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Lab Exploration of the Similarities of the Second-order Response of a Mechanical System and a Parallel RLC Electrical Circuit

Dr. Cuiling Gong, Texas Christian University

Dr. Cuiling (Sue) Gong earned her Ph.D. in Mechanical Engineering from the Massachusetts Institute of Technology (MIT) in 1999. She spent 11 years at Texas Instruments as a MEMS design engineer, where she contributed to the development of the Digital Micromirror Device. Following her industry career, she joined the faculty of the College of Science and Engineering at Texas Christian University. Leveraging her extensive experience in microelectromechanical systems (MEMS), Dr. Gong integrates real-world applications into her teaching, bridging mechanical and electrical systems through lectures and hands-on lab exercises.

Ms. Tammy Pfrang, Texas Christian University
Dr. R. Stephen Weis P.E., Texas Christian University

The Lab Exploration of the Similarities of the Second-order Response of a Mechanical System and a Parallel RLC Electrical Circuit

Abstract

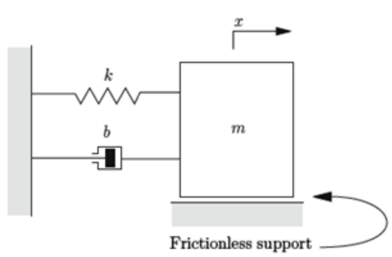
We developed a lab exercise to demonstrate that the second-order response of the RLC electrical circuit exhibits the same damped oscillation characteristics as those generated by a mechanical cantilever after its freestanding tip is deflected and released. Since the piezoelectric material generates an electrical signal when subjected to mechanical stress, the damped mechanical oscillation can be visualized by connecting an oscilloscope to the electrodes of the piezoelectric cantilever. Based on the damped oscillation curve, we can estimate the damped oscillation frequency and damping coefficient. We then design a parallel RLC electrical circuit by selecting appropriate resistor, capacitor and inductor to match the damped oscillation characteristics. This lab exercise allows students to gain a better understanding of second-order system responses through visualization and see the similarities of the mechanical and electrical systems. Learning opportunities include piezoelectric materials, cantilever functions, parallel RLC electrical circuit design with targeted design parameters, extraction of the damping coefficient and resonance frequency from a measured damped oscillation curve, and setup of the oscilloscope to capture transient, non-repetitive waveforms.

Keywords

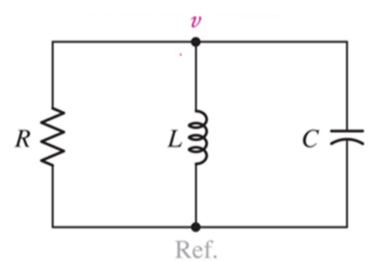
Second-order damped oscillation, RLC circuits, piezoelectric cantilever

Introduction

Second-order systems appear frequently in both mechanical and electrical engineering applications. Figure 1 shows a simple second-order mass-spring mechanical system and a source-free parallel RLC circuit. The equation that describes the voltage across the resistor, capacitor and inductor as a function of time is in the form of a second-order differential equation [1].



(a)



(b)

Figure 1. (a) A mass-spring mechanical system; (b) A source-free parallel RLC circuit

$$C\frac{d^2V}{dt^2} + \frac{1}{R}\frac{dV}{dt} + \frac{1}{L}V = 0. \tag{1}$$

where:

V = voltage
C = capacitance
R = resistance
L= inductance
t = time

The equation of motion of the mass-spring system is governed by a similar equation [2],

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0. \tag{2}$$

where:

m = mass
b = damping coefficient
k = spring constant
x = displacement
t = time

Since both equations are second-order differential equations, the transient responses of the two systems should be similar. We developed a lab exercise that directly compares the response of a mechanical cantilever beam with that of a parallel RLC electrical circuit. To help students develop an intuitive understanding of the dynamic response of the second-order system, we used a piezoelectric strip to represent a cantilever beam. When the free end of the strip is deflected and released, it undergoes a damped oscillation governed by its effective mass, stiffness, and damping. The mechanical strain induced by the vibration is converted into an electrical signal through the piezoelectric effect, which allows the mechanical response to be visualized using an oscilloscope.

