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Evaluating Student Concept Mastery through Critical Analysis of AI-Generated Engineering Mechanics Solutions

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WIP: Critical Evaluation of AI-Generated Solutions as an Assessment Strategy in Engineering Mechanics

Abstract

This work-in-progress (WIP) paper presents a novel assessment methodology for undergraduate engineering mechanics courses that leverages generative artificial intelligence (AI) outputs as objects of critique rather than sources of solutions. Students are provided AI-generated problem solutions of intentionally varied correctness and are tasked with evaluating the solutions. Their analysis will be accomplished using core mechanics principles, identifying errors, and justifying corrections. The approach is designed to strengthen conceptual understanding, reasoning, and engineering judgment, which are central to professional engineering practice. Additionally, this approach is designed to prepare students for professional careers by providing experience appropriately using tools that will be available, or even expected, to utilize. The methodology is piloted in a Strength of Materials course and aligned with learning outcomes. Ongoing data collection will inform future iterations and broader application across mechanics curricula.

1. Introduction

Generative artificial intelligence tools such as ChatGPT, Claude, and Gemini are increasingly capable of producing complete and convincing solutions to traditional engineering mechanics problems [1], [2]. This is especially true for commonly used problem sets often found in engineering textbooks. While these tools present opportunities for supporting student learning, they also challenge the validity of conventional assessments that rely on student-generated solutions as evidence of concept mastery. Studies have shown that students frequently use generative AI to generate solutions to problems and accepting the result without engaging in underlying reasoning processes [3]. This trend raises concerns regarding both academic integrity and the lack of critical thinking skills that are fundamental to the success of engineering students.

In professional engineering practice, engineers are rarely evaluated on their ability to generate solutions in isolation. Instead, they are routinely required to interpret, evaluate, validate, and critique analyses produced by others or by computational tools. Despite this reality, most engineering mechanics courses emphasize solving problems focused on building a solution from the ground up based on the fundamental engineering principles rather than evaluating a given solution. This mismatch as well as the desire to prepare students to appropriately use available tools motivates the development of assessment strategies that better align with authentic engineering practice [4], [5].

This paper introduces a work-in-progress assessment methodology that inverts the traditional problem-solving paradigm. Rather than asking students to solve mechanics problems from scratch, students are asked to critically evaluate AI-generated solutions whose correctness is intentionally varied and unknown to them. The primary research questions guiding this work are:

1. Does evaluating AI-generated solutions improve students' conceptual understanding in

engineering mechanics?
2. What types of errors do students most effectively identify, and where do misconceptions persist?
3. How does student performance and perception differ from traditional assessment formats?

2. Literature Review

Recent literature has documented the increasing capability of generative AI systems to solve engineering problems at a level comparable to or exceeding that of undergraduate students [1]. It has been demonstrated that ChatGPT could successfully complete a wide range of engineering assessment prompts, raising concerns about assessment integrity and learning validity. Other studies have emphasized the need to redesign assessments to assume AI use rather than prohibit it.

Research on student use of AI tools in STEM courses indicates that many students rely on AI as a shortcut to final answers. Students often submit these solutions without critically evaluating correctness or understanding the problem-solving rationale. This behavior has been associated with reduced engagement in course material and loss of the ability to deconstruct problems into fundamental components [6] [10]. To extrapolate this effect, students often overestimate the ability of AI in solving problems and blindly trust the solutions given [7]. In response, several authors have proposed integrating AI critique into coursework as a means of fostering critical thinking and AI literacy.

Despite these proposals, there remains a notable gap in domain-specific empirical studies examining AI-integrated assessments in engineering mechanics [7] [8]. Existing work largely focuses on programming, data analysis, or general STEM contexts. The present study addresses this gap by implementing and evaluating a structured AI-solution critique assessment within a core mechanics course.

3. Methodology

The assessment intervention was implemented in an undergraduate Mechanics of Materials course with approximately 30 students. The course covers the concepts of stress, strain and their effects on deformation and failure of mechanical bodies.

