Conceptual Errors in Dynamics: A Rubric-Based Analysis of Long Form Solutions and Concept Inventory Performance

Dr. Julian Ly Davis, University of Southern Indiana

Jul Davis is a Professor of Engineering at the University of Southern Indiana in Evansville, Indiana. He received his PhD from Virginia Tech in Engineering Mechanics where he studied the vestibular organs in the inner ear using finite element models and vibration analyses. After graduating, he spent a semester teaching at a local community college and then two years at University of Massachusetts (Amherst) studying the biomechanics of biting in bats and monkeys, also using finite element modeling techniques. In 2010, he started his career teaching in all areas of mechanical engineering at the University of Southern Indiana. He loves teaching all of the basic mechanics courses, and of course his Vibrations and Finite Element Analysis courses.

Dr. Amie Baisley, University of Florida

Amie Baisley is currently an Instructional Associate Professor at the University of Florida teaching primarily 2nd year mechanics courses. Her teaching and research interests are alternative pedagogies, mastery-based learning and assessment, student persistence in their first two years, and faculty development.

Dr. Geoffrey Recktenwald, Michigan State University

Geoff Recktenwald is a member of the teaching faculty in the Department of Mechanical Engineering at Michigan State University. Geoff holds a PhD in Theoretical and Applied Mechanics from Cornell University and Bachelor degrees in Mechanical Engineering and Physics from Cedarville University. His research interests are focused on best practices for student learning and student success. He is currently developing and researching SMART assessment, a modified mastery learning pedagogy for problem based courses. He created a multi-year integrated system design (ISD) project for mechanical engineering students. He is a mentor to mechanical engineering graduate teaching fellows and actively champions the adoption and use of teaching technologies.

WIP: Towards a standard rubric to compare concept inventory responses to long-form solutions

Abstract

A common philosophical discussion among STEM educators is if students are obtaining both good conceptual understanding and problem-solving skills in math, physics, and/or engineering principles. To better understand where students make strides or need help in both of these skills, three faculty collaborated to develop a 7-part rubric to quickly evaluate student long-form Dynamics solutions scoring each part for correctness, conceptual error, or procedural error.

In addition, each long-form exam problem was paired with a concept problem from the AIChE Concept Warehouse and assigned at the three different institutions. Each instructor assigned and graded the long form exams using their original grading format and then regraded each one using the newly created 7-part rubric to determine the transferability and adaptability for different instructors teaching and exam styles.

This paper presents the preliminary analysis of student long-form solutions paired with responses to concept problems testing similar topics. The concept problems are scored as correct or incorrect, and the long-form solutions are graded according to the standardized rubric.

For a small set of data, results indicate students can correctly answer concept problems while still showing conceptual misunderstandings in how to draw Free Body Diagrams or incorrectly setting up equilibrium equations. In addition, student "solution flow" indicates that students take many different paths to their long-form final answer and their concept problem answer. Along the way, there are procedural and conceptual errors in all sections of the rubric. The rubric shows a structured process to categorize errors and track student mistakes that can be related to student understanding. The next step will be to complete a full data analysis of each class to determine if and where students show connections between their long-form problem-solving process and conceptual understanding of in dynamics concept questions.

Introduction

There has been significant work done to create concept inventories and the vast areas they cover is an indication that engineering faculty would agree that their graduates should leave their programs with strong conceptual understanding of engineering topics. The concept inventories include the well known Force Concept Inventory (FCI) that was developed in 1992 [1]. There is also a Statics [2], [3] Strength of Materials, [4] and Dynamics concept inventory [5] that were created following the FCI. Currently, a new inventory is in development to focus on Rigid Body Dynamics [6]. A Heat and Energy concept inventory began development in 2009 [7] and is continually being adjusted as well [8]. There are also concept inventories in electrical engineering [9], [10], [11], civil engineering [12], engineering graphics [13], and engineering economics [14]. This list is hardly exhaustive and is always expanding.

These concept inventories are often used to try to assess student conceptual understanding which utilizes intuition and understanding often without the need to work through a solution. In addition to conceptual understanding, faculty are trying to support and understand the development of solid problem-solving skills for their students. However, the connection between these two forms of knowledge is not simple and are not always aligned. The two forms can develop to support each other, but they are not necessarily learned in parallel or consistently supported in every classroom.

Prior research has shown that students have an ability to learn pattern matching skills to successfully solve long-form problems without a clear understanding of the underlying principles they are applying [15]. However, conceptual understanding has been shown to provide students with the flexibility to apply their skills and transfer the knowledge to solve novel problems [16], [17], [18], [19]. The tension between conceptual understanding and problem-solving skills brings up important questions about how both are developed, assessed, and whether success in one relates to success in the other.

