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Vibration Mitigation Studies in a Mechanical Vibrations Course

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VIBRATION MITIGATION STUDIES IN A MECHANICAL VIBRATIONS COURSE

Abstract

The topics on the use of damping in controlling vibration, as well as on vibration suppression are typically covered in a junior (or senior) level course on mechanical vibration. We have created two student projects, addressing both topics. The first one is an analytical (numerical) project on the damping effects on the response of a two-degree-of-freedom system. The other is an experimental project to address vibration suppression. Both projects involve systems that require two independent coordinates and therefore are two-degree-of-freedom systems. We feel that these two so called "learn by doing" activities align very well with the associated lecture modules.

INTRODUCTION

Vibration mitigation strategies are the approaches, techniques, and technologies employed to reduce the intensity, amplitude or other adverse effects of vibrations in a structure. These efforts are fundamentally directed to reduce unwanted oscillatory motion to enhance system performance and longevity. In this paper we address two such studies: the first one involves dissipation of vibrational energy through a shock absorber in a car that uses damping to reduce the bouncing motion when it hits a bump. The second study uses auxiliary systems to resonate at specific frequencies to counteract and cancel out primary vibrations.

The first project involves analysis of car suspension systems. The process helps students to better understand the application of some fundamental concepts covered in this course. It also serves as a starting point for students to further apply their engineering knowledge in design analysis. Vehicle suspension systems which represent the connection between automobile body and road are important for road handling and ride comfort. The suspension system is located between the frame, and the wheels absorb bumps and other imperfections in the road leading to riding comfort. The springs and the dampening mechanisms are the basic construction components of a modern car suspension system. The quarter car model is the most useful practical model of a vehicle suspension system. This model is excellent to examine and optimize the body bounce mode of vibration. There have been several studies involving quarter car model [1-3]

The second project involves a study of a vibration absorbing system. The students are made aware of the fact that tuned mass dampers are employed in skyscrapers to reduce the sway caused by winds or earthquakes by acting as a vibration absorber. Vibration absorber is a component that has a mass and stiffness section attached to the main mass to protect it from vibration [2]. The combination of the absorber and the main mass yields a new system with two degrees of freedom and two natural frequencies. With the frequency of the input and the natural frequency of the original system, values can be selected for the mass and stiffness of the absorber so that the displacement of the mass is minimal. There exist several studies on tuned mass dampers [4-6].

PROJECT- 1

This project employs a quarter car model, a simplified yet effective representation of a vehicle's suspension dynamics. The model focuses on one-fourth of the vehicle to analyze the interaction between the tire, suspension and the chassis under various road conditions l7.8].

This project is first introduced through class and homework assignments as a base excited system as shown in Figure 1

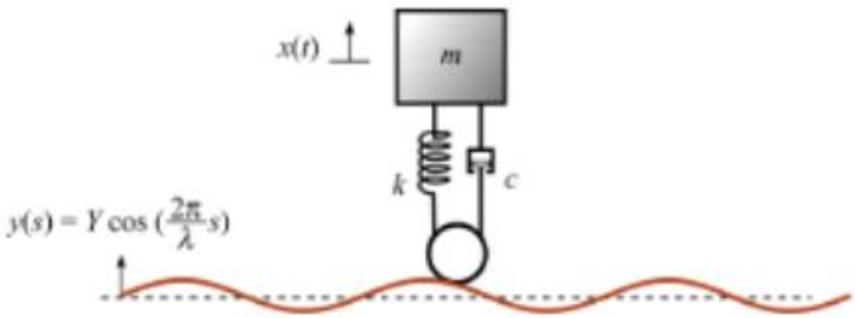

Figure 1 A one-degree-of-freedom quarter car model

The students soon realize that this model is not suitable to capture all the features of the suspension system of a car. Therefore, a two-degree-of-freedom model is introduced as shown in Figure 2. This model consists of two masses, the wheel mass (the unsprung mass) and the chassis mass (the sprung mass) connected by a spring and damper, and the tire modeled by a separate spring. This model allows for the study of various scenarios, such as, bump and rebound motions (the body bounce and the wheel hop), the ride comfort and handling, and helps in designing suspension systems that effectively isolate the chassis from road disturbances. The major objectives of this activity are:

- Explain how the various parameters affect the motion of a two-degree-of-freedom quarter car model.
- Create complete and legible plots of the input and output trajectories
- Numerically integrate ordinary differential equations of motion of the sprung and unsprung masses with MATLAB's ODE45 command.

