

Algebraic Derivation and Verification by Simulation of the Maximum Power Transfer Theorem for DC and AC Circuits

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Abstract

In both associate and baccalaureate engineering technology programs, the first circuit analysis course typically avoids the use of calculus since most students take the circuits course before taking a calculus course. One such example is the derivation and use of the maximum power transfer theorem for dc and ac circuits. This paper presents a detailed algebra-based derivation of the maximum power transfer theorem for both dc and ac circuits using the calculus-free arithmetic mean-geometric mean (AM-GM) inequality. For ac circuits, the maximum average power transfer theorem is presented for four types of loads: variable resistive and reactive elements, variable resistive element and fixed reactive element, variable resistive element only, and variable load impedance magnitude with fixed load impedance angle. All of the derivations purposefully avoid the use of calculus; however, the resulting equations match exactly with the ones typically derived using calculus in introductory circuits textbooks. Also included is the verification of the derived equations and proven theorems using the LTspice circuit simulation software. A topic-level outcome assessment data demonstrate effectiveness of the analysis approach presented in this paper.

Introduction

In both associate and baccalaureate engineering technology programs, most students take the first electric circuits course before taking a calculus course. Therefore, the first circuit analysis course typically avoids the use of calculus, instead using only algebra and trigonometry. One such example is the derivation and use of the maximum power transfer theorem for dc and ac circuits.

This paper presents a detailed algebra-based derivation of the maximum power transfer theorem for dc circuits using the calculus-free arithmetic mean-geometric mean (AM-GM) inequality [1]. A graphical interpretation of the AM-GM inequality is presented as well. For ac circuits, the paper presents detailed algebra-based derivations of the maximum average power transfer theorem using the AM-GM inequality for four types of loads: variable resistive and reactive elements, variable resistive element and fixed reactive element, variable resistive element with no reactive element, and variable load impedance magnitude with fixed load impedance angle. All of the derivations purposefully avoid the use of calculus; however, the resulting equations match exactly with the ones typically derived using calculus in introductory circuits textbooks [2], [3], [4].

Additionally, in support of broadening the pedagogical perspectives, verification of the algebra-based derived equations and proven theorems is presented using LTspice simulation. It is to be noted that LTspice [5] is a powerful yet freely downloadable circuit simulation software, and can be used effectively to add a graphical perspective to the understanding of key circuit equations and theorems.

The work included in this paper will aid the instructors in presenting the maximum power transfer theorem for electric circuits in a non-calculus approach to the entry level engineering technology students. It is worth mentioning that the idea for this paper started only after a student recently asked if the maximum power transfer theorem can be proved without using calculus, i.e. using only algebra and trigonometry. The ensuing literature search indicated that a very few papers addressed this topic. A couple of papers did address the non-calculus based derivation [6] [7], albeit with frequent use of algebraic trickery not commonly accessible to most engineering technology students. This paper presents a methodical and accessible approach to algebraic derivation using the principle of AM-GM inequality, supported by LTspice simulation verification.

The following sections present a topic-level assessment approach to ABET criterion-3 student outcomes and program criteria competency areas, detailed derivation of the maximum power transfer theorem for dc and ac circuits, an assessment summary, and a brief conclusion. For ac circuits, four different load configurations are analyzed in terms of load resistance and reactance variations. Each derivation is illustrated using calculated numbers as well as LTspice simulation.

Supporting Criterion-3 Student Outcomes and Program Criteria Competency Areas

The topic of an *algebraic derivation of the maximum power transfer theorem for dc and ac circuits* is part of the *Circuit Analysis* (ENGT141) course within the Electronics Engineering Technology program. This specific topic contributes to the assessment of one of the five ABET-ETAC Criterion-3 student outcomes [8], as shown in Table I. At the EET program level, the Criterion-3 *student outcome 1* is assessed using eight performance indices, encompassing multiple EET courses, including the circuit analysis course. The two topic-applicable performance indices (1-4 and 1-5) for *student outcome 1* are included in Table I. Similarly, the topic of interest in this paper contributes to the assessment of the first two of the five Program Criteria *competency areas* for the Electrical/Electronics Engineering Technology programs [8], as shown in Table II. At the EET program level, the Program Criteria topics *a* and *b* are assessed in multiple EET courses, including the circuit analysis course. The three topic-applicable competency areas (a-1, a-2, and b-1) for curriculum topics a and b are included in Table II.