Research in mathematics education has shown that procedural knowledge alone is not enough for developing conceptual understanding [19]. Instead, studies indicate that developing both conceptual and procedural knowledge together tends to support deeper learning and the ability to apply the knowledge across different contexts [20].

Interestingly, it has been shown that the order in which conceptual or procedural knowledge is taught does not have a major influence on the outcome. One example from a study of 10^{th} graders found the order in which the method was taught, conceptual first vs. procedural first, did not have a strong impact on post-test results [21]. This suggests that it is not the instructional order that impacts the development, but rather how the knowledge is integrated or assessed.

The overarching goal of this research is to connect the relationship between students' problem-solving skills and their conceptual understanding of different Dynamics concepts. This will include analyzing how students solve long-form problems created by the instructors paired with concept problems from the Dynamics Concept Inventory [22]. The objective of this work-in-progress paper is to present an *initial* step towards studying this relationship through the development of a standardized scoring rubric for evaluating student long-form problem solutions. This rubric allows consistency in evaluating these problems across instructors and provides preliminary analysis of student problem-solving skills paired with concept problem response [5], [22].

Rubric Development

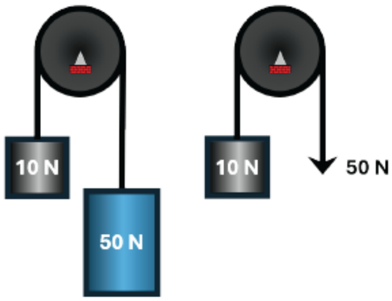
Over the last two academic years, three faculty at three different institutions met online to discuss the major elements that they use to assess homework and exam problems in their Dynamics courses. As discussions began the focus was narrowed to assessment of Newtonian based solutions, as opposed to solutions using energy methods. Three key solution elements were initially isolated as parts of the solution that should be evaluated as part of the developing rubric. These included the ability to:

1) Draw Free Body Diagrams
2) Write Equilibrium equations and
3) Solve the problem using proper algebra skills.

Next, they met to select long-form exam problems and paired these with previously developed concept problems[22] that examine students' understanding of the underlying concepts used in the long-form problem. An example of the paired concept and long-form problem are shown below in Figure 1. The concept problem examines the understanding of the difference between force and weight through a pulley mechanism. The long-form problem was selected because it requires students to illustrate an understanding of this same concept through their long-form solution. Note that the methods used to solve the long-form problem successfully could be used to solve the concept problem to arrive at the correct solution to the concept problem. Conversely, the intent of the concept problem is for students to be able to quickly answer it based on their intuition and knowledge of the system.

A) Selected concept problem adapted from Concept Warehouse (CW3244)

Given: **(CW3244)** Both systems shown have massless and frictionless pulleys. On the left, a 10 N weight and a 50 N weight are connected by an inextensible rope. On the right, a 50 N force pulls on the rope.

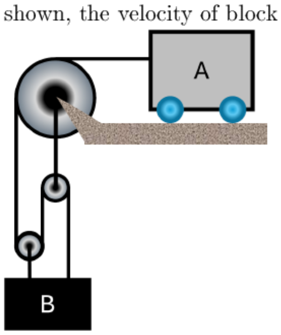


Find: Which of the statements is true immediately after unlocking the pulleys.

Sol'n:
1. In both cases the acceleration of the 10 N block will be zero.
2. The 10 block on the left will have a larger upward acceleration.
3. The 10 block on the right will have a larger upward acceleration.
4. The tension in the rope on the left is 40 N.
5. In both cases, the 10 N block will have the same upward acceleration.

B) Common exam problem statement

Given: **(GDR SP2025)** Blocks A ($m_A = 10\ kg$) and B ($m_B = 4\ kg$) are connected as shown. At the instant shown, the velocity of block A is $2\frac{m}{s}$ to the left. Consider the mass of the pulleys to be negligible.



Find: Determine

1. the tension in the rope at this instant,
2. the acceleration of each mass at this instant, and
3. how fast will blocks A and B be traveling 3 seconds later.

(Neglect the mass of the pulleys)

Figure 1: Paired Concept and Long-form problem

Figure 1A) is a recreation of a multiple-choice concept problem that evaluates students' understanding of the difference between force and weight. While Figure 1B) is the paired long-form problem from a faculty's exam.

