2026 ASEE ANNUAL CONFERENCE June 21–24, 2026 | Charlotte Convention Center, NC
Host Sponsor UNC Charlotte
Paper ID #51427

Predictive Validity of Online Un-Proctored and In-Person Proctored Precalculus Assessments in Calculus I and II

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1. Introduction

Calculus I and II are foundational courses in STEM education, often serving as gatekeepers to advanced study. Accurate placement into these courses is critical for student success and retention. At our institution, incoming engineering students complete an online un-proctored precalculus placement test (PT) a few weeks prior to the start of their first semester to place them into appropriate calculus courses. Starting from 2023, a mastery-based precalculus component has been incorporated into calculus courses. Our team developed a comprehensive list of precalculus learning objectives such as simplifying expressions, applying Laws of Logs, determining limits, etc. The PT is regulated by such list of learning targets with 30 multiple choice problems, one point awarded for each multiple-choice problem.

Realizing the critical role of precalculus skills in students' success in calculus, our team developed multiple highly comparable versions of a precalculus assessment (PA) and integrated these assessments into both Calculus I and Calculus II. Our aim was to help students reactivate precalculus skills they had previously learned but forgotten, as well as to acquire skills they may not have mastered earlier. Students were given multiple in-person proctored attempts—proctored by instructors and TAs—to achieve a threshold score of 80%, which contributed 5% toward their calculus course grade. Each attempt used a different version of the PA to ensure fairness and mastery. A previous study [1] examined students' perceptions of such PA incorporation and their precalculus skill development and the findings indicated that students viewed this mastery-based approach as beneficial for reinforcing their precalculus competencies.

Each version of the PA consists of thirty multiple-choice questions, with one point awarded for each correct response. All versions were carefully designed to align with the specific learning objectives outlined in PT (Table 1 [1]) and additional details regarding these alignments are provided in Appendix 1.

Table 1: Alignments for the learning objective, "can solve quadratic inequalities (N)"

PT	PA V1	PA V2
Q10: Solve for x: $x^2 - 2x > 3$ A. $(-\infty, -1) \cup (3, \infty)$ B. $(-\infty, -2) \cup (-1, \infty)$ C. $(-1, 3)$ D. $(-2, -1)$	Q10: Solve for x: $2x^2 + 6x - 20 < 0$ A. $(-\infty, -5) \cup (2, \infty)$ B. $(-\infty, -5) \cup (5, \infty)$ C. $(-\infty, \infty)$ D. $(-5, 2)$	Q10: Solve for x: $x^2 - 9x < -14$ A. $(-\infty, 9) \cup (14, \infty)$ B. $(-\infty, 2) \cup (7, \infty)$ C. $(2, \infty)$ D. $(2, 7)$
PA V3	**PA V4**	**PA V5**
Q10: Solve for x: $-x^2 - 9x < -14$ A. $(-7, -2)$ B. $(-\infty, 2) \cup (7, \infty)$ C. $(-\infty, -7) \cup (-2, \infty)$ D. $(-\infty, -2)$	Q10: Solve for x: $x^2 + 2x < 8$ A. $(-\infty, -4) \cup (2, \infty)$ B. $(-\infty, -2) \cup (4, \infty)$ C. $(-2, 4)$ D. $(-4, 2)$	Q10: Solve for x: $3x^2 + 21x + 36 > 0$ A. $(-\infty, \infty)$ B. $(-\infty, -4) \cup (-3, \infty)$ C. $(-4, -3)$ D. $(0, \infty)$

A study examining the reliability of the online un-proctored PT found that the PT does not accurately reflect students' precalculus skills prior to entering college [2]. To establish a more reliable baseline of students' precalculus knowledge before any institutional interventions occur, we require students in Calculus I and Calculus II to take the first proctored precalculus assessment (1stPA) at the very beginning of their first semester. This assessment is administered in class under instructor or TA proctoring. Since all of our precalculus assessments are proctored in person, and the 1stPA occurs before students receive any course-based support or review, we intend to believe that the 1stPA provides a more accurate representation of students' true precalculus knowledge than the PT before entering college.

The online un-proctored precalculus PT has been used at our institution for many years to place incoming engineering students; however, its effectiveness had not been examined until recently. A recent study demonstrated that relying solely on PT scores to place students into calculus courses resulted in a substantial number of misplacements [2]. However, that study did not explore how strongly prior precalculus knowledge predicts students' calculus performance, nor did it investigate how different modalities of precalculus assessment might vary in their predictive power. These unanswered questions naturally led us to consider whether the first proctored precalculus assessment (1stPA) might serve as a more accurate placement tool, and how well precalculus knowledge—measured across different assessment modalities—can predict students' calculus outcomes. This motivated the present study, which examines the predictive validity of both PT and 1stPA.

2. Literature Review

A substantial body of mathematics and engineering education research indicates that strong precalculus preparation is a critical factor for student success in introductory calculus courses. Precalculus courses emphasize foundational concepts such as functions, algebraic manipulation, and trigonometry, which are repeatedly identified as essential prerequisites for understanding limits, derivatives, and integrals.

Empirical studies consistently demonstrate a significant relationship between precalculus coursework and calculus performance. Ayebo et al. found that students with strong high school precalculus preparation were significantly more likely to succeed in introductory college calculus, whereas weak precalculus backgrounds were associated with higher rates of failure or withdrawal [3]. Similarly, Agatep reported that deficiencies in precalculus skills—particularly algebraic manipulation and trigonometric reasoning—were a primary contributor to students' difficulties in differential calculus [4].

At the postsecondary level, Hurdle and Mogilski compared different prerequisite pathways and showed that students who completed a dedicated precalculus course prior to Calculus I outperformed peers who entered calculus through alternative algebra–trigonometry pathways [5]. Their regression analyses further indicated that inadequate precalculus preparation had a lasting negative impact on calculus GPA. These findings suggest that precalculus is not merely remedial, but functionally preparatory for calculus success.

Evidence from engineering education research reinforces these conclusions. Ennis et al. emphasized that underprepared engineering students often struggle in calculus due to weak precalculus foundations, and that targeted precalculus for engineers' interventions significantly improve calculus readiness and retention in engineering programs [6]. Complementing this, multiple other papers document specific missing concepts. Ni *et al.* show that trigonometry is a persistent bottleneck for first-year engineering students and describe a targeted intervention to improve trigonometric fluency [7]. Kauffmann *et al.* analyze factors behind poor performance in pre-calculus, identifying concept gaps that propagate into early calculus outcomes [8]. Shryock *et al.* report an engineering-oriented preparatory approach designed to shore up algebra/trigonometry and functions before the first college calculus course, especially for students underprepared on entry [9]. Ancheta's error-analysis study found substantial conceptual gaps among engineering students, including misconceptions involving slope, line equations, graphing, and inverse trigonometric functions—concepts highly important for calculus learning [10]

Instructional strategies that directly target retention of prerequisite concepts also matter. Hopkins, Lyle, and Ralston (ASEE) show that retrieval practice and spacing produce meaningful gains in long-term retention of precalculus concepts among engineering students—an actionable approach to mitigating missing-concept barriers as students transition into calculus [11].

