Linking Risk and Scheduling through Unit-Price Equations: A Pedagogical Framework for Construction Students

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Abstract
Traditional approaches to teaching risk and scheduling in construction education often rely on qualitative risk matrices and deterministic activity durations, limiting students' understanding of how time and cost emerge from field operations. This paper presents a procedural, equation-based pedagogical framework that links scheduling and risk reasoning to unit-price formulations grounded in productivity. The framework decomposes a main construction activity into elemental activities and expresses each cost component through explicit material, labor, logistics, and equipment terms. Activity durations are derived consistently from the same productivity assumptions embedded in the unit-price equations. To introduce uncertainty, variability in operational parameters (e.g., productivity rates and consumption quantities) is modeled through simple stochastic formulations and propagated via simulation to obtain probabilistic cost and duration outcomes. The approach is illustrated using a worked construction example with documented deterministic inputs (Appendix A) and a focused stochastic demonstration on a selected elemental-activity branch (Appendix B), supported by a reference implementation (Appendix C). Learning objectives emphasize (1) causal understanding of productivity-driven cost–time relationships, (2) formulation of transparent unit-price equations, and (3) interpretation of probabilistic outputs. Results show that the framework supports these objectives within a controlled instructional setting, shifting instruction from tool execution toward procedural reasoning.

Keywords: *Construction Engineering Education, Unit-Price Modeling, Stochastic Scheduling, Productivity-based Estimation, Monte Carlo Simulation*

Introduction
Risk and scheduling are core topics in construction engineering education, yet they are commonly taught as largely independent subjects. Scheduling is often introduced through deterministic network techniques with fixed activity durations, see [1]. While risk is addressed separately using qualitative classifications or post hoc contingency adjustments. Although these approaches are practical for introducing basic concepts, they tend to obscure the procedural origins of time and cost in construction operations, making it difficult for students to understand how uncertainty emerges from field-level decisions and productivity assumptions.

In professional practice, however, construction schedules and costs are not independent abstractions. Activity durations are implicitly derived from the same productivity rates, crew compositions, material consumptions, and equipment efficiencies that define unit prices. Variability in these operational parameters directly affects both cost and time, yet this causal linkage is rarely made explicit in instructional settings. As a result, students may learn to manipulate scheduling tools or risk matrices without developing a clear understanding of how operational assumptions propagate through a project network, see [2] .

Several strands of the literature have attempted to address this gap by integrating cost, schedule, and risk through simulation-based approaches, stochastic scheduling, or probabilistic cost analysis [3]. While these methods are powerful, they often rely on abstract formulations or software-driven workflows that can be difficult for students who are still developing foundational reasoning skills in construction methods and estimating to contextualize. From a pedagogical perspective, there remains a need for examples that are sufficiently concrete to be traceable, yet rich enough to illustrate how uncertainty arises procedurally rather than being imposed externally [4].

This paper addresses this need by proposing a procedural, equation-based pedagogical framework that links unit-price formulations, productivity-based duration modeling, and basic uncertainty propagation within a single worked example. Rather than aiming to cover the full breadth of project-level risk analysis, the framework is intentionally limited to a single main construction activity (MCA) decomposed into elemental activities. This scoped approach allows students to follow, step by step, how material quantities, labor crews, equipment use, and productivity assumptions define both costs and durations.

The paper's contribution is not the development of a new risk analysis method, but the explicit alignment of cost estimation and scheduling logic at the instructional level. By expressing unit prices algebraically and deriving activity durations from the same productivity parameters, the framework provides a transparent basis for introducing variability and exploring its effects through simple simulation. The proposed approach is illustrated using a fully documented deterministic example (Appendix A), a focused stochastic demonstration on a selected elemental activity branch (Appendix B), and a reference computational implementation intended for instructional use (Appendix C).

By constraining the scope of the example while maintaining procedural completeness, the paper aims to support risk-informed reasoning in construction education without overloading students with methodological complexity. The framework is designed to be adaptable to different teaching contexts and to complement, rather than replace, existing tools and curricula.

Background and Literature Review

Cost, Productivity, and Scheduling in Construction

Unit-price analysis and productivity-based estimation are foundational components of construction cost engineering. Extensive literature documents how material quantities, labor crews, equipment usage, and productivity rates are combined to produce unit costs for construction activities, see [5], [6]. In practice, these same productivity assumptions implicitly determine activity durations, yet educational treatments often separate estimating from scheduling, introducing durations as fixed inputs rather than derived quantities.

Scheduling research, particularly around network-based methods such as the Critical Path Method (CPM) and its extensions, has focused on logical sequencing and time control, often assuming deterministic activity durations. While these approaches remain essential, they tend to obscure the operational origins of time, especially when taught in isolation from cost and productivity considerations, [7].

Risk and Uncertainty in Construction Planning

Risk and uncertainty have been addressed through a wide range of qualitative and quantitative approaches, including risk registers, sensitivity analysis, probabilistic scheduling, and simulation-based cost analysis. [8], [9]. Many studies emphasize the importance of capturing variability in productivity, material supply, and equipment performance, particularly for complex or large-scale projects.

However, from an educational perspective, these approaches frequently rely on abstract probabilistic constructs or software-driven workflows that can be difficult to reconcile with the procedural logic of construction operations. Risk is often introduced as an external modifier; through contingency factors or qualitative classifications, rather than as a consequence of variability embedded in the same parameters used for cost and duration estimation.

Integrated Perspectives and Production-Oriented Views

Several strands of research have attempted to bridge cost, schedule, and risk through integrated frameworks, including earned value management, production system modeling, and lean construction approaches. These perspectives emphasize flow, variability, and production efficiency as central drivers of project performance.

More recently, computational methods and data-driven techniques, such as simulation and machine learning, have been explored to enhance prediction and optimization capabilities. While these contributions advance the state of the art, their pedagogical applicability is often limited by complexity, data requirements, or lack of transparency in the underlying assumptions. [10].

Bibliometric Perspective and Research Clustering

To contextualize the study's thematic position, a bibliometric analysis was conducted using Scopus-indexed publications on construction project management, cost engineering, scheduling, and risk. The resulting keyword co-occurrence network, shown in Figure 1, highlights several dominant clusters centered around project management, cost engineering, uncertainty analysis, and production engineering.

