

Reversing Thermal-Fluid Problems to Enhance Conceptual Understanding

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Abstract

Undergraduate engineering instruction in the thermal-fluid sciences has long relied on forward problem-solving: students are given a scenario and tasked with deriving a solution from governing equations such as the conservation of energy. Widely adopted textbooks in thermal-fluid courses reinforce this approach by providing structured procedures that scaffold student analysis.

The forward problem-solving approach aligns well with problem-based learning and has proven effective in reinforcing procedural knowledge. However, this approach may limit opportunities for deeper conceptual understanding. Students frequently struggle to connect physical systems to the corresponding symbolic equations that govern system behavior in thermal-fluid courses.

To address this gap, this paper introduces a reverse problem-solving framework implemented across three mechanical engineering courses: thermodynamics, fluid mechanics, and heat transfer. In this approach, students are presented with a final symbolic equation and asked to construct a plausible problem statement with a description of the physical system, schematic, and set of simplifying assumptions that would lead to that result. This inversion of the traditional process draws on the Concrete–Representational–Abstract (CRA) model, rooted in Bruner’s theory of cognitive development, to strengthen connections between physical phenomena (concrete), system schematics (representational), and governing equations (abstract).

Reverse problem-solving encourages students to critically examine boundary conditions, assumptions, and system behavior, promoting systems-level thinking and conceptual integration. It also supports the development of metacognitive skills by shifting the instructional focus from solution execution to problem formulation. This paper includes samples for implementing reverse problem-solving in thermal-fluid courses and initial findings. Future work will further assess the impact of reverse problem-solving on student learning outcomes.

Introduction & Motivation

This paper introduces an adaptable teaching technique described as reverse problem-solving and provides examples used in thermal-fluid courses. However, the technique can be adapted for courses throughout the engineering curriculum. The technique was developed in response to common challenges faced by students in thermal-fluid courses where active learning, concept maps, and other pedagogical approaches had previously been adopted. The challenges included correctly identifying a system and connecting that system to the appropriate analytical model.

The idiom to “know something backwards and forwards,” indicates thorough knowledge of a topic. However, engineering education often emphasizes forward problem-solving, where students are given a scenario and tasked with deriving a solution from general governing equations. Widely adopted textbooks reinforce this approach by providing structured procedures

that scaffold student analysis. Educational tools, like flow charts and decision trees, outline a forward problem-solving approach for students to follow.

The forward problem-solving approach aligns well with problem-based learning and has proven effective in reinforcing procedural knowledge. However, this approach may limit opportunities for deeper conceptual understanding. Pedagogical tools like flow charts have been criticized as impeding student learning and encouraging over-reliance on a tool in place of true understanding. Reverse engineering, backwards thinking, backwards design, backwards reasoning, and reverse brainstorming are different applications of starting with an end result and working backwards. Such methods force a new perspective, spark creativity, and engage new neural pathways by disrupting the forward-thinking pattern. These methods can be found in many disciplines, including A.I., psychology, engineering, marketing, and education [1].

In courses such as Heat Transfer, Thermodynamics, and Fluid Mechanics, many analytical solutions require the manipulation of a small set of fundamental governing equations based on physical laws and principles. Students frequently struggle to connect physical systems to the corresponding symbolic equations that govern system behavior in thermal-fluid courses. This struggle is often evidenced either by students selecting a simplified form of an equation without considering the limitations of applicability, or by students incorrectly identifying a system which leads to an incorrect simplification of the governing equation.

To address this gap, this paper introduces a reverse problem-solving framework implemented across three mechanical engineering courses: thermodynamics, fluid mechanics, and heat transfer. In this approach, students are presented with a final symbolic equation and asked to construct a plausible problem statement with a description of the physical system, schematic, and set of simplifying assumptions that would lead to that result. This inversion of the traditional process draws on the Concrete–Representational–Abstract (CRA) model, rooted in Bruner’s theory of cognitive development, to strengthen connections between physical phenomena (concrete), system schematics (representational), and governing equations (abstract) [2].