Additionally, studies of placement and readiness systems have reported that grades in precalculus are strongly correlated with success in engineering calculus, with students earning less than a B in precalculus facing a markedly lower probability of passing Calculus I [12]. Meanwhile, some prior research has shown that placement tests can be moderately predictive of performance, but concerns remain about their reliability and equity. Belfield and Crosta found that placement test scores alone are weak predictors of college performance [13], while Scott-Clayton highlighted frequent misplacements and advocated for more holistic placement strategies[14] . Additionally, Newton [15] and Brallier & Palm [16] raised concerns about the integrity and score inflation associated with online un-proctored exams. In engineering contexts, longitudinal ASEE analyses underscore changing math readiness and high precalculus failure rates, strengthening the case for validated, proctored diagnostics and targeted scaffolding[17]. Recent engineering-education work further shows that placement cutoffs materially shape Calculus I enrollment and engineering persistence, highlighting the need to calibrate policy and supports with equity in mind [18].

Collectively, this literature underscores that precalculus competence is a key predictor of calculus performance, particularly for engineering students for whom calculus is a gateway course. Engineering students commonly arrive with missing precalculus concepts (notably algebraic manipulation, trigonometry, and function reasoning), and pedagogical interventions—from engineering-context precalculus and trigonometry supports to retrieval-spaced mastery work—can improve readiness and downstream calculus performance [6], [7], [8], [9], [11]. These findings provide strong justification for including precalculus-centered content in calculus placement exams and for evaluating students' mastery of precalculus concepts when making enrollment decisions.

3. Research Design and Study Method

This IRB (IRB #5362) approved study examines the predictive validity of two precalculus assessment modalities: the online un-proctored precalculus placement test (PT) and the first in-person proctored precalculus placement assessment (1stPA). Although direct comparisons of these two modalities remain largely absent from the existing literature, such an investigation is timely and important given the increasing reliance on online placement assessments. The findings of this study provide empirical evidence that may inform instructional and placement practices for mathematics educators and institutions navigating the expanding use of online evaluation tools.

3.1 Research Questions

Specifically, we address three research questions:
RQ1. To what extent do PT scores predict student performance in Calculus I and II?
RQ2. How well do 1stPA scores correlate with final grades or performance metrics in Calculus I and II?
RQ3. Which assessment—PT or 1stPA—serves as a stronger predictor of success in calculus courses?

3.2. Participants

The study draws on enrollment data from APMA 1090 (Calculus I) and APMA 1110 (Calculus II) across the Fall 2024 and Fall 2025 semesters from the University of Virginia School of Engineering & Applied Science. In Fall 2024, APMA 1090 enrolled about 170 students in five sections taught by three instructors, and APMA 1110 enrolled roughly 280 students in seven sections taught by four instructors. The following year, APMA 1090 grew to approximately 230 students across seven sections taught by five instructors, while APMA 1110 enrolled around 180 students in four sections taught by three instructors. Altogether, the dataset includes nearly 860 students across 23 sections taught by a diverse group of instructors, providing a broad and representative sample for analysis.

3.3. Data Sources & Collection

The primary data sources for this study encompass multiple cohorts and assessment instruments across two foundational calculus courses. Specifically, we draw on PT scores, 1stPA scores, and final exam results from students enrolled in APMA 1090 (Calculus I) and APMA 1110 (Calculus II) during the Fall 2024 and Fall 2025 semesters. These datasets provide a comprehensive view of students' mathematical preparation and subsequent performance in their respective calculus courses, allowing for comparisons both within and across academic years.

In addition to these quantitative measures of the objective artifacts, the study incorporates subjective students' perception insights gathered through a Fall 2025 end-of-semester survey administered via Qualtrics to students in APMA 1110. The survey consisted of a single Likert-scale, multiple choice question designed to capture students' perceptions of the predictive validity of precalculus in their calculus courses. Although brief, this survey component offers valuable context by highlighting how students themselves interpret the relationship between their precalculus skills and their eventual performance in calculus, complementing the statistical analyses derived from the assessment data.

3.4. Study Methods

All those data analysis were conducted in R programming. For RQ1, to evaluate the predictive power of the PT**,** the study treats overall PT scores as independent variables, while final grades in Calculus I and II to serve as the dependent variables. Statistical analysis with Pearson correlation coefficients to establish the direction and strength of the relationship. Linear regression models are then constructed to calculate the R^2 value, quantifying how much variance in calculus performance is explained by the un-proctored scores. For RQ2, we examine the 1stPA using a similar framework to ensure consistency. 1stPA scores function as the independent variable against the dependent variable of final calculus grades. Beyond standard correlation and regression modeling, this step also emphasizes predictive metrics such as R^2 to assess the precision of the in-person scores. By evaluating these metrics, the study identifies how closely the proctored environment correlates with actual student achievement in the rigorous Calculus I and II sequences. For RQ3, we conduct comparative analysis to determine which assessment serves as the stronger predictor of success. We build multiple regression model with both scores as simultaneous predictors and check p-values. Furthermore, hierarchical regression is performed to determine if the 1stPA adds significant predictive value beyond what is already captured by the PT, effectively highlighting the impact of proctoring on the integrity of placement data.

We also examined students' perceptions of whether precalculus serves as a strong predictor of calculus performance by administering a general Likert-scale survey question, independent of modality pathways.

In addition to these analytical approaches, we generated scatterplots with regression lines, boxplots, and violin distribution graphs to visually illustrate the relationships between precalculus scores and calculus performance. Descriptive analysis from tables were also produced to further summarize and contextualize the data.

4. Data Analysis and Results

In this section, we present a detailed analysis of the data and results for the four cohorts included in the study: Fall 2024 APMA 1090 (Calculus I), Fall 2024 APMA 1110 (Calculus II), Fall 2025 APMA 1090 (Calculus I), and Fall 2025 APMA 1110 (Calculus II). All graphs are displayed in a compact format due to space constraints and zooming is recommended for improved clarity.

4.1. Q1: To what extent do PT scores predict student performance in Calculus I and II?

Correlation Analysis. First, for each cohort, we conducted a correlation analysis and calculated the corresponding Pearson correlation coefficients. These results are summarized in Table 2 below.

Table 2: Correlation Analysis (final vs PT)

	Fall24 APMA 1090-Cal I	Fall24 APMA 1110-Cal II	Fall25 APMA 1090-Cal I	Fall25 APMA 1110-Cal II
Pearson Correlation Coefficient (Final Exam ~ PT)	0.2501415	0.2686561	0.2694069	0.2958945
Correlation Test p value	p-value = 0.002979	p-value = 5.424e-05	p-value = 0.0006193	p-value = 0.0003332

For all four cohorts, the Pearson correlation coefficients between Final Exam scores and PT (PT) scores fall within a narrow band—from approximately 0.25 to 0.30—indicating a consistent, positive, but modest linear relationship. In every cohort, students with higher PT scores tended to earn higher final exam scores, yet the strength of this association remains relatively small. This suggests that while the PT captures some aspects of students' readiness for calculus, it accounts for only a limited portion of the variation in final exam performance.

The statistical significance of the correlations is supported by the very small p-values (all well below 0.01), demonstrating that these relationships are unlikely to be due to chance. Notably, the correlations are remarkably stable across both years and course levels. The Fall 2025 APMA 1110 cohort shows the highest correlation ($r \approx 0.296$), but even this value reflects only a modest predictive relationship.

These results indicate that the PT provides reliable but weak predictive information about calculus performance. It consistently relates to final exam outcomes across different semesters and course levels, yet its predictive power is limited, implying that they should not be viewed as a strong standalone predictor of calculus success.

Linear Regression. Then for each cohort, we fit a simple linear regression model of the form $y = \alpha x + \beta$, where the final exam score served as the response variable and the PT score was used as the predictor. The resulting slope estimates, associated p-values, and R^2 values are summarized in the table below.

