

Analysis of Low Pass Filter in an Engineering Technology Course Using Monte Carlo Simulation

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Abstract

Engineers must deal with design parameters that need to be carefully selected to achieve design objectives. In electronic circuit design, for example, design engineers must choose the proper values for resistance, inductance, capacitance etc. to ensure the circuit performs certain tasks such as filtering, amplification, regulating voltages, and signal conditioning. In mechanical design, the design engineer must choose proper values for mass, geometrical dimensions, material strength, etc. Design engineers would do calculations and/or simulations to ensure the products work as designed. However, this is not enough, because when the designs are implemented through manufacturing processes variations in the design parameters will be introduced. For example, if a resistor is used in an electronic circuit design, the resistance value of the resistor populated on the circuit board will not be equal to the designed value exactly. Instead, resistance is the nominal value +/- a small variation. The design engineer needs to consider these kinds of variations in all design parameters.

Common practice in industry reveals an important aspect of verifying design requirements, that is, it involves the use of statistics. In theory, it is impossible to prove that requirements will be met 100% of the time. There are always failures related to any product. The real question is whether the requirements are met for a given confidence level. For example, can we prove that the requirement will be met 95% of time?

If we can conduct some statistical analysis during the early stage of designing products, for example during the feasibility study or before mass production of the products, the process of verifying the design requirement can be greatly improved. The cost will go down, and it will take less time to complete the verification. There is such a tool that can be used for exactly this purpose: Monte Carlo Simulation.

The concept of using Monte Carlo Simulation to verify an engineering design is important for Engineering Technology students. Since majority of Engineering Technology students graduated with a four-year BS degree will work as engineers, not technicians, they will have product design responsibilities.

In this paper, we present a simple Low Pass Filter circuit design to illustrate how to apply the Monte Carlo Simulation methodology. Monte Carlo Simulation was a Lean Six Sigma tool introduced in a course in the ESET program (ESET 329, Applied Statistics and Six Sigma) at Texas A&M University, and it worked well for students in the class.

Introduction

Engineers must deal with design parameters that need to be carefully selected to achieve design objectives. In electronic circuit design, for example, design engineers must choose the proper values for resistance, inductance, capacitance etc. to ensure the circuit performs certain tasks such as filtering, amplification, and complex mathematical operations. This is not enough though, because when the designs are implemented through manufacturing processes variations in the design parameters will be introduced. For example, if a resistor is used in an electronic circuit design, the resistance value of the resistor populated on the circuit board will not be equal to the designed value exactly. Instead, resistance is the nominal value $\pm$ a small amount. The design engineer needs to consider this kind of small variation in all design parameters. This is a topic discussed in a quality control course in an engineering technology program.

The challenge can be formulated as follows

Challenge: Assume there are design parameters, each has a tolerance band of $\pm d_i\%$ of the nominal value. The design requirement is that the response y must be between y_{min} and y_{max} . How do we verify that the design requirement is met for the design?

Because the response y can be a complex function of the design parameters, it is a difficult task to prove in theory whether y is between y_{min} and y_{max} . The common practice in industry is to use random sampling to select products and then for each product measure y and check if the values are between y_{min} and y_{max} based on statistical analysis of the collected data. However, this practice has some drawbacks. First, a significant number of products must be available for testing, which may not be possible during early stage of product development. Secondly, if the products are tested and the requirement is not met, then the design needs to be revised and whole process of design-fabrication-testing needs to be repeated until the requirement is met. This is an inefficient trial and error methodology. We need to find a better way.

Common practice in industry reveals an important aspect of verifying design requirements, that is, it involves the use of statistics. In theory, it is impossible to prove that requirements will be met 100% of the time. There are always failures related to any product. The real question is whether the requirements are met for a given confidence level, for example, can we prove that the requirement will be met 95% of time?

If we can conduct some statistical analysis during the early stage of designing products, for example during the feasibility study or before mass production of the products, the process of verifying the design requirement can be greatly improved. It is easy to change the tolerance bands in statistical analysis compared to the fabrication of many prototypes and testing of these prototypes. By using simulation and statistical analysis, the cost will go down, and it will take

less time to complete the verification. There is such a tool that can be used for exactly this purpose: Monte Carlo Simulation^{1,2}.