The temporal overlay presented in Figure 2 indicates a gradual convergence of these themes over time, particularly around topics such as uncertainty analysis, productivity, and integrated project controls. Despite this convergence, the clusters remain essentially connected through high-level methodological or managerial concepts, with limited emphasis on procedural, equation-based integration suitable for instructional use.

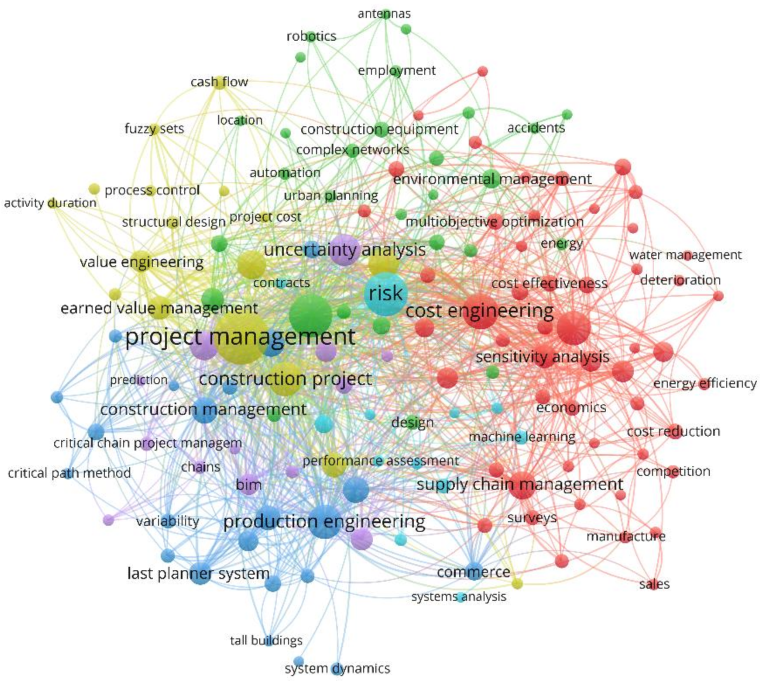

Figure 1. Cluster analysis image of the reference combination

Research Gap and Positioning of the Present Study

The literature reviewed above demonstrates strong theoretical and methodological foundations for cost estimation, scheduling, and risk analysis in construction. Nevertheless, a gap persists at the educational level between conceptual integration and procedural understanding. Specifically, there is limited guidance on how to introduce unit-price formulations, productivity-based duration modeling, and uncertainty propagation coherently within a single, teachable example.

This paper positions itself within this gap by focusing not on methodological novelty but on pedagogical clarity and procedural alignment. By grounding risk and scheduling reasoning directly in unit-price equations and productivity assumptions, the proposed framework seeks to complement existing literature with an instructional example that makes the causal links between cost, time, and uncertainty explicit and auditable. This paper addresses this gap by explicitly aligning unit-price formulation, productivity-based duration modeling, and uncertainty propagation within a single, traceable instructional framework.

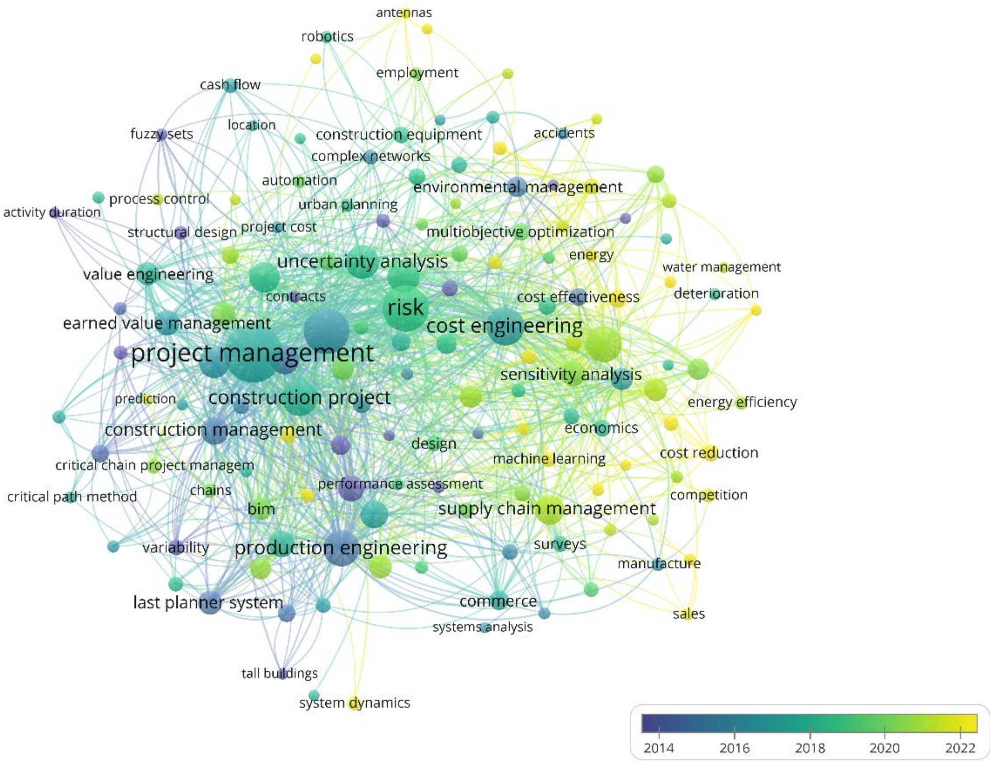

Figure 2 – Temporal overlay map from the bibliometric cluster

Proposed methodology

The proposed methodology is conceived as a procedural instructional framework that links unit-price formulation, productivity-based duration estimation, and basic risk reasoning within a single, traceable construction example. Rather than presenting cost estimation, scheduling, and risk analysis as separate analytical topics, the methodology integrates them through a shared set of operational assumptions derived from field execution.

The framework is intentionally scoped to a single Main Construction Activity (MCA), which is decomposed into a limited number of Elemental Activities (EAs). This constrained scope allows students to follow the complete reasoning chain, from resource definition to cost, time, and uncertainty; without the cognitive overload associated with full project-level models. The numerical reference for the methodology is documented in Appendix A, while uncertainty propagation is illustrated through a focused stochastic example in Appendix B.

