Exploring the Potential of Engineering Design in Promoting Mathematical Graph Sense (WIP)

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Introduction

Mathematical graph interpretation is fundamental not only for guiding actions that impact everyday situations but also for fostering the development of knowledge in STEM disciplines. Graphs serve as powerful tools for communicating information relevant to people's surroundings, and their interpretation is essential for making decisions that affect daily life [1], [2]. Moreover, graph interpretation is a critical component in developing knowledge within STEM fields [3]. However, graph interpretation is not a skill that develops automatically through mere exposure to graphs. Rather, it requires a process in which individuals participate in learning situations where making sense of graphs is necessary [4]. In addition, *graph sense* does not arise spontaneously; it "develops gradually as a result of one's creating graphs and using already designed graphs in a variety of problem contexts that require making sense of data" [5, p. 145]. Therefore, it is essential that students have learning opportunities to develop the abilities necessary for effective graph interpretation.

These abilities should be addressed early in students' educational trajectories. The Common Core State Standards (CCSS) emphasize that middle school students should participate in instructional processes that allow them to construct, interpret, and analyze mathematical graphs [6]. Similarly, the Next Generation Science Standards (NGSS) highlight that students at this educational level must be able to construct and interpret graphical representations to make sense of relationships that emerge in science and engineering [7]. Furthermore, the K–12 Framework for Science Education identifies graph interpretation as a relevant component of scientific and engineering practices. For instance, this framework notes that "engineers need to be able to express their ideas, orally and in writing, with the use of tables, graphs, drawings, or models and by engaging in extended discussions with peers" [3, p. 53]. Consequently, it is crucial that education provides middle school students with opportunities to develop graph interpretation abilities that enable them to advance in science and engineering practices

Student instruction at the middle school level should also extend beyond the construction and interpretation of static graphs. Technological advances have enabled engineers and other specialists to frequently use dynamic graphs in their professional practices, which allow them to make modifications and explore how changes in different parameters affect their designs. In this context, Rodger et al. [8] emphasize that the use of interactive graphs—tools that enable students to visualize and actively engage with data—can create more dynamic and meaningful learning experiences, thereby expanding students' opportunities to learn. However, Lai et al. [9] note that middle school instruction often leads many students to interpret graphs merely as pictures, due to the lack of interaction with dynamic graphs. In addition, these authors highlight that instruction at this level is frequently limited to constructing and describing linear functions and rarely includes representations comparable to those used in authentic STEM contexts. Therefore, advancing engineering instruction for middle school students requires promoting experiences that emphasize the interpretation of interactive graphs similar to those employed by engineers in professional practice.

Therefore, this work-in-progress (WIP) paper presents advances from a study aimed at expanding knowledge about the development of mathematical graph interpretation abilities of middle school students when they participate in learning experiences that foster the use of interactive graphs.

Literature Review

Glazer [4] emphasizes that graph interpretation is fundamental for understanding the world today and for developing competencies in science and engineering. However, this author notes that developing this understanding is a complex task that requires explicit guidance to support the development of graphing abilities. One common difficulty she observes is that students often "interpret the graph as a simple picture," rather than recognizing the relationships it represents. This tendency limits their ability to analyze patterns and relationships that are embedded in these forms of representation, thereby highlighting the importance of instructional approaches that foster students' graph comprehension.

Friel et al. [5] define *graph comprehension* as the "abilities of graph readers to derive meaning from graphs created by others or by themselves" (p. 145). Regarding instructional implications, these authors propose the construct of *graph sense*, along with its associated abilities, which allow for the characterization of the nature of students' graph comprehension. These abilities are: 1. To recognize the components of graphs, the interrelationships among these components, and the effect of these components. 2. To speak the language of specific graphs when reasoning about information. 3. To understand the relationships among a table, a graph, and the data being analyzed. 4. To respond to different levels of questions associated with graph comprehension. 5. To recognize when one graph is more useful than another [5, p. 146]. These authors suggest that such abilities can serve as indicators of graph sense among students of middle school age when they participate in learning experiences.

