Shaking Up Learning: An Interdisciplinary Earthquake Simulation Project

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Abstract - In the second semester of our undergraduate engineering program, students participate in interdisciplinary, team-oriented projects designed to connect core concepts from physics, mathematics, and computer programming. These projects serve as an integrative learning environment in which students apply theoretical knowledge—such as physical modeling, linear algebra, ordinary differential equations, and numerical methods—through the development of computational simulations in C#. The overarching goal is to support conceptual integration across foundational subjects while fostering engagement through application-oriented problem solving.

As an illustrative example, this paper presents the project “Earthquake Simulation of a Multistory Building,” in which students model the dynamic response of a simplified building structure subjected to seismic excitation. The building is represented as a system of coupled oscillators, with each floor modeled as a mass connected by springs and dampers. Students analyze natural frequencies to identify resonance effects and extend the model by integrating a tuned mass damper to explore vibration mitigation strategies used in real-world structures. The project culminates in an interactive simulation that enables real-time visualization and systematic variation of structural, absorber, and excitation parameters.

In this paper, the project framework, the modeling and computational approach, and the collaborative learning dynamics are presented. Student reflections and observed project outcomes indicate that the interdisciplinary project-based format supports students’ ability to integrate theoretical concepts with computational implementation, while also promoting collaborative problem solving and iterative refinement. In addition, it offers a transferable example of simulation-based project work in early-stage engineering education.

Introduction

A central challenge in engineering education is achieving an effective balance between conceptual understanding and practical application. Project-based learning (PBL) has proven to be a particularly suitable instructional approach for addressing this challenge, especially when it deliberately integrates methods and content from multiple disciplines. Over nearly three decades, our institution has developed and refined a structured, team-oriented PBL format that introduces project work in foundational subjects during the early phases of the Bachelor’s degree program (see, e.g., [1] - [5]).

The projects are designed to provide an interdisciplinary STEM learning experience, with a particular focus on physics, mathematics, computer programming as well as project management. Rather than treating these disciplines as isolated subjects, the project framework integrates them into a cohesive learning paradigm centered on applied, real-world problems. This approach is intended to support knowledge transfer across courses and to help students recognize the relevance of theoretical concepts in engineering practice. While recent shifts in engineering education have emphasized tightly framed, competency-based learning outcomes, our project format deliberately adopts a more holistic perspective that includes both technical proficiency and transferable professional skills.

The project-based learning process follows a clearly defined sequence (Figure 1). Project tasks are proposed by members of the faculty and sometimes by industrial partners in such a way that students need to combine their knowledge in physics, mathematics, engineering, and computer programming to find a suitable solution. If the scope of the project is suitable and deemed feasible for a period of one semester, it is included in the list of project proposals. At the beginning of the semester, students are introduced to all available projects during a kickoff meeting with the supervising faculty. In this session, supervisors present the project tasks, outline the expected scope and level of difficulty, and communicate the timetable, milestones, deliverables, and evaluation criteria. Each project is summarized in a concise, one-page task description. Students then form teams and select their preferred project topics. Typically, two or more teams of three to four students work simultaneously and competitively on the same task.

Allowing students to choose their project topic supports intrinsic motivation and encourages engagement with areas of individual interest. The tasks are intentionally designed to exceed students' initial solution competence at the beginning of the project. Required knowledge and skills are introduced progressively through accompanying lectures and exercises during the project phase. This slightly delayed just-in-time alignment between project demands and course content has been observed to increase students' attention to and engagement with underlying theoretical material.

Following an initial consultation with the supervisors, teams assign roles and responsibilities and begin their project work. This phase includes literature research, acquisition of relevant background knowledge, development of mathematical models, and the design and implementation of software solutions. Throughout the semester, supervisors act as mentors, providing guidance and targeted feedback while maintaining student ownership of the problem-solving process.