The project deals with a car suspension system, wherein a quarter model of an automobile is employed for its motion over a sinusoidal road surface. Each individual group is assigned a particular amplitude and wavelength of the assumed road surface as well as a specific damping parameter for the suspension system, while all groups use identical values of suspension and tire stiffnesses. The students are asked to derive the equations of motion for the two-degree-of-freedom system, obtain the mode shapes and the modal frequencies, the force and displacement transmissibility using MATLAB. In addition, they will obtain the responses of the system with and without damping. As the different groups present their work the students can clearly observe the effect of varying stiffness and damping parameters on the dynamic responses. This activity thus helps develop computational thinking and makes the students better equipped to tackle real world vibration problems.

A quarter car model of an automobile is shown in Figure 2. The vehicle is travelling with a maximum velocity v = 100 km/h on a sinusoidal road surface with amplitude Y, and a wavelength of □. Assume m_1 = 1000 kg, m_2 = 75 kg, k_1 = 30,000 N/m, and k_2 = 300,000 N/m. The values of Y, □ and c will be different for different students or groups. Assume zero initial conditions.

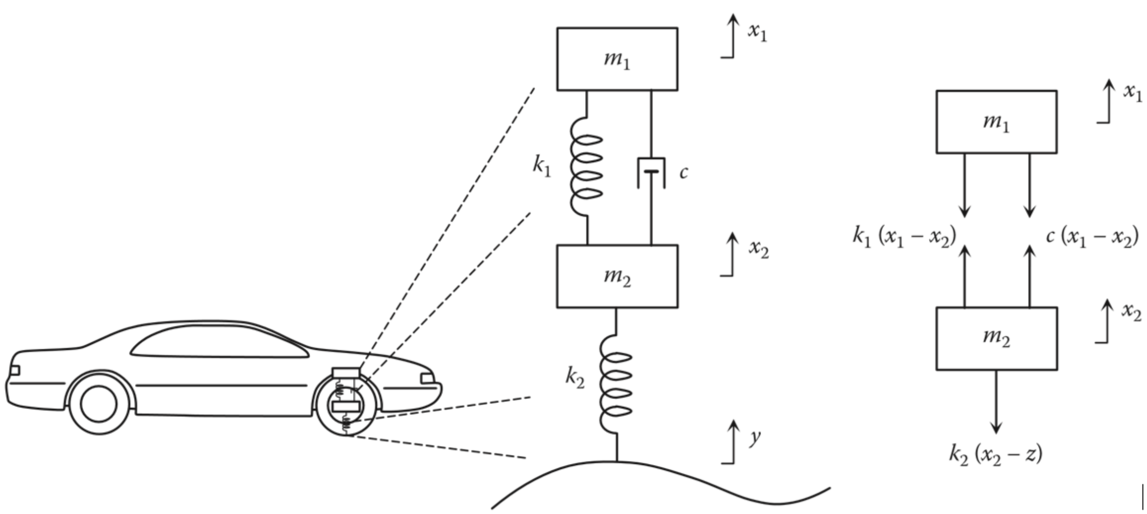

Figure 2 A two-degree-of-freedom quarter car model

The students are assigned the following tasks:

1) Derive the differential equations of motion and obtain the mass, stiffness, and damping matrices, and the force vector.
2) Determine the displacement transmissibility, force transmissibility, for the system using MATLAB [10]
3) Use MATLAB to compute the responses (i.e. x_1 versus time and x_2 versus time) of the system with and without the damper. Afterwards, they can vary the values for c and k_1 to examine the effects and the quality of the response (including displacement transmissibility, and force transmissibility), to input road surface. Determine the values for c and k_1 that would produce the "best" suspension system

For the car with mass m_1, stiffness k_1, and the damping coefficient c, we have:

$$-k_1\;(x_1 - x_2) - c\;(\dot{x_1} - \dot{x_2}) = m_1\ddot{x_1} \qquad (1)$$

For the tire with mass m_2, stiffness k_2, we have:

$$k_1\ (x_1 - x_2) + c\ (\dot{x_1} - \dot{x_2}) - \ k_2(x_2 - y) = \ m_2\ddot{x_2} \qquad (2)$$