Step 1: Procedural Decomposition of the Main Construction Activity

The methodology begins by decomposing the MCA into a set of elemental activities that reflect actual execution logic, rather than contractual or accounting-based breakdowns. Each EA represents a distinct operational process, such as formwork installation, steel reinforcement placement, or concrete production and placement. Elemental activities are indexed by their procedural order i, and when necessary, by an additional execution level j to capture differences in resource coupling (e.g., material supply versus on-site fabrication). This indexing scheme is illustrated in Appendix A through a Directed Acyclic Graph (DAG) and its tabular representation, which together define the structural backbone of the example.

At this stage, no durations or uncertainty assumptions are introduced. The objective is to ensure that students clearly identify materials, labor crews, equipment, and auxiliary resources associated with each EA before any numerical modeling is performed.

Step 2: Unit-Price Formulation Based on Productivity

Once the procedural structure is defined, each elemental activity is economically modeled using an explicit unit-price equation. Instead of relying on predefined composite prices, costs are assembled from first principles by combining material consumption, labor effort, logistics, and equipment operation.

A key feature of the methodology is that labor costs are expressed as a function of productivity, making production rates an explicit modeling variable rather than an implicit assumption. Equipment-associated labor is treated separately to preserve transparency and avoid double-counting of operational variability.

The deterministic values used to construct the unit-price equations, for example, MCA, are provided in Appendix A. These values establish a transparent baseline that can be directly traced to the algebraic formulation introduced in the main text.

Step 3: Introduction of Uncertainty as Operational Variability

After the deterministic unit-price model is established, uncertainty is introduced by allowing selected operational parameters—such as productivity rates or material consumption—to vary. Rather than defining risk in terms of discrete events or qualitative classifications, uncertainty is treated as the inherent variability of field operations.

For instructional clarity, uncertainty modeling is intentionally limited to a single EA branch (steel reinforcement placement) documented in Appendix B. Simple probability distributions are assigned to selected parameters, and Monte Carlo simulation is used to propagate variability through the unit-price equation. This allows students to observe how cost and performance distributions emerge naturally from operational assumptions.

Step 4: Consistent Derivation of Activity Durations and Scheduling Interpretation

Activity durations are derived directly from the same productivity parameters used in the cost model, ensuring consistency between cost and time reasoning. These durations can then be incorporated into a simplified activity network to assess schedule implications.

Rather than focusing on detailed network optimization, the emphasis is on interpretation: students analyze duration distributions, identify percentile-based outcomes, and relate schedule variability to the underlying productivity assumptions. This step reinforces the idea that schedule risk is not an independent construct, but a direct consequence of operational variability encoded in the unit-price formulation.

Methodological Positioning

Overall, the proposed methodology does not aim to introduce new analytical techniques. Its contribution lies in pedagogical alignment: cost, time, and risk are derived from a

shared procedural foundation and explored through a single worked example. By anchoring abstraction in a concrete MCA and explicitly referencing deterministic (Appendix A) and stochastic (Appendix B) layers, the framework supports transparent, teachable reasoning about construction risk and scheduling without exceeding the intended instructional scope.
Each step of the methodology is directly aligned with the intended learning objectives. The procedural decomposition and unit-price formulation support equation development; the introduction of uncertainty enables the identification of variability sources; and the derivation of durations reinforces the causal relationship between productivity and scheduling outcomes.

Pedagogical Rationale and Learning Philosophy

The pedagogical rationale of the proposed framework is grounded in the idea that students develop a more profound understanding of construction risk and scheduling when abstract concepts are derived from explicit operational procedures rather than introduced as independent analytical layers. In many instructional settings, cost estimation, scheduling, and risk analysis are taught as separate topics, often relying on predefined unit prices, fixed durations, or qualitative risk classifications. While effective for introducing tools, this separation can limit students' ability to reason about how time and cost actually emerge from construction operations.

The framework adopts a procedural learning philosophy, in which students are guided to construct activity costs and durations from first principles. By decomposing a Main Construction Activity into elemental activities and expressing each cost component algebraically, the framework encourages learners to focus on the causal relationships between materials, labor, equipment, productivity, and performance. Risk is not introduced as an external adjustment, but as variability affecting the same operational parameters already used to define cost and time. [11], [12], [13].

A key pedagogical choice is the intentional limitation of scope. Rather than modeling an entire project, the framework centers on a single, fully documented MCA. This allows students to follow the complete reasoning chain, from resource definition to unit-price formulation, duration derivation, and uncertainty propagation; without excessive cognitive load. The deterministic baseline presented in Appendix A serves as a reference case, while Appendix B introduces controlled, focused uncertainty for a selected activity branch.

The learning philosophy emphasizes interpretation over optimization. Students are not expected to produce optimal schedules or probabilistic forecasts, but to explain why variability in productivity or resource use leads to variability in cost and duration. The accompanying computational implementation (Appendix C) is used to support transparency and reproducibility, reinforcing conceptual understanding rather than software proficiency.

Overall, the pedagogical approach seeks to shift instruction from tool-centered execution to procedural reasoning, helping students connect estimation, scheduling, and risk within a single coherent example that reflects how construction activities are actually performed in the field.

Proposed Equation-Based Pedagogical Framework

Overview of the Framework

When a main construction activity (MCA) is explicitly decomposed into a set of preliminary activities, each elemental activity cost, p_i can be economically evaluated using the following formulation:

$$p_{i,j} = \left(\rho_{i,j}^{(M)} \cdot M_{i,j}\right) + \left(\rho_{i,j}^{(L)} \cdot L_{i,j}\right) + \left(\rho_{i,j}^{(F)} \cdot F_{i,j}\right) + \left(\rho_{i,j}^{(O)} \cdot O_{i,j}\right) + A_{i,j} \tag{1}$$