Table I: Supported ABET-ETAC *Criterion-3* Student Outcomes [8]

ABET-ETAC <i>Criterion-3</i> Student Outcome		Relevant <i>Circuit Analysis</i> Course-specific Performance Indicators for Student Outcomes Assessment	
1	An ability to apply knowledge, techniques, skills and modern tools of mathematics, science, engineering, and technology to solve broadly-defined engineering problems appropriate to the discipline	1-4	1-5
		Use of mathematical analysis, modeling, or simulation tools	Use of theoretical and/or applied procedure to solve engineering problems

Table II: Supported ABET-ETAC *Program Criteria* Topics for Electrical/Electronics Engineering Technology Programs [8]

ABET-ETAC Program Criteria Curriculum Topics		Relevant <i>Circuit Analysis</i> Course-specific Program Criteria Competency Areas	
a	Application of circuit analysis and design, computer programming, associated software, analog and digital electronics, microcontrollers, and engineering standards to the building, testing, operation, and maintenance of electrical/electronic(s) systems	a-1	a-2
		Application of circuit analysis and design	Computer programming, simulation, and associated software
b	Application of natural sciences and mathematics at or above the level of trigonometry to the building, testing, operation, and maintenance of electrical/electronic systems	b-1	
		Application of mathematics to the operation of electrical/electronic systems	

Students are assessed for specific performance indices and curriculum competency areas linked to the topic of interest within the circuit analysis course, thereby supporting the program-level assessment of the student outcomes and curriculum competency areas.

Algebraic Derivation of the Maximum Power Transfer Theorem for DC Circuits

Consider the variable load resistance connected to a dc Thevenin equivalent circuit, as shown in Fig. 1. Using the I^2R formula, power delivered to the load is expressed in (1).

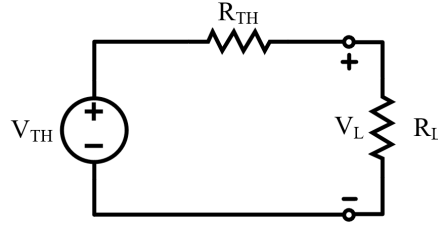


Figure 1: Thevenin equivalent of a dc circuit connected to a load resistor

$$P_L = \left[\frac{V_{TH}}{(R_{TH} + R_L)} \right]^2 R_L \quad (1)$$

Next, a parameter named k is defined as the ratio of load resistance to Thevenin resistance.

$$k \equiv \frac{R_L}{R_{TH}} \quad (2)$$

Combining (1) and (2), load power is expressed in (3) in terms of k and the known Thevenin voltage and resistance.

$$P_L = \frac{V_{TH}^2 k R_{TH}}{R_{TH}^2 (1 + k)^2} \Rightarrow P_L = \frac{V_{TH}^2}{R_{TH}} \left[\frac{1}{2 + k + \frac{1}{k}} \right] \quad (3)$$

The maximum value of load power and the corresponding load resistance value can be established algebraically using the arithmetic mean (AM) and geometric mean (GM) inequality. The AM-GM inequality states that the arithmetic mean of a list of positive real numbers is greater than or equal to the geometric mean of the same list of numbers [1]. Applying the principle of AM-GM inequality to the list consisting of only k and $1/k$ results in (4).

$$\frac{k + \frac{1}{k}}{2} \geq \sqrt{k \left(\frac{1}{k} \right)} \Rightarrow k + \frac{1}{k} \geq 2 \quad (4)$$

Using (4), it can be stated that the minimum value of $(k + 1/k)$ is 2, and it occurs at $k = 1$. Using this information in (3), the maximum load power is expressed in (5).

$$P_{L,max} = \frac{V_{TH}^2}{4R_{TH}}. \quad (5)$$

Additionally, using the definition of k in (2), the maximum load power occurring at $k = 1$ corresponds to $R_L = R_{TH}$. This proves the well-known maximum power transfer theorem that the load receives maximum power when the load resistance (R_L) is exactly the same as the Thevenin resistance (R_{TH}), and the resulting maximum power is as given in (5).