In a simple linear regression model, the slope (α) represents the expected change in the final exam score associated with a one-point increase in PT; a larger slope therefore indicates a stronger positive relationship between PT and exam performance. The intercept (β) is the predicted final exam score when PT equals zero. The p-value for the slope tests whether the relationship between PT and final exam scores is statistically distinguishable from zero; a small p-value (typically less than 0.05) provides evidence that PT is a meaningful predictor of exam performance. Finally, the coefficient of determination, R^2, measures how much of the overall variation in final exam scores is explained by PT. For example, an R^2 of 0.07 indicates that approximately 7% of the variability in final exam scores can be attributed to differences in PT. The resulting slope estimates, associated p-values, and R^2values are summarized in Table 3 below.

Table 3: Linear Regression (final vs PT: $final = \alpha * PT + \beta$)

	Fall24 APMA 1090-Cal I	Fall24 APMA 1110-Cal II	Fall25 APMA 1090-Cal I	Fall25 APMA 1110-Cal II
Linear Regression slope	1.2951	2.7691	0.9424	2.9008
Slope p value	0.00298	5.42e-05	0.000619	0.000333
Linear Regression (LR) R^2	0.06257	0.07218	0.07258	0.08755
LR overall p values	0.002979	5.424e-05	0.0006193	0.0003332

Across all cohorts, PT is a statistically significant predictor of final exam performance: all slopes are positive, and all p-values are well below 0.01. This indicates a consistent positive association between PT scores and final exam outcomes. The strength of the relationship varies by cohort. For Calculus I (APMA 1090), slopes range from 0.94 to 1.30, while Calculus II (APMA 1110) shows notably larger slopes (2.77–2.90), suggesting that PT scores are more strongly related to performance in Calculus II. Despite this, the R^2 values remain modest (about 6–9%), meaning PT explains only a small portion of the overall variability in final exam scores.

In sum, PT provides statistically significant and meaningful—but limited—predictive information about final exam performance, with stronger effects observed in Calculus II than in Calculus I.

Linear Regression Visualization. The scatterplots below illustrate the relationship between PT scores and final exam performance, with fitted linear regression lines shown for each of the four cohorts. In each panel, the x-axis represents the PT score (maximum 30 points), with the number of participants indicated, and the y-axis represents the final exam score. The total possible points on the final exam vary by cohort: 150 points for Fall 2024 and Fall 2025 APMA 1090 (Calculus I), 250 points for Fall 2024 APMA 1110 (Calculus II), and 280 points for Fall 2025 APMA 1110 (Calculus II). These visualizations further confirm the patterns identified in the preceding table-based analysis.

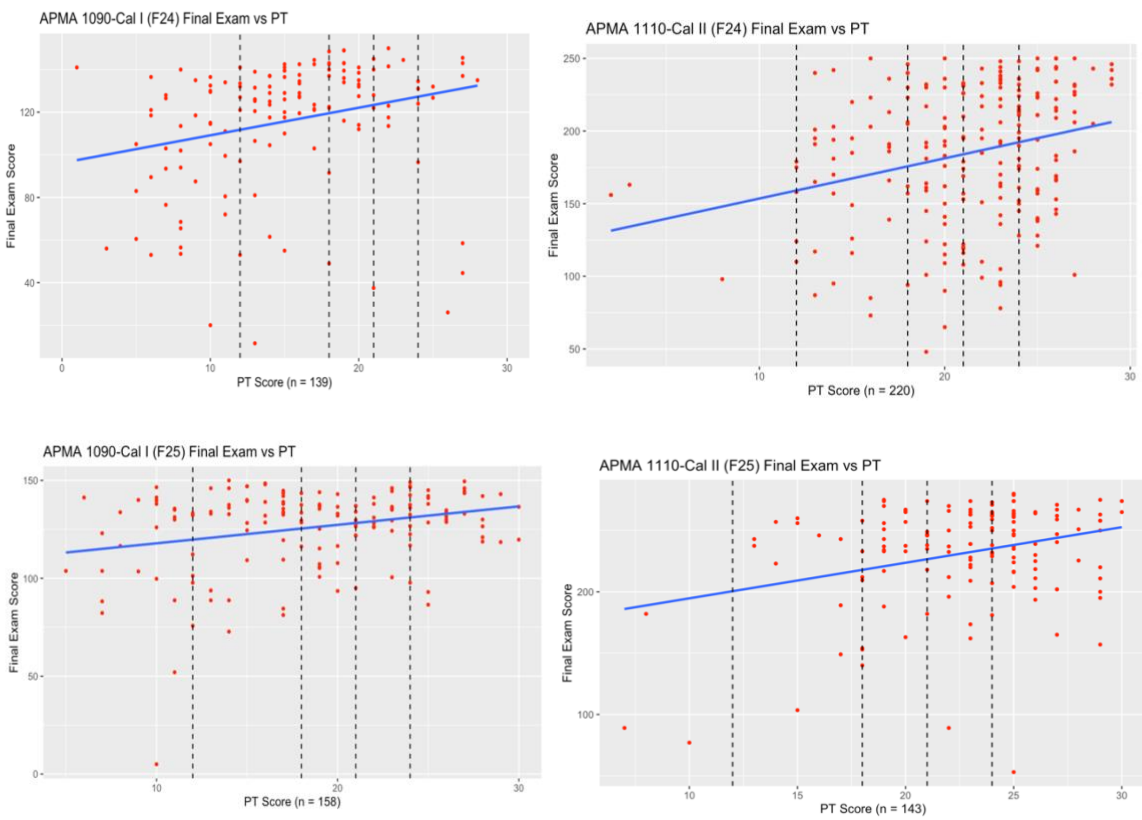

Figure 1: Linear Regression final vs PT (PT)

4.2. RQ2. How well do 1stPA scores correlate with final grades or performance metrics in Calculus I and II?

To investigate this research question, we conduct similar analysis as section 4.1.

<u>*Correlation Analysis.*</u> We summarize correlation analysis results between final exams score and 1stPA scores in Table 4 below. The table reports Pearson correlation coefficients measuring the linear association between final exam scores and 1stPA scores across four cohorts: APMA 1090 (Calculus I) and APMA 1110 (Calculus II) in Fall 2024 and Fall 2025.

Across all cohorts, the correlations between 1stPA and final exam scores are positive and statistically significant. This means students who demonstrate stronger PA performance at the very beginning of the semester generally perform better on the final exam. The magnitude of the correlations ranges from moderate to strong (approximately 0.40–0.59), suggesting that early precalculus skills account for a meaningful portion of student differences in calculus performance.

Table 4: Correlation Analysis (final vs 1stPA)

	Fall24 APMA 1090-Cal I	Fall24 APMA 1110-Cal II	Fall25 APMA 1090-Cal I	Fall25 APMA 1110-Cal II
Pearson Correlation Coefficients (Final Exam ~ 1stPA)	0.5932348	0.4000273	0.4861583	0.4297466
Correlation Test p value	p-value = 1.413e-14	p-value = 7.337e-10	p-value = 9.445e-11	p-value = 8.499e-08

The strongest association occurs in Fall 2024 Calculus I ($r \approx 0.59$), indicating that nearly 35% of the variance shared between the two measures is linear. In Calculus II, correlations are slightly smaller (around 0.40–0.43), but still clearly significant. This pattern suggests that while precalculus mastery continues to matter in Calculus II, additional factors—such as algebraic fluency, conceptual understanding of derivatives, or semester-long learning behaviors—likely contribute as well to final exam outcomes. The consistently small p-values (all $< 10^{-7}$) confirm that these relationships are not due to chance. Even in the cohorts with weaker correlations, the evidence strongly supports a real and reliable linear association.