In this expression, the index i refers to the procedural order of the elemental activity (EA) inside MCA. In contrast, the index j, denotes the execution level or subprocess associated with that elemental activity. For example, in slab construction, the third procedural activity (*i*=3) may correspond to concrete placement. However, this activity can be further decomposed into distinct execution levels such as material fabrication (*j*=1), transportation or pumping (*j*=2), and casting (*j*=3). Not all elemental activities necessarily include multiple execution levels; in such cases, the index j reduces to a single value. $M_{i,j}$ is the materials cost, $L_{i,j}$ is the direct labor cost, $F_{i,j}$ is the freight and logistics cost, $O_{i,j}$ is the equipment operation cost (non-crew-related) and $A_{i,j}$ is the equipment-associated labor cost. The adjustment coefficients $\rho_{i,j}$ links each EA component to the MCA. It accounts for unit coupling, construction waste, and other operational inefficiencies that systematically affect base costs, e.g., when the material is measured in kg within an EA. The MCA unit is m3; therefore, a conversion factor is required to convert the material to kg per m^3. This formulation allows the economic structure of a construction activity to be expressed procedurally, rather than through aggregated unit prices or fixed durations. By explicitly separating cost components that are exposed to general operational variability from those already conditioned by equipment–labor interactions, the equation provides a transparent basis for subsequent productivity modeling, uncertainty propagation, and schedule derivation. When MCA contains different sequentially arranged EAs, the final cost of the MCA can be evaluated as follows:

$$P_k = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{i,j} \tag{2}$$

Where n is the number of construction activities, m is the number of levels in the different activities, and k is the number of EAs. The costs in equation (1) can be evaluated as follows:

$$M_{i,j} = \sum_{q=1}^{Q} \theta_{i,j,q} \cdot \delta_{i,j,q} \tag{3}$$

$$L_{i,j} = \frac{\nu_{i,j}\left(1 + \tau_{i,j}\right)}{R_{i,j}} \tag{4}$$

$$A_{i,j} = \left(\prod_{m=1}^{i} \varphi_{i,j,m}\right) \times \left(\frac{\nu_{i,j}\left(1 + \tau_{i,j}\right)}{t_w \cdot \psi_{i,j}}\right) \tag{5}$$

Where $\theta_{i,j,q}$ is the unit price of the material input q for the preliminary activity i at level j; $\delta_{i,j,q}$ is the material quantity (consumption rate) of material input q required by i-elementary activity (i-EA) at level j. $\nu_{i,j}$ represents the crew cost rate (wages per daily 8-hour journey) for the i-EA at the level j, considering that Q is the total number of materials (auxiliary material in scaffolding, composed material as in concrete mix, or simple materials), and $\tau_{i,j}$ is the additional costs of personal safety equipment and hand-tools of workers in terms of a percentage of labor. $R_{i,j}$ stands for the productivity rate of the crew for the i-EA at the level j, e.g., units/8-hours standard work journey; making $L_{i,j}$ the direct labor cost per MCA-unit after being multiplied by the respective $\rho_{i,j}$; and $A_{i,j}$ as the equipment-associated labor cost contains the products of $\varphi_{i,j,m}$ A factor that considers the coupling between the equipment production unit, productivity rate, and time unit to the upper EA's units at different j levels, e.g., A concrete mixer produces in terms of m^3, but a 10-cm slab construction can be estimated in m^2, meaning that if one considers

the 0.1 m^3 of concrete with 4% of considered waste, the initial factor is $\varphi_{i,j,1}$= 0.104 m^3/m^2 of slab. When the production unit of the mixer is m^3, and the upper EA is m^3, the second factor is $\varphi_{i,j,2}$= 1.0 m^3/m^3; and the hourly productivity rate is 2.14 m^3/hr, then $\varphi_{i,j,3}$=0.46667 hr/m^3. For $A_{i,j}$, the variable $\nu_{i,j}$ refers to the operational crew cost rate for the specific machinery or major equipment. t_w is generally equal to eight hours in a standard daily work journey, and $\psi_{i,j}$ is the efficiency coefficient. The equipment operational costs $O_{i,j}$ is generally expressed in terms of cost per hour, and by multiplying by $\rho_{i,j}^{(O)}$ the hourly rate of production is matched with the upper EA's unit. The freight and logistics cost $F_{i,j}$ depends on what is considered within the EA.
From the scheduling perspective, $A_{i,j}$ and $L_{i,j}$ are the temporal variables directly impacting in the time (performance) and resources, and the other terms and part of the equation (1) influence only material resources.

Illustrative Example: Application of the Framework to a Main Construction Activity
To demonstrate the proposed equation-based pedagogical framework, an illustrative example based on a single Main Construction Activity (MCA) is presented. The objective of this example is not to provide a comprehensive project analysis, but to demonstrate how cost, productivity, scheduling, and uncertainty reasoning can be integrated procedurally within a controlled, traceable instructional setting.

The example begins with the explicit definition and decomposition of the MCA into a limited set of elemental activities. This deterministic baseline, including resource definitions, unit-price components, and procedural relationships, is fully documented in Appendix A. The appendix provides the numerical and structural reference required to formulate the unit-price equations and to derive activity durations consistently from productivity assumptions. [6].

Building on this baseline, uncertainty is introduced selectively to illustrate how operational variability propagates through the framework. Rather than modeling uncertainty across all activities, the example focuses on a single elemental-activity branch related to reinforcing steel placement. This targeted approach, documented in Appendix B, enables exploration of variability in selected parameters through simple stochastic simulation, yielding probabilistic cost and duration outcomes that can be interpreted pedagogically.

To support transparency and reproducibility, a reference computational implementation is provided in Appendix C. The code mirrors the structure of the equations and examples discussed in the paper and is intended solely as an instructional aid. Together, the example and its supporting appendices demonstrate how the framework can be applied step by step, from deterministic unit-price formulation to basic risk-informed interpretation, without exceeding the intended educational scope.

Learning Objectives and Assessment Strategy

The learning objectives of the proposed framework are intentionally scoped to align with the illustrative nature of the example and the paper's instructional focus. Rather than aiming at full professional competency in risk or scheduling analysis, the framework is designed to support foundational reasoning skills that connect cost estimation, productivity, and scheduling through explicit procedural logic.

Specifically, upon completion of the instructional sequence based on the framework, students are expected to be able to:

1. Formulate transparent unit-price equations for a construction activity by explicitly identifying material, labor, logistics, and equipment components.

2. Explain the causal relationship between productivity assumptions and activity durations, recognizing that time and cost are derived from shared operational parameters.
3. Identify key sources of operational variability at the elemental-activity level and describe how this variability affects both cost and schedule outcomes.
4. Interpret basic probabilistic results (e.g., distributions or percentiles) generated through simple simulation, without relying on advanced statistical techniques.

