

GIFTS: An Activity to Enhance Engineering Modeling with an Entrepreneurial Mindset

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GIFTS: An Activity to Enhance Engineering Modeling with an Entrepreneurial Mindset

This GIFTS paper details the design and implementation of a single-class activity to enhance students' engineering modeling skills. This paper activity has students understand model assumptions by positioning objects inside/outside a system boundary. The objectives of this activity are partly inspired by the literature on entrepreneurial mindset: awareness of one's limitations, and a willingness to challenge accepted solutions.

Motivation

Mathematical analysis is core to engineering practice. This is evident from studies of practicing engineers [1] and from the ABET requirements for engineering programs [2]. However, the systems that engineers design are sufficiently complicated that no mathematical analysis is ever exact—the math that engineers use comes from imperfect *models*. Thus, practicing engineers develop skepticism towards the mathematical analyses used in design [1], and there are formal processes for verifying and validating a mathematical model [3, pp. 31--33].

Students do not naturally develop skepticism towards mathematics. For example, in a study of civil engineering students, Gainsburg [4] found no awareness of model assumptions, "Most students in this study expressed no recognition of the subjectivity and fallibility of mathematical methods for solving engineering problems."

Furthermore, not all faculty teach students to build mathematical models. In a separate study, Gainsburg [5] found that civil engineering faculty did not teach modeling explicitly. She wrote,

"As a result, the students in this study possessed weak knowledge about mathematical models, probably even weaker than the veterans imagined."

The faculty in Gainsburg's study did not introduce modeling because they believed students did not have sufficient domain knowledge to engage in legitimate modeling. Thus, introductory modeling activities must be designed to minimize domain knowledge requirements.

At Olin College we teach a first-year course on computational modeling. This course has students engage in all phases of a modeling *process*: defining, building, assessing, and revising a model [3, p. 19]. A key step of this process is defining the system to be modeled, first by drawing a *system boundary* [3, p. 26]. Since students often struggle to define a system [6], we have developed the *tenchi* diagramming technique to have students visualize both the system and the model [7]. Figure 1 shows an example *tenchi* diagram in the context of a baseball, while Appendix A is a primer on *tenchi* diagrams.

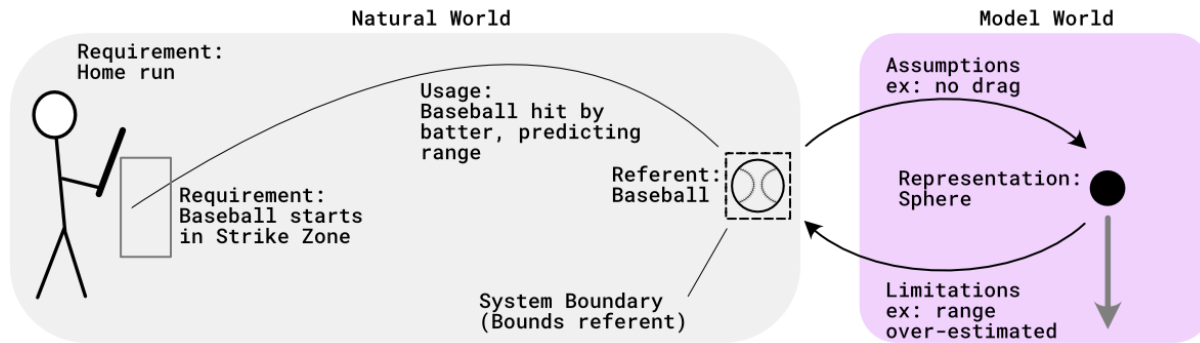


Figure 1. A tenchi diagram depicts the system to be modeled using a system boundary.

Often, engineering analysis ideas are introduced as "laws." This language may suggest to students that math is unquestionable—a mindset inappropriate for modeling. However, ideas from entrepreneurship are well-aligned with the modeling process, particularly the 3Cs framework [8]. In particular, we draw inspiration from "Curiosity: Willingness to challenge accepted solutions," and "Connections: Aware of one's own limitations in knowledge and skills."