The Concrete-Representational-Abstract (CRA) model was widely adopted as a scaffolded approach to teaching mathematics. While most of the CRA research focuses on k-12 math instruction [3,4], there are examples of a similar approach being used in engineering education at the undergraduate level [5-7], including references to constructivist theory [8,9]. In its true form, the CRA model begins with hands-on learning for students using physical objects (concrete) to explore a concept. Similarly, hands-on activities, classroom demonstrations, and laboratory exercises in engineering courses provide a concrete example to enhance student learning [10-12]. The representational step includes drawing a schematic or simple sketch that communicates the concrete example. The abstract step introduces equations and values. The reverse problem-solving approach provides the abstract model and challenges students to both create a representational model and describe an appropriate concrete example.

Framework

In the forward problem-solving approach, students are given a description of a physical system and required to solve for an unknown variable. Students begin by identifying/defining the system

using keywords that may be listed as simplifying assumptions. Then a fundamental governing equation, such as the First Law of Thermodynamics, can be reduced by applying the assumptions. The result is a simplified governing equation specific to the defined system. If the system has been correctly defined, the remaining steps in the analysis usually include mathematical calculations and identifying appropriate inputs, perhaps from reference tables, as depicted in Figure 1.

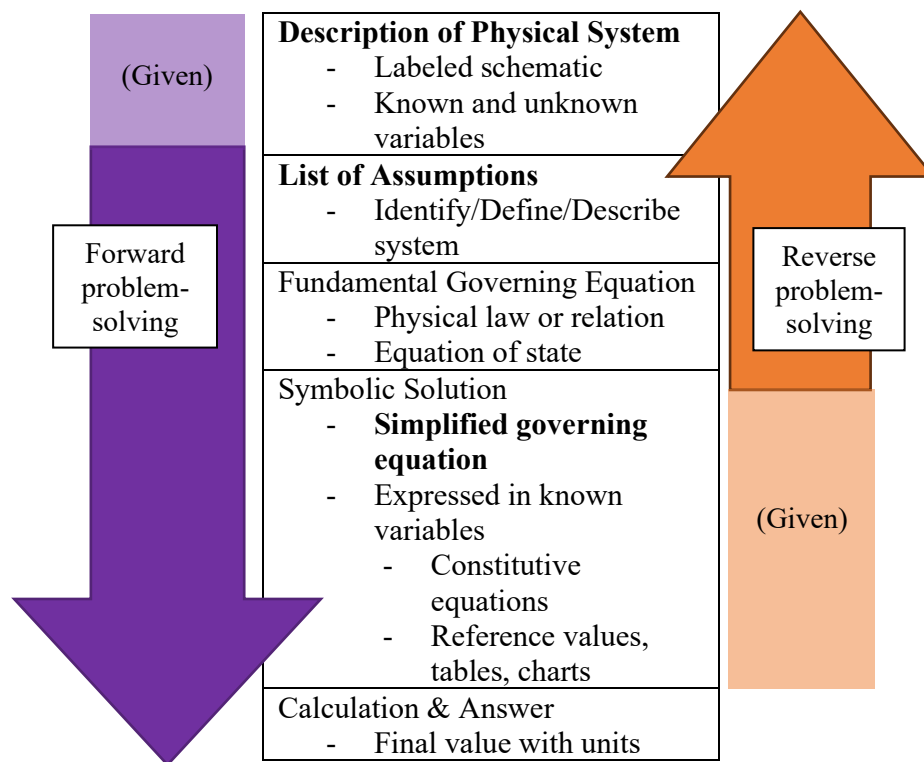


Figure 1: Reverse Problem-Solving Approach

Using the reverse problem-solving approach, students are given a simplified equation and asked to generate a description of a physical system described by the provided equation. They must first work backwards to identify the limitations of the equation validity, noting simplifying assumptions and identifying key indications of geometry, material, and more.