Assessment within this framework emphasizes reasoning and interpretation rather than numerical optimization or software proficiency. Student performance can be evaluated through structured exercises that require formulating equations, explaining modeling assumptions, and qualitatively interpreting probabilistic outputs derived from the illustrative example. The focus is placed on students' ability to justify modeling choices and to articulate how changes in productivity or resource assumptions propagate through cost and time calculations.

By aligning learning objectives and assessment strategies with a single, fully documented example, the framework supports coherent instruction while remaining adaptable to different course formats and instructional constraints, see [4].

Educational Application and Classroom Implementation

The proposed framework is intended for use as a guided instructional example within undergraduate or early graduate courses in construction engineering, cost estimating, or project controls. Its classroom application is designed to complement existing course content rather than replace established curricula or software-based instruction.

Course Context

The framework can be integrated into courses in which students already have a basic familiarity with construction methods, unit-price analysis, and deterministic scheduling concepts. The example is suitable for classroom settings in which the objective is to strengthen conceptual understanding of how cost, productivity, and time are procedurally linked, rather than to train students in advanced optimization or specialized risk software.

Implementation Strategy

In a typical classroom implementation, instruction begins with the deterministic definition of the Main Construction Activity and its elemental activities, using the data provided in Appendix A. Students are guided to interpret the unit-price equations, identify cost components, and relate labor and equipment inputs to productivity assumptions.

Once the deterministic baseline is established, variability is introduced selectively through the focused example presented in Appendix B. Rather than analyzing the entire activity network, students examine how uncertainty in a small set of operational parameters affects cost and duration outcomes. This staged approach allows instructors to control cognitive load while maintaining procedural continuity.

The computational reference provided in Appendix C may be used to support transparency and repeatability, but it is not required for conceptual understanding. Instructors may choose to implement calculations using spreadsheets or simpler scripts, depending on course constraints and students' backgrounds.

Development Metrics by Students

Student engagement with the framework is evaluated by their ability to formulate equations, articulate modeling assumptions, and interpret results, rather than solely by numerical accuracy. Typical instructional artifacts include annotated unit-price formulations, short explanatory reports, and qualitative interpretations of probabilistic outputs.

These artifacts allow instructors to assess whether students can connect productivity assumptions to cost and time outcomes and whether they can reason about variability as an operational phenomenon. The framework thus supports incremental skill development while remaining adaptable to different instructional formats and time allocations.

Discussion

The discussion focuses on the pedagogical implications of the proposed framework and its illustrative application, rather than on empirical validation or benchmarking of methodological performance. By constraining the example to a single Main Construction Activity and a limited set of elemental activities, the framework prioritizes procedural clarity and instructional traceability over analytical completeness.

One of the primary contributions of the framework is to make explicit the causal links between productivity, cost, and time that are often implicit in traditional teaching approaches. By deriving activity durations from the same productivity assumptions used in unit-price formulations, the framework helps students recognize that schedule outcomes are not independent inputs but consequences of operational decisions. This explicit linkage supports a more coherent mental model of construction planning, particularly in early stages of professional formation.

The introduction of uncertainty as variability in operational parameters, rather than as abstract risk events or external contingencies, further reinforces this procedural perspective. Limiting the stochastic demonstration to a single elemental-activity branch allows students to focus on interpretation rather than on model complexity. While this choice limits the analytical scope, it aligns with the pedagogical objective of illustrating how probabilistic outcomes arise from familiar construction assumptions, rather than overwhelming learners with project-wide simulations.

At the same time, the framework has apparent limitations. The example does not address interactions among multiple activities, project-level resource constraints, or the optimization of schedules and costs. These omissions are intentional and reflect the instructional focus of the paper. The framework is not proposed as a replacement for established project management tools or advanced risk analysis methods, but as a complementary approach that supports conceptual understanding before students engage with more complex models.

From an instructional standpoint, the use of fully documented appendices is critical. Appendix A provides a deterministic reference to anchor the discussion and avoid ambiguity. At the same time, Appendices B and C allow instructors to extend the example incrementally based on course objectives and students' backgrounds. This modularity supports adaptation across different educational contexts without requiring substantial restructuring of existing curricula.

Overall, the discussion highlights that the value of the proposed framework lies not in analytical novelty, but in pedagogical alignment. By grounding cost, schedule, and risk reasoning in a shared procedural foundation, the framework offers an accessible pathway to introduce integrated thinking into construction education while remaining transparent about its scope and limitations.

These limitations define the scope within which the framework is pedagogically effective and highlight the conditions under which transferability to other contexts should be considered.

Conclusions

This paper presented an equation-based pedagogical framework designed to support integrated reasoning about cost, productivity, scheduling, and uncertainty in construction education. Rather than proposing new analytical methods, the framework emphasizes procedural transparency by deriving both unit prices and activity durations from shared operational assumptions.

The contribution of the work lies in its explicit alignment with the instructional objectives defined for the framework. The proposed approach enables students to (1) formulate transparent unit-price equations, (2) understand the causal relationship between productivity assumptions and activity durations, and (3) interpret probabilistic outputs derived from operational variability. The deterministic baseline documented in Appendix A, together with the focused stochastic illustration in Appendix B, provides a coherent pathway to achieve these learning outcomes within a single, traceable example.

Assessment within this framework is centered on students' ability to articulate modeling assumptions, construct equations, and interpret variability-driven results. In this sense, the framework supports not only computational understanding but also conceptual reasoning, reinforcing the intended shift from tool-based execution toward procedural comprehension.

The framework intentionally limits its analytical scope to maintain pedagogical clarity. It does not address project-level optimization, multi-activity interactions, or advanced risk modeling. These limitations are consistent with its instructional purpose and define the boundaries within which the framework is expected to be effective.

From a transferability perspective, the approach is adaptable to other construction activities that can be decomposed into elemental operations with traceable resource definitions. It is particularly suited for undergraduate and early graduate courses where deterministic baselines can be established and used as a reference for introducing uncertainty. However, its applicability may be limited in contexts that require high-fidelity project-level modeling or complex interdependencies among activities.

Overall, the proposed framework offers a structured and accessible way to connect estimation, scheduling, and risk concepts within a single teachable example. It is intended to complement existing curricula by reinforcing procedural understanding and supporting risk-informed reasoning grounded in construction operations. Future work may extend the framework to broader activity networks or integrate it with more advanced analytical tools, while preserving its core pedagogical emphasis.