These paired problems were incorporated into each faculty's exams and exam schedule – no alteration was made to an individual's course schedule or grading schema. The concept problems were treated as extra credit for low point values relative to the rest of the exam: generally, 2-4 points. After administering the exam, each exam was graded by the corresponding faculty, and then re-graded using the rubric to assess common errors. After everyone had assigned and graded the problem, the faculty met to discuss the results of grading with the initial rubric. During the discussion, expectations for what students should include to solve the problem successfully were discussed based on the three sections of the developing rubric: Free Body Diagrams, Equilibrium Equations, and Algebra Skills. Topics discussed included: 1) what constitutes a procedural error versus a conceptual error in each section, and 2) if a procedural error was made in one part of the problem, how should OTHER parts of the problem be assessed (i.e. does the error carry forward?). After discussion, the initial three rubric sections were expanded into seven categories to provide more granularity and clarity when identifying areas of procedural and conceptual error.

The final rubric used for grading the long-form solutions includes seven key categories. These are:

1) Coordinate Systems (CS),
2) Free Body Diagrams (FBD),
3) Acceleration Diagrams (AD),
4) Equilibrium Equations (EqEqn),
5) Other Equations (OthEqn),
6) Algebra (ALG) and
7) Final Answer (FA).

For each category, the scoring was done using the following classifications:

a) Correct
b) Procedural error (such as math, unit, transcription, or calculation errors)
c) Conceptual error

During the rubric development phase, discussions on how to categorize different types of errors came up. For example, not following coordinate systems or acceleration diagrams when setting up equilibrium equations constituted a conceptual error for that category. However, writing equilibrium equations that followed an incorrectly declared coordinate system did NOT count as a conceptual error – because the students are consistent with their own work. Other examples of conceptual errors included:

1) Coordinate systems not starting from fixed reference
2) Incorrect number of tensions on the FBD
3) Not including weight on the FBD

After these discussions and the rubric creation, each instructor re-graded their exams following the seven rubric categories using the three-scoring options. The instructors kept track of the type of procedural errors made and included notes for the conceptual errors that were made. The results in this paper will include the totaled results for some of the rubric categories and an illustration of tracking the flow of these errors to the conceptual problem answers.

Results

The results come from three universities, including two large public R1 universities, in the South and Midwest, and a public M1 institution in the Midwest. Due to the small class size in one of the instructors' courses, only a small (randomly selected) number of student exams were selected from the other two instructors' classes for the analysis to support the proof-of-concept for the rubric. The results include a preliminary analysis of a small data set from the problem shown in Figure 1 that will consider student performance on the concept problem and their corresponding solution to the long-form problems using the rubric.

Also included in this section is an illustration of the flow of a long-form solution from rubric category 1 to 7 ending at the concept problem response to illustrate how errors in one section of the rubric "track" to other sections of the rubric as a student solves a long-form problem. This flow illustration starts to track how those errors may be connected to correctly answering a concept problem.

The three instructors assigned the long-form problem and paired concept problems on their normal in-class exams in an introductory second year Dynamics course. The students were not required to answer the questions in a particular order, so it is possible students could answer the concept problem first and then work out the long-form problem, or they could work out the problem first and then answer the concept problem.

Preliminary Analysis of Concept Answer and Rubric Category Scoring

Initial analysis of the connections between students' concept answers and different rubric categories is ongoing, but the preliminary analysis will show this relationship for some of the rubric categories. In the tables below, results are organized in terms of students having answered the concept problem correctly or incorrectly and then those groups are broken into the three score options for one rubric category (Category 7 – Final Answer is shown in Table 1). Currently, the results have been quantified for 4 categories of the rubric: Final Answer, Coordinate systems, Free Body Diagrams, and Equilibrium Equations. This provides an overview of how students are performing in different areas of the problem-solving process and attempts to draw connections to their response to the concept problems.

The use of the rubric provides a systematic way to break down errors in long-form solutions and relate those errors to concept problem performance. One of the central questions to this study is whether students who correctly answer a concept problem also arrive at the correct numerical solution on a long-form problem, or conversely, whether students who correctly answer the long-form problem demonstrate correct conceptual understanding.

Category 7: Concept-Final Answer Analysis

Table 1 presents the preliminary results for the final answer relationship. It shows that for our data set, 23.8% of our students were able to answer the concept problems correctly, while 76.2% answered incorrectly – close to guessing. Of those that answered the concept problem correctly, 20% got the correct final answer to the long-form problem without making any procedural errors. However, of the 76.2% of students who answered the concept problem incorrectly, 12.5% were able to arrive at the correct long-form problem answer, but the remaining 68.8% made procedural errors in arriving at their final answer. So, the other rubric categories will allow us to track where those errors are made to see the places of errors that are leading to a poor performance.

Table 1: Comparing concept inventory performance to calculating the correct final answer

Shown here are the results of the types of errors students made in the Final Answer section based on the students' performance in answering the concept problem. Either the final answer was correct, or there was a procedural error. In this category, there was no classification for conceptual errors, since any errors here are typically from calculations, transcriptions, or other procedural errors.