The learning outcomes of this lab include understanding how piezoelectric materials work, how a simple mechanical cantilever functions, and how to design a parallel RLC electrical circuit with targeted design parameters for oscillation frequency and damping characteristics. In addition, students learn to calculate the damping coefficient from a measured damped oscillation curve and use an oscilloscope to capture transient, non-repetitive waveforms.

The details of the lab exploration of the second-order response of a mechanical system and a parallel RLC electrical circuit are presented in three sections. In the first section, the experimental procedures are discussed including extraction of the damped resonance frequency and damping coefficient from the measured damped oscillation waveform, design and construction of a source-free parallel RLC circuit using the measured parameters as design targets, setup of the function generator and oscilloscope to capture the transient response of the parallel RLC circuit, and extraction of the oscillation frequency and damping characteristics from the RLC circuit to compare with those from the mechanical system. In the second section, we discuss the learning opportunities this lab exercise offers. In the third section, we provide the resources that this lab requires so that the readers can develop similar lab exercises.

Experimental Procedures

Generate an underdamped oscillation waveform using a piezoelectric strip

1. Connect the piezoelectric strip to the oscilloscope

A piezoelectric strip configured as a cantilever beam was connected to Channel 1 of a Rohde & Schwarz RTE oscilloscope using a coaxial cable, a BNC banana split with alligator clips attached at the split ends as shown in Figure 2.

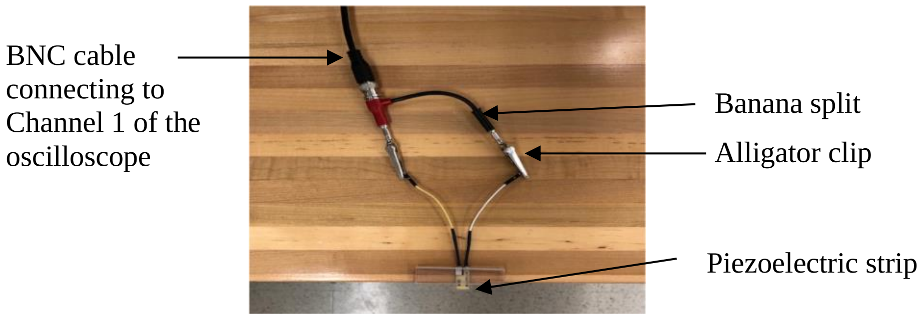


Figure 2. Connection of the piezoelectric strip to the BNC banana split and BNC cable to the oscilloscope.

2. Configuration of the oscilloscope

The oscilloscope was configured for single-shot (NxSingle) acquisition with a horizontal scale of 2 ms/div, vertical scale of 4 V/div with no offset, and **Normal** edge triggering at **Trigger level** of 2V on Channel 1. Press the **RUN Nx SINGLE** button on the upper right corner of the oscilloscope.

3. Set up the piezo vibration sensor [3]

The piezoelectric strip is held in place at the edge of the desk using a plastic block with 5~6mm of the strip freestanding. The length of the freestanding cantilever can be adjusted to obtain the waveform with a desired resonance frequency. A shorter cantilever will have a higher oscillation frequency. For this lab, we target a damped frequency ~500Hz.

A damped oscillation of the cantilever can be generated by deflecting the tip of the freestanding piezoelectric strip a few millimeters using a fingertip and releasing quickly. The oscilloscope should display the oscillation waveform as shown in Figure 3. If the waveform does not look like a damped sinusoidal function, try it again (reset the RUN Nx SINGLE button) until a clean damped sinusoidal response was captured.

