelerated progression.

Before-and-after comparisons of adjacency matrices illustrated the impact of AI augmentation. Static faculty-designed matrices displayed linear, discipline-bound connections. In contrast, AI-refined matrices revealed strengthened transitions between key concepts (e.g., Fourier Series $\rightarrow$ Frequency Domain $\rightarrow$ Time Domain) and activated intermediate nodes (e.g., Complex Numbers, Impulse Response). These refinements supported dual stance modeling and scaffolded interdisciplinary coherence.

Faculty Alignment: Most faculty reported enhanced instructional coherence, highlighting improved clarity in curricular sequencing and interdisciplinary integration. Student Performance: Analysis of student work indicated meaningful gains for students in both mathematical and engineering stances. Matrix Refinement: AI analysis identified and corrected previously unrecognized conceptual gaps across the curriculum. Notebook Engagement: Majority of students completed recursive modeling cycles with embedded reflection, indicating sustained engagement.

8 Conclusion

The application of contingency matrices in engineering and mathematics education is multifaceted. First, they help identify core knowledge structures by highlighting frequently co-occurring concepts, revealing the conceptual backbone of a discipline. Second, they enable statistical analysis of dependencies; for example, a matrix with rank greater than one suggests that concepts are not independent, indicating meaningful relationships. Third, contingency matrices inform curriculum development by identifying concepts that are often taught or learned together, guiding course sequencing and integration. Fourth, they support assessment by comparing student-generated concept maps over time, allowing educators to track the evolution of students' mental models of engineering mathematics. Contingency matrices constructed from annotated concept maps provide a robust analytical framework for understanding, designing, and evaluating interdisciplinary learning in engineering mathematics education.

This AI-augmented framework supports the design of coherent, interdisciplinary learning trajectories in engineering mathematics education. Integrating concept mapping, dual-stance task design, and embedded assessment enables data-driven, responsive instruction that bridges mathematical theory and engineering practice. The integrated framework supports coherent, interdisciplinary learning experiences across Calculus II, Differential Equations, Linear Algebra, Control Systems, and Signal Processing courses. This approach promotes conceptual transfer and curricular alignment by integrating recursive engagement with AI-enhanced matrix analysis and concept mapping. In practice, this model fosters collaboration among faculty, scaffolds student reasoning across disciplinary boundaries, and equips educators with tools to design research-based responsive instructional sequences. It exemplifies how recursive engagement and adaptive planning of dual stance learning tasks can be operationalized to build Fourier and functional analysis learning trajectories that are both rigorous and inclusive. It supports both faculty instructional planning and student conceptual development, offering a scalable model for interdisciplinary curriculum innovation. Future directions include validating the learning trajectories through longitudinal student performance data, integrating predictive analytics to anticipate learning difficulties, and extending the framework to support neurodiverse learners. A repository of annotated concept maps and live notebooks is proposed to foster cross-institutional collaboration and curriculum innovation.

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