Objectives

The activity occurs in the first week of a first-year course on computational modeling. The activity introduces compartmental models [9] and the tenchi diagram technique [7]. While compartmental models are a specialized approach, they have broad applicability to epidemiology [9], biological systems [10], and chemical engineering [11]. The activity topic—the interaction of rabbits and foxes—is not engineering per se, but for first years it is key to minimize requirements of domain knowledge [5]. As a result of the activity, students will:

- Develop their understanding of compartmental models and tenchi diagrams
 - Specifically, students will manipulate a representation of the natural world to align it with a given model.
- [Connections] Become more aware of one's own limitations in knowledge and skills
 - Specifically, students will become more aware of the limitations of the quantitative tools they use by studying the limitations of a given model.
- [Curiosity] Build a greater willingness to challenge accepted solutions
 - Specifically, students will learn to challenge accepted models, particularly their assumptions, by modifying a given model.

Practical Implementation Details

The activity is designed to last ~45 minutes and proceeds in three phases (see Appendix B):

0. Presentation: A short introduction to compartmental models and tenchi diagrams
1. Normative system: Students use materials to define a system that matches a given model
2. Self-defined system: Students define their own system and modify the model to match

The activity presents students with a visual representation (Fig. 2a) of a classic compartmental model for the interaction of predator and prey species [12]. Rather than re-derive

or simulate with the model, students are tasked with understanding the system that the model represents. To show their interpretation of the system, students use paper materials to specify what elements of the world are inside the system by placing them inside a provided system boundary. Figure 2 (b, c) illustrates two configurations of the materials.

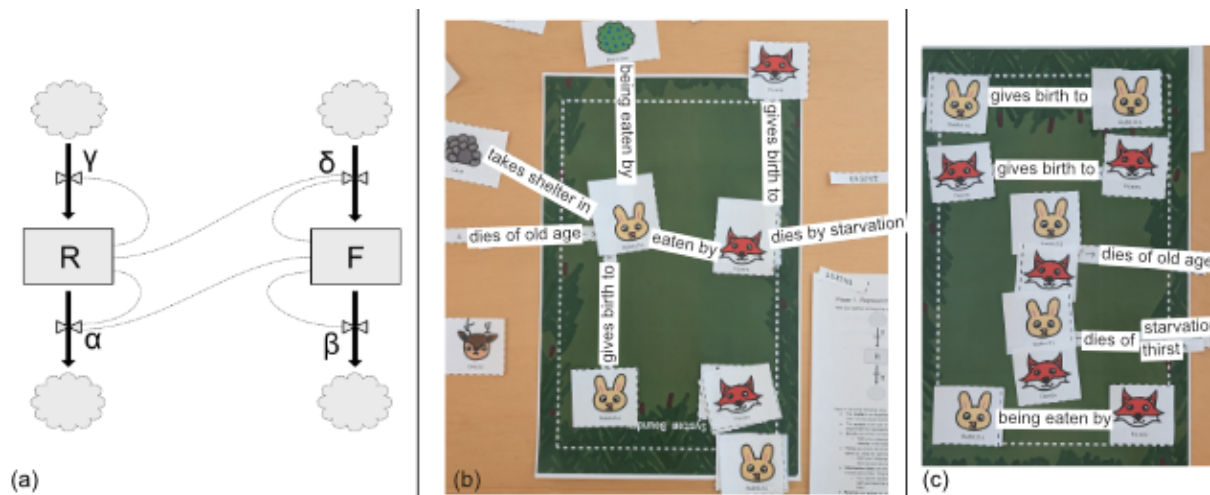


Figure 2. (a) Provided compartmental model of rabbit (R) and fox (F) populations, (b) configuration of materials with some objects (correctly) outside the system, (c) near match with correct object inclusion, but some incorrect interactions (e.g., rabbits die of old age). This predator-prey model is used to introduce modeling ideas used in chemical engineering [11].