Percent of Each Category								
Concept	Correct				Incorrect			
	23.8%				76.2%			
Final Answer	Correct	Incorrect		DNA	Correct	Incorrect		DNA
	20.0%	80.0%		--	12.5%	68.8%		18.8%
Error Type		Procedural	Conceptual			Procedural	Conceptual	
		100.0%	--			100.0%	--	

These results indicate that the answer to our central question: "Is it possible for students who correctly answer concept questions to arrive at the correct numerical solution to a long-form problem?" would be "Yes"– but not many do. In addition, some students who incorrectly answer the concept problem can still arrive at the correct numerical answer to the long-form problem. With the limited data that we analyzed, the results are mixed. We cannot conclude that there is a one-to-one correlation between getting the correct answer and answering the concept question correctly. In fact, this is exactly what we might expect from an average Dynamics class.

This leads to the next questions for this study: "1) What type and where are the mistakes students are making such that they are not getting to the correct long-form problem answer?" and "2) How do those mistakes relate to answering the concept problem?"

Category 1: Concept-CS Analysis

The first category in the rubric includes selecting a coordinate system which is a foundational step for setting up correct directions and descriptions of the motion. Table 2 includes the correlation between students' response to the concept problem and their ability to select an appropriate coordinate system to solve this problem.

Table 2: Comparing concept inventory performance to selecting the correct coordinate system

Shown here are the results of the types of errors made in the Coordinate System section in comparison to a students' performance in answering the concept problem. In addition, incorrect answers in the rubric category are further broken down into procedural or conceptual errors

Percent of Each Category								
Concept	Correct				Incorrect			
	23.8%				76.2%			
CS	Correct	Incorrect		DNA	Correct	Incorrect		DNA
	60.0%	0.0%		40.0%	62.5%	37.5%		0.0%
Error Type		Procedural	Conceptual			Procedural	Conceptual	
		--	--			16.7%	83.3%	

This table illustrates that 23.8 % of the students answered the concept problem correctly– the same as shown in Table 1. However, of those students, 60% were able to correctly select an appropriate coordinate system to help them solve the problem. The other 40% did not even attempt to indicate a coordinate system. An error here might have been to have a coordinate system that did not start from a fixed reference frame. In addition, 76.2% of the students who incorrectly answered the concept problem, 62.5% were still able to select an appropriate coordinate system. Those who made errors in selecting a coordinate system, 83.3% made conceptual errors and the remaining 16.7% made procedural errors.

The relationship between coordinate system and concept problem response shows that the small percentage of students who answered the concept problem correctly, did not seem to have issues in selecting an appropriate coordinate system. In addition, though a large percentage of students answered the concept problem incorrectly, they were still able to select a coordinate system appropriate to solving the long-form problem. This is encouraging because this is one of the key items necessary to solve long-form problems correctly and communicate a solution to others.

Category 2: Concept-FBD Analysis

Next, the free-body-diagram category seemed to be a clear step in the solution process where students could make conceptual errors that lead to misunderstandings of dynamics concepts. The results relating error types in the FBD category to concept problem performance are summarized in Table 3.

Here, of the 23.8% of students answering the concept problem correctly, only 40% of the students are drawing FBDs properly. A similar percentage of the students that answered the concept problem incorrectly were able to draw a proper FBD. The majority in both groups had incorrect FBDs, and it seems most of the errors were conceptual, versus procedural. Some of the conceptual errors regarding this problem include:

- Wrong number of tensions
- Doesn't use pulley FBD to relate tension in ropes
- Did not count number of ropes
- 1 Normal force
- Only 1 tension on Block B

Table 3: Comparing concept inventory performance to free body diagram performance
Shown here are the results of the types of errors made in the Free Body Diagram section in comparison to a students' performance in answering the concept problem. In addition, incorrect answers in the rubric category are further broken down into procedural or conceptual errors

Percent of Each Category								
Concept	Correct				Incorrect			
	23.8%				76.2%			
FBD	Correct	Incorrect		DNA	Correct	Incorrect		DNA
	40.0%	60.0%		0.0%	37.5%	62.5%		0.0%
Error Type		Procedural	Conceptual			Procedural	Conceptual	
		33.3%	66.7%			30.0%	70.0%	

Category 4: Concept-Equilibrium Equations Analysis

The next rubric category that was analyzed is one of the most critical steps in the long-form solution process, but also in illustrating a conceptual understanding of a problem – transferring those vectors on the FBD into equations of equilibrium. Interestingly, most students were able to get the equations correct regardless of how they answered the concept problem. It should be reiterated that the rubric does not carry forward errors that may have happened in a previous rubric category. However, the error type made in this category are all conceptual showing that the students who are making mistakes are having a hard time conceptually transferring the vectors from the FBD into a complete set of properly formed equilibrium equations. Some examples of the descriptions of errors include:

- Did not follow FBD
- Sign on T and a_x do not match FBD
- Set each FBD equal to accel for Block A

Table 4: Comparing concept inventory performance to the ability to write a complete set of equilibrium equations

Shown here are the results of the types of errors made in the Equilibrium Equations section in comparison to a students' performance in answering the concept problem. In addition, incorrect answers in the rubric category are further broken down into procedural or conceptual errors

Percent of Each Category								
Concept	Correct				Incorrect			
	23.8%				76.2%			
EqEQNs	Correct	Incorrect		DNA	Correct	Incorrect		DNA
	60.0%	40.0%		0.0%	62.5%	18.8%		18.8%
Error Type		Procedural	Conceptual			Procedural	Conceptual	
		0.0%	100.0%			0.0%	100.0%	

Solution Flow Diagram

The flow diagram shows how student-work progresses through each part of the long-form solution process as defined by the rubric and how this path relates to concept problem response. To visualize how one error may have influenced another, each type of error (procedural & conceptual) was tracked from one rubric category to the next: see Figure 2. The ideal relationship would show that students who answered each section of the dynamics rubric correctly would be able to answer the concept problem correctly, but the results indicated that this relationship does not exist as shown in the tables above, particularly Table 3. Correct answers on the different rubric items does not necessarily result in a correct answer on the concept problem. Figure 2 shows an overall view of how students collectively progress from one part of the long-form solution process to the next. This is a high-level view of the flow, but a more detailed look at the flow for an individual student offers more insight into the effects of errors throughout the process.

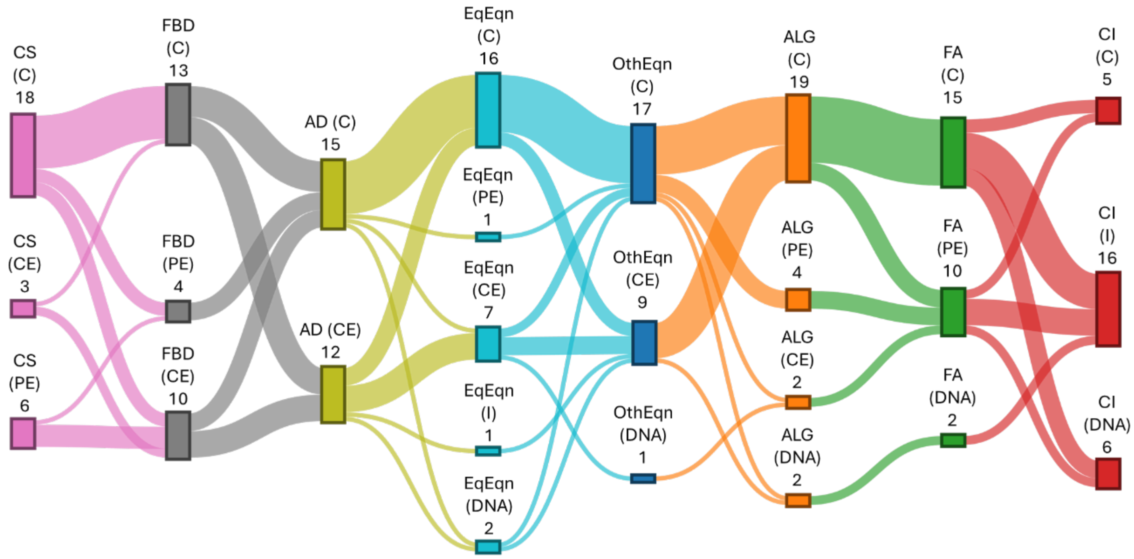


Figure 2: Flow diagram of scores for each rubric category.

Each of the seven sections of the grading rubric (CS, FBD, AD, EqEqn, OthEqn, ALG, FA) are shown in the general flow (left to right) that students take from one rubric category to the next, and then to the concept inventory (CI) question answer.

Each rubric category is scored as (C) - Correct, (PE) - Procedural Error, (CE) - Conceptual Error, (I) – Incorrect, or (DNA) - Did not Attempt.

Figures 3 and 4 show the same image in Figure 2 with the paths of two students (solid and dashed lines) through the solution process overlaid. Figure 3 illustrates that there are multiple ways to get to the correct Final Answer (FA) and correct answer on the Concept Inventory (CI) problem.