Overall, 1stPA appears to function as a practical and informative early indicator of student readiness in both Calculus I and II. Although it does not capture all dimensions of calculus learning, its moderate-to-strong correlations with final exam performance highlight its value as a diagnostic tool—useful for early alerts, placement verification, and identifying students who may benefit from additional support early in the semester of these relationships highlight 1stPA's potential value as a diagnostic or early-alert measure.

Linear Regression. Similar with RQ1's investigation, for each cohort, we fit a simple linear regression model of the form $y = \alpha x + \beta$, where the final exam score served as the response variable, but the 1stPA score was used as the predictor this time. The resulting slope estimates, associated p-values, and R^2values are summarized in Table 5 below.

For all four cohorts, slopes are positive (3.51, 3.9,4 2.17, 3.77) and highly significant ($p<0.01$), showing that stronger 1stPA performance consistently predicts higher final exam scores. Effect sizes vary, but the predictive relationship is robust in every model.

Table 5: Linear Regression (final vs 1stPA: $final = \alpha * 1stPA + \beta$)

	Fall24 APMA 1090-Cal I	Fall24 APMA 1110-Cal II	Fall25 APMA 1090-Cal I	Fall25 APMA 1110-Cal II
Linear Regression slope	3.5053	3.9447	2.1655	3.769
Slope p value	1.41e-14	7.34e-10	9.44e-11	8.5e-08
Linear Regression (LR) R^2	0.3519	0.16	0.2363	0.1847
LR overall p values	p-value: 1.413e-14	p-value: 7.337e-10	p-value: 9.445e-11	p-value: 8.499e-08

In Fall 2024 Calculus I, the slope of 3.51 indicates that each additional point on 1stPA corresponds to roughly a 3.5-point gain on the 150-point final exam. The R^2 of 0.35 demonstrates substantial explanatory power. Fall 2024 Calculus II shows an even larger slope (3.94), though with a smaller R^2 (0.16), suggesting that more factors influence performance in Calculus II. The Fall 2025 cohorts show similar patterns: Calculus I has a slope of 2.17 ($R^2 = 0.24$), and Calculus II has a slope of 3.77 ($R^2 = 0.18$). All models remain highly significant.

Such analysis indicates that 1stPA is a strong and consistent predictor of final exam outcomes, with much higher R^2 values than models based on PT. Although the strength of the relationship varies slightly by course and term, 1stPA provides a far more informative measure of student readiness and early-semester preparedness.

Linear Regression Visualization. The scatterplots along with the linear regression lines below illustrate the relationship between 1stPA scores and final exam performance, which further confirm the patterns identified in Table 5.

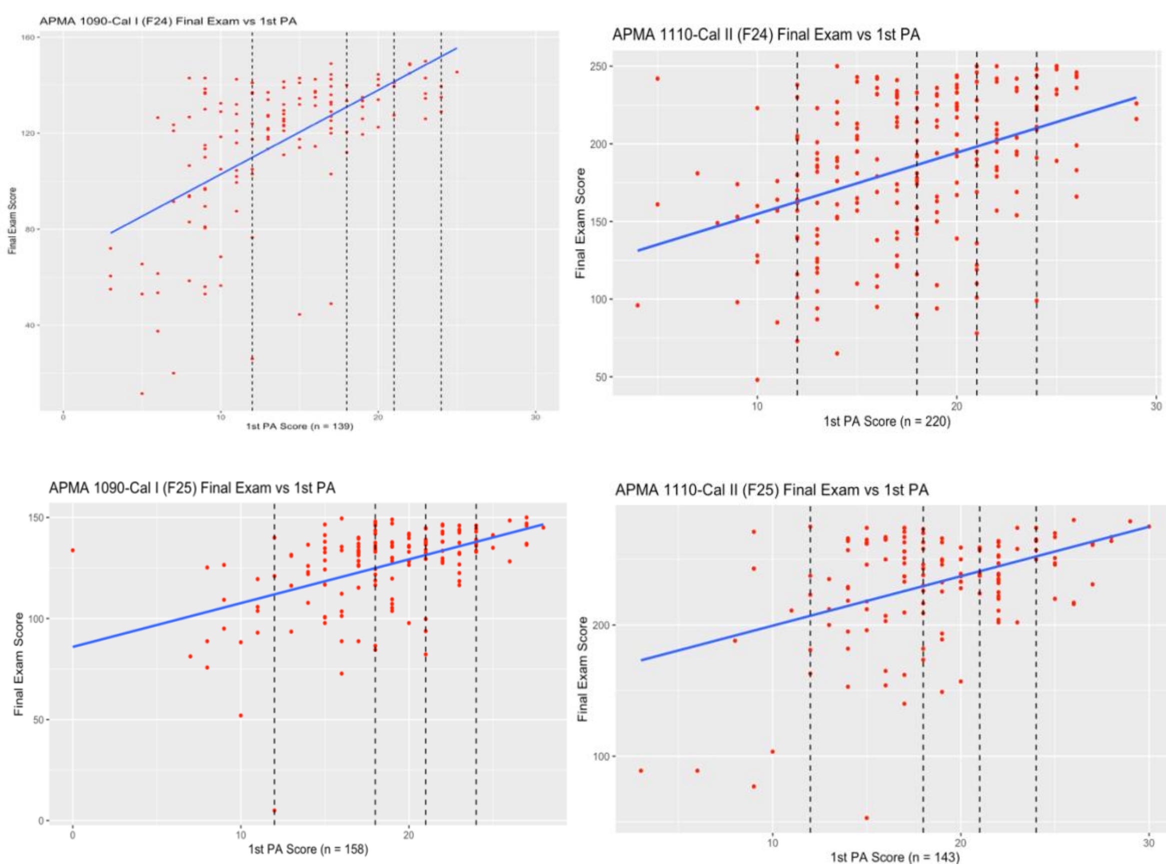

Figure 2*: Linear Regression final vs 1st PA (1stPA)*

Boxplots Visualizations. Given that 1stPA scores were shown to be strongly associated with calculus performance, we further investigate the relationship between 1stPA performance and final exam outcomes. 1stPA scores are stratified into five proficiency bands — <40%, 40–59%, 60–69%, 70–79%, and ≥80%—and final exam score distributions are examined across these categories.

1stPA (1st PA) scores were grouped into five proficiency bands: 1stPA < 40%**,** 40% ≤ 1stPA < 60%**,** 60% ≤ 1stPA < 70%**,** 70% ≤ 1stPA < 80%, and 1stPA **≥** 80%**.** For each group, distributions of calculus final exam scores are summarized using box plots (Figure 3) and violin plots (Figure 4) for APMA 1090 and APMA 1110.

In Figure 3, across all panels of boxplots, median final exam performance increases monotonically with 1stPA group, with students in the 1stPA < 40% category consistently exhibiting the lowest median scores and those in the 1stPA ≥ 80% category achieving the highest medians. This monotonic relationship is observed in both APMA 1090 and APMA 1110 and is consistent across semesters, underscoring 1stPA's role as a robust early indicator of calculus performance.