Methodology

The reverse problem-solving approach was incorporated as a supplemental tool designed by the instructor to encourage students to look at the concepts from a different viewpoint. Reverse problems were presented in class after students had received traditional instruction emphasizing the forward-solving approach and practiced applying the governing equations on multiple homework assignments. Reverse problems were intentionally presented after students gained experience applying key equations to help identify gaps in deeper understanding versus

procedural knowledge. Reverse problems reviewed understanding rather than introducing new material.

The course framework was not altered by the inclusion of reverse problems. Reverse problems were incorporated as in-class activities to engage students in discussions of equations, assumptions, and system identification, connecting the equations (abstract) back to physical phenomena (concrete) and system schematics (representational). As a first introduction, reverse problem-solving activities replaced a portion of a class meeting that would have previously been spent on forward problem-solving examples as continued review of key concept or fundamental governing equation. The general course framework is summarized in Figure 2.

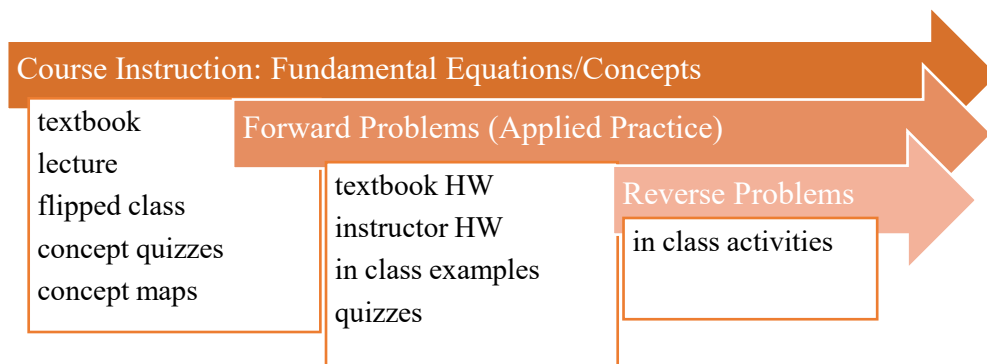


Figure 2: Course Framework

Samples of reverse problem prompts and solutions for the three courses are included here.

Heat Transfer

Reverse problems were presented after covering the following topics: (a) one-dimensional, steady-state conduction with no generation, (b) steady-state conduction, (c) transient conduction, and (d) convection.

After gaining practice applying the heat equation and thermal resistance networks for one-dimensional, steady-state heat transfer in different coordinate systems, the first reverse problems were introduced in class. The instructor wrote a simplified equation on the board. Working in small groups, students were tasked with identifying the required assumptions, followed by drawing a schematic of a physical system that could be represented by the equation. As student groups shared their identified assumptions, the instructor listed them on the board with the equation. The instructor reinforced the concept of assumptions as limitations for determining when the specific equation can be applied.

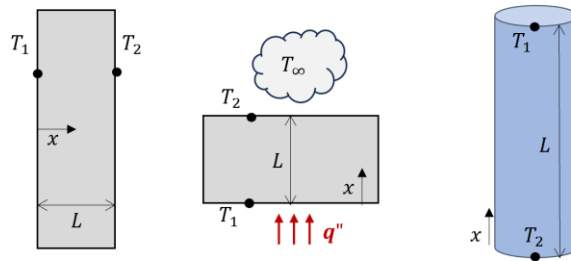
Once a complete list of assumptions had been identified, the student groups attempted to sketch a physical system corresponding to the equation either on small whiteboards or on paper. Similar to think-pair-share activities, groups were then asked to hold their sketches up for everyone to see. The instructor led a discussion highlighting some similarities and differences in the physical

systems and added a few sketches to the board to emphasize how a variety of physical systems can be described by the same equation.

With a simplified equation, list of required assumptions, and multiple physical representations on the board, the reverse problem was used to underscore the importance of properly identifying a system as the first step in an analytical solution. Initial feedback from students indicated a stronger understanding of the equations and a higher level of comfort with identifying systems. A sample reverse problem and accompanying solution is shown in Figure 3.