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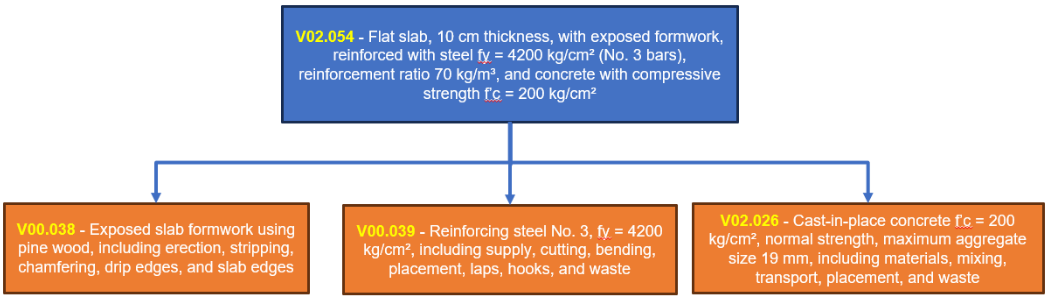
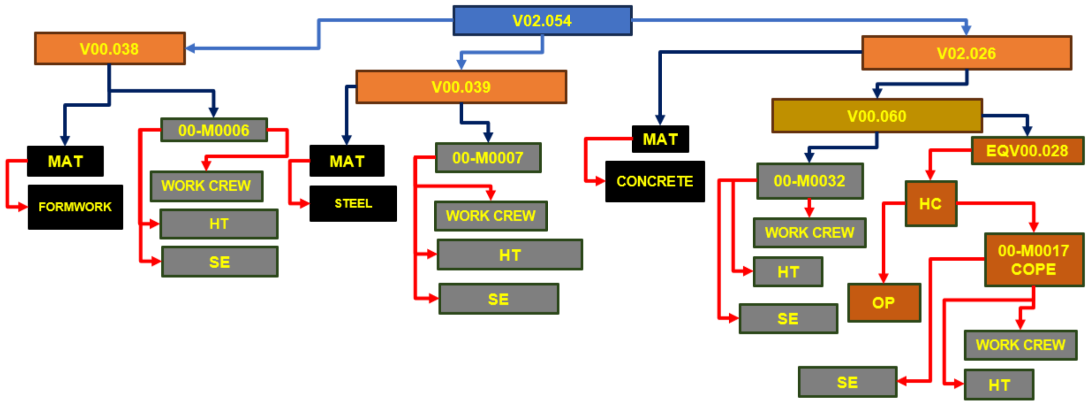
Appendix A – Preliminary Information on the Selected Main Construction Activity

This appendix documents the procedural and economic decomposition of the Main Construction Activity (MCA), which serves as the reference example for the proposed equation-based pedagogical framework. Its purpose is to provide a transparent and traceable description of the elemental activities, labor compositions, material inputs, and equipment parameters that support the unit-price formulations introduced in the main body of the paper.

The information presented in this appendix is not intended to represent optimized costs or benchmark values. Instead, it serves as a worked instructional example that allows readers to follow the complete chain from operational definition to unit-price formulation and activity aggregation.



(a)



(b)

Figure A.1 – (a) Directed Acyclic Graph (DAG) of the Main Construction Activity (MAC) of the example. (b) Complete structure of MAC where MAT=material, HT=hand tools, SE=safety equipment, OP=Operational costs, HC=hourly costs, and the item codes can be found in the following tables.

A.1 Description of the Main Construction Activity

The MCA considered in this example corresponds to the construction of a 10 cm thick reinforced concrete flat slab with exposed formwork, including:

- formwork installation and removal,
- reinforcing steel placement, and
- cast-in-place concrete production and placement.

The activity is defined per unit area (m²) of slab and reflects a common construction operation used in undergraduate construction engineering courses. The MCA and its associated elemental activities are summarized in Table A.1. At the same time, their procedural relationships are represented graphically by the Directed Acyclic Graph (DAG) shown in Figure A.1, and its tabular representation in Figure A.2.

	MCA		
	EA-1	**EA-2**	**EA-3**
LEVEL 1	$p_{1,1}$ **i=1, j=1** V00.038 – Formwork	$p_{2,1}$ **i=2, j=1** V00.039 – Steel	$p_{3,1}$ **i=3, j=1** V02.026 – Concrete
LEVEL 2			$p_{3,2}$ **i=3, j=2** V00.060 - Fabricarionn

Figure A.2 – Tabler version of the DAG in Figure A.1

Table A.1 - Main Construction Activity and elemental activities, see figure A.1						
Item Code	**Type**	**Description**	**Unit**	**Unit Cost (USD, 2026)**	**Quantity**	**Amount (USD, 2026)**
V02.054	MCA	Flat slab, 10 cm thickness, with exposed formwork, reinforced with steel fy = 4200 kg/cm² (No. 3 bars), reinforcement ratio 70 kg/m³, and concrete with compressive strength f'c = 200 kg/cm²	m²			
V00.038	EA	Exposed slab formwork using pine wood, including erection, stripping, chamfering, drip edges, and slab edges	m²	31	1.04	32.24
V00.039	EA	Reinforcing steel No. 3, fy = 4200 kg/cm², including supply, cutting, bending, placement, laps, hooks, and waste	kg	2.12	7.49	15.88
V02.026	EA	Cast-in-place concrete f'c = 200 kg/cm², normal strength, maximum aggregate size 19 mm, including materials, mixing, transport, placement, and waste	m³	137	0.104	14.25

A.2 Procedural Decomposition of the MCA

The MCA is decomposed into three primary Elemental Activities (EAs), each corresponding to a distinct operational process:

- EA-1 ($i = 1, j = 1$): Formwork installation
- EA-2 ($i = 2, j = 1$): Reinforcing steel placement
- EA-3 ($i = 3$): Concrete production and placement

The third EA is further decomposed into execution sub-levels to reflect differences in operational logic and resource coupling:

- Concrete materials ($j = 1$)
- Concrete fabrication using on-site equipment ($j = 2$)

This decomposition follows the procedural execution logic rather than the contractual work breakdown structure. Each EA is defined by its actual field operations, including material consumption, labor crews, hand tools, safety equipment, operational costs, and machinery requirements.