AI-generated solutions were created using structured prompts designed to produce three distinct levels of correctness: fully correct solutions, partially correct solutions containing subtle conceptual errors, and incorrect solutions exhibiting systemic reasoning flaws. The design of AI prompts was informed by prior work, which demonstrates that prompt structure significantly influences the quality and nature of AI-generated responses [9]. AI-generated solutions were produced using Claude Sonnet 4.5 via the Perplexity.ai interface (accessed February 2026). Default generation parameters were used, and outputs were not edited for correctness or clarity prior to student distribution. The solutions provided were evaluated and deemed acceptable without modification. Examples of problems, prompts, and solutions are given in ***Appendix A***.

Students were given access to a sample exam with a sample problem containing this type of question along with a sample rubric of how the problems would be scored. The assessment methodology was explained to students 10 days prior to the exam to allow time for familiarization and preparation with a likely new examination format.

All midterm and final exams follow the same structure and are equally weighted. Each exam contains four questions, each worth equal points. Students were provided with an exam containing two AI-generated solutions and two traditional engineering mechanics problems. They were asked to:

1. Identify specific errors if present
2. Explain correct reasoning using course concepts.
3. Note the correct solution or approach

It is critical to note that students were not informed of the correct level of the solution that they were given. These types of problems were included in each of the mid semester exams as well as the final. The exam design employed counterbalanced versions to control for item-position effects Two versions of the exam were given. Each version will contained two AI solutions for evaluation and two traditional-type problems. Version A contained AI questions of topics 1 and 3, while version B included AI evaluations on topics 2 and 4. AI correctness levels were assigned to problems randomly. At this time of this work, AI provided solutions were limited to text and numerical formats.

Student performance on question type was measured by evaluating grades on questions while perceptions will be measured by survey. These surveys will use Likert scale items evaluating perceived difficulty, cognitive demand, and confidence in solution. The surveys will be completed after each exam to track changes as students' comfort with the question type increases.

This methodology mirrors the way professional engineers in industry engage with AI. In many cases, AI may be thought of as a tool extending the capabilities of a calculator. Engineers have been trained to conduct a "sanity check" of the number provided by the calculator before committing that number to the solution. Similarly, current engineering students must be trained to conduct the same sort of evaluation on AI solutions before assigning them to the solution [10].

4. Assessment Instruments and Rubric

Student submissions were evaluated using an analytic rubric designed to assess reasoning and critical thinking rather than procedural execution. The rubric focuses on error identification accuracy, conceptual explanation quality, and clarity of justification. To promote grading consistency, rubric criteria were defined prior to exam administration and applied uniformly across both exam versions. All AI-critique questions were graded using the same analytic rubric. All exams were graded by the instructor to ensure consistent application of rubric criteria.

Table 1. Evaluation Rubric

Criterion	4 - Exemplary	3 - Good	2 - Satisfactory	1 - Developing	0 - None
Error Identification	All errors correctly identified	Most errors identified	Some errors missed or incorrect	Many errors missed or incorrect	No errors identified correctly
Reasoning/ Justification	Logical and well-structured argument	Strong justification provided	Partial justification, parts missing	Little or no justification	No justification provided
Solution/Approach	Shows the correct solution or pathway	Mostly correct containing minor errors	Partially correct	Limited	No solution or pathway

5. Preliminary Results and Discussion

Preliminary results are available based on results from a two midterm examinations. Exam grades were collected and compared for the exam as well as by individual question. Survey results are not yet available due to the timing of the exam. The exams were given to the 24 students in the class. 12 students received version A with traditional problems posed for questions 1 and 3 and 12 students received version B with the questions swapped. AI-critique questions were graded via the rubric and then scaled proportionally so allow the total question value to be 25. Preliminary analysis is provided in Table 2. Numerical analysis of the scores is intended to be descriptive at this stage of this work in progress.