Given: $T(x) = (T_2 - T_1) \frac{x}{L} + T_1$

Solution: Assumptions
 1D conduction
 Steady state
 No internal generation
 Plane wall (slab)



Temperature distribution in solid slab or along length of solid rod
Ex: concrete wall, foamboard insulation, aluminum rod

Figure 3: Heat Transfer Reverse Problem – sample 1

Similar in-class reverse problem-solving activities were incorporated after covering additional topics. In addition to simplified equations, prompts took the form of thermal resistance network diagrams and Nusselt correlations. A sample problem with a thermal resistance network is shown in Figure 4. The prompt could be modified to include equations for any of the thermal resistance terms, which would specify geometric characteristics to be included in the solution.

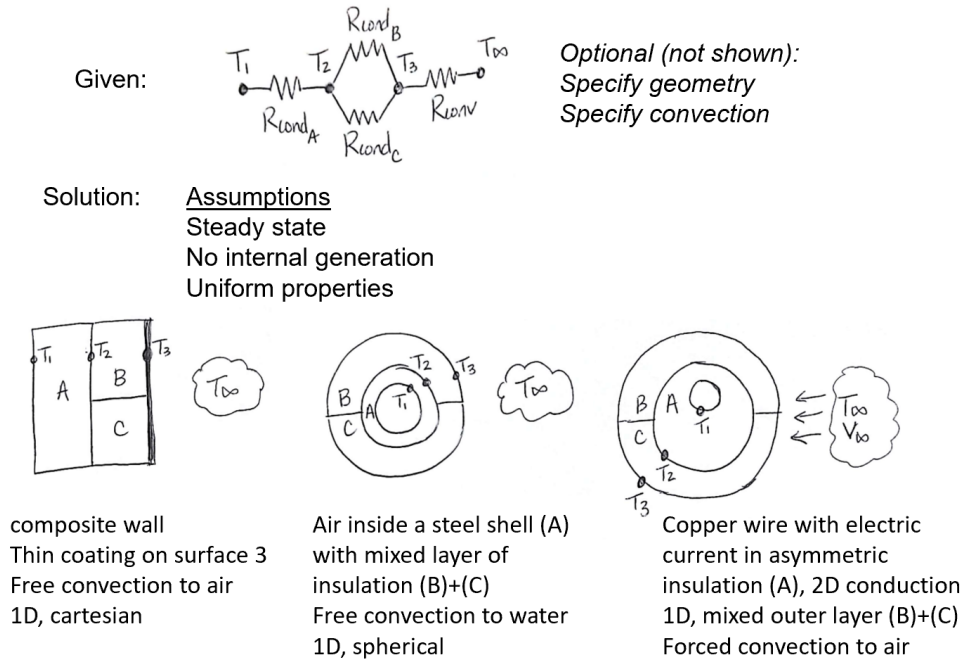
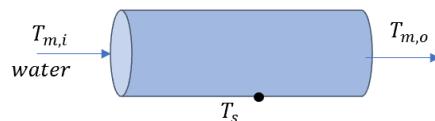


Figure 4: Heat Transfer Reverse Problem – sample 2

A sample reverse problem for internal convection is shown in Figure 5.

Given: $q'' = \left(\frac{3.66k_{\text{water}}}{D} \right) LMTD$

Solution: Assumptions
Laminar, fully developed internal flow
Circular pipe
Constant surface temperature
Thin-walled pipe
Uniform properties



Average heat flux for water flowing through a thin copper tube

Figure 5: Heat Transfer Reverse Problem – sample 3

While these sample problems provide students with a single equation, a system of equations could also be provided to share a more detailed model without requiring students to determine which algebraic substitutions had been made to form a combined equation. Reverse problem-solving activities were introduced in a similar framework in Thermodynamics and Fluid Mechanics, with an emphasis on key conservation equations and frequently used applications.

Thermodynamics

Reverse problems were presented after covering the following topics: (a) First Law, (b) Second Law, and (c) power cycles.