Equations (1) and (2) can be expressed in matrix form as:

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{matrix} \ddot{x_1} \\ \ddot{x_2} \end{matrix} + \begin{pmatrix} c & -c \\ -c & 0 \end{pmatrix} \begin{matrix} \dot{x_1} \\ \dot{x_2} \end{matrix} + \begin{pmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{matrix} 0 \\ k_2 y \end{matrix} \qquad (3)$$

Using numerical values (typical) of amplitude *Y*= 0.02 m and a wavelength of □ = 5m,

The frequency of the exciting force is given by,

f = v/□ = 100/(3.6x5) = 5.556 Hz which leads to ω = 34.9 rad/ss x1 and x2

The road profile then is given by,

y = 0.02 sin (34.9t) (4)

Equation (3) with the assumed numerical values is then,

$$\begin{pmatrix} 1000 & 0 \\ 0 & 75 \end{pmatrix} \begin{matrix} \ddot{x_1} \\ \ddot{x_2} \end{matrix} + \begin{pmatrix} 5000 & -5000 \\ -5000 & 5000 \end{pmatrix} \begin{matrix} \dot{x_1} \\ \dot{x_2} \end{matrix} + \begin{pmatrix} 30000 & -30000 \\ -30000 & 330000 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{matrix} 0 \\ 6000\sin(34.9t) \end{matrix} \qquad (5)$$

We note that the gravity term does not appear, because the static equilibrium position is taken as the coordinate origin.

With the state variables, x_1, x_2, x_3, and x_4 introduced as, we have

$$x_3 = \dot{x_1}$$

$$\boldsymbol{x_4 = \dot{x_2}}$$

$$\dot{x_3} == x_1{}^{\cdot\cdot} = -300\,x_1 + 400\,x_2 - 5x_3 + 667x_4 \qquad (6)$$

$$\dot{x_4} == x_2{}^{\cdot\cdot} = 4000\,x_1 - 4400\,x_2 + 667x_3 - 667x_4 + 400\sin(34.9t)$$

MATLAB [10]is used to solve Equations (6) using the command ODE45. Zero initial conditions are applied.

The displacements x_1 and x_2 are determined as functions of time. The maximum sprung displacement is found to decrease with increased damping. The displacement and force transmissibility's are also determined. The low values obtained for these two means that the displacement and force transmitted due to road irregularity to the driver are small compared to the road, which ensures comfortable driving of the car. Typical displacement time histories are shown in Figure 3, which shows that the sprung mass (chassis) displacements are small compared to that of the unsprung mass (tires)

The velocities are obtained by differentiating the displacements with respect to time as shown in Figure 3. The accelerations are obtained by differentiating the displacement response twice with respect to time. Velocities and accelerations follow the same trend as displacements as in Figure 3. Acceleration values are also found to be low ensuring driver comfort.