Table 2: Scores by Question Type

Question Type	Number of questions (n)	Mean (Max 25)	Standard Deviation
Total	184	20.13	4.7
AI-Critique	92	21.15	3.8
Procedural	92	19.1	5.26

Comparisons between question types across all 24 students show similar results between question types. This early data suggests that AI-critique questions do not place students at a disadvantage, as the average score is higher on those types of questions. Early data suggests that this assessment is a feasible approach. Performance was comparable between question types. A further comparison between question types categorized by question and concept is shown in Table 3. Sample student AI critique responses are provided in ***Appendix B.***

An important note is that students were not yet intentionally exposed to an AI-critique type problem with systematic conceptual flaws needing identification. It is feasible that all students are not careful enough to identify errors and are reliant on the model's mostly correct answers.

However, Question 2 was generated using the same prompt intending to create a solution containing partially correct solutions as an analogy to Question 1 on the opposite version. The solution would be categorized by the instructor as containing systematic errors with fundamental flaws. The solution was left unmodified for consistency. It should be noted that this question was the only one in which students scored lower on the AI-critique compared to the procedural question. This highlights the need for additional data to clarify results.

Table 3: Scores by Question

Question	AI			Procedural		
	Mean	Median	SD	Mean	Median	SD
1. Statically Indeterminate	17.67	18	2.19	14	16	6.42
2. Stress Concentration	18	15	5.08	21.25	21.5	3.33
3. Strain/Poisson Ratio	23.25	24.5	3.58	18.83	20.5	6.04
4. Shear Strain	22.41	25	3.58	20.33	21	4.12
5.Torsional Shear Stress	22.8	22.5	2.20	22.088	24	3.9
6. Bending Stress	23	23	2	19.7	20.5	2.87
7. Composite Beam Stress	21.9	22.5	2.66	19.4	21	2.68
8. Beam Deflection	20.4	21	4.1	17.2	17	5.47

One emerging observation is that students score universally higher on the AI-critique questions. It was noticed during the evaluation of the completed exams that the AI-style question may have the effect of raising the exam score floor. Students lacking sufficient understanding of a concept may not be able to progress in the traditional style problem. In contrast, those same students have a path to a solution in the AI-critique version of the question.

6. Limitations and Future Work

As a work-in-progress study, this implementation is limited by sample size and scope. Additional data will be collected in this course to given a larger sample size and allow for exposure to all types of AI questions.

Additional versions of the exam can explore the feasibility of incorporating graphical solutions such as free body diagrams not the AI critique solutions. The importance of diagramming problems in mechanics courses is beyond question. The current AI capability is currently reliable enough to provide satisfactory solutions.

Future work will expand the methodology to additional mechanics courses, incorporate longitudinal measures of learning, and explore optimal scaffolding strategies.

7. Conclusion

The AI-solution critique approach begins to provide insight into student understanding that is not readily observable in traditional problem-solving tasks. By requiring students to articulate why a solution is incorrect, instructors gain visibility into conceptual gaps and reasoning patterns. This approach also aligns assessment with professional engineering practice, where validating analyses is a critical skill.

This paper presents an assessment methodology that leverages AI-generated solutions as tools for developing analyzing solutions in engineering mechanics education. It also offers an alternative approach to develop conceptual mastery in engineering students who may become over-reliant on AI-based tools. This approach may encourage students to engage more deeply with underlying concepts in order to identify errors within a solution. It also has a secondary benefit of subtly reinforcing the idea that AI solutions are not inherently correct. Finally, this methodology introduces students to appropriate usage of available tools which will become advantageous when they begin their professional practice.