The value of $(k + 1/k)$ expression as a function of k can be visualized graphically as shown in Fig. 2 (created using the LTspice circuit simulation software). It shows the exact same information obtained above algebraically using the AM-GM inequality, i.e., the minimum value of $(k + 1/k)$ is 2 and it occurs at $k = 1$. This basic concept will be used for proving the maximum power transfer theorem for ac circuits as well.

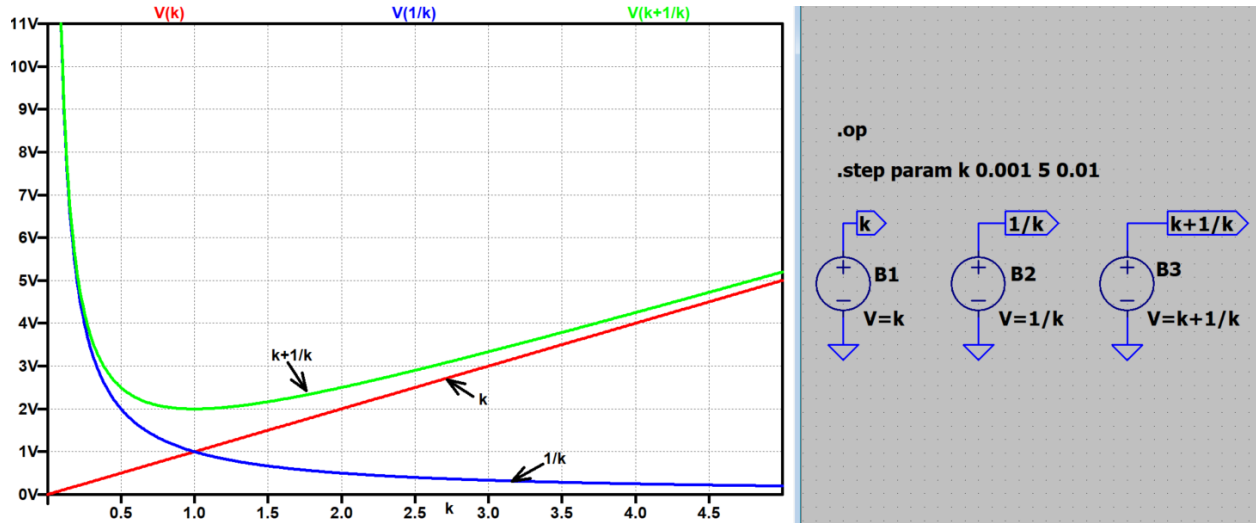


Figure 2: Plots of k , $1/k$, and $(k+1/k)$ as a function of k

Finally, LTspice simulation of load power for a variable load resistance is shown in Fig. 3. It is clearly evident from the simulation plot that for the given circuit, the maximum load power of 250 mW is occurring when the load resistance is 1Ω , i.e., when the load resistance exactly

equals the Thevenin resistance. The calculated maximum load power using (5) is 250 mW, exactly same as the simulated value.