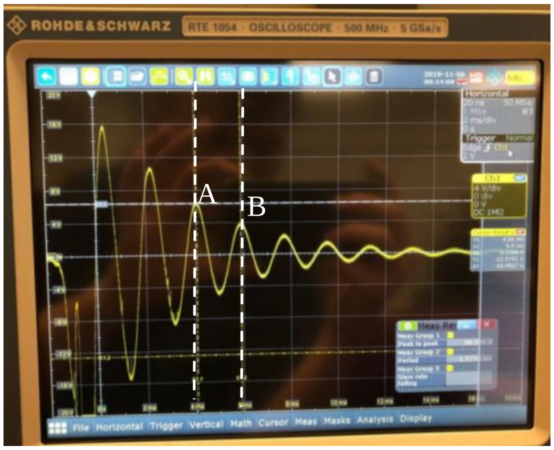


Figure 3. A damped sinusoidal waveform was generated by the piezoelectric strip oscillation and captured by the oscilloscope.

Estimate damped frequency and damping coefficient

1. Find damped frequency

Using the Oscilloscopes caliper system, the damped frequency can be determined by measuring the time difference between points A and B that represents the period *T* of the waveform as shown in Figure 3.

$$T = t_B - t_A = 5.9 \text{ ms} - 4.06 \text{ ms} = 1.84 \text{ ms}; \qquad f = 1/T = 1/0.00184 = 543 \text{ Hz} \tag{3}$$

where:

t_A = time at point A of the waveform
t_B = time at point B of the waveform
T = period of the damped waveform
f = frequency of the damped waveform

2. Find damping coefficient

The damping curve can be described by an exponential decay function

$$V = Ae^{-\alpha t} \tag{4}$$

where:

V = voltage value
A = initial value of the damped waveform
α = damping coefficient
t = time

Use two data points on the exponential decay curve: A (4.06ms, 6.37V) and B (5.90ms, 3.88V) to find α. We don't need to find A. Using the voltage and time values at points A and B in Equation 4, we obtained the following equations.

$$\begin{cases} 6.37 = Ae^{-\alpha \cdot 0.00406} \\ 3.88 = Ae^{-\alpha \cdot 0.00590} \end{cases} \tag{5}$$

Solve for α,

$$\alpha = \frac{l_n(\frac{6.37}{3.88})}{(0.00590-0.00406)} = 269\ 1/s \quad (6)$$

Design a second-order underdamped parallel RLC circuit

The parallel RLC circuit as shown in Figure 4 consists of a resistor, a capacitor and an inductor. To simplify the design work, a premade inductor with ~40mH is provided. The resistance R and capacitance C are determined based on the measured damping coefficient and damped frequency from the waveform obtained from the piezoelectric cantilever.

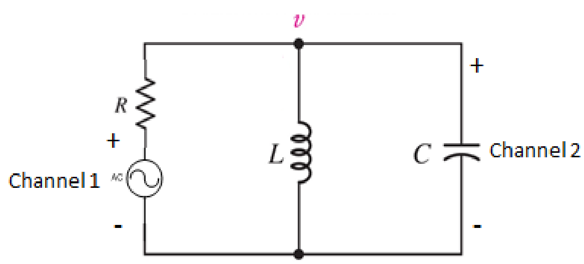


Figure 4. The parallel RLC circuit and the locations of the oscilloscope probes

For a parallel RLC circuit, the damping coefficient and damped resonance frequency are defined by the following equations:

$$\alpha = \frac{1}{2RC} \quad (7)$$

$$\omega_d = 2\pi f = 2\pi \cdot 543 = 3414\ rad/s \quad (8)$$

$$\omega_o = \sqrt{{\omega_d}^2 + \alpha^2} = \sqrt{3414^2 + 269^2} = 3424\ rad/s \quad (9)$$

$$\omega_o = \frac{1}{\sqrt{CL}} \quad (10)$$

where:

ω_o= natural resonance frequency;

ω_d= damped resonance frequency.