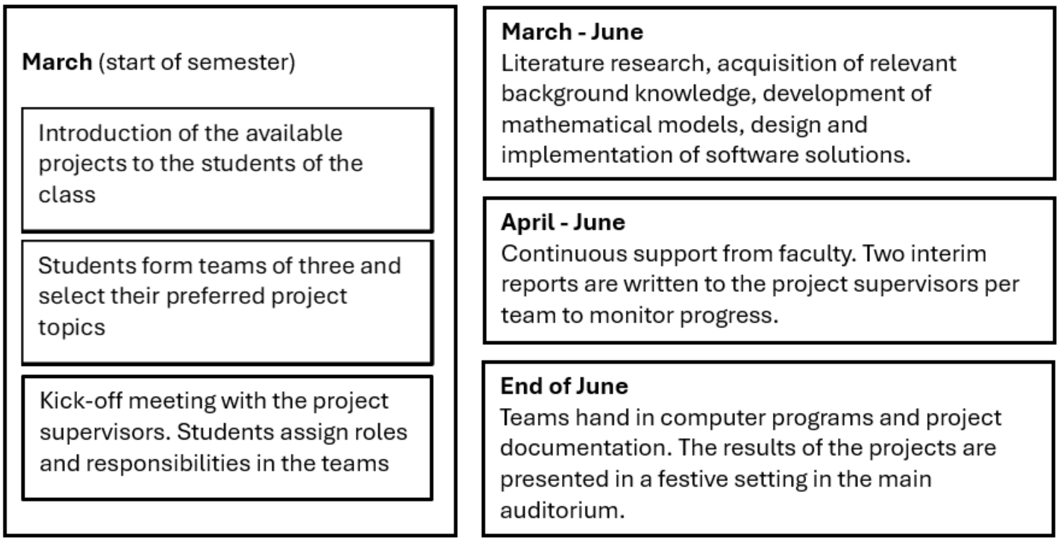

Figure 1: Visual representation of the team-oriented project-based learning workflow

The earthquake simulation project presented in this paper serves as a representative example of how this pedagogical design translates into concrete learning experiences. The project required students to develop a stand-alone software application simulating the dynamic response of a multistory building under seismic excitation, including the optional integration of a tuned mass absorber. Completing this task required the combined application of vibration theory, numerical methods, and software engineering principles.

Two teams of three students each have chosen this project, hereinafter referred to as "Team A" and "Team B".

Model design for the simulation of building vibrations

To simulate the dynamic response of multistory buildings subjected to seismic excitation, a simplified structural model was developed and implemented by the student teams. A schematic representation of the model is shown in Figure 2. The building is represented as a system of rigid floors with lumped masses m_i, interconnected by load-bearing walls modeled through linear stiffness elements k_i and viscoelastic dampers. The stiffness parameters approximate the bending behavior of the walls, while the damping terms c_i account for material-related energy dissipation (see, e.g., [6] - [8]).

When the floors undergo horizontal displacements x_i, shear forces arise within the supporting walls. In the model, this behavior is approximated by assuming that shear forces are uniformly distributed over the wall height and proportional to the relative floor displacements. The resulting bending deformation is therefore modeled analogously to the elongation of linear springs. This abstraction yields a tractable representation of building dynamics while retaining the essential characteristics required for vibration analysis.

The motion of the building is described relative to a horizontally moving foundation. Ground motion is introduced through a prescribed foundation displacement $x_0(t)$ derived from the double integration of a given earthquake acceleration signal. The equation of motion of the first floor relative to x_0 is

$$m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_0) + k_1(x_1 - x_0) = 0, \quad (1)$$

where $\dot{x}_0(t)$ represents the corresponding velocity of the foundation. Each floor is dynamically coupled to its neighboring floors, resulting in a system of coupled ordinary differential equations that describe the building's horizontal motion

$$m_i\ddot{x}_i + c_i(\dot{x}_i - \dot{x}_{i-1}) + k_i(x_i - x_{i-1}) - c_{i+1}(\dot{x}_{i+1} - \dot{x}_i) - k_{i+1}(x_{i+1} - x_i) = 0, \quad (2)$$

as represented in Figure 2.

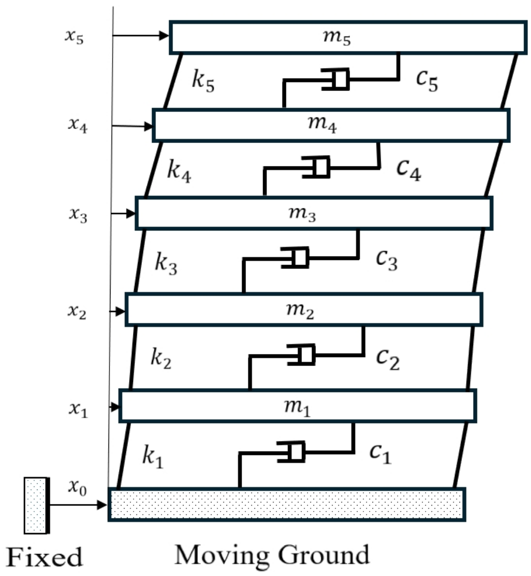

Figure 2: Schematic representation of the simulation model of a five-story building

This leads to a coupled system of ordinary differential equations, which is summarized in Equation (3) for a building with N floors

$$\begin{pmatrix} m_1 & 0 & 0 & \dots & 0 \\ 0 & m_2 & 0 & \dots & 0 \\ 0 & 0 & m_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & m_N \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \vdots \\ \ddot{x}_N \end{pmatrix} + \begin{pmatrix} c_1 + c_2 & -c_2 & 0 & \dots & 0 \\ -c_2 & c_2 + c_3 & -c_3 & \dots & 0 \\ 0 & -c_3 & c_3 + c_4 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & c_N \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_N \end{pmatrix} + \begin{pmatrix} k_1 + k_2 & -k_2 & 0 & \dots & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \dots & 0 \\ 0 & -k_3 & k_3 + k_4 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & k_N \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{pmatrix} = \begin{pmatrix} k_1 x_0 + c_1 \dot{x}_0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \tag{3}$$