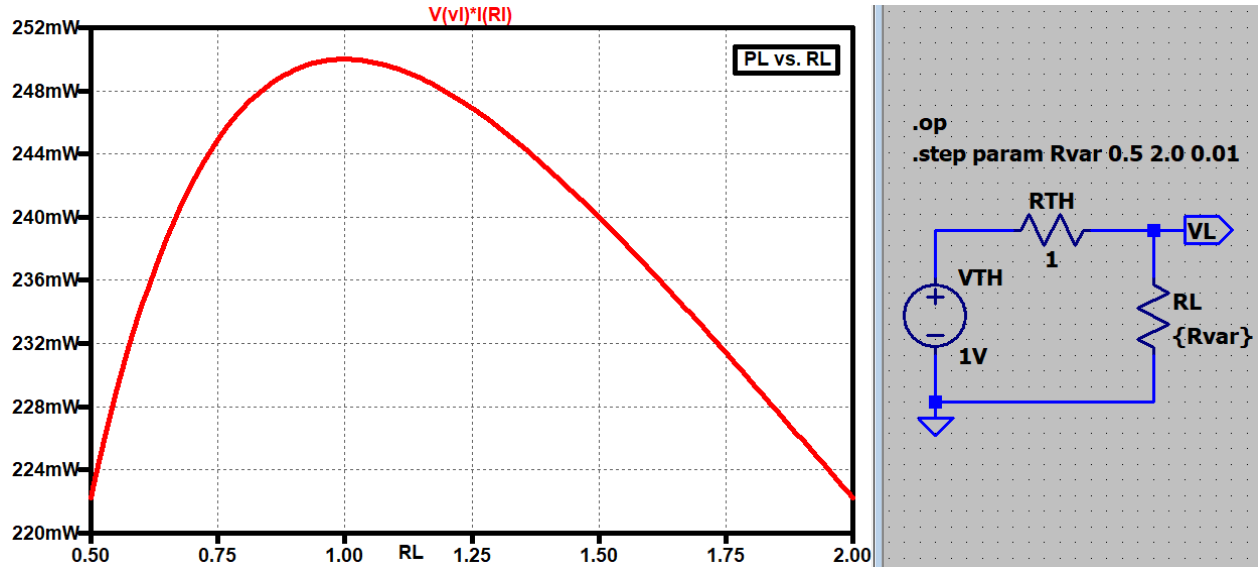


Figure 3: Load power as a function of load resistance ($V_{TH} = 1$ V and $R_{TH} = 1 \Omega$)

Algebraic Derivation of the Maximum Power Transfer Theorem for AC Circuits

For ac circuits, consider a Thevenin ac source connected to an ac load, as shown in Fig. 4.

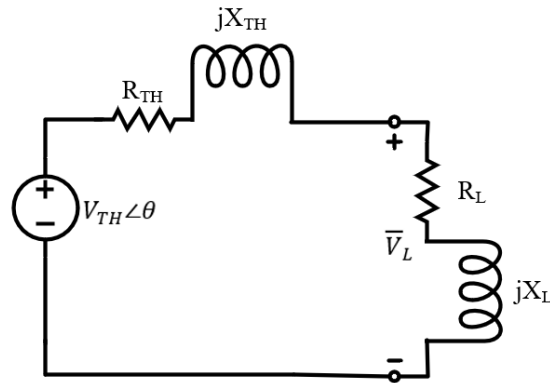


Figure 4: Thevenin equivalent of an ac circuit connected to a load impedance

For the variable load, multiple configurations are possible. In this paper the following four configurations are algebraically analyzed, including the derivation of maximum power transfer condition and the associated maximum power.

- A. Both the load resistance and the load reactance are adjustable.
- B. Load reactance is fixed but the load resistance is adjustable.
- C. Load reactance is zero, i.e., the load is purely resistive and adjustable.
- D. Load impedance angle is fixed but the load impedance magnitude is adjustable.

Using the I^2R formula, average power delivered to the ac load is expressed in (6). This equation will be the starting point for algebraic analysis of maximum average power for all four configurations.

$$P_L = \left[\frac{V_{TH}^2}{(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2} \right] R_L \quad (6)$$

A. Both the load resistance and the load reactance are adjustable

Looking at (6), it can be concluded that for maximizing the average power against a variable X_L , its value simply needs to be set to $-X_{TH}$. This simplifies (6) to (7) as shown.

$$P_L = \left[\frac{V_{TH}}{(R_{TH} + R_L)} \right]^2 R_L \quad (7)$$

It can be noted that (7) is identical to (1), and therefore, per the previous analysis, average power in the load will be maximum against a variable R_L when its value is equal to R_{TH} . Using this load condition in (7), the corresponding maximum value of the average power is given by (8), which is identical to the maximum average power in a dc circuit as derived in (5).

$$P_{L,max} = \frac{V_{TH}^2}{4R_{TH}} \quad (8)$$

In summary, the maximum power in an ac circuit with adjustable resistance and reactance occurs when the load impedance is complex conjugate of the Thevenin impedance, verifying the well-known maximum power transfer theorem for ac circuits. In introductory circuits textbooks, this derivation uses differential calculus, whereas an algebraic derivation is presented herein.