Rodgers [8] emphasizes the importance of incorporating dynamic and interactive learning experiences in engineering contexts, such as those provided by interactive graphs. Rodgers notes that the integration of these experiences "allow students to visualize and interact with data in a way that is more engaging and meaningful than traditional static graphs" (p. 80). In addition, Kaput et al. [10] have highlighted that the dynamic components of digital tools expand opportunities for creating richer and more diverse learning environments, in which students themselves construct dynamic objects based on conjectures and argumentation.

Nelson and Hjalmarson [11] highlight that engineering education requires students not only to develop abilities to interpret graphs but also to make comparisons and draw interpretations of phenomena across different representations. Building on Cramer's [12] work, these authors recognize that mathematical representations go beyond written expressions alone. Cramer emphasizes that "elementary mathematical ideas can be represented in five different modes: manipulatives, pictures, real-life contexts, verbal symbols, and written symbols" [12, p. 450]. Consequently, preparing students in the interpretation of graphs relevant to engineering practices should be expanded beyond written expressions to incorporate these forms as relevant mathematical representations. Thus, Nelson and Hjalmarson [10] emphasize that, in the development of engineering practices, it is necessary to provide learning experiences in which students work with multiple representations of the same phenomenon.

Therefore, providing middle school students with opportunities to develop graph sense abilities relevant to engineering practices requires engaging them in learning experiences that mirror the challenges faced by engineers. Thus, building on this idea, the present study is grounded in the Integrated STEM through Design approach, which is outlined in the following section.

Theoretical Framework

The STEM education approach emphasizes that students' education is enriched by moving from a vision in which disciplines are taught in isolation toward a more integrated perspective [13], [14]. Although disciplinary knowledge is essential, it is equally important to offer students opportunities to apply this knowledge and their disciplinary abilities in meaningful contexts [15]. Integrated STEM education is particularly relevant in this regard, as it involves "an effort to combine some or all of the four disciplines of science, technology, engineering, and mathematics into one class, unit, or lesson that is based on connections between the subjects and real-world problems" [16, p. 38]. However, one of the challenges that emerges from this perspective is the design of integrated learning experiences.

One way to develop integrated learning experiences is through design [17]. Engineering can serve as a natural connector among STEM disciplines, as solutions that require engineering design also demand the use of knowledge from engineering, mathematics, and science [16]. According to Atman et al. [18], the five basic themes of design are "problem scoping, project realization, alternative solutions generation, distribution of activity over time, and solution quality" [18, p. 360]. On this basis, STEM learning experiences through engineering design "consist of an open-ended, highly iterative process that can provide a meaningful context for learning scientific, mathematical, and technological concepts and thus increase students' systems thinking, modeling, testing, evaluating, modifying, and other higher-order thinking abilities" [19, p. 108]. Thus, when students participate in integrated learning experiences through design, they have the opportunity to engage in processes similar to those that engineers use to solve engineering problems.

Methods

This WIP paper reports on advances in an exploratory qualitative case study [20] involving a group of nine students between 11 and 13 years old. The participants had not previously directly addressed the topic of interactive graph interpretations; however, in the context of this study, they had been engaged in a learning experience designed to foster reflection on why airplanes fly. Therefore, with the aim of exploring students' graph sense processes, an Integrated STEM through the Design experience was designed and implemented within the same context. The research question guiding this study is: *How does a design-based integrated STEM learning experience contribute to the development of graph sense abilities in middle school-aged students?*

The *Mission Flight Designers* Integrated STEM through the Design experience was developed using the free-access simulation tool FoilSim [21] and was implemented in a homeschool setting by the first author of this WIP and one parent of the participating students. This learning experience was structured to first familiarize students with the context of airplane flight. Subsequently, the implementation team guided students through a brief exploration of the simulator tools, including the schematic wing view, the controls for adjusting parameters such as airfoil shape, angle of attack, camber, and thickness, and the graphs depicting changes in air pressure and airflow velocity. Following this, all students worked collaboratively in teams of two or three to design what they considered the most effective wing. Finally, they presented their models to the group, where the group participated by asking questions. The implementation took place in a single session of approximately 90 minutes, with each team having access to a computer equipped with the simulator.