Using a premade inductor with L = 41.57 mH, capacitance C can be obtained using Equation 10,

$$C = \frac{1}{L{\omega_o}^2} = \frac{1}{41.57 \cdot 10^{-3} \cdot 3424^2} = 2.05\mu F \quad (11)$$

Resistance R can be obtained using Equation 7 after capacitance C is determined,

$$R = \frac{1}{2\alpha C} = \frac{1}{2 \cdot 269 \cdot 2.05 \cdot 10^{-6}} = 906\ \Omega \quad (12)$$

Build the parallel RLC circuit

After determining the capacitance and resistance for the capacitor and resistor, we will build the RLC circuit on the breadboard. The function generator will supply a positive pulse waveform with 1 cycle to power the circuit. The waveform of the power source is connected to Channel 1 of the oscilloscope. The voltage across the inductor and capacitor is connected to Channel 2 of the oscilloscope as shown in Figure 5.

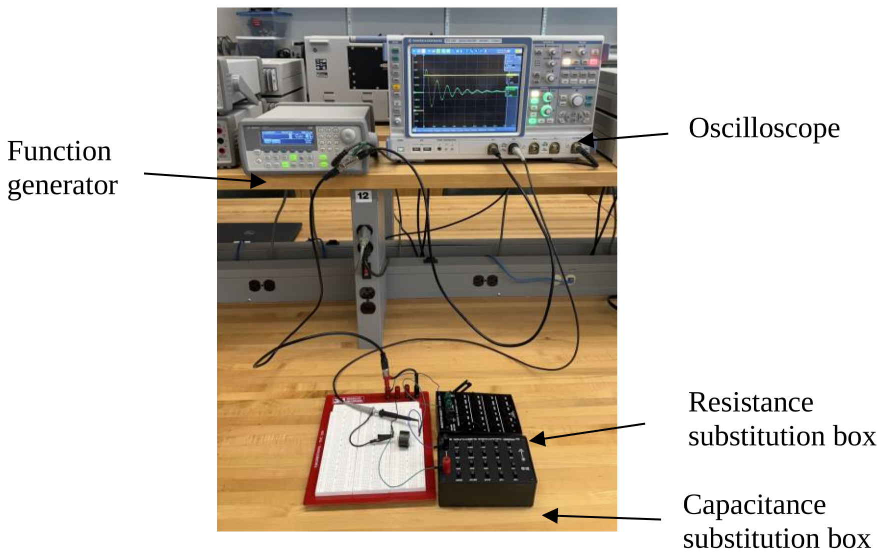


Figure 5. The RLC circuit built on a breadboard with Oscilloscope connections.

Capture the transient response of the second-order parallel RLC circuit

1. Function generator setup [4]

Select the positive pulse waveform ⎍ on the function generator. Set the output of the function generator at High Z with 10 Vpp, a frequency of 1Hz, an offset voltage = 5V and a duty cycle of 50%. Select BURST by pressing the BURST button and set the number of cycles to 1. This will create a 10V pulse that will last 0.5 second. Use the Trigger Setup menu to choose the Trigger source as Manual.

2. Oscilloscope set up [5]

Horizontal: Set the horizontal scale to 2ms/div and set the Reference point to 10%.
Vertical: Set Channel 1 to 2V/div with a 5V offset and Channel 2 to 400mV/div with no offset.
Trigger: Set the trigger source of the scope to Channel 1. Edge trigger with a trigger level at 5V, positive slope. Set Mode to NORMAL and push the RUN Nx SINGLE button.

Push the Trigger button on the front panel of the function generator, you should see both the source waveform and the response transient waveform from the RLC circuit. Figure 6 shows an example of the transient response of the parallel RLC circuit.