Monte Carlo Simulation

Monte Carlo Simulation uses random numbers generated by computers as inputs to an abstract model for a system, which can produce the output based on the input values. The output values are then used to predict the result of some uncertain event based on statistical analysis of the output values. It is an effective tool commonly used in industry³.

To use this methodology, one must know the probabilistic properties of the inputs, such as input A is a random variable with a normal distribution and the mu and sigma values for the distribution are known. An accurate math model for the system is also required. The accuracy of the model directly determines the reliability of prediction coming out of the Monte Carlo Simulation. A model can be simple algebraic equations, or any other way that can produce the output value given the values of the inputs. Depending on the nature of the system model, the simulation can be done using Excel, Minitab⁴, and other software.

Despite the fact that there have been many successful application of statistics in engineering, researchers realized decades ago the weakness in statistical education in engineering majors⁵ and improvements were badly needed^{6,7}. There have been constant efforts in the engineering education community to enhance statistical education in their curricula^{8,9,10}.

However, more needs to be done in teaching statistics to engineering students. Statistics used to be a required course for ESET program at Texas A&M University. Students took the statistics course but never applied the knowledge to other courses. Even for capstone design courses where there were many opportunities to apply statistics in their design effort, students still chose to design their product based on the performance of a single prototype without considering the impact of design parameter variations. One of the authors presented challenges being discussed in this paper to students in his class, students had no clue how to tackle this challenge.

There are many examples for illustration of the application of Monte Carlo Simulation. Estimating the value of π is selected mainly for its unique way of developing a model¹¹.

Let (x, y) be a pair of randomly and independently generated numbers from a uniform distribution, with the x, y values between 0 and 1. The probability of the (x, y) falling inside the quarter unit circle is $\pi/4$, which is calculated as the ratio between the area of the quarter unit circle and the unit square as illustrated in Fig. 1.

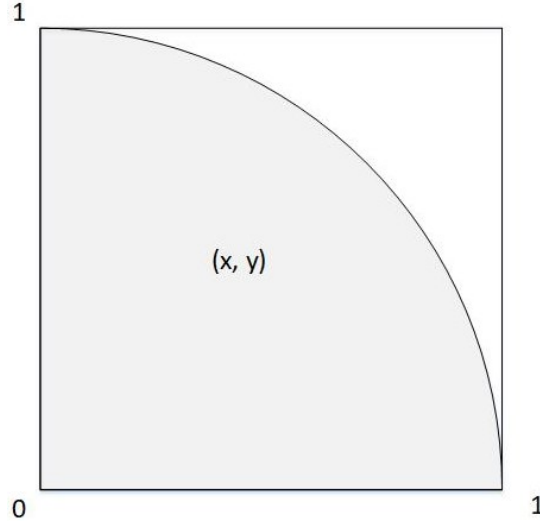


Figure 1. Randomly generated coordinates (x,y)

If we generated many such coordinates (x, y), the probability of the (x, y) falling inside the quarter unit circle is approximated by

$$P \approx \frac{N_c}{N} \quad (1)$$

where N_c is the number of coordinates inside the quarter unit circle and N is the total randomly generated coordinates. But the probability of the (x, y) falling inside the quarter unit circle is $\pi/4$ as discussed earlier. Therefore, we have the following approximation for the value of π

$$\pi \approx \frac{4N_c}{N} \quad (2)$$

In this example, the inputs are the (x, y) coordinates. Their values are randomly generated from uniform distribution. The estimation of value of π is simply Equation (2). The accuracy of the estimation relies on the value of N , the larger the value the better the estimation.

This Monte Carlo Simulation example can be implemented in Excel as illustrated in Fig. 2. First, we need to check that Data Analysis option is available in the Data tab in Excel. If it is not available, we need to change the setting by going to File>Options>Add-ins and select Analysis ToolPak, then click Go... and OK. After this, you should be able to click on Data>Data Analysis and select Random Number Generation>OK. Then type 2 in Number of Variables since we are generating values for x and y. Next, choose a large value for Number of Random Numbers, for example 10,000, as shown in Fig. 2. Then select Uniform from the pulldown menu of Distribution. Select Output Range and type \$A\$2, then click OK. The random values for x and y will be created and stored in Columns A and B in Rows 2-10,001 (only 7 rows of data are shown). The first row in Fig. 1 is used as a header. In Column C, the values of x^2+y^2 are calculated. In Cell D2, the Excel function COUNTIF is used to count how many of the coordinates are inside the quarter unit circle. E2 and F2 are the calculation of the estimation of π based on Equation (2).