Data Collection and Data Analysis

Data were collected through audio and video recordings, as well as images documenting participants' problem-solving processes during the learning experience. The recordings were transcribed and systematically organized.

The analysis was conducted using an *a priori coding process* [22] focused on identifying students' graph interpretation processes. Specifically, the codes were aligned with the graph sense framework and its associated abilities proposed by Friel et al. [5]. In addition, following the recommendations of Nelson and Hjalmarson [11], the analysis included students' interpretation of phenomena across different representations. Specifically, we examined their interpretations across a schematic drawing of the wing and a plot of pressure variations.

The study is currently undergoing data analysis, with future work aimed at completing this phase. In this WIP paper, we present the preliminary findings from one of the participating teams. In this team (Team 1), Participant 1 and Participant 2 collaborated. Participant 1 was a 12-year-old girl, while Participant 2 was a 12-year-old boy. Both participants received homeschooling instruction aligned with middle school-level content and had no familial relation to each other. They were part of the same homeschooling community–"a collective of like-minded families from diverse backgrounds who provide socialization opportunities to their homeschooled students through parent-led schooling that takes place in the home" [23, p. 2]–and had previously participated in earlier phases of the study.

Preliminary Results and Discussion

The preliminary results indicate that the Mission Flight Designers learning experience supported participants in developing relevant abilities related to graph sense. In this section, we present preliminary findings from Team 1.

One of the findings was that communication and interaction among participants played a critical role in advancing their graph sense abilities. Although the implementation team had previously introduced the main features of the software to the group, some team members initially encountered difficulties in interpreting the graphs. Peer interaction proved essential in overcoming these challenges. The following excerpt illustrates a key moment of interaction within Team 1:

> Participant 1: I don't know how to read the graph. What does it mean that there is more pressure?
>
> Participant 2: Look, in this graph it says Upper Surface, which is the blue line showing the pressure on the top of the wing, and Lower Surface, the orange line, for the bottom. So, if we draw a straight line where they are equal, that would

be equal pressure. They would be the same, but what we want is less pressure on top so the wing goes up and more pressure underneath. So here [sliding his finger along the y-axis and pointing to the Upper Surface line on the graph] it's 14, and here it's 15 [pointing to the lower surface line]. Which one is greater, 14 or 15?

Participant 1: 15.

Participant 2: So this one is higher [pointing to the lower surface line], then the pressure here is higher than 14, which is the blue one, and that's what we want.

This interaction first allows us to identify that social interaction between the students sharing a common dynamic display provided by the software was fundamental for students to advance their understanding of graph interpretation. It also highlights how students' explanations were strongly connected to the context of the airplane wing and the possibility to modify the pressure distribution in the Upper and Lower Surfaces. This dynamic process experimenting with different pressure variations and interpreting patterns of possible combination of pressure differences allowed this team to move forward in their wing design process while interpreting the pressure graph.

Following this interaction, the team began constructing their wing design. The students experimented with multiple simulator parameters while simultaneously observing changes in the pressure graphs. Specifically, Team 1 explored variables related to wing design, including *airfoil shape, angle of attack, camber, and thickness.* Their design process was iterative, with successive modifications informed by ongoing discussions about pressure distributions shown in the graphs. For instance, the team tested different airfoil shapes, including airfoil, ellipse, and flat plate configurations, and adjusted the angle of attack across both positive and negative values. By the end of the activity, this team presented a final design with the following configuration: shape type airfoil, camber of 13.5%, thickness-to-chord of 12.5%, and an angle of attack of 5 degrees; where camber percentage is a measure of the wing's curvature (maximum distance between mean line and chord divided by the chord length), thickness-to-chord percentage is a measure of the ratio between the thickness of the wing and the chord, and the angle of attack is the measure at which the chord meets the relative wind.

During their presentation, Participant 1 continuously used the cursor to make references that moved between the views of the schematic drawing of the wing and the pressure variation plot. In addition, she used the cursor dynamically by making adjustments to the angle of attack as she explained the behavior of the wing and the graphs. At the beginning of her explanation, she pointed with the cursor to the upper surface of the wing and then to the lower section in the schematic drawing of the wing view of the simulator (Figure 1). The following excerpt from her presentation illustrates these explanations.