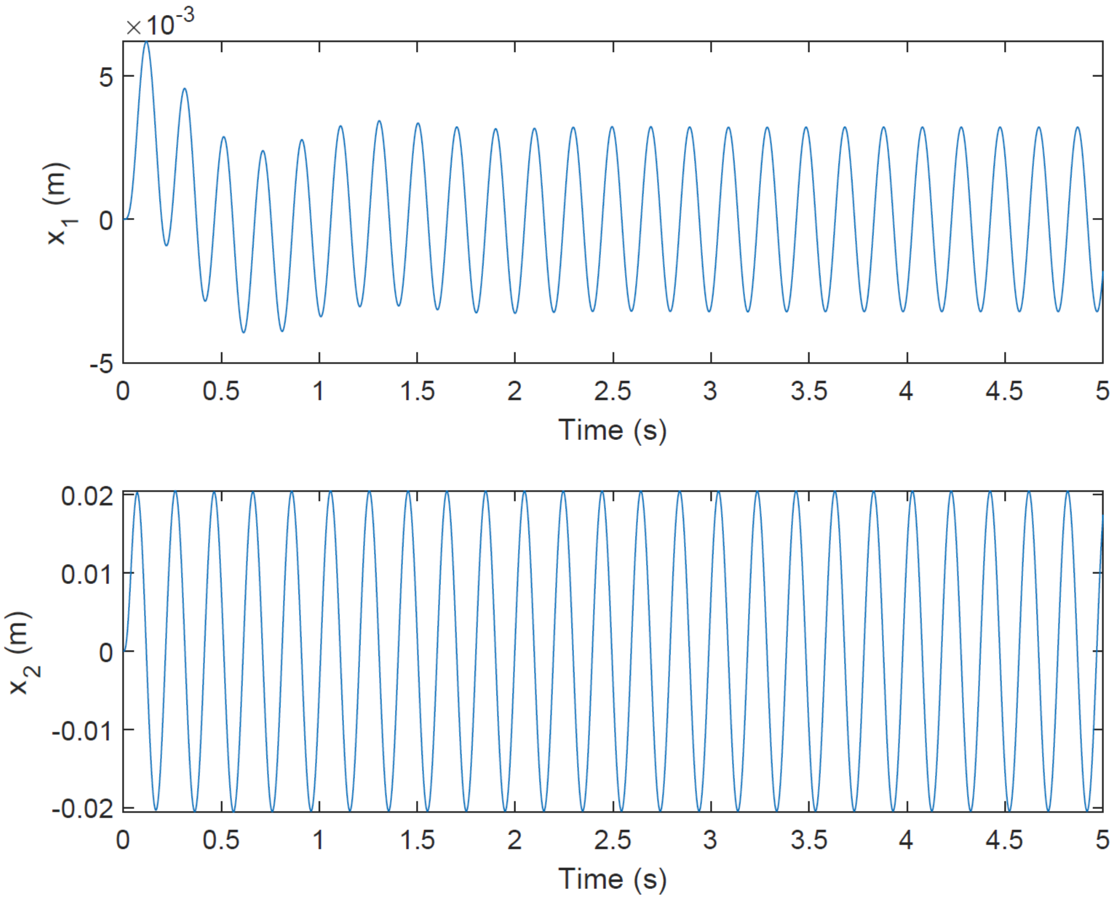

Figure 3 Displacement time histories for the car and the tire

PROJECT 2

The second project is to design and build a vibration absorber demonstration system [7.8]. Figure 4 shows a simple vibration absorbing system.

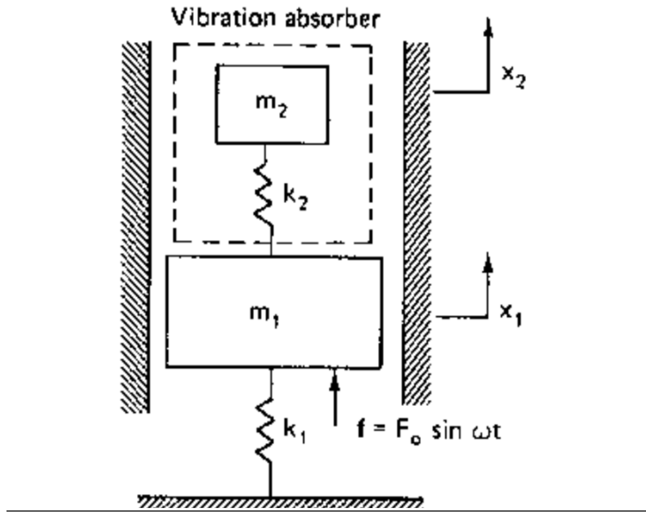

Figure 4 Vibration Absorber

When a structure is externally excited, it will have undesirable vibrations. The amplitude of vibration will be maximal when the system gets excited close to its natural frequency, and this can cause rapid catastrophic failure. Hence it becomes necessary to neutralize these vibrations. One of the methods for neutralizing these vibrations is by coupling a vibrating system to it so that the amplitude can be brought down to zero. The system known as dynamic vibration absorber (DVA) consists of another mass and stiffness attached to the main (or original) mass. Thus, the main mass spring system with the DVA constitutes a two-degree-of-freedom system, and therefore the combined system will have two natural frequencies.

The objectives of this project are:

- Recognize that reducing vibration is not always by subtracting but also by addition.
- Learn how to absorb vibrations in a device and recognize that vibration control applies not just to machinery and can apply to the structure of the tallest building in the world.
- Tune the DVA by varying dynamic system parameters.