In matrix form, the system is expressed using mass (**M**), damping (**C**), and stiffness matrices (**K**), yielding a compact representation suitable for numerical treatment

$$\mathbf{M}\ddot{\boldsymbol{x}} + \mathbf{C}\dot{\boldsymbol{x}} + \mathbf{K}\boldsymbol{x} = \boldsymbol{F}(t). \tag{4}$$

From a structural dynamics perspective, the resulting model represents, to a first approximation, a multi-degree-of-freedom oscillator with multiple natural frequencies and corresponding vibration modes. Such systems are susceptible to resonance excitation when external loading frequencies coincide with their natural frequencies, potentially leading to excessive vibrations and structural damage. To mitigate these effects, vibration control devices such as tuned mass dampers are commonly employed in real-world engineering practice (see, e.g., [9] – [12]).

A prominent example is the Taipei 101 skyscraper, which incorporates several tuned mass dampers to reduce wind- and earthquake-induced sway. The largest of these devices is a pendulum-based damper with a mass of approximately 700 tons, suspended near the top floors of the building. Inspired by this engineering solution, the student teams were tasked with extending their simulation model to include a pendulum-type tuned mass damper mounted on the top floor of the structure. Figure 3 shows the pendulum with the pendulum length l_d, the mass of the pendulum bob m_d and the angular displacement of the pendulum θ.

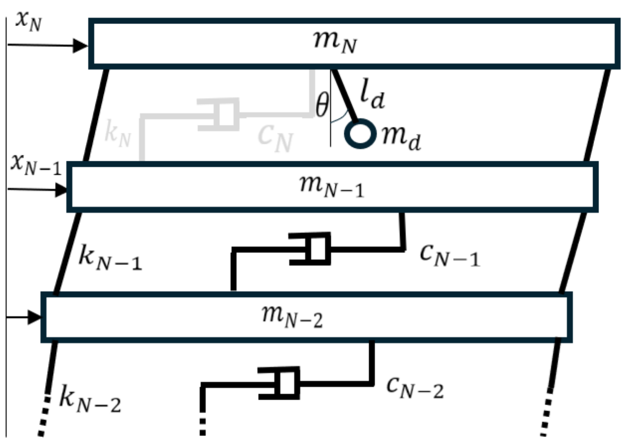

Figure 3: Schematic representation of the pendulum damper mounted on the top ceiling

This extension required augmenting the coupled system of equations (3) by incorporating the dynamic interaction between the building and the pendulum absorber. The equation of motion of the N^{th} story is extended by the pendulum reaction

$$(m_N + m_d)\ddot{x}_N + c_N(\dot{x}_N - \dot{x}_{N-1}) + k_N(x_N - x_{N-1}) = m_d l_d \ddot{\theta}\cos(\theta), \tag{5}$$

and the equation of motion of the pendulum hinged to the top floor

$$m_d l_d^2 \ddot{\theta} + c_d \dot{\theta} + m_d g l_d \sin(\theta) = -m_d l_d \ddot{x}_N \cos(\theta) \tag{6}$$

is added to Equation (3).

To estimate the natural frequencies of the undamped building model, the governing equations (4) were analyzed without damping and external excitation, i.e.,

$$\mathbf{M}\ddot{\boldsymbol{x}} + \mathbf{K}\boldsymbol{x} = \mathbf{0}. \tag{7}$$

By multiplying (7) by the inverse mass matrix $\ddot{\boldsymbol{x}} + \mathbf{M}^{-1}\mathbf{K}\boldsymbol{x} = \ddot{\boldsymbol{x}} + \mathbf{A}\boldsymbol{x} = \mathbf{0}$, the reduced formulation leads to the eigenvalue problem $(\mathbf{A} + \lambda^2\mathbf{I})\boldsymbol{x} = \mathbf{0}$ with the necessary condition

$$|\mathbf{A} + \lambda^2\mathbf{I}| = 0 \tag{8}$$

for nontrivial solutions.

The eigenvalues λ obtained from equation (8) represent the system's natural angular frequencies and the eigenvectors **x** the corresponding mode shapes. These quantities provide first-order estimates of resonance conditions and form the basis for both analytical interpretation and targeted excitation within the simulation.

Implementation into a computer program

Following the formulation of the mathematical model, the student teams implemented the simulation as a stand-alone software application. At an early stage, tasks were distributed within each team to ensure balanced workloads and clearly defined responsibilities. A modular structure was adopted, with distinct work packages focusing on (i) numerical implementation of Equations (3), (5), (6) and (8), (ii) development of the graphical user interface and visualization components, and (iii) generation of excitation signals, documentation, and project coordination. Clearly specified interfaces between these components facilitated parallel development.

Both teams selected C# as the implementation language, as it supports efficient numerical computation, object-oriented design, and the creation of interactive graphical user interfaces. C# is also part of the regular curriculum, enabling students to directly apply previously acquired programming knowledge while extending their skills in a project-based context.