Appendix A is a primer on tenchi diagrams for instructors, Appendix B has the student-facing activity instructions, and printable materials and slides are in a public repository.¹

Assessment Methods

As an introductory activity, assessment is focused on formative feedback rather than summative assessment [13]. There is a tension in coaching students through this activity: The content learning outcome (compartmental models, tenchi diagrams) requires correctness, while the mindset outcomes (Connections and Curiosity) require student agency. Put differently, steering students to a pre-selected configuration of materials is at odds with developing their mindset to challenge accepted solutions. Thus, instructors running this activity should adopt a constructivist approach [14]: building on students' prior knowledge and work to coach them towards a fuller realization of the learning outcomes.

This activity is used in a studio course [15], so most instructional days involve students working independently and in groups to complete exercises and self-directed work. In this setting, an instructor works as a coach, circulating around the classroom to inspect student work, answer questions, and intervene to help students notice more details [16]. The activity is designed to be highly visual: Circulating instructors can quickly and unobtrusively inspect

¹ URL for anonymous peer review: https://osf.io/7dngr/overview?view_only=6bd4cd9786da4f0f839db73feb68d8db

student work simply by looking over their shoulder. Key points to look for include checking that all objects tracked in the model (rabbits and foxes) are inside the system boundary, objects not tracked in the model (e.g., food sources, shelter, sunlight) are outside the system boundary, and the interactions between objects can plausibly match the model.

While checking inclusion and exclusion of objects is easy, checking that interactions are "plausible" is more difficult. As a concrete example, notice that Figure 2b includes the interaction rabbit "takes shelter in" cave. We did not anticipate this interpretation of the model. However, one could argue that reducing the rate of rabbit deaths (reducing the alpha parameter) could represent the abundance of shelter in the environment. Instructors can support the learning outcomes of the activity by not immediately "correcting" student work, but rather by asking questions, both to understand students' current understanding, and to coach them towards fuller understanding—a Socratic dialogue [17].

Tips for Running the Activity

As of writing, we have run this activity once for a class of ~100 students in 4 sections. We first piloted the activity as a teaching team (faculty and course assistants), which helped us identify possible material configurations and plan our coaching approach.

Our approach to feedback is constructivist [14], [18]: Rather than prioritize catching student work that is "wrong", we assume that non-normative student work is motivated by a partial understanding. Constructivism seeks to build on students' partial understanding.

As a concrete example of constructivist coaching: Aligned with prior work on defining thermodynamic systems [6], we have seen students struggle to define the system. In this activity, this manifests as incorrect inclusion or exclusion of objects. Rather than stating "that's wrong", we use questions such as "Why are berries inside your system?" This is often sufficient to help students notice the contradiction between the given model and their system definition.

Summary, Later Assignments, and Future Work

We designed and implemented a paper activity introducing compartmental models that do not require domain knowledge. The activity introduces the *tenchi* diagramming technique and focuses on defining a system boundary. Later assignments build on this by assessing whether students correctly sort items inside/outside a system boundary for their own models. Given the documented difficulties [6], students need repeated practice with this skill. Future work could generalize the activity to other topics, such as other dynamical systems or statistical modeling.

Acknowledgements

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Appendix A Tenchi Diagrams - A Primer

Tenchi diagramming is a visual technique for making sense of models. The basic idea is to draw the thing we’re trying to model (*referent*) alongside our model (*representation*), and note the relationships between the two. Figure 1 shows an example.²