The project consists of a motor with a rotating imbalance mounted on a cantilever beam as shown schematically in Figure 6. At the free end of the cantilever a motor is attached. Four bolts are connected to the motor by single nut each, except for one bolt which has two extra nuts. This results in an imbalance and will cause the beam to vibrate. Ther. r. p.m. of the motor is controlled by a power supply; The vibration absorber is a combination of spring and mass as shown. The absorber is attached by a hook under the beam. An accelerometer records the voltage on an oscilloscope as the beam vibrates.

The combination of the vibration absorber and the main mass leads to a two-degree-of-freedom system. An oscilloscope is used to observe the vibration of the beam and the accelerometer attached to the beam gave the acceleration data. The detailed view of the cantilever beam carrying the unbalanced motor is shown in Figure 7. Figure 8 shows the motor imbalance,

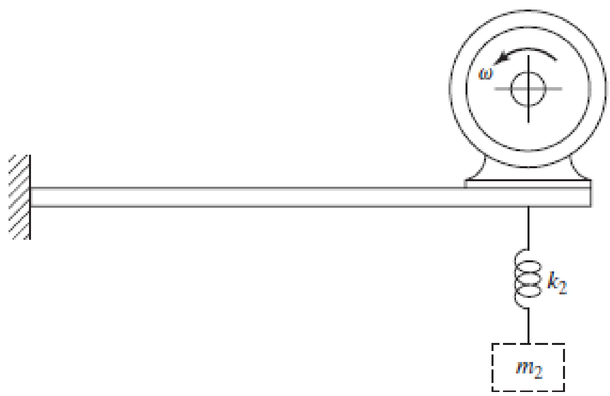

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Figure 5. Vibration Absorber Setup Schematic