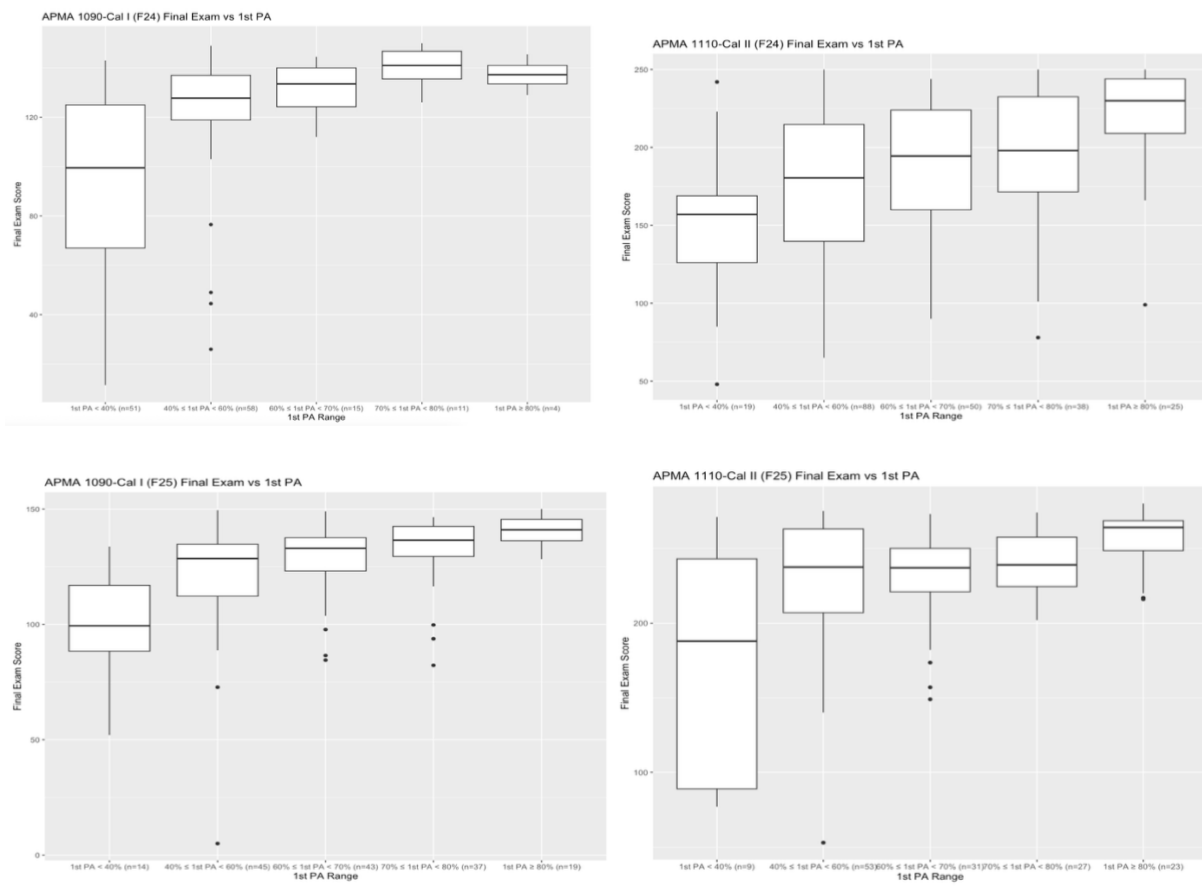

Figure 3: Boxplots final vs 1stPA groups

In addition to shifts in central tendency, score variability decreases systematically with increasing 1stPA. The interquartile range is largest among students in the 1stPA < 40% and 40% ≤ 1stPA < 60% groups, indicating substantial heterogeneity in final exam outcomes. In contrast, students in the higher 1stPA groups—particularly those with 1stPA ≥ 80%—exhibit markedly narrower interquartile ranges, suggesting more stable and predictable final exam performance as incoming proficiency increases.

The distributions also reveal an asymmetry in outlier behavior across 1stPA groups. Lower-1stPA categories contain a nontrivial number of pronounced low outliers, corresponding to students who perform very poorly on the final exam. The frequency and magnitude of these outliers decline steadily with increasing 1stPA and are nearly absent in the highest proficiency group. This pattern indicates that the risk of failure or extremely low performance is concentrated among students with weaker incoming preparation, whereas students with high 1stPA scores rarely experience severe negative outcomes.

Although APMA 1090 and APMA 1110 share these overall trends, course-level differences in variability are evident. For comparable 1stPA groups, APMA 1110 exhibits greater dispersion in final exam scores, particularly in the intermediate ranges (60% ≤ 1stPA < 70% and 70% ≤ 1stPA < 80%). This suggests that, even among students with similar incoming proficiency, performance outcomes in Calculus II are less tightly constrained, likely reflecting the increased conceptual and technical demands of the course.

Violin plots Visualizations. The violin plots (Figure 4) provide additional insight into the relationship between 1stPA (1st PA) performance and calculus final exam outcomes by revealing the full distributional shape of scores within each 1stPA group. Consistent with the box-plot summaries, the distributions shift upward as 1stPA increases, indicating higher overall final exam performance among students with stronger incoming proficiency. However, the violin plots further elucidate within-group heterogeneity, skewness, and tail behavior that are not evident from quartiles alone.