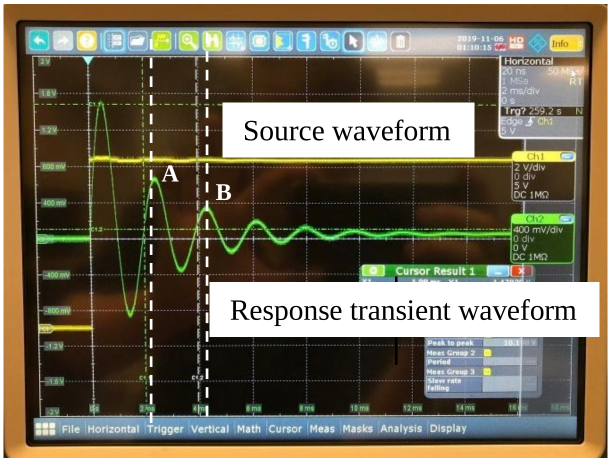


Figure 6. The transient response from the parallel underdamped RLC circuit.

Verify the oscillation frequency and damping characteristics

Estimate the damped resonance frequency and damping coefficient using the same approach as described for the mechanical waveform using two data points A and B. Since the inductor is not ideal that has a small series resistance, the damped resonance frequency and damping factor are f = 526 Hz and α = 304 1/s respectively that deviate slightly from the designed value. However, the similarity of the characteristics of the second-order damped waveforms between the mechanical and electrical systems are demonstrated.

Learning Opportunities of the lab

In addition to see the similarities of the second-order response between a mechanical system and a parallel RLC electrical circuit, this lab exercise offers several other key learning opportunities including properties of the piezoelectric materials, impact of the freestanding length of a cantilever on its resonance frequency, how to estimate the resonance frequency and damping coefficient of a damped sinusoidal waveform, and how to design and build an underdamped parallel RLC circuit, as well as oscilloscope functions to capture special non repeating waveforms.

The learning from this lab exercise help students gain a better understanding of similar contents in subsequent engineering courses. For example, piezoelectric materials are discussed in more detail in the Introduction to MEMS course when piezoelectric sensing and actuation are introduced. The second-order damped sinusoidal oscillation are covered in both Dynamics and Vibrations and Advanced Engineering Dynamics courses.

Resources

This lab uses standard lab equipment and common circuit components that are listed below. The piezo vibration sensors were ordered specifically for this lab. The vender and cost of the sensors are provided for reference.

- Rohde & Schwarz RTE Oscilloscope with probes
- Agilent 33220A Function generator
- coaxial cable with BNC at each end
- BNC-to-banana plug adapter
- Protoboard
- **Piezo Vibration Sensor**
  Vender: Sparkfun (www.sparkfun.com), part # SEN-09196, $14 per sensor
- Capacitance substitution box
- Resistor substitution box
- Philips PM 6304 RLC bridge: it is used to measure the actual capacitance and inductance.
- **A 40mH pre-made inductor**

The inductor was made by students in a previous lab. It is constructed using two ferrite core halves (All Star Magnetics: https://allstarmagnetics.com), 1 nylon bobbin, 1 nylon bolt, washer and nut, and #26 enameled copper wire. Figure 7 shows the components used to build the inductor and the assembled inductor.

Figure 7. Components used to build the inductor and the assembled inductor.

Summary

This lab exploration demonstrated the similarities between second-order mechanical and electrical systems through visualization using a piezoelectric strip functioned as a cantilever. By measuring the transient response of the piezoelectric strip, a parallel RLC circuit was designed to exhibit the same oscillation and damping characteristics. This lab exercise also accomplishes many learning opportunities that help students to have a better understanding of the second-order engineering systems and to build a strong foundation for future engineering courses. This lab is easy to setup using components and instruments in electronic labs with a $14 piezoelectric strip for each setup.

References

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[5] Rohde & Schwarz RTE Digital Oscilloscope Getting Started, Rohde & Schwarz USA, Inc.