A sample reverse problem for the First Law of Thermodynamics is shown in Figure 6, with two options for the prompt. The solution includes three possible schematics and descriptions of physical systems that could be modeled by the equation.

$$\text{Given: } \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$\text{or } \dot{m}_1(h_1 - h_3) = \dot{m}_2(h_3 - h_2)$$

Solution: Assumptions

No change in kinetic energy

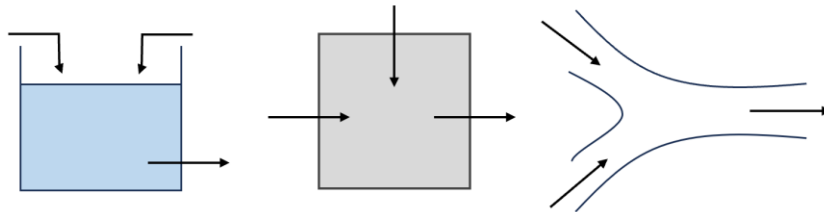
No change in potential energy

Steady state

Open system (two inlets, 1 exit)

No work

No stray heat



mixing chamber, open feedwater heater, or merging flows

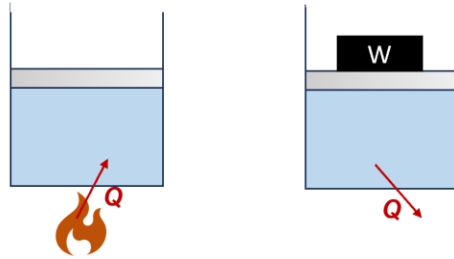
Figure 6: Thermodynamics Reverse Problem – sample 1

The sample reverse problem shown in Figure 7 was more challenging for students because the simplified First Law equation included a substitution using the definition of enthalpy.

Given: ${}_1Q_2 = m(h_2 - h_1)$

Solution: Assumptions

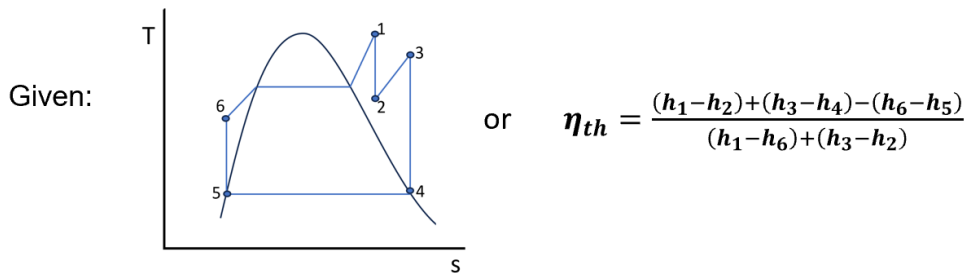
- No change in kinetic energy
- No change in potential energy
- Transient
- Closed system (mass is constant)
- Isobaric process



Fluid in a piston-cylinder assembly being heated or cooled at a constant pressure

Figure 7: Thermodynamics Reverse Problem – sample 2

Reverse problems in thermodynamics could also take the form of a cycle or process represented on a property diagram, as shown in Figure 8.



Solution: Assumptions

ideal Rankine Cycle with reheat and superheat

- complete condensing
- no pressure drop
- steady-state, open components
- no stray heat
- isentropic pump and turbine
- no changes in KE or PE