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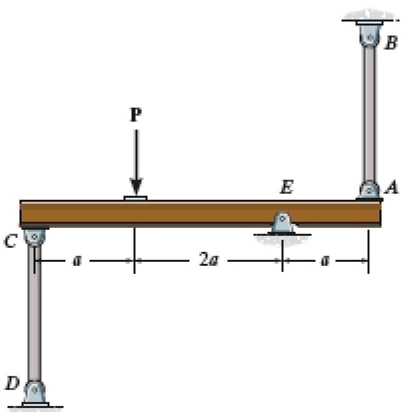
Appendix A: Examples of Problems, Prompts, and Solutions

Example 1: Partial Correctness

Two identical rods AB and CD each have a length L and diameter d, and are used to support the rigid beam, which is pinned at E. If a vertical force P is applied to the beam, determine the normal stress developed in each rod. The rods are made of material that has a modulus of elasticity of E.



Prompt:

Solve this problem to be 85% correct but contain several errors. List those errors separately. (Completed Using Claude Sonnet 4.5, 2/20/26)

Solution:

Static Equilibrium Analysis

Starting with force equilibrium in the vertical direction:

$$F_{AB} + F_{CD} = P(1)$$

Taking moments about pin E (with the erroneous sign convention):

$$-F_{CD}(a) + P(2a) - F_{AB}(4a) = 0$$

Simplifying:

$$F_{CD} + 4F_{AB} = 2P(2)$$

Solving equations (1) and (2) simultaneously:

- $F_{AB} = \frac{P}{3}$
- $F_{CD} = \frac{2P}{3}$

Stress Calculation

Using the incorrect area formula $A = d^2$ (instead of the correct circular cross-section area):

Normal stress in rod AB:

$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{P}{3d^2}$$

Normal stress in rod CD:

$$\sigma_{CD} = \frac{F_{CD}}{A} = \frac{2P}{3d^2}$$

List of Errors

Error 1: Incorrect moment equilibrium equation

- What was done: Used $-F_{CD}(a) + P(2a) - F_{AB}(4a) = 0$
- What's correct: Should be $F_{CD}(a) - P(2a) + F_{AB}(4a) = 0$ (upward support forces create positive moments when taken counterclockwise about E)
- Impact: This led to incorrect force distribution between the rods

Error 2: Wrong cross-sectional area formula

- What was done: Used $A = d^2$
- What's correct: For circular rods, $A = \frac{\pi d^2}{4}$
- Impact: Final stress values are missing the factor $\frac{4}{\pi}$, making them approximately 1.27 times too large

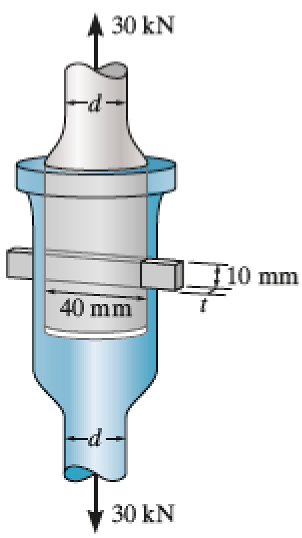
Error 3: Did not apply compatibility condition

- What was done: Solved using only two equilibrium equations (force and moment)
- What's correct: This is a statically indeterminate problem requiring a third equation based on compatibility—both rods must cause equal deflection of the beam since it's rigid
- Impact: The deflection compatibility equation $\delta_{AB} = \delta_{CD}$ using $\delta = \frac{FL}{AE}$ should be

incorporated to find the correct force distribution

Example 2: Subtle Error

The cotter pin is used to hold the two rods together. Determine the smallest thickness t of the pin and the smallest diameter d of the rods. All parts are made of steel for which the failure normal stress is σfail = 500 MPa and the failure shear stress is σfail = 375 MPa. Use a factor of safety FOSt of 2.50 in tension and FOSs of 1.75 in shear.