Participant 1: [Pointing to the upper part of the wing in the schematic drawing of the wing, she mentioned] On the top part, from what I understand, it has to have less pressure, so this is the top part and it really has much less pressure [pointing to the blue line on the plot]. And the orange part really has a lot of pressure [pointing to the orange line on the plot], which is where we want to have a lot of pressure so that it goes up [pointing to the wing in the schematic drawing of the wing]. That's why we chose it.

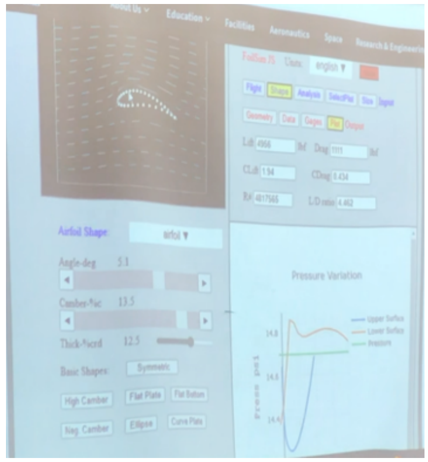

Fig. 1. Wing design of Team 1

Analyzing this excerpt through the lens of the graph sense framework [5] allows us to identify that the Mission Flight Designers learning experience supported students in developing relevant graph sense abilities. For example, students recognized the components of graphs and the interrelationships among these components [5], as they identified that the blue and orange lines were related to distinct pressure variables of the wing. In addition, they identified that the y-axis represented pressure on the wing and reasoned that the upper portion of the graph corresponded to higher pressure than the lower portion of the axis. Although students did not use technical language, they were able to communicate their ideas and reasoning based on the graphs and in relation to the context of wing design. Furthermore, students demonstrated that they "understand the relationships" among different types of representations, as their explanations included transitions between representations—specifically, between the pressure graph and the schematic drawing of the wing. This finding is also significant in light of the arguments presented by Nelson and Hjalmarson [11], who emphasize that in engineering it is necessary to move across and make connections among interpretations of different representational modes.

Subsequently, Participant 1 explained that, based on a conversation with another team, they wanted the airplane to descend. Accordingly, they conducted tests and adjusted the wing parameters to explore this possibility. During their presentation to the other teams, Team 1 used the angle of attack slider, changing it from an initial value of 5.1 to –20, to demonstrate that this configuration caused both pressure curves to decrease. Thus, consistent with the work of Kaput et al. [10], these findings highlight the value of dynamic software environments in supporting the construction of dynamic objects through hypothesis generation, argumentation, and social interaction. In this case, the use of a simulator with dynamic graphs allowed students to manipulate design variables while simultaneously developing interpretations of graphical representations, thereby fostering a deeper understanding of the graphs and of the aerodynamics phenomenon related to wing design.

Preliminary Conclusions and Future Direction

The preliminary findings indicate that the Mission Flight Designers learning experience broadens opportunities for students in middle school age to advance their graph sense abilities. For example, students demonstrated processes of identifying and recognizing graph components and their interrelationships. They constructed arguments and responded to relevant questions about their design processes, grounded in their interpretation of graphs within a meaningful design context. Furthermore, the simulator with dynamic graphs enabled students to create dynamic designs and engage in interpretive processes across multiple forms of representation, including schematic drawings of the wing and graphs showing pressure variations. As a result, the *Mission Flight Designers* experience brings students closer to graph analysis and design processes similar to those used in engineering, as well as to a better understanding of fundamental ideas of aerodynamics in the context of wing design.

This research is significant in the field of mathematics education in engineering because it supports the development of students in middle school age by broadening their graph interpretation abilities in ways that more closely align with authentic engineering practices and move beyond static graphs, which are often perceived merely as objects. Future directions of this research will focus on analyzing data from all student teams to gain a deeper understanding of the processes through which students develop their graph sense abilities.

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