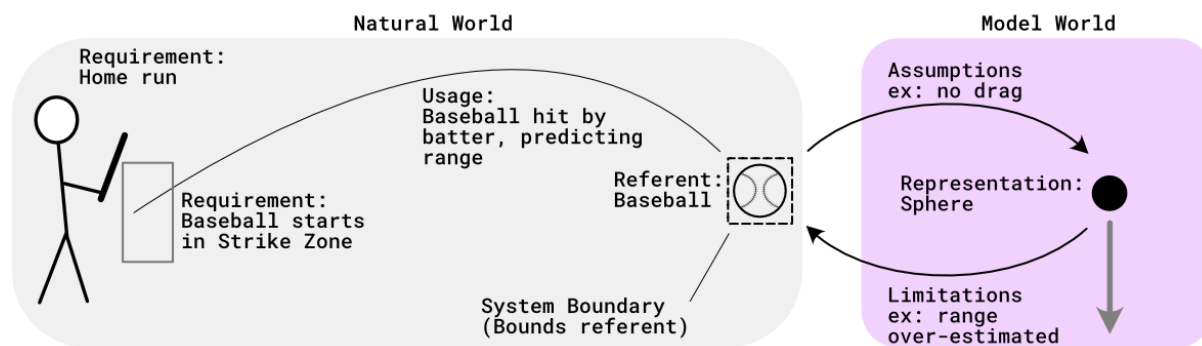


Figure 3. Example tenchi diagram depicting a baseball hit by a batter. The referent (in the natural world) is connected to the representation by assumptions and limitations.

Notice that a tenchi diagram is split in two regions: the *natural world* and the *model world*. The natural world is the world we ordinarily experience, with plants, animals, people, games, and goals. The model world is where we build representations of those things. For example, Figure 3 depicts a model for a baseball. A real baseball has many details: the texture of the leather, the length and pattern of the stitching, the subtle asymmetry from deformation when we hold it. In Figure 3, we’ve chosen to ignore all those details by representing the baseball as a rigid, perfect sphere. Any time we ignore details, or make any kind of simplification, we are making an *assumption*.

Assumptions introduce *limitations*—ways in which the model fails to represent the natural world. Assuming our baseball is a rigid, perfect sphere means we can’t describe how it feels to hold the ball, and we can’t accurately predict how the baseball will react to being hit by a bat.³ Assumptions are necessary to build models, and limitations are an unavoidable consequence of assumptions. We depict assumptions and limitations as labelled arrows between the natural and model worlds. It can help to start the assumption arrow where it happens, and end it where it affects the model.

Whenever we build a model, we always do so in the context of some *usage*. The usage helps us determine if the limitations are acceptable. For instance, if we want to predict how far the ball will go when hit with a bat, the rigid sphere will limit our accuracy. However, if we already know how fast the ball is going initially (say, by taking a measurement), we might be OK

² The word “tenchi” (天地) means “heaven and earth” or “above and below” in Japanese.

³ A truly rigid sphere would preserve energy in a collision, while a real baseball would deform, which would lose energy (therefore, some speed) in a collision.

using a rigid sphere model. If we're trying to get a rough estimate of distance, we might be OK ignoring drag. However, for a precise estimate, we'd want to account for drag too.

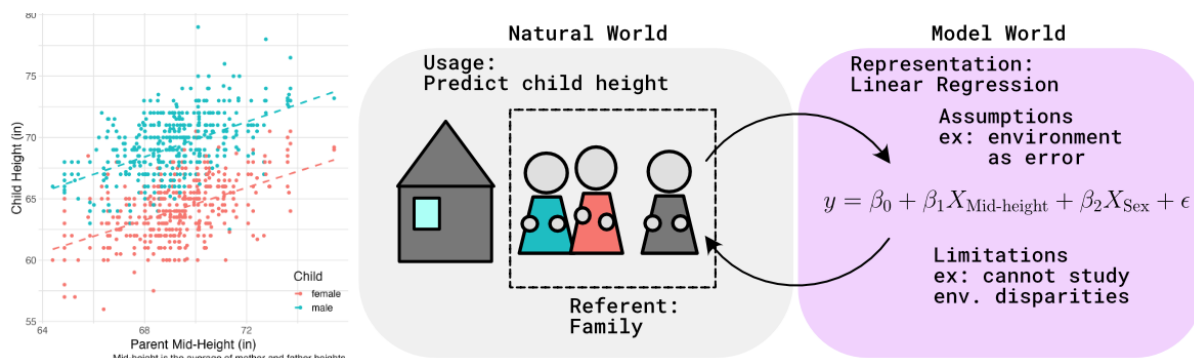
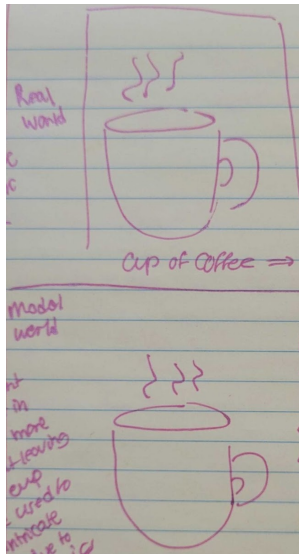


Figure 4. Example tenchi diagram in a linear regression context. Environmental factors (depicted as a house outside the system boundary) are one set of confounds, though careful consideration of the natural world could reveal more confounds (e.g., genetic factors).