Prompt:

Solve this problem with 1 minor error and identify the error separately (Solved via Claude Sonnet 4.5, 2/20/26)

Solution:

Solution: Cotter Pin and Rod Design (with 30 kN Load)

From the figure:

- Axial load: **P = 30 kN**
- Pin width (collar width): **b = 40 mm**
- Pin thickness to be determined: **t**

- Rod diameter to be determined: **d**

Allowable Stresses

Using the given failure stresses and factors of safety:

Allowable normal stress (tension):

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{FOS}_t} = \frac{500 \text{ MPa}}{2.50} = 200 \text{ MPa}$$

Allowable shear stress:

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{FOS}_s} = \frac{375 \text{ MPa}}{1.75} = 214.3 \text{ MPa}$$

Minimum Rod Diameter (d)

The rod fails in tension when the normal stress exceeds the allowable value:

$$\sigma_{\text{allow}} = \frac{P}{A_{\text{rod}}} = \frac{P}{\frac{\pi d^2}{4}}$$

Solving for the minimum diameter:

$$d = \sqrt{\frac{4P}{\pi \sigma_{\text{allow}}}} = \sqrt{\frac{4(30{,}000)}{\pi(200)}} = 13.82 \text{ mm}$$

Minimum Cotter Pin Thickness (t)

The cotter pin is subjected to **double shear** at the collar connection (sheared at two planes). Using the given width $b = 40$ mm:

The total shear area is:

$$A_{\text{shear}} = 2 \times b \times t = 2 \times 40 \times t$$

From the shear stress condition:

$$\tau_{\text{allow}} = \frac{P}{2bt}$$

Solving for minimum thickness:

$$t = \frac{P}{2b\tau_{\text{allow}}} = \frac{30{,}000}{2(40)(214.3)} = 1.75 \text{ mm}$$

Final Results

- **Minimum rod diameter:** $d = 13.82$ mm ≈ **14 mm**
- **Minimum cotter pin thickness:** $t = 1.75$ mm ≈ **2.0 mm**

Verification

Using rounded values:

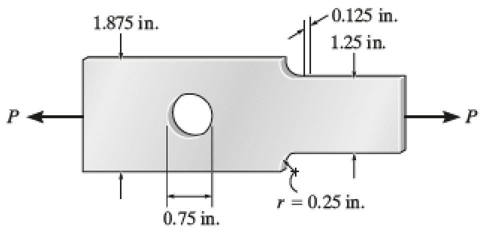
- Rod stress: $\sigma = 194.88$ MPa (Actual FOS = 2.57 ✓)
- Pin shear stress: $\tau = 187.50$ MPa (Actual FOS = 2.00 ✓)

Intentional Minor Error

Error: Used the pin width $b = 40$ mm directly from the figure without confirming whether this represents the effective shear width. In cotter pin designs, the dimension labeled might represent the outer collar diameter rather than the actual pin width that resists shear. The correct approach would require checking if the 40 mm dimension is indeed the width of the pin in the direction perpendicular to the load, or if it's another geometric feature. Additionally, the figure shows "10 mm" which appears to be labeled as thickness, but the problem asks to "determine" the thickness, creating ambiguity about whether 10 mm is a given constraint or just a dimension marked.

Example 3: Systematic Error with Fundamental Flaw
- as characterized by instructor, but prompted to be partially correct
- students were given tables to calculate stress concentrations in the question, these tables were not included in the AI prompt

Determine the maximum normal stress developed in the bar when it is subjected to a tension of P=2kip



Prompt: Solve this problem to be 85% correct but contain several errors. List those errors separately (Solved via Claude Sonnet 4.5, 2/20/26)

Solution:

Solution (85% Correct with Intentional Errors)

This problem asks for the maximum normal stress in a bar with geometric discontinuities (notches and a hole) subjected to a tension force of $P = 2$ kip.