For the numerical core of the program, the coupled second-order differential equation system describing the building dynamics was transformed into an equivalent first-order state-space representation. If, for example, only two of the coupled differential equations from equation (3) are considered, i.e.,

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{pmatrix} + \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{pmatrix}\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} + \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{pmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} F_1(t) \\ F_2(t) \end{pmatrix}, \tag{9}$$

the transformation of this system into the state space, with $z_1 = x_1,\ z_2 = x_2,\ z_3 = \dot{x}_1,\ z_4 = \dot{x}_2$, leads to

$\dot{z}_1 = z_3$, $\dot{z}_2 = z_4$, $\dot{z}_3 = \frac{1}{m_1}[F_1 + c_2(z_4 - z_3) - c_1 z_3 + k_2(z_2 - z_1) - k_1 z_1]$, and

$$\dot{z}_4 = \frac{1}{m_2}[F_2 + c_2(z_3 - z_4) - c_3 z_4 + k_2(z_1 - z_2) - k_3 z_2]. \tag{10}$$

This formulation allowed the application of standard numerical integration techniques, such as the explicit Euler method and the classical fourth-order Runge–Kutta method, which are covered in accompanying undergraduate mathematics courses. Eigenvalue calculations required for natural frequency analysis were performed using freely available linear algebra libraries [13], which the students integrated into their code.

In contrast to the mathematically driven implementation tasks, the development of the graphical user interface offered greater freedom for creative design. The resulting programs differ in layout and functionality while adhering to the same underlying physical model.

Figures 4 and 5 show the graphical interface developed by Team A. Their default configuration displays a 15-story building with predefined parameter values that provide users with reasonable orders of magnitude for masses, stiffnesses, and damping coefficients. Users can configure excitation parameters, select numerical integration schemes, and activate a tuned mass damper. Calculated natural frequencies are accessible via a pull-down menu and displayed alongside the visualization.

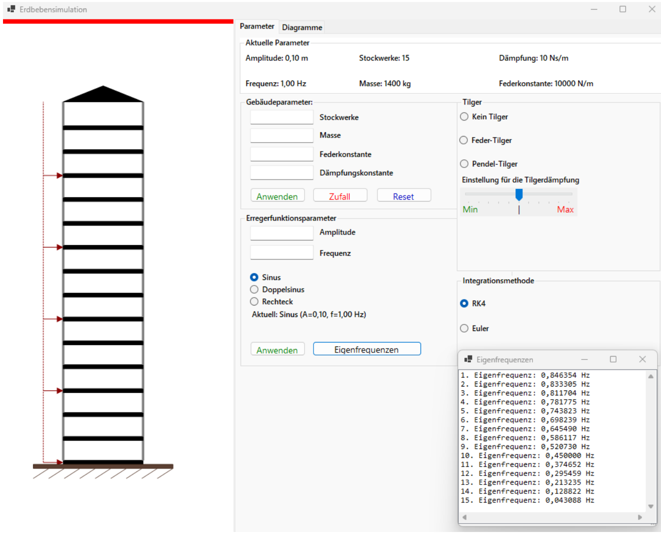

Figure 4: Graphical user interface of the computer program created by Team A

A second interface tab provides additional analytical tools, including time histories of floor deflections and phase-space representations with Poincaré sections (Figure 5).

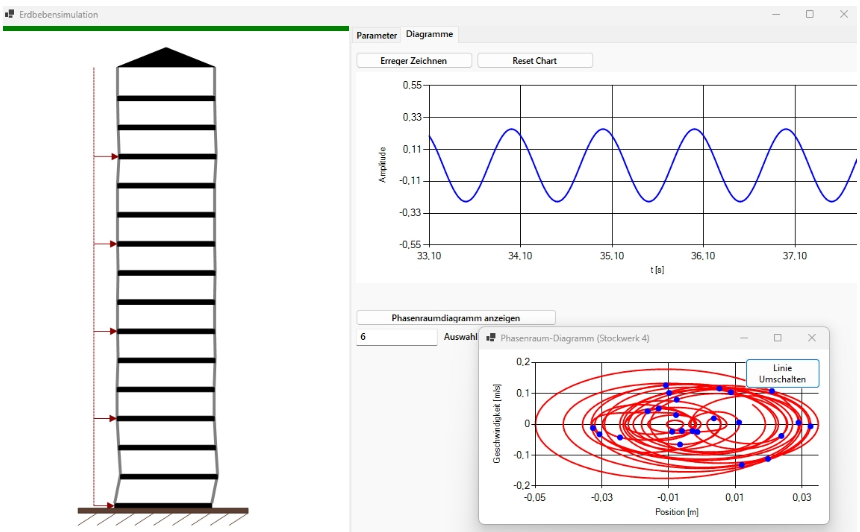

Figure 5: Second tab of Team A’s graphical user interface

When the tuned mass damper option is enabled, users may choose between a pendulum absorber and a mass–spring–damper absorber, each with freely adjustable parameters. This functionality allows systematic exploration of vibration mitigation effects under resonant excitation conditions. Figure 6 shows both types of absorbers, with the building excited in resonance with the third natural frequency.