One of the most important elements of a tenchi diagram is the *system boundary*. As the name implies, this boundary defines the system—things inside the system boundary are represented (or "tracked") in the model, while things outside the system boundary are not. We call the thing(s) inside the system boundary the *referent*, which is a word meaning "thing being represented." The word "referent" is paired with "representation", the "portrayal of the thing."

When we draw the natural world, we must take care to draw as much as we can, including things inside *and* outside the system boundary. Accounting for more in the natural world helps us more fully understand our assumptions and limitations. Figure 4 depicts a statistical model for the height of a child, based on the heights of the parents. Note that environmental factors (drawn as a house) are left outside the system boundary, and are hence not represented in the model. Because environmental factors actually *do* impact a child's height (say, through nutritious food to help with growth), and since the model doesn't represent those factors directly (through an "environmental" X factor), they will effectively be absorbed by an "error term" (epsilon). This isn't a "clean" representation of environmental factors, as all the other neglected factors (e.g., genetic factors) are also absorbed by the error term. Statisticians often make assumptions about the structure of an error term, such as assuming it is random with a particular distribution.



red = liquid in
our system
boundary



Our model is just
the liquid in the cup.
This kind of looks silly but
I guess it's accurate
to what the referent is

Figure 5. A progression of diagrams. *Left:* An initial tenchi diagram, with nearly identical referent and representation. Note that the system boundary (solid line enclosing cup) encloses both liquid and solid. *Right:* Revised diagram, depicting a refined understanding of the model, which only includes the liquid.

To start, our understanding of a model may be vague or incomplete. Often, we have to iterate on a tenchi diagram to pin down exactly what the model is. Figure 5 shows an example with a thermal system model for the liquid in a coffee cup. The initial diagram (left) depicts the referent and representation in almost exactly the same way: It's hard to see what assumptions are being made. In actuality, since the model is considering the liquid only, the referent is only the liquid inside the cup. The updated diagram (right) shows the representation as a mass of liquid floating in space. This may look silly, but it's a better picture of the assumption we've made to build the model.

When making a tenchi diagram, there are several required elements, listed in Table 1 (next page). We've talked about a few: the natural and model worlds, the referent and representation, assumptions and limitations. We also talked about *usage*—this refers to the broader context of how the referent is being used. This is in contrast with *engineering requirements*, which we often talk about in Design classes. Requirements are not the same as usage; for example, our usage might be predicting the range of a baseball, but a requirement might be to hit a home run.

Table 1. Required elements of a tenchi diagram, modified from [7].

Element	Description	Rules
Natural World	The home of physical objects, a subset of which are modeled.	The natural world and model world are fully separated by a boundary (distinct from the system boundary).
Model World	The home for representations.	
Referent	A subset of the natural world that is represented by the model. The referent is everything inside the system boundary.	Must be in the natural world, fully contained in the system boundary.
System boundary	A boundary whose interior defines the referent. Things in the natural world that lie outside the system boundary are not tracked in the model.	Fully contained in the natural world.
Usage	A broader context for the referent in the natural world. The system boundary separates the referent from the broader context of its usage.	Fully contained in the natural world.
Engineering Requirements	Rules for judging success of the engineering task, in the context of the usage.	Fully contained in the natural world. Distinct from "success" of the model.
Representation	A portrayal of the referent, defined by assumptions and possessing limitations.	Fully contained in the model world. Can be represented using equations, diagrams, and/or text.
Assumptions	Differences from the referent that define the representation. E.g., idealization of the referent, neglect of natural phenomena outside the system boundary.	Represented as a directed connection from the referent to the model. All assumptions lead to some limitation(s).
Limitations	Aspects of the referent that cannot be represented by the model.	Represented as a directed connection from the model to the referent. All limitations arise from some assumption(s).