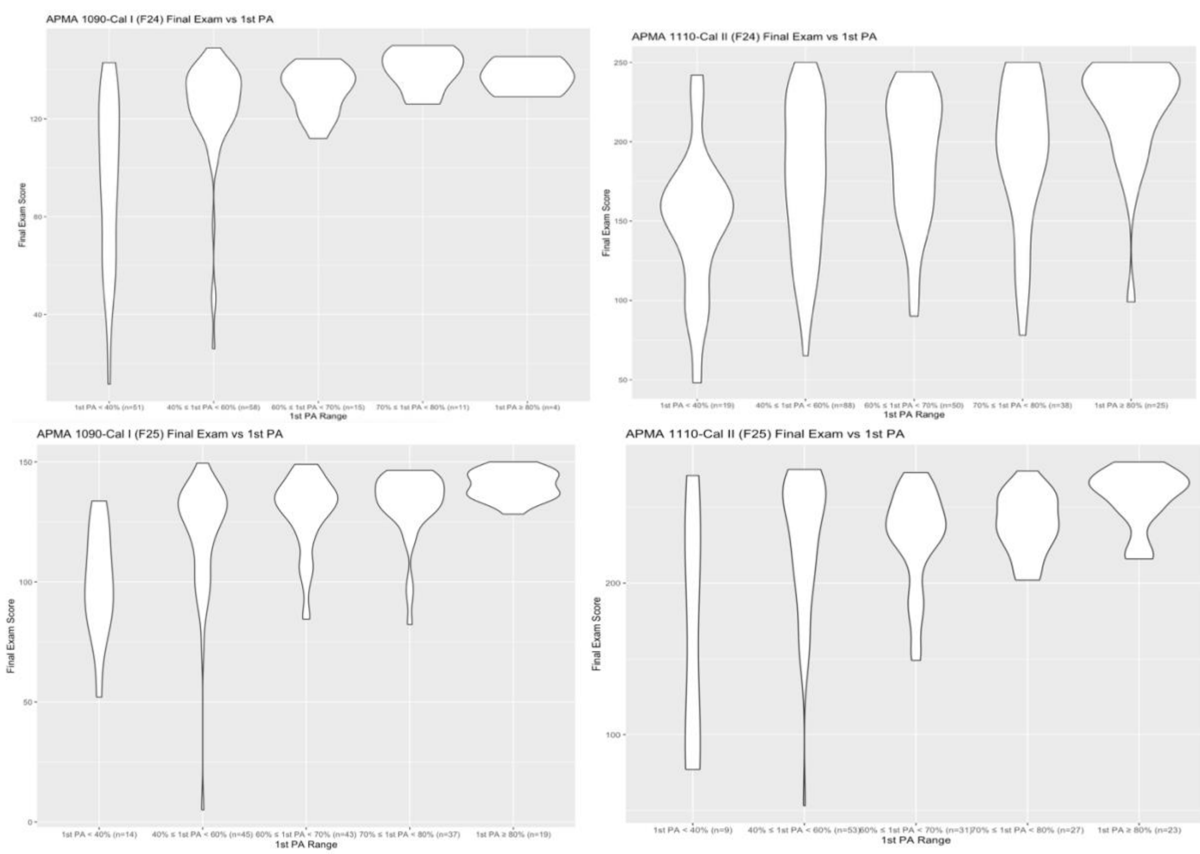

Figure 4: *Violin plots final vs 1stPA groups*

The violin plots in Figure 4 provide additional detail on distributional shape. Lower 1stPA groups show wide, strongly left-skewed distributions with substantial density at low final exam scores and long lower tails, indicating high risk of poor performance. Some students in these groups do perform well, but such upward deviations are relatively uncommon. Intermediate groups (60–69% and 70–79%) show the most heterogeneous distributions, sometimes with evidence of multiple performance clusters. These bands represent a transitional zone where outcomes vary widely despite similar incoming proficiency, suggesting opportunities for targeted instructional support. Students in the ≥80% group exhibit compact, top-heavy distributions concentrated near the upper end of the final exam range. In APMA 1090 especially, the density near the maximum score indicates a mild ceiling effect. Variability is much lower for this group, consistent with the boxplot findings.

Across courses, APMA 1110 again shows broader violins—especially in the middle ranges—indicating higher performance variability in Calculus II. Still, both courses show the same upward shift and tightening of distributions with increasing 1stPA, reinforcing 1stPA's value as a robust early diagnostic measure across the calculus sequence.

Wilcoxon Rank–Sum Tests. To further assess differences in final exam performance across 1stPA groups, we conducted additional statistical analyses. Across all four cohorts, several 1stPA groups contained small sample sizes, making parametric approaches such as analysis of variance (ANOVA) inappropriate. Consequently, we employed non-parametric pairwise Wilcoxon rank–sum tests to compare final exam score distributions between 1stPA groups; the resulting pairwise p-values are presented in the following Table 6.

Table 6: Pairwise Wilcoxon Rank-Sum Tests Summary

APMA 1090-Cal I (F24) Pairwise Comparisons of Final Exam Scores by 1st PA (Wilcoxon)

	1st PA < 40%	40% ≤ 1st PA < 60%	60% ≤ 1st PA < 70%	70% ≤ 1st PA < 80%
40% ≤ 1st PA < 60%	4.46e-05	-	-	-
60% ≤ 1st PA < 70%	2.39e-04	0.277	-	-
70% ≤ 1st PA < 80%	6.03e-05	0.003	0.045	-
1st PA ≥ 80%	0.014	0.130	0.408	0.601

APMA 1110-Cal II (F24) Pairwise Comparisons of Final Exam Scores by 1st PA (Wilcoxon)

	1st PA < 40%	40% ≤ 1st PA < 60%	60% ≤ 1st PA < 70%	70% ≤ 1st PA < 80%
40% ≤ 1st PA < 60%	0.035	-	-	-
60% ≤ 1st PA < 70%	0.003	0.124	-	-
70% ≤ 1st PA < 80%	0.002	0.054	0.466	-
1st PA ≥ 80%	4.28e-05	6.98e-05	0.002	0.028

APMA 1090-Cal I (F25) Pairwise Comparisons of Final Exam Scores by 1st PA (Wilcoxon)

	1st PA < 40%	40% ≤ 1st PA < 60%	60% ≤ 1st PA < 70%	70% ≤ 1st PA < 80%
40% ≤ 1st PA < 60%	0.003	-	-	-
60% ≤ 1st PA < 70%	9.22e-05	0.063	-	-
70% ≤ 1st PA < 80%	2.94e-05	0.005	0.241	-
1st PA ≥ 80%	2.38e-05	2.94e-05	0.003	0.023

APMA 1110-Cal II (F25) Pairwise Comparisons of Final Exam Scores by 1st PA (Wilcoxon)

	1st PA < 40%	40% ≤ 1st PA < 60%	60% ≤ 1st PA < 70%	70% ≤ 1st PA < 80%
40% ≤ 1st PA < 60%	0.053	-	-	-
60% ≤ 1st PA < 70%	0.069	0.628	-	-
70% ≤ 1st PA < 80%	0.053	0.903	0.604	-
1st PA ≥ 80%	0.006	0.010	0.006	0.007

In APMA 1090, the lowest proficiency group (1stPA < 40%) performs significantly worse than nearly all higher groups in both semesters. Differences among adjacent middle groups (40–60% vs. 60–70%) are less consistent, but contrasts involving larger gaps—especially comparisons with the ≥80% group—are consistently significant. In Fall 2025, the ≥80% group is statistically distinct from all other groups, indicating clearer separation at the high-proficiency end. In APMA 1110, the ≥80% group again outperforms all lower groups in both semesters. However, many comparisons among the lower and middle bands are not significant, especially in Fall 2025, where no differences emerge among the bottom

four groups. This indicates greater overlap in outcomes for students with moderate or low incoming proficiency in Calculus II.
Such results support a monotonic relationship between incoming proficiency and final exam performance, while revealing course-level differences in how clearly 1stPA distinguishes outcomes. 1stPA separates performance more sharply in Calculus I than in Calculus II, where higher variability reduces distinctions among intermediate groups.

Taken together, the Wilcoxon tests reinforce the boxplot and violin-plot patterns: 1stPA is most predictive at the extremes—identifying students at high risk (<40%) and those likely to excel (≥80%)—while performance among intermediate groups is more variable and less predictable, particularly in Calculus II.

4.3. RQ3. Which assessment—PT or 1stPA—serves as a stronger predictor of success in calculus courses?

Analyses addressing RQ1 and RQ2 indicate that 1stPA serves as a substantially stronger predictor of success in calculus courses than PT. To further substantiate this conclusion, we next employ multiple linear regression and hierarchical regression analyses to evaluate the predictive contribution of 1stPA relative to PT while ignoring other relevant factors.

<u>Multiple Linear Regression.</u> For each cohort, we fit a multiple linear regression model of the form $y = \alpha_1 X_1 + \alpha_2 X_2 + \beta$, where the final exam score served as the response variable, PT score was included as predictor X_1, and 1stPA score was included as predictor X_2. The estimated regression coefficients, corresponding p-values for each cohort are reported in Table 7 below.