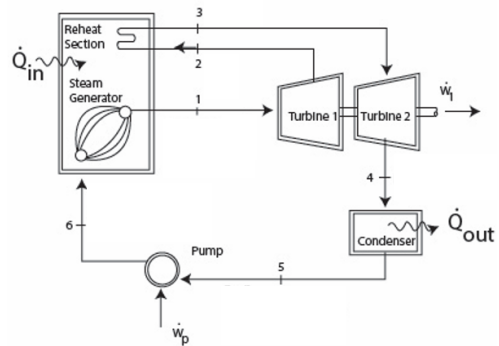


Figure 8: Thermodynamics Reverse Problem – sample 3

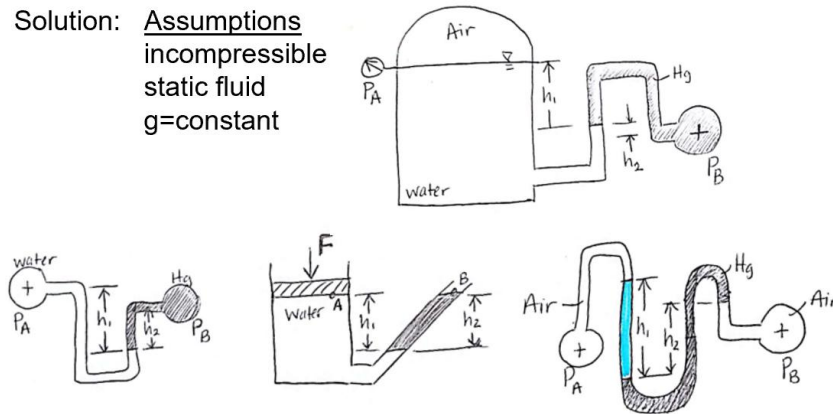
Fluid Mechanics

Reverse problems were presented after covering the following topics: (a) hydrostatic forces, (b) conservation of mass, (c) conservation of mechanical energy, (d) conservation of momentum, and (e) pipe flow.

Figure 9 shows a reverse problem using the manometer rule with multiple schematics as possible solutions. Students were asked to solve this reverse problem in class. After allowing time to sketch a system, the instructor encouraged students to trade drawings with a classmate and apply the manometer rule to verify it met the given equation.

$$\text{Given: } P_A + \gamma_{H_2O} h_1 - \gamma_{Hg} h_2 = P_B$$

Solution: Assumptions
incompressible
static fluid
 $g = \text{constant}$



Various manometer and tank configurations

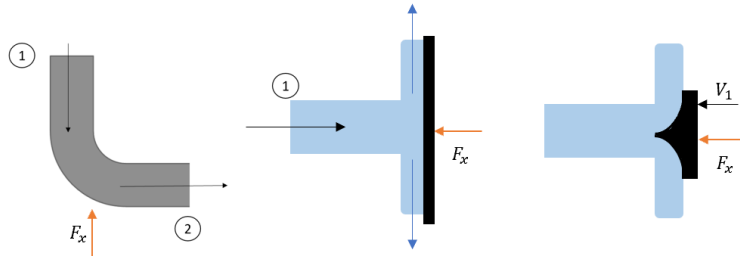
Figure 9: Fluid Mechanics Reverse Problem – sample 1

A sample reverse problem using the conservation of linear momentum is shown in Figure 10.

Given: $F_x = \rho AV_1^2$

Solution: Assumptions

Steady, incompressible flow
(conservation of linear momentum)



Force (x-component) required to:

- hold a pipe elbow in place,
- hold a plate stationary against impinging water jet
- push a flared object through stationary water

Figure 10: Fluid Mechanics Reverse Problem – sample 2

The reverse problem-solving approach was introduced in three courses taught by the same instructor: Thermodynamics (15 students), Fluid Mechanics (23 students), and Heat Transfer (12 students). Heat Transfer and Thermodynamics were offered in the fall semester, while Fluid Mechanics was offered the following spring semester. 13 of the students enrolled in Fluid Mechanics had been exposed to the reverse problem-solving approach the previous semester in Thermodynamics. All students enrolled in Heat Transfer and Thermodynamics were majoring in mechanical engineering. Both mechanical and civil engineering majors were enrolled in Fluid Mechanics. 4th-year students were enrolled in Heat Transfer, while 2nd- and 3rd-year students were enrolled in both Thermodynamics and Fluid Mechanics.

Assessment and Future Work

Formal assessment was not conducted in the fall semester Heat Transfer and Thermodynamics courses as part of the initial introduction of reverse problem-solving. Enrollment in the Fluid Mechanics course included some students with prior exposure to reverse problems from the Thermodynamics course. Student solutions to the Fluid Mechanics problems were evaluated for correctness. Performance of the group with prior reverse problems exposure was compared to the group with no prior exposure. When comparing exam performance with a previous course offering where reverse problem-solving was not introduced, no significant difference was found.