The complete hierarchical structure of the MCA is illustrated in Figure A.1(b), where each node corresponds to a cost component explicitly used in the unit-price formulation. This structure makes visible how material flows, labor resources, and equipment operations are coupled at different execution levels.

Table A.2 – EA of Reinforcing steel, i=2, j=1					
Item Code	**Description**	**Unit**	**Quantity**	**Unit Cost (USD, 2026)**	**Amount (USD, 2026)**
V00.039	Reinforcing steel No. 3, fy = 4200 kg/cm², including supply, cutting, bending, placement, laps, hooks, and waste	kg	—	—	—
	MATERIALS				
	Steel tie wire, gauge 18 (Item 1201)	kg	0.03	1.75	0.0525
	Reinforcing bar, Grade 42, No. 3 (3/8") (Item 30764)	t	0.00105	1,451.40	1.52397
	LABOR				
00-M0007	Crew 07 (ironworker + helper)	day	0.00667	81.32	0.542404

A.3 Main Construction Activity and Elemental Activities

Table A.1 presents the MCA and its associated elemental activities, including their units, unit costs (expressed in USD, 2026), quantities, and resulting amounts. These values establish the deterministic baseline for the example and serve as the numerical reference for applying Equation (1).

The table intentionally avoids aggregation beyond the elemental activity level to preserve transparency and allow direct correspondence between tabulated values and the terms of the unit-price equations.

Table A.3 – Labor into EA, i=2

Item Code	Description	Unit	Quantity	Unit Cost (USD, 2026)	Amount (USD, 2026)
00-M0007	Crew 07 (ironworker + helper)	day	—	—	—
	LABOR				
	Skilled ironworker	day	1	42.87	42.87
	Assistant worker	day	1	28.69	28.69
	Foreman/lead tradesperson	day	0.1	59.03	5.9
	EQUIPMENT AND TOOLS				
	Safety equipment	%-labor	0.02	77.47	1.55
	Hand tools	%-labor	0.03	77.47	2.32

A.4 Elemental Activity Cost Breakdown

Detailed cost breakdowns for selected elemental activities are provided in Table A.2. These tables document the materials, labor crews, and equipment associated with each EA or sub-elemental activity, expressed in USD (2026) for consistency across the example.

As an illustrative case, Table A.2 documents EA-2 (Reinforcing steel placement), capturing both material inputs (reinforcing steel and tie wire) and labor composition (Crew 07). The table provides the numerical basis for evaluating the corresponding cost components in Equation (1), without aggregation or simplification.

A.5 Labor Composition and Equipment Parameters

To further enhance traceability and pedagogical clarity, labor compositions are disaggregated in Table A.3, which explicitly defines crew structures, individual roles, unit costs, and auxiliary percentages for hand tools and safety equipment.

This level of detail allows readers to trace how labor costs propagate from individual workers to elemental activities and, ultimately, to the MCA. It also supports direct linkage between labor-related parameters and productivity-based modeling introduced later in the paper.

A.6 Role of Appendix A within the Pedagogical Framework

Appendix A functions as the deterministic reference layer of the proposed pedagogical framework. Documenting the complete procedural and economic structure of the MCA provides a concrete foundation for the introduction of uncertainty modeling and stochastic extensions presented in Appendix B.

The explicit representation of activities, resources, and unit-price components enables replication, adaptation, and instructional reuse of the example across different educational contexts. Rather than serving as a prescriptive cost model, the appendix supports learning by making assumptions, units, and operational logic explicit and auditable.

Appendix B – Example and stochastic models

The example uses the MCA-DAG branch V00.039: Steel reinforcement placement and fabrication. This branch only contains material and associated labor. The equation (1) then remains as follows:

$$p_{i,j} = \left(\rho_{i,j}^{(M)} \cdot M_{i,j}\right) + \left(\rho_{i,j}^{(L)} \cdot L_{i,j}\right) \qquad \text{(B.1)}$$

The EA components are shown in Figure B.1.

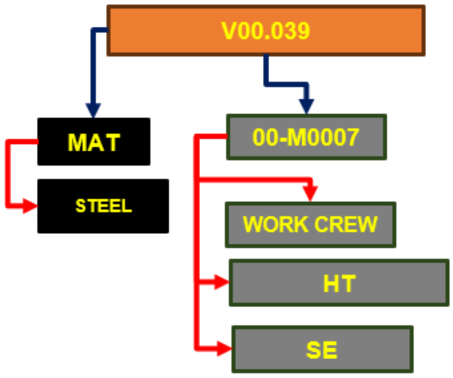


Figure B.1 – Branch in the MAC-DAG related to steel reinforcement, where MAT=material, HT=hand tools, SE=safety equipment, and the item codes can be found in Appendix A tables.

	Table B.1 – Assigned Stochastic models to EA, i=2				
Item Code	**Description**	**Unit**	**Stochastic models**	**Quantity**	**Unit Cost (USD, 2026)**
V00.039	Reinforcing steel No. 3, fy = 4200 kg/cm², including supply, cutting, bending, place-ment, laps, hooks, and waste	kg	--	--	--
	MATERIALS				
	Steel tie wire, gauge 18 (Item 1201)	kg	U	0.03-0.032	1.75-1.80
	Reinforcing bar, Grade 42, No. 3 (3/8") (Item 30764)	ton	U	0.00103 – 0.00106	1,451.40 – 1,460.3
	LABOR				
00-M0007	Crew 07 (ironworker + helper)	day	LN	μ=(1/140) and σ=(1/15)	See table B.1 (work day/kg)
Journey of 8 hours per day, U = Uniform distribution, LN=Log-normal distribution					

	Table B.1 – Labor into EA, i=2				
Item Code	**Description**	**Unit**	**Stochastic model**	**Quantity**	**Unit Cost (USD, 2026)**
00-M0007	Crew 07 (ironworker + helper)	day	--	—	—
	LABOR				
	Skilled ironworker	day	D	1	42.87
	Assistant worker	day	D	1	28.69
	Foreman/lead tradesperson	day	D	0.1	59.03
	EQUIPMENT AND TOOLS				
	Safety equipment	%-labor	U	0.015 – 0.02	77.47
	Hand tools	%-labor	U	0.02 -0.04	77.47
D = Deterministic, U=Uniform distribution					

In this case, the coefficients $\rho_{i,j}^{(M)}$ and $\rho_{i,j}^{(L)}$ are equal to 7.49, as shown in Table A.1. In Appendix C, the code and estimates are shown, with a mean cost of 16.19 USD per reinforced steel installation (including material) per square meter of slab and a standard deviation of 0.4812 USD. The mean time to finish the task is 7.22 days (8 working hours per day) with a standard deviation of 0.77 days.