Appendix B Paper Activity Instructions

Learning Outcomes

As a result of this activity, students will:

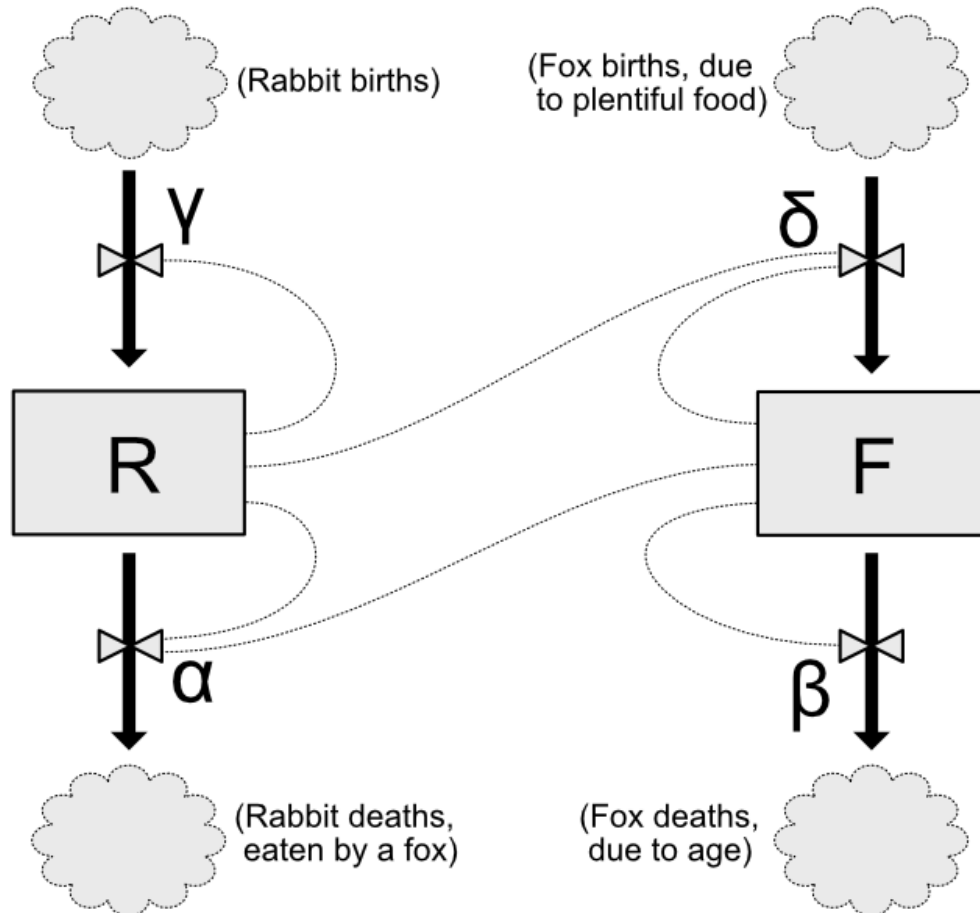
- Develop their understanding of compartmental models and tenchi diagrams
 - Specifically, students will manipulate a representation of the natural world to align it with a given model.
- [Connections] Become more aware of one's own limitations in knowledge and skills
 - Specifically, students will become more aware of the limitations of the quantitative tools they use by studying the limitations of a given model.
- [Curiosity] Build a greater willingness to challenge accepted solutions
 - Specifically, students will learn to challenge accepted models, particularly their assumptions, by modifying a given model.