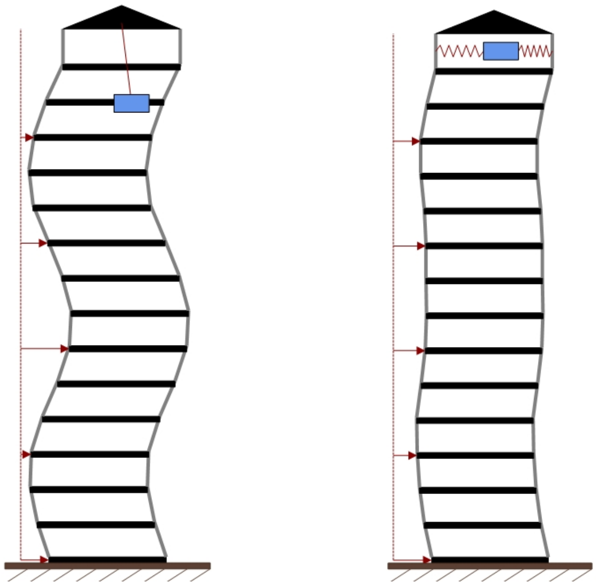

Figure 6: Pendulum (left) and mass-spring (right) tuned mass dampers

This team allows the user to excite the building with a sinusoidal oscillation with a predetermined amplitude and frequency. With the help of the pre-calculated natural frequencies of the structure, the pure, associated vibration modes can thus be excited. Figure 7 shows screenshots of the vibration states at the first, second, fourth and eighth natural frequencies of the building in a steady state.

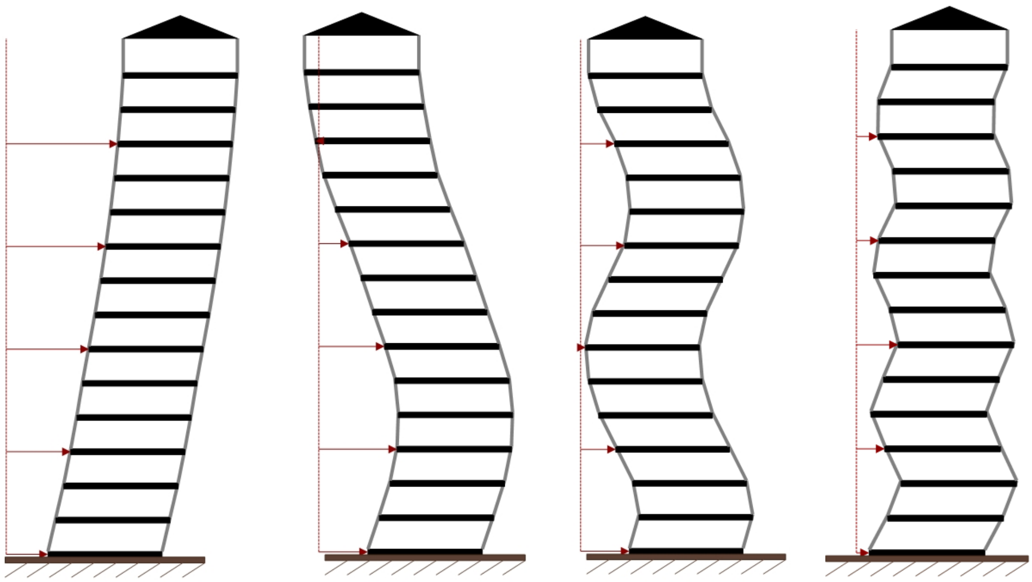

Figure 7: Structure excited with first, second, fourth and eighth eigenfrequency

Team B implemented a different interface concept, shown in Figure 8, which includes a comparison mode enabling simultaneous visualization of two buildings subjected to identical excitation. This feature facilitates direct comparison of structural responses with and without a tuned mass damper. As in Team A's implementation, natural frequencies can be selected from a pull-down menu to deliberately excite specific vibration modes.