	A	B	C	D	E	F
1	N= 10000		x^2+y^2	How many less than 1?	Probability	solve for pi
2	0.528885	0.84487441	=A2^2+B2^2	=COUNTIF(C2:C10001, "<1")	=D2/B1	=4*E2
3	0.283761	0.15567491	=A3^2+B3^2			
4	0.426557	0.09604174	=A4^2+B4^2			
5	0.715170	0.25379192	=A5^2+B5^2			
6	0.113437	0.61711478	=A6^2+B6^2			
7	0.830378	0.60151982	=A7^2+B7^2			
8	0.804986	0.89529099	=A8^2+B8^2			

Figure 2. Excel implementation of the π value estimation

Design of a Low Pass Filter

The circuit in Fig. 3 is a simple passive Low Pass Filter consisting of a resistor and a capacitor. Let's say we want to design the filter such that the cut-off frequency is 159.15 Hz. The cut-off frequency can be calculated as follows

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 10^3 \times 10^{-6}} = 159.15 \text{ Hz} \quad (3)$$

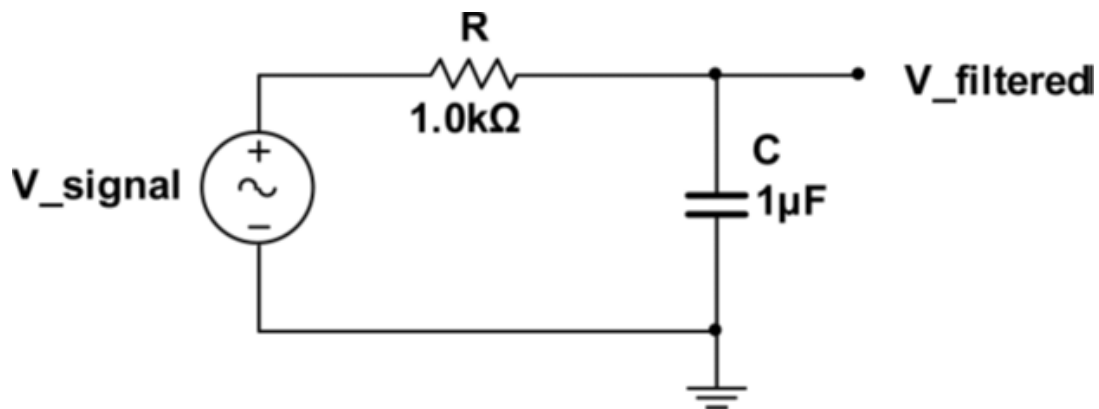


Figure 3. Low Pass Filter Design

Therefore, with the values of the design parameters R and C selected to be 1 K Ω and 1 μ F, the design objective of having 159.15 Hz cut-off frequency has been met.

When this Low Pass Filter design is implemented, we must choose an actual resistor and capacitor. Let's say the following Vishay parts are selected: Resistor: CRCW08051K00JNEA, 5% tolerance; Capacitor: VJ0805Y105MXQTW1BC, 20% tolerance. Now let's consider the challenge presented in Section 1. If the design requirement is to have the cut-off frequency between 159.15 $\pm$ 10%, i.e., 143.235Hz-175.065Hz, does the product meet the design requirement?

One can ask more questions related to the Low Pass Filter design, for example,

What is the defective rate if the Lower Specification Limit (LSL) and Upper Specification

Limit (USL) are given for the cut-off frequency for certain confidence level (e.g., 97%)?

This question is not easy to answer, however, using Monte Carlo Simulation, it can be answered.