Preparing the materials

- Determine the number of groups (two students each)
- For each group assemble a packet of materials:
 - Print one instruction sheet, 8.5" x 11" paper
 - The instruction sheet comprises the last two pages of this document.
 - Print and separate one of each cutout sheet [Sheet 1, Sheet 2], 8.5" x 11" paper
 - This contains elements that students will use to build a representation of the system in the natural world
 - Print one Background Sheet, 11" x 17" paper
 - This provides a system boundary, which students will use to specify what is in their system (and by exclusion, what is not)
- Distribute the materials among groups (2x packets per table, assuming 4 students per table)

Before the activity (~15 minutes)

- [~15 minutes] Present the Slides
 - Land on the slide showing the following image



Activity Instructions (~30 minutes)

This activity proceeds in two phases. Phase 1 has students create (a representation of) the system in the natural world that best matches the provided model. Phase 2 has students choose their own system (by changing what's in the system boundary), and modify the model to represent their new system. Phase 1 is more constrained (though there are still multiple reasonable answers), while Phase 2 is much less constrained. Figure 6 shows some example arrangements of the materials.

Reference Images

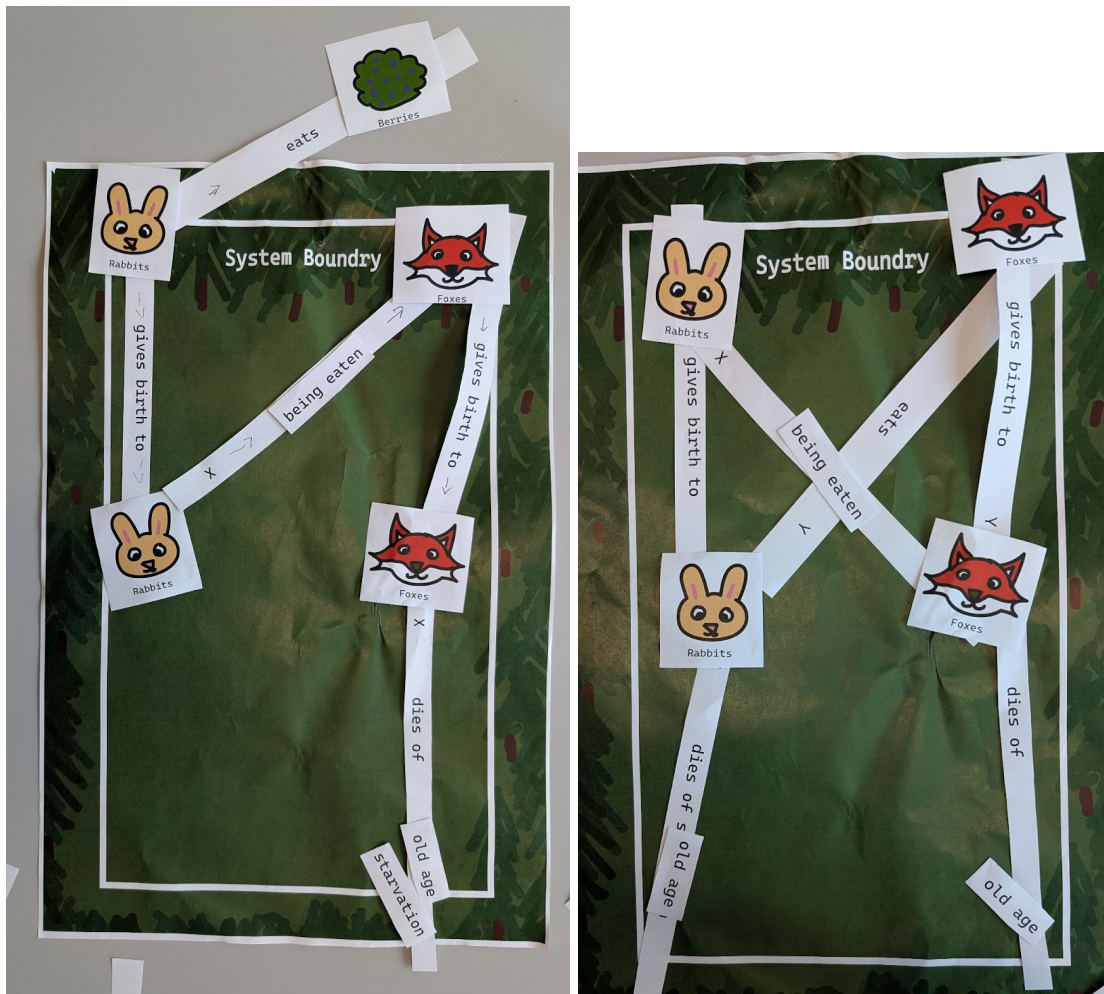


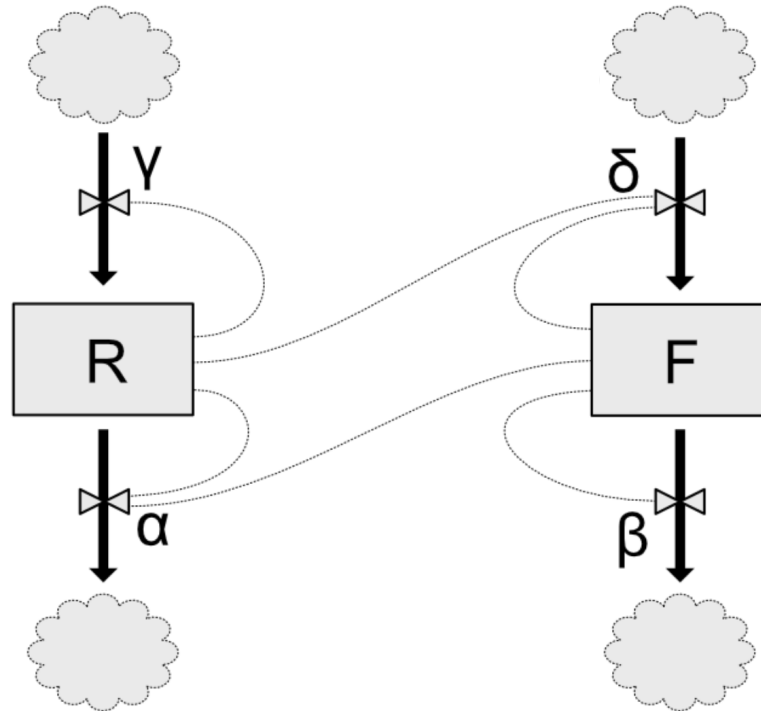
Figure 6. (Left) One possible correct answer for Phase 1. Since berries are not tracked in the model, it is appropriate to place them outside the system boundary. (Right) An incorrect answer for Phase 1 (rabbits do not have a linear death term), but a possible answer for Phase 2.

Student-facing instructions

The next page (double-sided) is the student-facing instruction sheet, for inclusion in the group packet.

Phase 1 - Represent this model