Table 7: Multiple Linear Regression- final vs (PT+1stPA): $final = \alpha_1 * \text{PT} + \alpha_2 * \text{1stPA} + \beta$

	Fall24 APMA 1090-Cal I	Fall24 APMA 1110-Cal II	Fall25 APMA 1090-Cal I	Fall25 APMA 1110-Cal II
Predictor	p values of the predictor			
PT	0.973	0.00677	0.0153	0.0334
1st PA	.52e-12	7.75e-08	2.07e-09	6.88e-06

Here, each column corresponds to a cohort (APMA 1090–Calc I F’24; APMA 1110–Calc II F’24; APMA 1090–Calc I F’25; APMA 1110–Calc II F’25), and the entries are p-values from a multiple linear regression in which final-exam score was regressed on two predictors: PT and 1stPA. Across all four cohorts, 1stPA is a uniformly strong predictor: its p-values are exceedingly small—5.2×10^{-13}, 7.75×10^{-8}, 2.07×10^{-9}, and 6.88×10^{-6}—indicating a robust association with final-exam performance even when PT is included in the model. By contrast, PT shows inconsistent and weaker evidence: non-significant in APMA 1090 F’24 ($p = 0.973$) but nominally significant in the other three cohorts ($p = 0.0068,\ 0.0153,\ 0.0334$). If family-wise error is controlled across the eight tests (two predictors × four cohorts) using, for example, a Holm adjustment, the $p = 0.0334$ result would likely lose significance, whereas all 1stPA effects remain significant. Taken as a set, the table demonstrates that 1stPA provides a stable, cross-cohort predictive signal for calculus outcomes**,** while PT contributes modest, cohort-dependent incremental information at best**.**

*Hierarchical Regression***.** To evaluate the relative and incremental predictive contributions of PT and 1stPA we conducted a series of hierarchical regression analyses. For each cohort, we first fit a reduced model with final exam score regressed on PT alone (final ~PT) and then fit an expanded model that added 1stPA as a second predictor (final ~ PT+1stPA). Improvements in model fit were assessed by changes in adjusted R^2. We then reversed the order of entry, first fitting a model with 1stPA as the sole predictor (final ~1stPA) and subsequently adding PT (final ~1stPA +PT) to examine whether PT provided additional explanatory power beyond 1stPA. An increase in adjusted R^2 following the addition of a predictor was taken as evidence of improved model performance.

The PT-only models explain a relatively small proportion of variance in final exam scores across cohorts, with adjusted R^2 values ranging from 0.056 to 0.081, whereas the 1stPA-only models account for substantially more variance (0.156–0.347). This pattern indicates that 1stPA captures a much larger share of the variability in calculus outcomes than PT when each is considered independently. Consistent with these results, the combined models (1stPA + PT) yield higher overall explanatory power, particularly for APMA 1090, where adjusted R^2 exceeds 0.25 in both semesters. These differences suggest that 1stPA reflects course-relevant competencies more directly aligned with final exam performance than the placement-oriented PT.

Table 8: Hierarchical Regression Summary

	Fall24 APMA 1090-Cal I	Fall24 APMA 1110-Cal II	Fall25 APMA 1090-Cal I	Fall25 APMA 1110-Cal II
Predictor(s)	Adjusted R^2			
PT	0.05573	0.06792	0.06664	0.08108
PT -> PT+1stPA	0.05573 -> 0.3424	0.06792 -> 0.1805	0.06664 -> 0.2554	0.08108 ->0.1994
1stPA	0.3472	0.1562	0.2315	0.1789
1stPA->1stPA+PT	0.3472->0.3424	0.1562->0.1805	0.2315 ->0.2554	0.1789 ->0.1994

Hierarchical model comparisons provide stronger inferential support for the incremental validity of 1stPA. When 1stPA is added to a baseline PT-only model, the nested-model ANOVA yields highly significant improvements in model fit for all four cohorts ($p = 1.52 \times 10^{-12}$, 7.75×10^{-8}, 2.07×10^{-9}, and 6.88×10^{-6}). These results align with the substantial increases observed in adjusted R^2 and indicate that 1stPA explains a large amount of variance in final exam scores beyond what is captured by PT. In contrast, when PT is added to a baseline 1stPA-only model, the improvement in fit is modest and cohort-dependent. The nested-model ANOVA is not significant for Fall 2024 APMA 1090 ($p = 0.9725$)

and only marginally significant for the remaining cohorts ($p = 0.0068, 0.0153$, and 0.0334), consistent with the small gains in adjusted R^2.

Table 9: *Hierarchical Regression Model Comparisons Using Nested ANOVA*

	Fall24 APMA 1090-Cal I	Fall24 APMA 1110-Cal II	Fall25 APMA 1090-Cal I	Fall25 APMA 1110-Cal II
Models	p-value of Nested ANOVA (model1, model2)			
Model1=lm(Final~ PT) Model 2= lm(Final~ PT+1stPA)	1.518e-12	7.745e-08	2.074e-09	6.877e-06
Model1=lm(Final~ 1stPA) Model 2= lm(Final~ 1stPA+PT)	0.9725	0.006768	0.0153	0.0334

The adjusted R^2 values and nested-model ANOVA results (Table 9) lead to a consistent conclusion: 1stPA contributes substantial and robust explanatory power beyond PT, whereas PT provides limited additional information once 1stPA is known. This asymmetry is observed across both calculus courses and semesters, though it is more pronounced in APMA 1090 than in APMA 1110, reflecting greater outcome variability in Calculus II. These findings reinforce the interpretation that 1stPA is a more instructionally aligned and diagnostically effective measure of calculus readiness, while PT plays, at best, a secondary and context-dependent role in predicting final exam performance.

4.4. Students' perceptions of whether precalculus serves as a strong predictor of calculus performance, independent of OPPPT or 1stPA placement pathways.

In the survey, students were asked one Likert-scale question: *"Precalculus understanding is an important predictor of how well I will perform in my calculus classes."* Response options were coded as Strongly agree (5)**,** Agree (4)**,** Neutral (3)**,** Disagree (2), and Strongly disagree (1). The short survey was administered only to Fall 2025 APMA 1110 (Calculus II) students. A total of 111 students completed the survey, of whom 89 provided valid responses to this item.

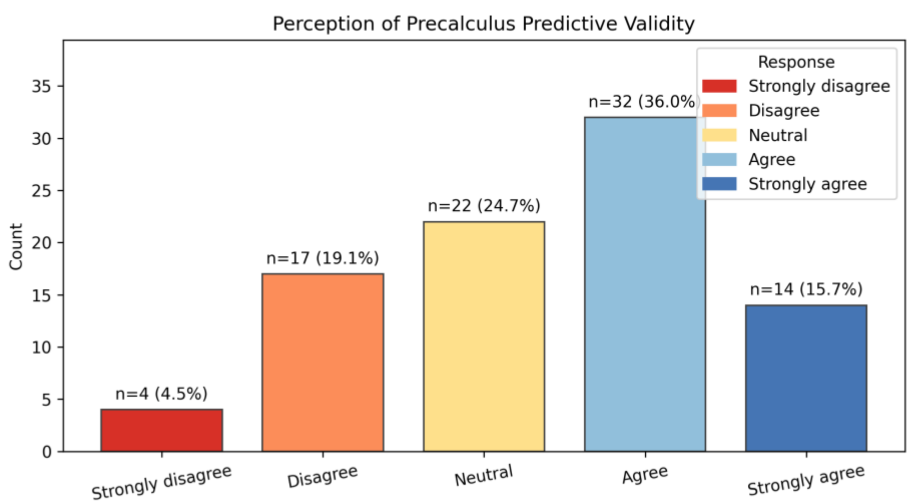

Figure 5: Student Perception of Precalculus Predictive Validity (n=89)

In Figure 5, bars show counts; labels show counts and percentages among valid responses (N=89). On-bar percentages use valid responses as the denominator.

Most APMA 1110 students view precalculus understanding as an important contributor to success in calculus. The mean rating on the five-point Likert scale was 3.39, slightly above the neutral midpoint, indicating a moderately positive perception. In terms of distribution, 51.7% of students agreed or strongly agreed that precalculus knowledge predicts calculus performance, 24.7% were neutral, and 23.6% disagreed. Thus, while a majority recognize the value of precalculus preparation, a meaningful minority remain uncertain or skeptical. The relatively high proportion of neutral responses suggests varied student experiences—some students may believe that success in Calculus II depends more on course-specific instruction or study habits than on prior preparation. Even so, the overall positive skew indicates that students generally regard precalculus as a meaningful foundation for calculus success.