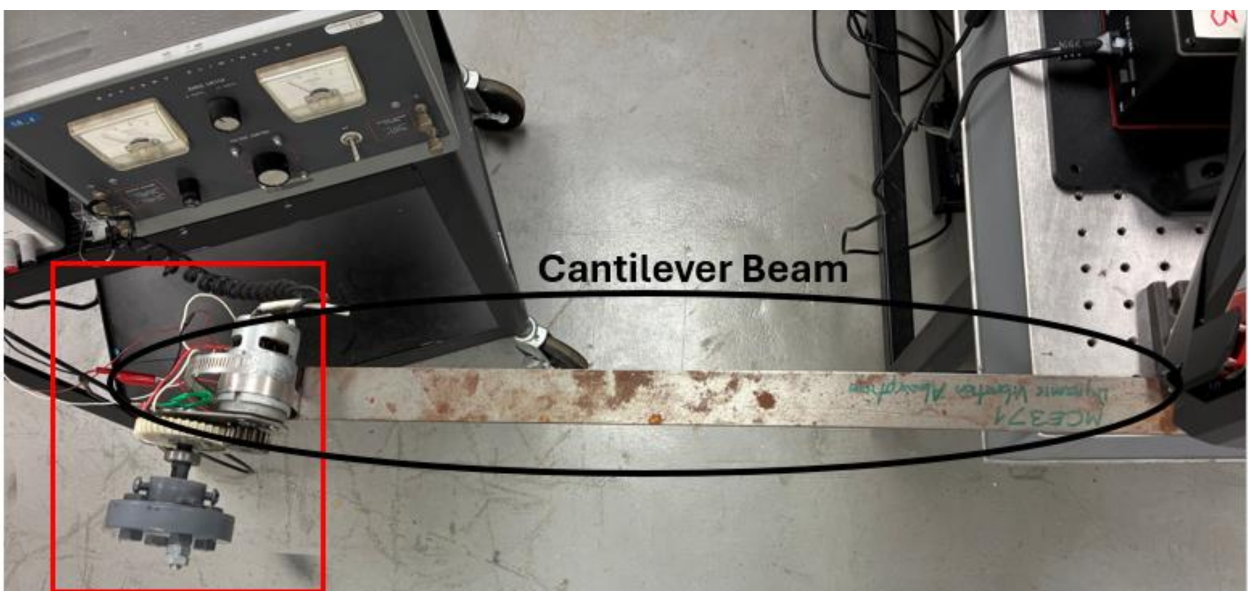

Figure 6 Detail of the Unbalanced Motor on the Cantilever Beam

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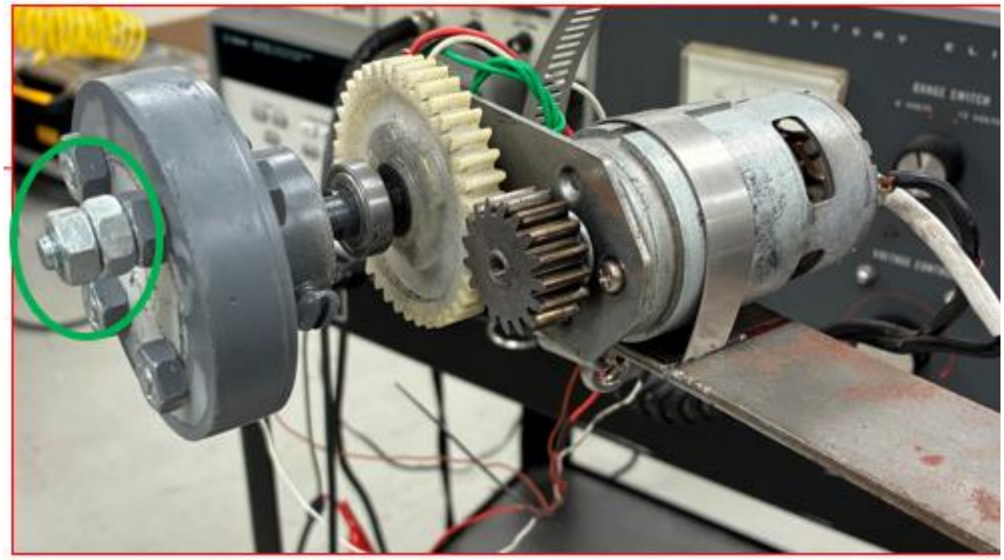

Figure 7 Creation of Rotary Imbalance

Figure 8 is a display of the setup of the experiment for vibration absorption studies**.**