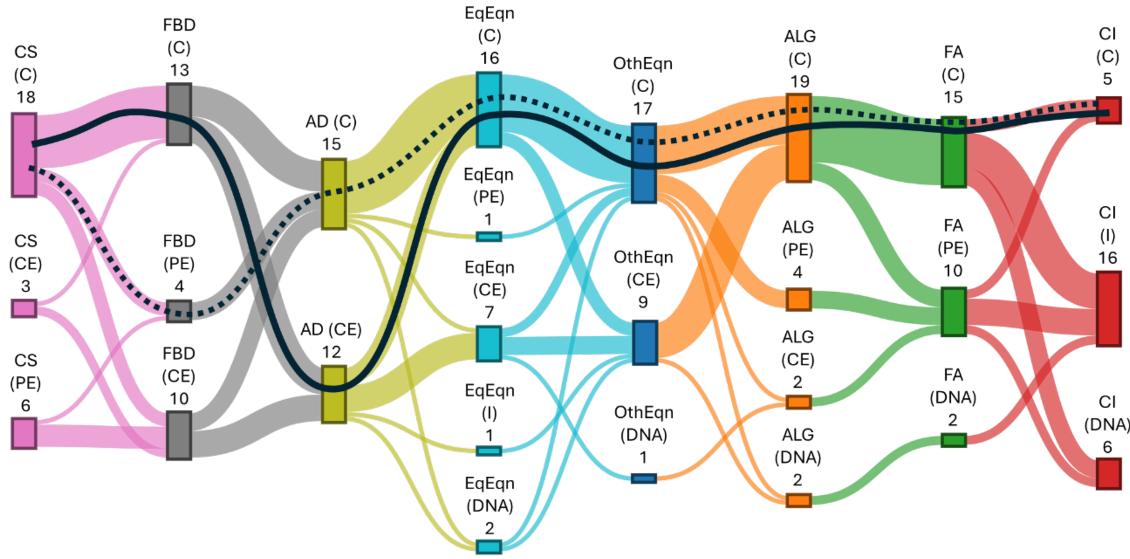


Figure 3: Two paths to answering the concept problem correctly

The two lines here indicate paths of two different students through solving the problem and answering the concept inventory question correctly. Notice the divergence in drawing the FBDs but then the convergence in writing equilibrium equations.

Interestingly, the student path indicated with the dashed line shows that students can make errors in drawing their FBDs while still arriving at the correct answer for the concept inventory problem. In addition, the student path indicated with the solid line indicates that an error in drawing acceleration diagrams does not influence their understanding of the concept.

In Figure 4, two paths are shown for students who arrived at the incorrect concept problem answer. One student illustrated that they could arrive at the correct numerical answer to the long-form problem while still showing that they did not quite grasp the concept fully. Alternatively, the path from the student indicated with the dashed line shows that the student had trouble from the start and could probably use a little help with both the concept and the problem-solving process.

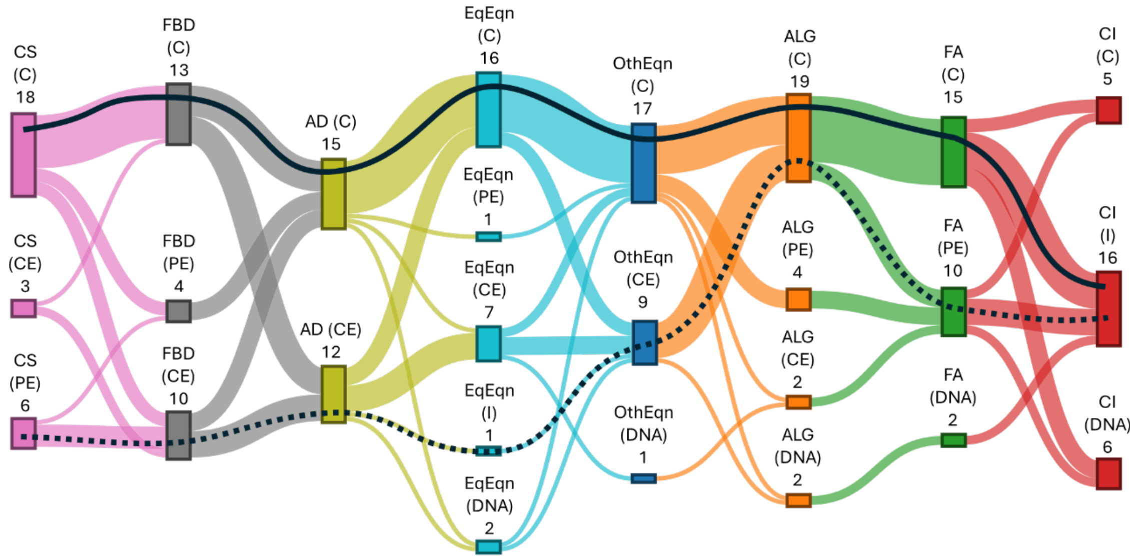


Figure 4: Two paths to answering the concept problem incorrectly

Two paths to answering the concept problem incorrectly. The student solution indicated with the solid line was able to answer each part of the rubric correctly, including solving for the final answer, however, the student could not answer the concept problem correctly. Another student, dashed line, made many conceptual and procedural errors and attempted the concept problem unsuccessfully.