Overall, the survey results support the instructional premise that precalculus concepts play a consequential role in later performance and highlight the value of examining their predictive validity that presented in this paper.

5. Discussion and Conclusion

Across distributional visuals, nonparametric tests, and regression analyses (including hierarchical comparisons and nested-ANOVA), our results consistently show that 1stPA is a stronger and more stable predictor of calculus performance than PT, and that 1stPA provides substantial incremental explanatory power beyond PT, and 1stPA has stronger predictive validity for Calculus I than Calculus II. Interpreted through a modern validity lens, this supports the intended interpretation and use of 1stPA scores for early risk identification and advising, consistent with the *Standards for Educational and Psychological Testing*, which emphasize assembling convergent evidence to justify score uses for specific decisions rather than treating validity as a property of the test itself [19]. The argument-based perspective further underscores that our claim concerns the use of 1stPA as an early diagnostic in these courses, with evidence spanning relations to external criteria (final exams), replication across cohorts, and analyses of distributional shape and variability, aligning with Messick's unified view of validity that integrates meaning and consequences of score use [20].

A salient interpretive factor is test-administration integrity. Although PT and 1stPA target comparable competencies, PT is completed in minimally proctored settings (online un-proctored), which can enable unauthorized assistance and thus attenuate the association with downstream performance. Empirical studies consistently report lower scores under proctoring relative to un-proctored conditions—an effect most parsimoniously explained by integrity controls removing sources of artificial score inflation. For example, Alessio et al. found sizable score reductions when online proctoring was introduced in otherwise similar online assessments, implying that un-proctored conditions overstate performance [21]. In a randomized field experiment, webcam monitoring produced significantly lower scores than identical un-proctored testing, strengthening the causal case that monitoring suppresses misconduct and improves score fidelity [22]. Within our context, this helps explain why PT contributes little unique variance once 1stPA (administered under controlled, course-embedded conditions) is in the model.

At the same time, the literature is not monolithic. Chan and Ahn report strong per-student correlations between in-person proctored and un-proctored online exams across many courses during the 2020 pivot, arguing that un-proctored exams can, in some settings and designs, provide meaningful assessment [5]. Viewed together with the integrity-sensitive findings, the prudent reading is contextual: un-proctored exams may sometimes approximate proctored performance, but their validity is contingent on incentives, item design, and integrity safeguards. For policy, we recommend treating PT as advisory and 1stPA as confirmatory/diagnostic for early support and placement corrections; where PT remains in use, institutions should bolster integrity (e.g., human/AI proctoring, randomized item pools, or two-stage un-proctored-then-proctored verification) to protect classification accuracy [19], [20], [21], [22]. Within courses, 1stPA thresholds can drive early alerts (e.g., <40%) and targeted scaffolding for the heterogeneous mid-bands. While our evidence is observational and single-institution, the pattern is robust across semesters and courses. Future work should add covariates (prior prep, section effects), test quantile/ordinal models, and evaluate causal impacts of 1stPA-triggered interventions—thereby extending the validity argument for using 1stPA in placement and advising decisions [19], [20].

6. Departmental Implications and Recommended Actions

The results of this study provide clear, actionable guidance for departmental decision-making around placement, instructional support, and early-semester interventions. Because the in-person, proctored 1stPA consistently demonstrates stronger predictive validity than the online un-proctored PT, the department/instructors can draw on these findings to consider several concrete policy and instructional actions.

Communicate findings to students. Provide students with clear messaging about why precalculus competence matters and how 1stPA results relate to course performance. Student-friendly graphics or brief explanations can be used to highlight the connection between strong precalculus skills and positive outcomes in calculus, thereby encouraging students to refresh/review/learn precalculus knowledge seriously.

Make PT proctored. Because the primary difference between PT and 1stPA is proctoring modality—and the proctored 1stPA demonstrates substantially stronger predictive validity—the department may increase the reliability of placement by transitioning the PT to a proctored format. This could take place either in person or through secure remote-proctoring systems.

Adopt a two-stage placement protocol. If immediately proctoring the PT is infeasible, a two-stage placement system can be implemented: students complete the PT online before arrival, followed by the proctored 1stPA during the first week of class. Large discrepancies between PT and 1stPA scores can trigger placement review, with 1stPA serving as the preferred basis for re-placement decisions.

Implement targeted early-semester support for at-risk students. Students in the lowest 1stPA proficiency bands (<40% and 40–59%) exhibit lower averages and greater variability in performance. The department can take early, structured steps to support these students, including: i) automatically flagging low 1stPA scores in the SIS for advisor outreach; ii) requiring an early-semester check-in with a math advisor or learning specialist; and iii) referring flagged students to tutoring, math-learning-center sessions, or short "skill clinic" interventions.

Develop a short, required pre-semester refresher (bridge module). Offer a brief (5–7 hour), self-paced "Precalculus Refresher Module" focused on core algebra, function, and trigonometry concepts. Completion may be required for students scoring below a placement threshold (e.g., <60%). This module could be implemented as a Canvas-based unit, a short August boot camp, or a "Week 0" preparatory assignment.

Provide enrichment and acceleration opportunities for the ≥80% group. Students scoring ≥80% on the 1stPA exhibit strong foundational readiness and reduced performance variability. To maintain appropriate challenge, instructors may offer optional enrichment experiences such as advanced problem-solving sessions, early exposure to selected Calculus II topics, or participation in honors-style recitation groups.

Encourage leadership and peer-support roles for the ≥80% group. High-proficiency students can contribute meaningfully to the learning environment. The department and instructors may encourage these students to serve as peer mentors in guided study groups, assist with collaborative problem-solving activities, or participate in early-semester review sessions, reinforcing their own mastery while supporting peers.

Collectively, these actions demonstrate how the department and instructors can leverage 1stPA-based evidence to improve placement accuracy, support students more effectively, and strengthen the transition from precalculus to calculus across the engineering curriculum.

Acknowledgment

This study was reviewed and approved by the Institutional Review Board (IRB) at the University of Virginia under protocol number 5362. All procedures and data collection methods were conducted in accordance with the approved IRB guidelines. We are grateful to our APMA 1090 and APMA 1110 colleagues for providing their class grades to support this project, and we thank the reviewers for their thoughtful comments and suggestions, which substantially enhanced the quality of this manuscript.

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Appendix 1

	A	B	C	D	E	F	G
1	**Learning Target**	**Code**				**PreCalculus Test Problem in PT and PAs**	**Learning Targets**
2	Able to simplify expressions.	S				1	S, I, E, L
3	Understands function inverses.	I				2	S, L
4	Can use Laws of Logs	L				3	S, E, L
5	Can use Laws of Exponents	E				4	S, P
6	Can perform polynomial division	P				5	S, E
7	Can solve equations.	Q				6	QA
8	Can solve equations that include absolute value.	QA				7	Q
9	Can solve inequalities.	N				8	Q
10	Can solve inequalities that include absolute valu	NA				9	NA
11	Can quickly determine a trig function value.	T				10	N
12	Can use the unit circle.	TU				11	T
13	Knows and can use important trig identities.	TD				12	T
14	Can determine limits to infinity.	LF				13	T
15	Can determine limits at discontinuity	LC				14	T
16	Can Graph Trigonometric Functions	TG				15	T
17	Can graph power functions.	GP				16	TU
18	Can graph exponential functions	GE				17	T, TD
19	Can graph log functions.	GL				18	T
20						19	T, TD
21						20	TD
22						21	LF
23						22	LF
24						23	LC
25						24	LC
26						25	LF
27						26	TG
28						27	TG
29						28	GP
30						29	GE
31						30	GL