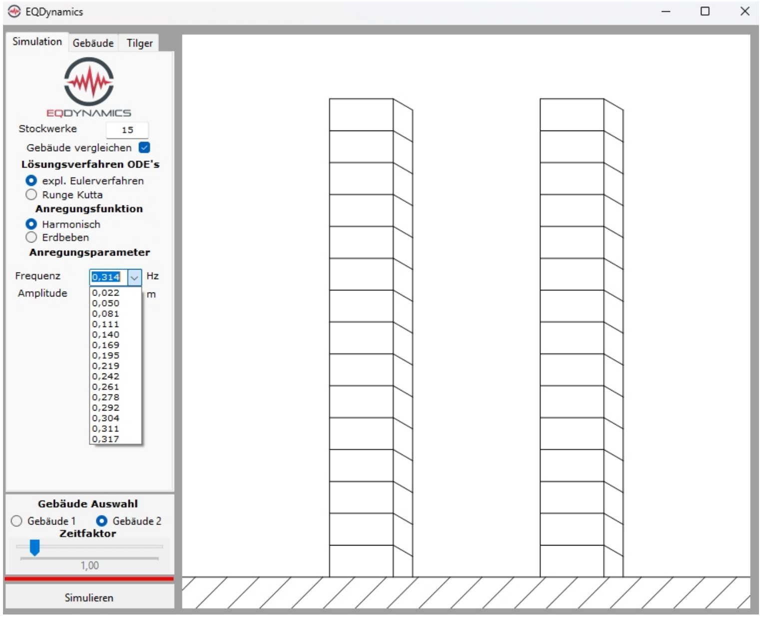

Figure 8: Graphical user interface of the computer program created by Team B

Figure 9 shows a comparison of the building swaying with the same structures and excitation functions, but once with and once without a pendulum damper. The buildings were excited with their first natural frequency.

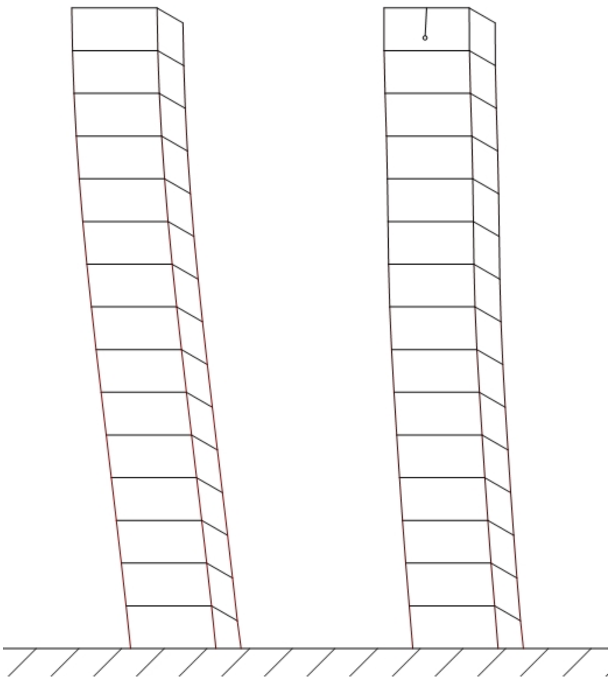

Figure 9: Building without (left) and with (right) pendulum damper

In addition to sinusoidal excitation, Team B incorporated real earthquake acceleration records obtained from the Center for Engineering Strong Motion Data (CESMD) [14], [15]. Users can select from several recorded earthquakes and adjust visualization speed and scaling factors during the simulation (see Figure 10). An additional interactive feature allows users to inspect motion data of individual floors by clicking directly on the animated structure.

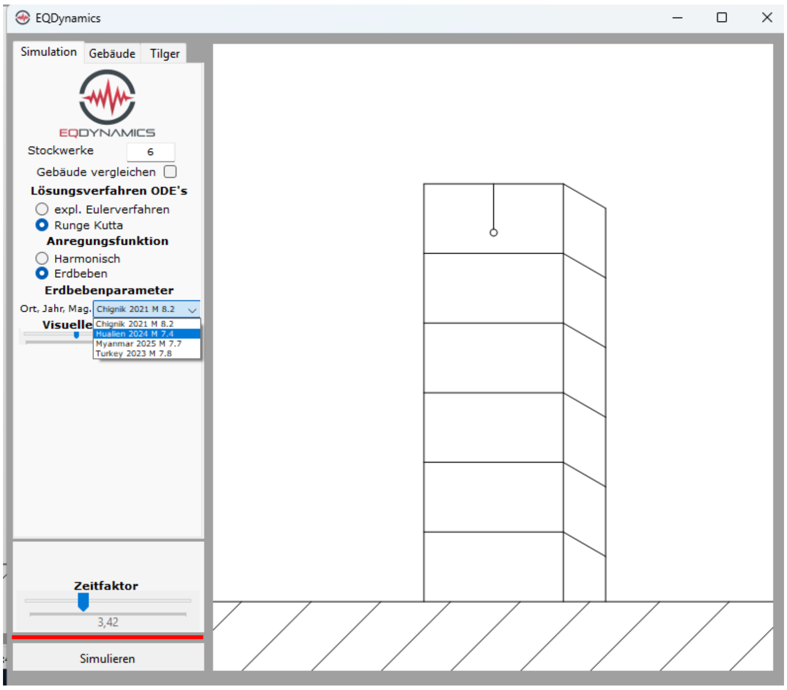

Figure 10: Pull-down menu for the selection of real earthquake accelerations

As a further distinguishing feature from the competing team, Team B has implemented a step size control for the integration of the equations of motion, which achieves an optimal compromise between computational accuracy and fluidity of the motion sequence of the earthquake animation by taking into account the current CPU load.