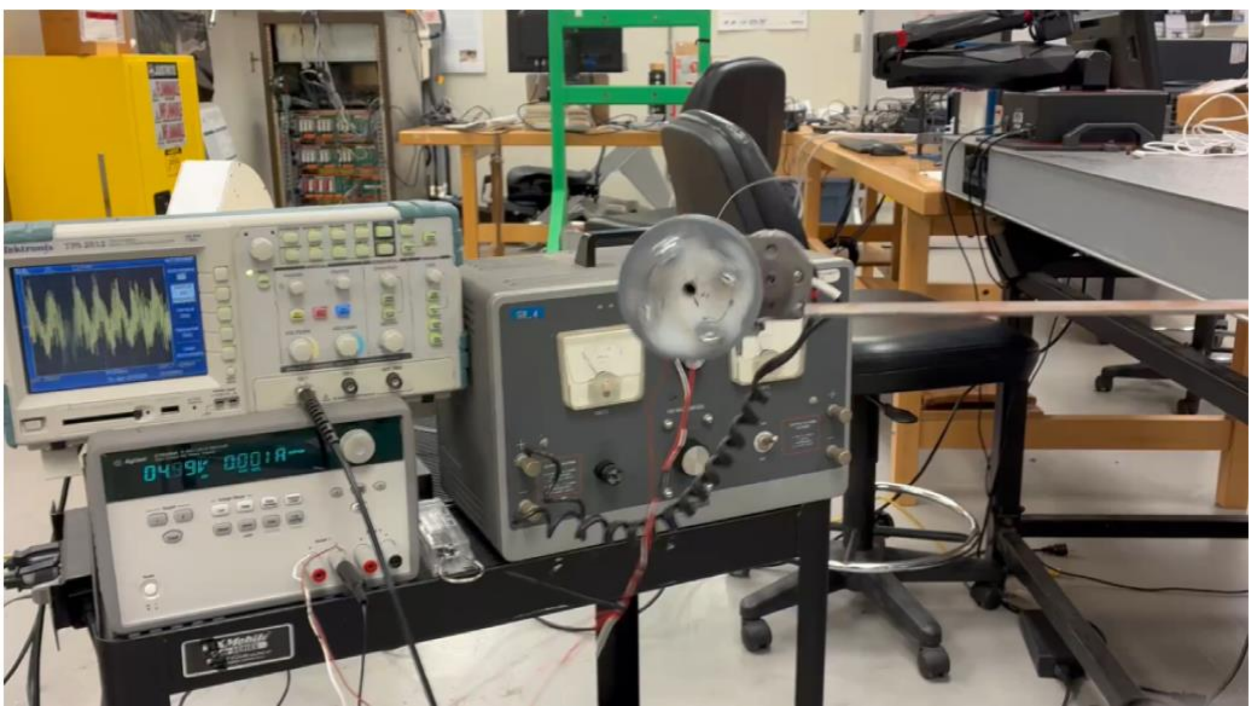

Figure 8 Experimental Setup for the Vibration Absorption Studies

The student groups are each assigned a different cantilever beam length, L

The spring constant of the cantilever beam is calculated both analytically ($k = \frac{3\,EI}{L^3}$ $with\ EI\ as\ flexural\ rigidity$ [9]) and experimentally and a comparison is made. The free vibration of the cantilever beam is recorded and shown in Figure 9. For the underdamped system, the damped natural frequency is evaluated by counting the number of peaks over a certain period. The undamped natural frequency is calculated by logarithmic amplitude decrement to determine the damping ratio.

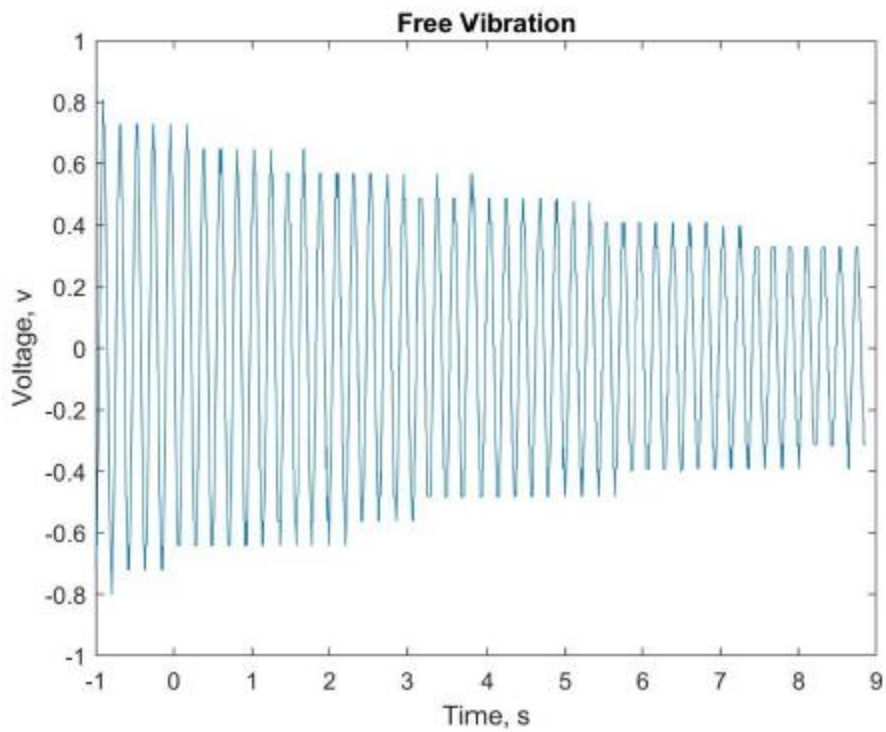

Figure 6 Free Vibrations of the cantilever beam on which the motor was mounted

The absorber mass and spring are adjusted to match with the beam natural frequency of the beam.

The second mass is found to absorb most of the vibratory motion of the motor. This activity thus enables the students to witness how the designed system redirects the vibrations from the unbalanced motor to another mass connected to the system, thereby suppressing the vibration of the unbalanced motor.

Euler Lagrange derivation of the equation of motion of the two degree-of-freedom system is carried out. The force and distance transmissibility's are calculated and a good correspondence with the experimental results is observed.