With your partner, arrange the materials to show the **system** the following **model** represents:



Keep in mind the following rules:

- The **model** is an imperfect representation of the system. We're providing the model for now—it's the stock-and-flow diagram above.
- The **system** is the part of the real world inside our **system boundary**. You are responsible for representing the system using your materials.
- **Stocks** are shown as boxes. Here we have Rabbits (R) and Foxes (F).
 - With your materials, the only objects that belong inside the system boundary are **stocks** in the model.
- **Flows** are shown as arrows with valves. We have four flows, controlled by *parameters* alpha (α), beta (β), gamma (γ), and delta (δ)—the worksheet will discuss parameters.
 - With your materials, you can represent flows with actions. These are thin strips that connect two objects with verbs.
- **Information links** are shown as thin, dashed lines. Information links always go between a stock and a flow. They tell us which stocks affect the rate of the flow.
 - You cannot represent information links directly using your materials; instead, we'll just take the objects connected by actions to be hooked up with information links.
- **Sources** and **sinks** are shown as clouds. We only track the *rate* of stuff coming in from a source or leaving into a sink.
 - You can represent a source/sink by having an action cross outside the system boundary.

Phase 2 - Build your own model

Now re-arrange your materials to define a system you're interested in. When you're satisfied, take a picture of your materials.

Using the whiteboard, update the Rabbit and Fox model to match the system you've defined. When you're satisfied, take a picture of your materials.

Write a short description of your system, and how your model represents that system. What assumptions have you made in defining your model?