The basic idea of using Monte Carlo Simulation to check whether the design requirement is met is relatively simple. Random numbers for R and C are generated first. For each pair of R C values, the cut-off frequency can be calculated using the formula in Equ. 3. Based on the results, one can calculate the Lower Specification Limit (LSL) and Upper Specification Limit (USL) for a given confidence level. The LSL and USL are calculated such that 97% of the values fall between LSL and USL. Note that the confidence level of 97% can be changed to other values if so desired. Simple comparison of the calculated LSL and USL with the given LSL and USL in the design requirement will determine if the design requirement is met.

To generate the random numbers for R and C, one needs information about the distributions of R and C. In this example, we assume that both R and C belong to Normal distributions with standard deviation equal to $9.2\text{E-}8$ and 22, respectively. These values are determined based on the tolerance bands of R (5%) and C (20%).

	A	B	C	D	E	F
1	R(random seed1)	C(random seed2)	cut-off frequency			
2	929.2614521	7.2448E-07	236.5244922	mu	160.9831439	
3	1003.74553	1.11364E-06	142.4528705	sigma	15.50558647	
4	979.7406454	1.05876E-06	153.5083453	confidence	97%	
5	1020.434401	1.00995E-06	154.5090553		0.015	
6	1005.024562	1.1178E-06	141.7420718	LSL	127.3346199	
7	998.8189456	1.02323E-06	155.8040322	USL	194.6316679	
8	991.001896	9.87338E-07	162.7421901			
9	1029.457724	1.07963E-06	143.2703186			
10	1021.674087	9.58688E-07	162.5738339		In the Example: f_c1=126.31; f_c2=209.41	
11	1015.53394	1.01365E-06	154.6883587			
12	978.0495104	9.12068E-07	178.5057599			
13	1025.168674	1.09951E-06	141.2689379			
14	1012.982511	9.72271E-07	161.1951821			

Figure 4. Excel implementation of Monte Carlo Simulation for Low Pass Filter

One can use random number generation function in Excel to generate 2000 pairs of values for R and C in Columns A and B. The cut-off frequencies are calculated in Column C. The parameters mu and sigma for the cut-off frequency are calculated in E2 and E3 in Fig. 4. The LSL and USL for the cut-off frequency are calculated in E6 and E7, they are 127.33 Hz and 194.63 Hz. Compare this with the requirement of 143.235Hz-175.065Hz, the conclusion is that the design requirement is not met.

With this Excel program, one can answer the questions raised earlier.

What is the defective rate if the Lower Specification Limit (LSL) and Upper Specification Limit (USL) are given for the cut-off frequency for certain confidence level (e.g., 97%)?

The answer to this question can be found by calculating

$$=NORM.DIST(143.235, 160.98314, 15.505586, TRUE)+1-NORM.DIST(175.065, 160.98314, 15.505586, TRUE)$$

in Excel, which turns out to be 30.8%.

This Excel program allows the user to study the effects of changing the tolerance bands of the resistance and capacitance. Changing the tolerance bands usually means changing the cost of the components. Therefore, one can conduct the cost and performance trade off analysis using this Excel program. The impact of each design parameter's tolerance can be studied as well. Once the initial Excel program is developed, the impact of tolerance bands can be easily studied.

4. Conclusions

In this paper, we present a simple example of Low Pass Filter design to illustrate the application of Monte Carlo Simulation. Monte Carlo Simulation provides a low cost, efficient way of conducting statistical analysis in engineering design and allows us to answer questions related to design requirement and failure rate.

Monte Carlo Simulation provides design engineers with an opportunity to analyze the impact of parameter variations on the design requirement. The analysis can be done without the availability of large number of actual products. Instead of the trial-and-error method, one can predict during feasibility study whether design requirements will be met with a particular design. Once the analysis is set up in Excel, it is easy to make changes to the design parameters, and the analysis will be automatically updated. This allows quick evaluation of the design requirements.

In principle, this methodology can be applied to many other engineering designs. The process discussed in this paper can be easily modified to be applied to other engineering designs. All we need is a model that relates the output to the inputs. In the Low Pass Filter example, it is the calculation of the cut-off frequency in Equ. 3.

This method is also helpful for Engineering Technology students to understand that modeling and simulation tools are just as important as the laboratory work. It is well-known that Engineering Technology students are good at hands-on learning. The addition of Monte Carlo Simulation can contribute to the minds-on learning aspect in Engineering Technology programs.

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