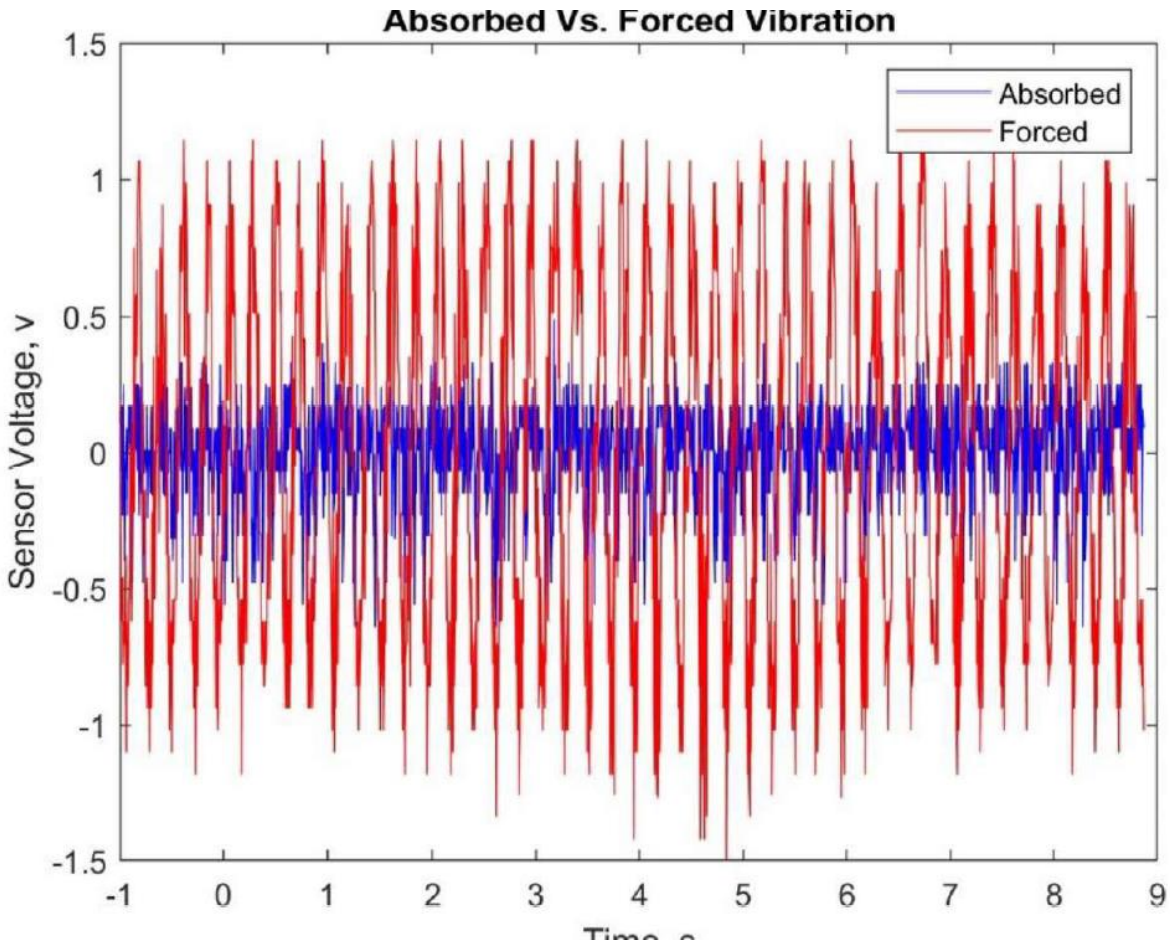

Figure 7 Absorbed vs Forced Vibration

Figure 7 shows the experimental validation of the vibration absorbing system. The vibration absorber appears to reduce the amplitude of vibration of the system of the order of 75% to 80%.

It is notable how the dynamic absorber cancels vibration at the resonance frequency. This was observed in the experiment. While the cantilever beam carrying the motor vibrated at its natural frequency (by adjusting the motor rpm), when the tuned vibration absorber system was attached the beam came to a standstill. With the reduction of excitation frequency another resonance was observed. This corresponded to one of the two resonant frequencies. Note that with the two-degree-of-freedom system there are two new natural frequencies, one below and one above the original natural frequency.

CONCLUSION

The projects on vibration mitigation have proved to be effective tools in enhancing student concepts on vibration control as well as other key areas of mechanical vibration. The in-depth integration of analytical and computational methods and tools through these two projects is a noteworthy takeaway for this very important course. Integration of a comprehensive analytical/computational project and a hands-on experimental project contributes to a unique student-centered learning process.

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