Overall, the two independently developed implementations demonstrate how a common physical and mathematical model can be translated into distinct, fully functional simulation tools. The diversity of interface designs and visualization features provided students with opportunities to explore both the technical and communicative aspects of engineering software development while reinforcing their understanding of structural dynamics.

Reflections on the impact on student learning

The description of the student projects is complemented by reflections on the impact of our team-oriented, project-based learning environment on student learning outcomes. While this learning format has been established at our institute for nearly three decades, systematic attention to learning outcomes has intensified only in recent years. This shift reflects broader changes in engineering education, including a move from broadly formulated educational aims toward explicitly defined, competency-based learning outcomes. At the same time, universities remain responsible for preparing students for professional practice, which requires not only the acquisition of technical knowledge but also the development of transferable skills relevant to the workplace.

Against this background, our cross-course projects deliberately adopt a holistic PBL approach that places equal emphasis on disciplinary knowledge and broader professional competencies. Each year, faculty members design new project topics to ensure that students work on authentic problems that have not yet been set in previous years. This design choice prevents reliance on prior solutions and requires students to engage independently in problem analysis, research, and implementation. Although the diversity of project topics leads to variation in the specific technical skills developed, all projects share a common structure that emphasizes teamwork, documentation, and adherence to deadlines—competencies consistently identified by industry partners as essential for engineering graduates.

Evidence of the learning impact of this format is drawn from multiple sources. First, student performance is assessed through the functionality and robustness of the developed computer programs, as well as through written project reports that document problem analysis, methodological choices, and implementation decisions. These artifacts provide concrete evidence of students' ability to integrate prior coursework in physics, mathematics, and computer science into a coherent solution. Second, students conduct structured self-evaluations within their teams, reflecting on individual contributions, collaboration processes, and skill development. These self-assessments are incorporated into the final course grade and provide qualitative insight into perceived learning gains.

In the specific project discussed in this paper, students reported that the interdisciplinary nature of the task and its clear connection to a real-world application were particularly motivating. In reflective feedback collected at the end of the project, students emphasized the iterative interaction between initial literature research, the conceptual understanding and application of underlying physical principles, and the formulation of appropriate mathematical models. Several students also highlighted incremental improvements in their programming proficiency, describing the project work as a low-threshold but sustained opportunity to practice algorithmic thinking and software development in an applied context.

A comparative, quantitative evaluation of learning outcomes across different instructional formats is currently not feasible. Project teams are small, project topics vary annually, and no control groups exist that complete equivalent coursework without participating in the project-based format. Nevertheless, longitudinal observations across cohorts provide indicative evidence of effectiveness: students who have completed these early-stage projects demonstrate greater readiness for subsequent, large-scale team projects in advanced semesters. This preparedness is particularly evident in complex extracurricular and curricular "lighthouse projects," such as the institute's long-standing participation in international Formula SAE/Formula Student competitions [16], [17], where student teams consistently achieve top rankings. While such outcomes cannot be attributed solely to a single course, they suggest that the sustained integration of team-oriented PBL contributes meaningfully to students' technical competence, collaborative skills, and confidence in tackling open-ended engineering problems.

Summary and Conclusions

Based on the goal of educating engineering students through an interdisciplinary and applied approach, we have implemented a project-based learning environment within our undergraduate degree program that emphasizes active, team-oriented learning and the integration of foundational STEM subjects. Beginning in the second semester, students engage in project work embedded within a project- and problem-based learning framework. Software development projects, accompanied by lectures that provide the necessary physics and mathematics background, form the basis of these early undergraduate research experiences.

Within the earthquake simulation project presented in this paper, student teams developed interactive software tools for analyzing the vibration behavior of multistory buildings subjected to seismic excitation. The applications are based on linear elastic multi-degree-of-freedom systems with viscous damping, with equations of motion solved using numerical integration techniques. The simulations were implemented in C# using Windows Forms technology. Both explicit Euler and fourth-order Runge–Kutta methods were employed for time integration, while natural frequency analysis was performed through the solution of an eigenvalue problem.

The resulting programs allow users to freely configure key model parameters, including the number of floors, floor masses, stiffness and damping coefficients, excitation characteristics, and the optional inclusion of a tuned mass damper. Real-time visualization of building motion, complemented by interactive plots of excitation functions and floor deflections, enables users to directly observe dynamic effects such as resonance, damping behavior, and vibration mitigation. These visualization features proved particularly valuable for linking theoretical concepts from lectures to observable system behavior within the simulation.

From a software engineering perspective, the modular architecture adopted by both teams—combined with structured collaboration via a shared GitHub repository—supported efficient teamwork and parallel development. The clear separation between physical modeling, numerical computation, visualization, and user interaction facilitated debugging, maintainability, and extensibility, while also exposing students to professional development practices commonly used in engineering projects.