Students were asked to respond to a survey near the end of the semester to evaluate student perceptions of the effectiveness of reverse problem solving. The survey questions are listed in Table 1, which includes both Likert scale and open-ended response prompts. The three

highlighted prompts examine the perceived impact on ability forming concrete-representational-abstract connections.

Table 1: Student Survey Prompts

Likert Scale Prompts (1-strongly disagree, 2-disagree, 3-neutral, 4-agree, 5-strongly agree)					
1. The reverse problem-solving approach was presented clearly	1	2	3	4	5
2. I understood the purpose of applying the reverse approach.	1	2	3	4	5
3. Using the reverse approach improved my understanding of course concepts.	1	2	3	4	5
4. The reverse approach strengthened my ability to identify appropriate simplifying assumptions for a given physical system.	1	2	3	4	5
5. The reverse approach improved my ability to identify simplifying assumptions (limitations) in governing equations.	1	2	3	4	5
6. The reverse approach strengthened my ability to connect physical systems to appropriate equations.	1	2	3	4	5
7. The approach encouraged me to think more deeply about the concepts involved when solving problems for the course.	1	2	3	4	5
8. I used the method even when it was not explicitly required.	1	2	3	4	5
9. Overall, I see value in the reverse problem-solving approach as a learning tool.	1	2	3	4	5
10. This method has influenced how I think about problems in other courses.	1	2	3	4	5
11. I would recommend future sections of the course include this approach.	1	2	3	4	5
Open-ended response questions					
What did you find most helpful about the reverse approach? Least helpful?					
Any additional comments about your experience with reverse problem-solving?					

A summary of student feedback from the Fluid Mechanics course is shown in Figure 11 as average responses from the Likert scale prompts with standard error indicated by the error bars. Of the 22 students enrolled in the course, 19 responded to the survey resulting in an 86% response rate.

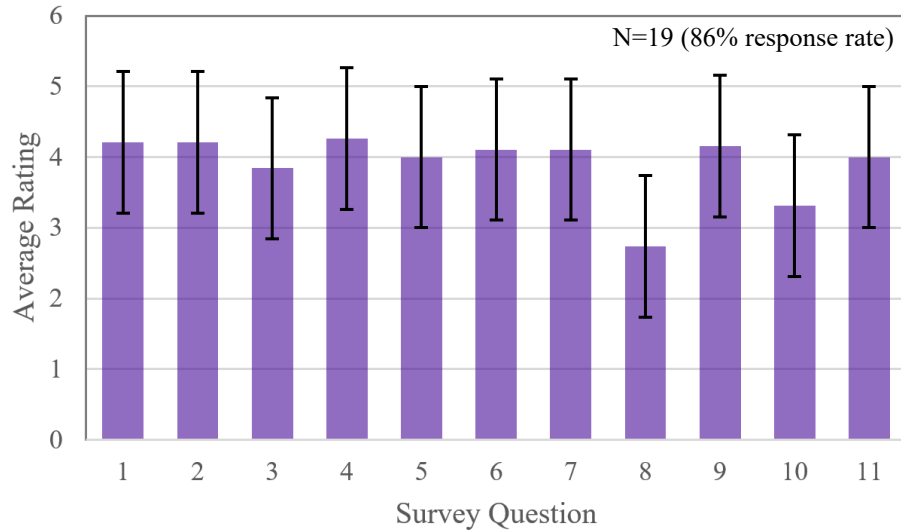


Figure 11: Averaged results from Fluid Mechanics

Overall, students agreed most strongly (avg. 4.26) with Question 4, “The reverse approach strengthened my ability to identify appropriate simplifying assumptions for a given physical system.” The average rating was 4.00 or higher (agree or strongly agree) for eight of the eleven questions, indicating that students had a positive perception of the reverse problem-solving approach. Students disagreed most (avg. 2.74) with Question 8, “I used the method even when it was not explicitly required.”