Table III below shows the maximum power values, calculated using (7), for a variable load resistance, with the load reactance set to cancel the Thevenin reactance. As expected, when the load and Thevenin resistances equal, the highest value of maximum power (250 mW) is achieved since the load and Thevenin impedances become complex conjugate of each other. Under this specific condition, (7) and (8) become identical.

Table III: Load maximum power for various load resistance values

$V_{TH} = 1 \text{ V}, R_{TH} = 1 \text{ } \Omega, X_{TH} = 1 \text{ } \Omega, X_L = -1 \text{ } \Omega$		
$R_L \text{ (}\Omega\text{)}$	$(R_{TH} + R_L) \text{ (}\Omega\text{)}$	$P_{L,max} \text{ (mW)}$
0.5	1.5	222.2
0.75	1.75	244.9
1.0	2.0	250.0
1.5	2.5	240.0
2.0	3.0	222.2

The LTspice simulation schematic and the associated power plot, shown in Fig. 5, clearly indicates that maximum power occurs when the load reactance and Thevenin reactance cancel each other, and the maximum power is highest when the load impedance is complex conjugate of

the Thevenin impedance. Additionally, as expected, the maximum power values of Fig. 5 are consistent with values calculated in Table III.

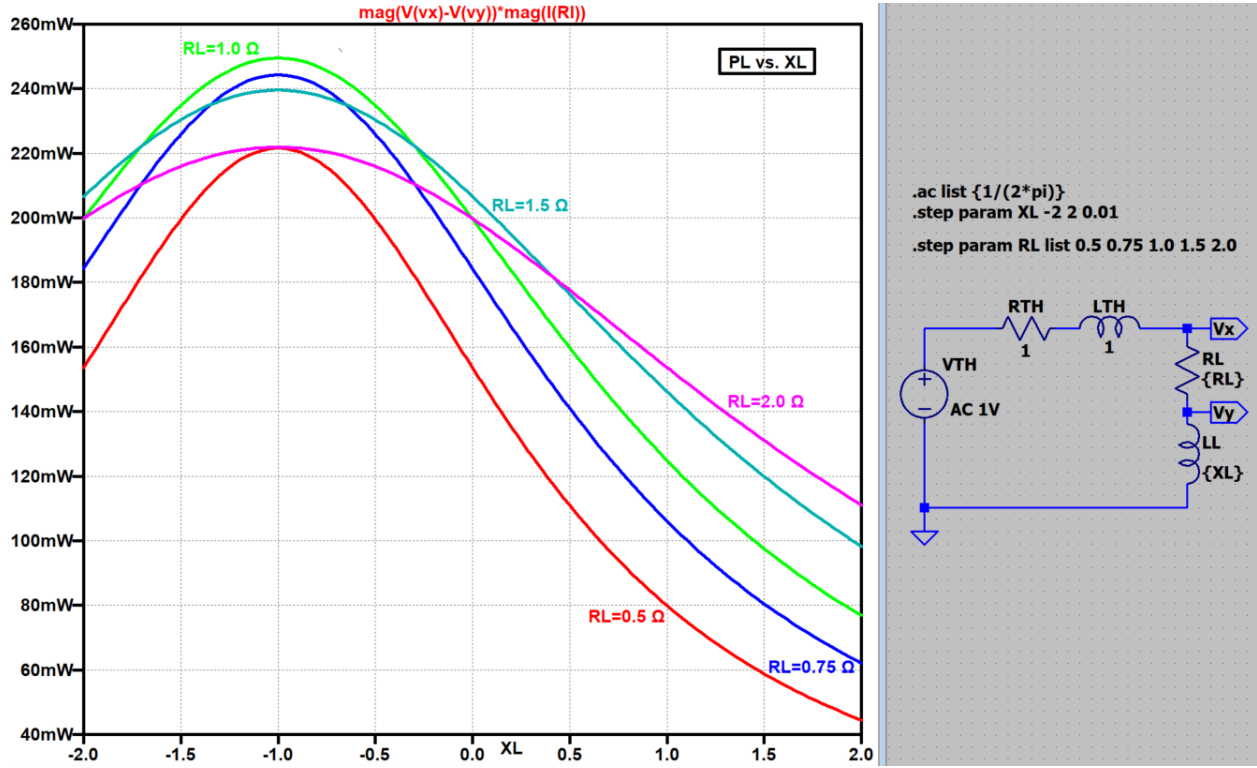


Figure 5: Load power as a function of load reactance ($V_{TH} = 1$ V, $R_{TH} = 1$ Ω , $X_{TH} = 1$ Ω)

B. Load reactance is fixed but the load resistance is adjustable

For this configuration, since the load reactance is fixed, it can be added to the Thevenin reactance to find the total reactance (X_T) of the circuit, as defined in (9).

$$X_T \equiv X_{TH} + X_L \quad (9)$$

Combining (6) and (9), results in (10).

$$P_L = \left[\frac{V_{TH}^2}{(R_{TH} + R_L)^2 + X_T^2} \right] R_L \quad (10)$$

Next, a parameter named m is defined as the ratio of load resistance to the impedance magnitude of the rest of the circuit.

$$m \equiv \frac{R_L}{\sqrt{R_{TH}^2 + X_T^2}} \quad (11)$$