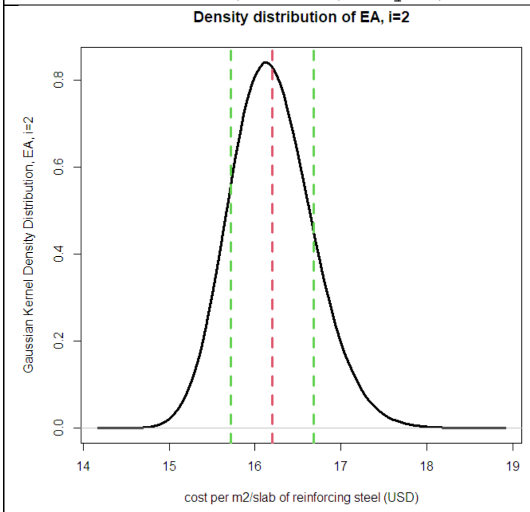
Appendix C - R-code for example and graphical output

This appendix provides a reference implementation of the computational procedures supporting the pedagogical framework presented in the paper. The code illustrates how the unit-price equations, productivity-based activity durations, and uncertainty propagation described in the methodology can be implemented transparently and reproducibly using simple scripting tools. Its purpose is instructional rather than prescriptive: the implementation is intended to help readers understand the logical structure of the calculations, the flow of information between cost, productivity, and schedule components, and the basic mechanics of simulation, rather than to serve as an optimized or production-ready software solution.

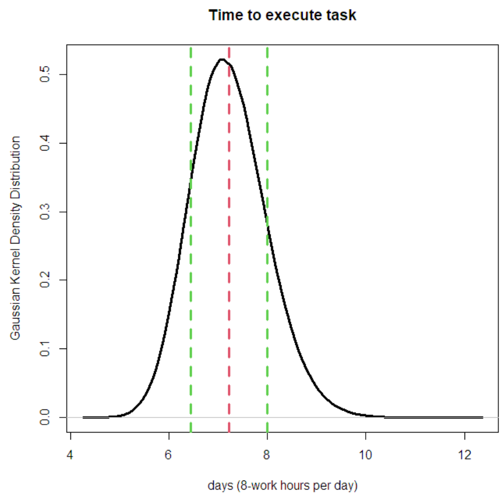
C.1 – (a) R-code to simulate the selected EA (i=2). (b) Density distribution of EA, i=2

```
# Erase variables in the environment
rm(list=ls())

# number of simulations
nsim <- 1000000
given_kg <- 1000

### Labor into EA, i=2
# vector of Labor
 vec_labor_quantity <- c(1,1,0.1)
 vec_labor_unitcost <- c(42.87, 28.69, 59.03)
# Stochastic simulations : vector of equipment and tools (%-labor)
 safety_equipment_simulations <- runif(nsim, min=0.015, max=0.02)
 hand_tools_simulations <- runif(nsim, min=0.02, max=0.04)
# labor costs per day (8-hour journey a day)
labor_plus_eq_tools <- array(0, dim=c(nsim))
eq_tools            <- array(0, dim=c(nsim))

for (i in 1:nsim){
 # equipment and tools
   eq_tools[i] <- safety_equipment_simulations[i] +
hand_tools_simulations[i]
 # only labor costs
  labor_plus_eq_tools[i] <-
sum(vec_labor_quantity*vec_labor_unitcost)*(1+eq_tools[i])
}

## EA estimation
 steel_wire_tie   <- runif(nsim, min=0.03, max=0.032)
 reinfocing_bar   <- runif(nsim, min=0.00103, max=0.00106)
 mean_performance <- 140 # mean kg/day (8 hours work = journey)
 sd_performance   <- 15 # standard deviation of mean kg/day performance
 cov_performance  <- sd_performance / mean_performance
 log_sd_performance   <- sqrt(log(1 + cov_performance^2 ))
 log_mean_performance <- log(mean_performance) -
(0.5*(log_sd_performance^2))
 crew_performance  <- exp( rnorm(nsim, mean=log_mean_performance,
sd=log_sd_performance )  )
 crew_performance_r <- 1/crew_performance # journey/kg reciprocal
# material and labor costs
```

```
 cost_steel_wire        <- runif(nsim, min=1.75, max=1.80)
 cost_reinfocing_bar    <- runif(nsim, min=1451.4, max=1460.3)
 cost_crew_performance <- labor_plus_eq_tools #journey/kg reciprocal

final_cost <- array(0, dim=c(nsim))
rho <- 7.49
for (i in 1:nsim){
    va <- c(steel_wire_tie[i], reinfocing_bar[i], crew_performance_r[i] )
      vb <- c(cost_steel_wire [i], cost_reinfocing_bar[i],
cost_crew_performance[i])
      final_cost[i] <- sum(va*vb)*rho
}
mf <- mean(final_cost)
sdf <- sd(final_cost)
mf ; sdf
plot(density(final_cost),
       xlab="cost per m2/slab of reinforcing steel (USD)",
         ylab="Gaussian Kernel Density Distribution, EA, i=2",
         main="Density distribution of EA, i=2", lwd=3)
abline(v=mf, col=2, lty=2, lwd=3)
abline(v=mf-sdf, col=3, lty=2, lwd=3)
abline(v=mf+sdf, col=3, lty=2, lwd=3)

# Scheduling information for Gantt Diagram
Performance_per_hour <- (1/crew_performance_r)/8
mean(Performance_per_hour)
time_execution <- (given_kg / Performance_per_hour)/8 # hours
md <- mean(time_execution)
sdd <- sd(time_execution)
md ; sdd
plot(density(time_execution), main="Time to execute task", xlab = "days (8-
work hours per day)",
     ylab="Gaussian Kernel Density Distribution", lwd=3)
abline(v=md, col=2, lwd=3, lty=2)
abline(v=md-sdd, col=3, lty=2, lwd=3)
abline(v=md+sdd, col=3, lty=2, lwd=3)
```



(b)



(c)