Discussion

With only a small sample size from each university, we are cautious to make any hard conclusions from this data. Nevertheless, the rubric proved to be useful as a tool to systematically identify where and what type of errors show up in the problem-solving process. As expected, students that had a hard time with certain rubric categories, particularly the FBD often had a hard time answering the concept problem correctly. However, for this preliminary data set, an unexpected finding was that a similar percentage of students who answered the concept problem correctly were also making conceptual errors when drawing their free body diagrams, suggesting that conceptual understanding as assessed by concept problems may not fully capture how students apply concepts when solving a long-form problem. However, more data and analysis are necessary to make any hard conclusions, to tease apart fundamentals in the long-form problem solving process and understanding of concepts.

These preliminary results also show that the errors in developing the equations of equilibrium were all classified as conceptual errors rather than procedural. This finding is not surprising since this step requires students to accurately transfer the vectors from the FBDs into the equilibrium equations which can be a challenge for students. The consistency in error type for this category indicates that the rubric is robust enough to support similar results from three different faculty grading independently.

Finally, these preliminary results suggest that students take all sorts of different paths to finding the correct or incorrect final answer to a long-form problem and a concept problem. This variability reflects that students are still developing a strong procedural process for solving problems early in their academic careers. It may also indicate that conceptual understanding may take a while to develop and does not always happen at the same time as procedural problem-solving skills. Additional data is needed to make any strong correlations to the relationship between procedural problem-solving and conceptual understanding, and to identify specific areas in the problem-solving process where faculty can reinforce ideas to support conceptual understanding. These findings can help inform instructional practices to ensure that our students graduate with both strong problem-solving skills and a deep conceptual understanding of topics.

Conclusion

This study highlights that there are many different paths to arrive at the correct numerical answer to a long-form problem, and these pathways can reflect different levels of conceptual understanding. To better evaluate the complexity of these pathways, we created a rubric to identify different key parts of the solution process and the types of errors that are being made. The rubric enables for a method of analysis of student reasoning throughout the long–form solution process to be correlated to student responses on concept problems.

The initial application of the rubric shows promise as a systematic tool to track down errors in long-form problems. The results were consistent with the places of error that would be expected for Dynamics students including conceptual difficulty in drawing FBDs and developing equilibrium equations. This was consistent for multiple instructors at different institutions, suggesting that the rubric supports consistent evaluation of student work. The team will keep moving forward with more data analysis and further testing of the robustness of the rubric with a larger data–set.

Future Work

Ultimately, this study discusses the development of a rubric used to investigate the connection between long-form problems and concept inventory questions to understand the development of each form of knowledge and how to evaluate student understanding for each type. The *initial* analysis showed conceptual understanding can manifest both explicitly in concept problems but also implicitly in the problem-solving process. Since it has been shown that both conceptual and procedural learning can enhance a student's ability to apply knowledge to new contexts [16], [17], future analysis will be valuable in identifying points in the problem-solving process where misconceptions occur. This can begin to identify the connections between problem-solving skills and conceptual understanding, and when the two forms of knowledge develop for students. The results will be useful to inform instructional strategies and support more targeted feedback for students.

REFERENCES

[1] D. Hestenes, M. Wells, and G. Swackhamer, "Force concept inventory," *Phys. Teach.*, vol. 30, no. 3, pp. 141–158, 1992, doi: 10.1119/1.2343497.

[2] P. S. Steif, A. Dollar, and J. a. J. Dantzler, "Results from a Statics Concept Inventory and their Relationship to other Measures of Performance in Statics," *Proceedings Frontiers in Education 35th Annual Conference*, pp. T3C-5-T3C-10, 2005, doi: 10.1109/FIE.2005.1611927.

[3] P. S. Steif, "Initial data from a statics concept inventory," *Proceedings of the 2004 American Society of Engineering Education Conference & Exposition*, 2004, doi: 10.1111/j.1365-2427.2011.02613.x.

[4] J. Richardson, P. Steif, J. Morgan, and J. Dantzler, "Development of a concept inventory for strength of materials," in *33rd Annual Frontiers in Education, 2003. FIE 2003.*, 2003, pp. T3D–T3D.

[5] G. Gray, F. Costanzo, and D. Evans, "The dynamics concept inventory assessment test: A progress report and some results," *American Society for Engineering Education*, 2005, [Online]. Available: http://www.esm.psu.edu/dci/papers/ASEE-DCI-Portland.pdf

[6] J. L. Davis, A. Baisley, G. Recktenwald, B. P. Self, and P. J. Cornwell, "A Modified Concept Inventory for Dynamics Work in Progress: A Rigid Body Dynamics Concept Inventory," in *2024 ASEE Annual Conference & Exposition*, Portland, Oregon, 2024. doi: 10.18260/1-2--46457.