While a quantitative comparison with traditional instructional formats was not feasible due to the small cohort size and the absence of control groups, multiple qualitative indicators point to the educational value of the approach. Assessment of project artifacts, student self-evaluations, and supervisory observations suggest that students developed an improved ability to integrate physics, mathematics, and programming concepts, as well as greater confidence in tackling open-ended, interdisciplinary problems. Moreover, the skills practiced in these early projects appear to contribute to students' preparedness for more advanced team-based projects in later semesters.

In conclusion, the presented project-based learning framework demonstrates how simulation-centered, interdisciplinary projects can be effectively integrated into the early stages of an engineering curriculum. By combining theoretical instruction with applied modeling, numerical analysis, and software development, the approach supports both conceptual understanding and the development of transferable professional skills. Future work will focus on refining assessment instruments and exploring opportunities for more systematic evaluation of learning outcomes across cohorts, with the aim of further strengthening the evidence base for interdisciplinary project-based learning in foundational engineering education.

Authorship declaration and acknowledgments

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References

[1] G. Bischof, E. Bratschitsch, A. Casey, and D. Rubeša, “Facilitating engineering mathematics education by multidisciplinary projects,” in *Proceedings of the ASEE Annual Conference and Exposition*, Honolulu, Hawaii, USA (2007)

[2] E. Bratschitsch, A. Casey, G. Bischof, D. Rubeša, “3 Phase Multi Subject Project Based Learning as a Didactical Method in Automotive Engineering Studies,” in *Proceedings of the ASEE Annual Conference and Exposition*, Honolulu, Hawaii, USA (2007)

[3] G. Bischof, and C. Steinmann, “Visualizing the kinetic theory of gases by student-created computer programs,” in *Proceedings of the ASEE Annual Conference and Exposition*, Columbus, Ohio, USA (2017)

[4] G. Bischof, M. Klatzer, C. Müller, D. Reifer, C. Steinmann, „Investigation of the transition from order to chaos by a numerical simulation of Pohl’s wheel,” in *Proceedings of the ASEE Annual Conference and Exposition*, Tampa, Florida, USA (2019)

[5] G. Bischof, K. Dobetsberger, M. Ensbacher, C. J. Steinmann, and A. Strutzenberger, “Exploring Swarm Behavior: An Undergraduate Project in Physics and Computer Programming,” in *Proceedings of the ASEE Annual Conference and Exposition*, Portland, Oregon, USA (2024)

[6] L. Munteanu, V. Chiroiu, & T. Sireteanu, “On the response of small buildings to vibrations,” *Nonlinear Dynamics*, 73(3), 1527-1543 (2013)

[7] J. Yang, S. Sun, T. Tian, W. Li, H. Du, G. Alici, & M. Nakano, “Development of a novel multi-layer MRE isolator for suppression of building vibrations under seismic events,” *Mechanical systems and signal processing*, 70, 811-820 (2016)

[8] Y. Zhang, Z. Guo, D. Liu, & W. Sun, “Seismic response analysis of super-high-rise building structures with three-layer isolation systems,” *Scientific Reports*, 13(1), 19165 (2023)

[9] V. J. García, E. P. Duque, J. A. Inaudi, C. O. Márquez, J. D. Mera & A. C. Rios, “Pendulum tuned mass damper: optimization and performance assessment in structures with elastoplastic behavior,” *Heliyon*, 7(6) (2021)

[10] J. S. Love, & T. C. Haskett, “The practical effects of friction for tuned mass dampers installed in tall buildings,” *Engineering Structures*, 265, 114495 (2022)

[11] P. Soltani, & A. Deraemaeker, “Pendulum tuned mass dampers and tuned mass dampers: Analogy and optimum parameters for various combinations of response and excitation parameters,” *Journal of vibration and control*, 28(15-16), 2004-2019 (2022)

[12] V. Y. Janga, P. K. Gourishetty, B. Carboni, G. Quaranta, & W. Lacarbonara, “Hysteretic tuned mass damper with bumpers for seismic protection: Modeling, identification, and shaking table tests,” *Journal of Sound and Vibration*, 597, 118816 (2025)

[13] C. Rüegg, M. Cuda, et al., *Math.NET Numerics*, https://numerics.mathdotnet.com/. [Accessed Jan. 21, 2026].

[14] H. Haddadi, A. Shakal, C. Stephens, W. Savage, M. Huang, W. Leith, J. Parrish & R. Borcherdt, “Center for engineering strong-motion data (CESMD),” in *Proceedings of the 14th World Conference on Earthquake Engineering*, Beijing, 12-17 (2008)

[15] Center for Engineering Strong Motion Data (CESMD) https://www.strongmotioncenter.org/ [Accessed Jan. 21, 2026].

[16] Formula SAE, https://www.fsaeonline.com/ [Accessed Jan. 21, 2026].

[17] Formula Student, Institution of Mechanical Engineers, http://www.formulastudent.com/ [Accessed Jan. 21, 2026].