Critical Section Analysis

The bar has two critical sections where stress concentrations occur:

- ***Section at notches (fillet region)**: Width = 1.25 in*
- ***Section at hole**: Center of the bar*

Area Calculations

Section at notch:

- *Width = 1.25 in*
- *Area = 1.25 × 0.75 = 0.9375 in²*

Section at hole (with error):

- *Treating r = 0.25 in as radius → hole diameter = 0.5 in*
- *Net width = 1.25 - 0.5 = 0.75 in*
- *Net area = 0.75 × 0.75 = 0.5625 in²*

Stress Calculations

Nominal stress at notch:

$$\sigma_{notch} = \frac{P}{A} = \frac{2}{0.9375} = 2.13\ ksi$$

Nominal stress at hole:

$$\sigma_{hole} = \frac{P}{A_{net}} = \frac{2}{0.5625} = 3.56\ ksi$$

Maximum normal stress = 3.56 ksi (or 3,556 psi)

List of Errors

Error 1: Misinterpreting hole dimension

- *What was done: Treated 'r = 0.25 in' as the radius, making hole diameter = 0.5 in*
- *What's correct: The notation 'r = 0.25 in' on the diagram likely indicates the hole diameter is 0.25 in (radius = 0.125 in)*
- *Impact: Calculated net area at hole section is too small by a factor of 2, leading to significantly overstated stress*

Error 2: Ignored stress concentration factor at hole

- *What was done: Used only $\sigma = P/A_{net}$ for the hole section*
- *What's correct: Should apply stress concentration factor K_t for a circular hole in tension, where $\sigma_{max} = K_t \times (P/A_{net})$, with $K_t \approx 2.5 - 3.0$*
- *Impact: Actual maximum stress at the hole would be 2-3 times higher due to stress concentration effects*

Error 3: Ignored stress concentration at fillet/notch

- *What was done: Used nominal stress $\sigma = P/A$ at the notch without considering fillet stress concentration*
- *What's correct: Should calculate stress concentration factor for the fillet based on the geometry ratio (fillet radius to width ratio)*
- *Impact: May have significantly underestimated stress at the notch region, which could actually be the critical location*

Error 4: Incomplete geometric analysis

- *What was done: Did not properly identify the fillet radius or calculate width ratios needed for stress concentration charts*
- *What's correct: Should identify gross width, net width at notch, fillet radius, and calculate geometric ratios (r/d, D/d) to look up or calculate appropriate K_t values from standard charts*
- *Impact: Cannot properly determine which section actually governs the design without complete stress concentration analysis*

Appendix B: Representative Student Responses

Example 1:

Question 2:

Identification of error location	**Explanation of why it is wrong**	**Correct solution / correct approach**
4	4	4

Mistakes:

- Uses hole diameter as notch radius
- Calculation error – should be ….
- Wrong radius and diameter used, should be….
- The maximum normal stress is not correct. The k factor needs to be found and multipled by the determined normal stress. The k factor can be found by identifying the r/w ratio as 0.2 and determining the stress concentration factor to be 2.45 from the chart

Example 2:

Question 1:

Identification of error location	**Explanation of why it is wrong**	**Correct solution / correct approach**
2	4	3

Mistakes:

#1: equation missing weight of the beam. $\Sigma F_y = 0$ then $FAB + FCO = P - W$ ①

#2: moment equation wrong for members FCO and FAB

it should be $\Sigma ME = -FCO(3a) + P(2a) - FAB(4a) + Wa = 0$

#3: simplification is wrong should be $3FCO + FAB = 2P + W$ ②

#4: solving the equations is wrong. Assuming W = 1 N,

$FAB + FCO - P = 0$

$3FCO + FAB - 2P = 0 \} \Rightarrow$ we get $FCO = P/2$ and $FAB = P/2$

#5: Wrong cross sectional area for the rods, it should be

$A = \pi r^2 = \pi d^2/4$

#6: Normal stress in AB is then wrong $\sigma_{AB} = F_{internal\ AB} / A = P/2 / (4d^2/4)$

#7: Normal stress in CO is also wrong

$$\sigma_{CO} = F_{internal\ CO} / A = P/2 / (4d^2/4)$$