[7] M. Prince, M. Vigeant, and K. Nottis, "Development of a Concept Inventory in Heat Transfer," in *2009 ASEE Annual Conference & Exposition*, 2009.

[8] M. Prince, M. Vigeant, and K. Nottis, "Development of the Heat and Energy Concept Inventory: Preliminary Results on the Prevalence and Persistence of Engineering Students' Misconceptions," *Journal of Engineering Education*, vol. 101, no. 3, pp. 412–438, 2012, doi: https://doi.org/10.1002/j.2168-9830.2012.tb00056.x.

[9] G. Rowe and C. Smaill, "Development of an Electromagnetics course concept inventory," in *2008 ASEE Annual Conference & Exposition*, 2008. [Online]. Available: https://engineering.purdue.edu/SCI/workshop

[10] K. E. Wage, J. R. Buck, T. B. Welch, and C. H. G. Wright, "The Signals and Systems Concept Inventory," *2002 ASEE Annual Conference & Exposition*, 2002, doi: 10.18260/1-2--10670.

[11] G. L. Herman and M. C. Loui, "Administering a Digital Logic Concept Inventory at Multiple Institutions," in *2011 ASEE Annual Conference & Exposition*, 2011.

[12] S. Sengupta, J. A. Cunningham, S. J. Ergas, R. K. Goel, D. Ozalp, and T. Reed-Rhoads, "Development of a Concept Inventory for Introductory Environmental Engineering Courses," in *2013 ASEE Annual Conference & Exposition*, 2013.

[13] S. Sorby and M. Sadowski, "Engineering Graphics Concept Inventory," in *2018 ASEE Annual Conference & Exposition*, 2018. [Online]. Available: https://tinyurl.com/EGCI-2018

[14] K. M. Bursic, "Work in progress: An Engineering Economy Concept Inventory," *2017 ASEE Annual Conference & Exposition*, 2017.

[15] S. Li, K. Raza, A. Ghasemloonia, and C. Chua, "Pattern recognition as a learning strategy in the study of engineering dynamics," *International Journal of Mechanical Engineering Education*, vol. 52, no. 4, pp. 550–569, Oct. 2024, doi: 10.1177/03064190231203692.

[16] R. Taraban, E. E. Anderson, A. DeFinis, A. G. Brown, A. Weigold, and M. P. Sharma, "First Steps in Understanding Engineering Students' Growth of Conceptual and Procedural Knowledge in an Interactive Learning Context," *Journal of Engineering Education*, vol. 96, no. 1, pp. 57–68, Jan. 2007, doi: 10.1002/j.2168-9830.2007.tb00915.x.

[17] B. Rittle-Johnson, R. S. Siegler, and M. W. Alibali, "Developing conceptual understanding and procedural skill in mathematics: An iterative process.," *J. Educ. Psychol.*, vol. 93, no. 2, pp. 346–362, Jun. 2001, doi: 10.1037/0022-0663.93.2.346.

[18] D. W. Braithwaite and L. Sprague, "Conceptual Knowledge, Procedural Knowledge, and Metacognition in Routine and Nonroutine Problem Solving," *Cogn. Sci.*, vol. 45, no. 10, Oct. 2021, doi: 10.1111/cogs.13048.

[19] B. Rittle-Johnson and M. W. Alibali, "Conceptual and procedural knowledge of mathematics: Does one lead to the other?," *J. Educ. Psychol.*, vol. 91, no. 1, pp. 175–189, Mar. 1999, doi: 10.1037/0022-0663.91.1.175.

[20] B. Rittle-Johnson, M. Schneider, and J. R. Star, "Not a One-Way Street: Bidirectional Relations Between Procedural and Conceptual Knowledge of Mathematics," *Educ. Psychol. Rev.*, vol. 27, no. 4, pp. 587–597, Dec. 2015, doi: 10.1007/s10648-015-9302-x.

[21] G. F. L. Asilo-Ebisa and L. S. L. Lomibao, "Procedural Knowledge-to-Conceptual Knowledge or Conceptual Knowledge-to-Procedural Knowledge: Which Order of Knowledge is Better for Conceptual Understanding and Procedural Fluency," *Journal of Innovations in Teaching and Learning*, vol. 4, no. 1, 2024.

[22] G. L. Gray, D. Evans, P. Cornwell, F. Costanzo, and B. Self, "Toward a Nationwide Dynamics Concept Inventory Assessment Test," *American Society for Engineering Education*, 2003.