A histogram showing student responses to the three highlighted prompts assessing the perceived impact on concrete-representational-abstract connections is shown in Figure 12. There was only one response of “Disagree” by a single student on Question 6. The portion of students responding “Agree” or “Strongly Agree” was 89.5%, 73.7%, and 84.2% for Questions 4, 5, and 6, respectively. These results support the success of reverse-problem solving in strengthening student ability for form connections between physical phenomena (concrete), system schematics (representational), and governing equations (abstract).

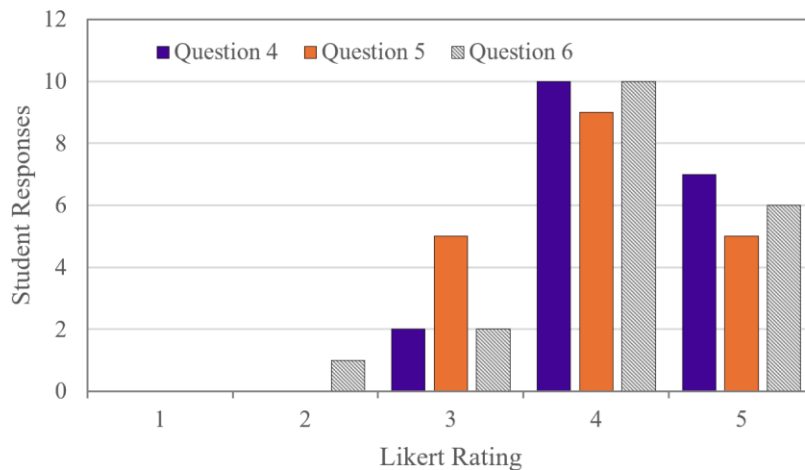


Figure 12: Likert rating for direct CRA prompts

In the open response results, there were two students that appeared to confuse reverse problems with some of the flipped classroom materials used in the course. The first prompt results in fourteen student responses, while only six students responded to the second prompt. Representative comments are included below.

What did you find most helpful about the reverse approach? Least helpful?

“I found that there are many very different problems that can be represented by very similar or identical equations.”

“It forced students to think about the problem compared to just waiting for the answer. Also felt like a more active classroom than example problems.”

“It linked physical systems to equation models more clearly and reinforced assumptions.”

“The most helpful was the physical intuition of understanding assumptions. However, the nature of the open-ended possibilities based off the assumptions left more questions than answers.”

“The most helpful aspect was getting to make your own system gives you a better feel for what are dealing with, better than just words or a provided schematic.”

“It helps to see the bigger picture of a problem and prevents getting tunnel vision.”

“I think the least helpful aspect was just that it was a first time thing, so it was challenging at times.”

Any additional comments about your experience with reverse problem-solving?

“If reverse problem solving is continued, I would clearly specify the desired apparatus or mechanism being designed. Giving an equation and assumptions provides for an infinite number of potential systems, some of which are beyond the scope of this class.”

“I think it could be beneficial to start with simple systems, then add additional factors and keep solving to see the effects of the additions.”

Overall the comments were positive and reflected the intended outcomes of the approach. One goal was to help students recognize the same equation can represent many different physical systems. The critical comments seemed more indicative of student discomfort with open-ended problems.

The framework described in this paper was limited to in-class activities. The reverse problem-solving approach was not extended to homework and exams when first introduced in the course, but could be included as part of these assignments in the future. In the first iteration, less than 10% of class time was dedicated to reverse problem-solving activities. To evaluate student

learning, future iterations will include reverse problem-solving prompts as course concepts are introduced, in addition to acting as review tools as described in the current framework.

Conclusions

This paper describes an initial approach for reversing Thermo-Fluids problems with a goal of enhancing conceptual understanding. Reverse problems were developed and introduced in three courses in a limited manner. The new teaching technique served as an additional tool for reviewing concepts and connecting governing equations (abstract) with physical phenomena (concrete) and system schematics (representational). While the current work is limited to examples of in-class activities in thermo-fluids courses, the framework could be applied to other courses and concepts in the broader curriculum. The results presented in this study are limited to a small sample size, but indicate positive student outcomes from reverse problem solving. Additional data should be collected to further evaluate the effectiveness of the approach.

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