Combining (10) and (11) results in (12), and further simplification results in (13).

$$P_L = \frac{V_{TH}^2 \left(m \sqrt{R_{TH}^2 + X_T^2} \right)}{R_{TH}^2 + 2R_{TH} \left(m \sqrt{R_{TH}^2 + X_T^2} \right) + m^2 (R_{TH}^2 + X_T^2) + X_T^2} \quad (12)$$

$$P_L = \frac{V_{TH}^2}{2R_{TH} + \left(m + \frac{1}{m} \right) \sqrt{R_{TH}^2 + X_T^2}} \quad (13)$$

Again, the principle of AM-GM inequality is applicable to (13). Using (4), it can be stated that the minimum value of $(m + 1/m)$ is 2, and it occurs at $m = 1$. Using this information in (13), the maximum average load power can be expressed as in (14). Additionally, using the definition of m in (11), the load resistance value corresponding to maximum power in the load is given by (15).

$$P_{L,max} = \frac{V_{TH}^2}{2 \left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_L)^2} \right]} \quad (14)$$

$$R_L|_{\max_power} = \sqrt{R_{TH}^2 + (X_{TH} + X_L)^2} \quad (15)$$

Table IV below shows the calculated maximum power values and the corresponding load resistance values using (14) and (15), for a few specific values of load reactance. It can be observed that the highest value of maximum power (250 mW) is obtained when the load impedance is complex conjugate of the Thevenin impedance.

Table IV: Load maximum power for various load reactance values

$V_{TH} = 1 \text{ V}, R_{TH} = 1 \text{ } \Omega, X_{TH} = 1 \text{ } \Omega$			
$X_L \text{ (}\Omega\text{)}$	$(X_{TH} + X_L) \text{ (}\Omega\text{)}$	$R_L _{\max_power} \text{ (}\Omega\text{)}$	$P_{L,max} \text{ (mW)}$
1.5	2.5	2.693	135.4
1.0	2.0	2.236	154.5
0	1.0	1.414	207.1
-1.0	0	1.0	250.0
-1.5	-0.5	1.118	236.1

The LTspice simulation schematic and the associated power plot, shown in Fig. 6, clearly indicates that maximum power values and the corresponding load resistance values are consistent with the values calculated in Table IV.

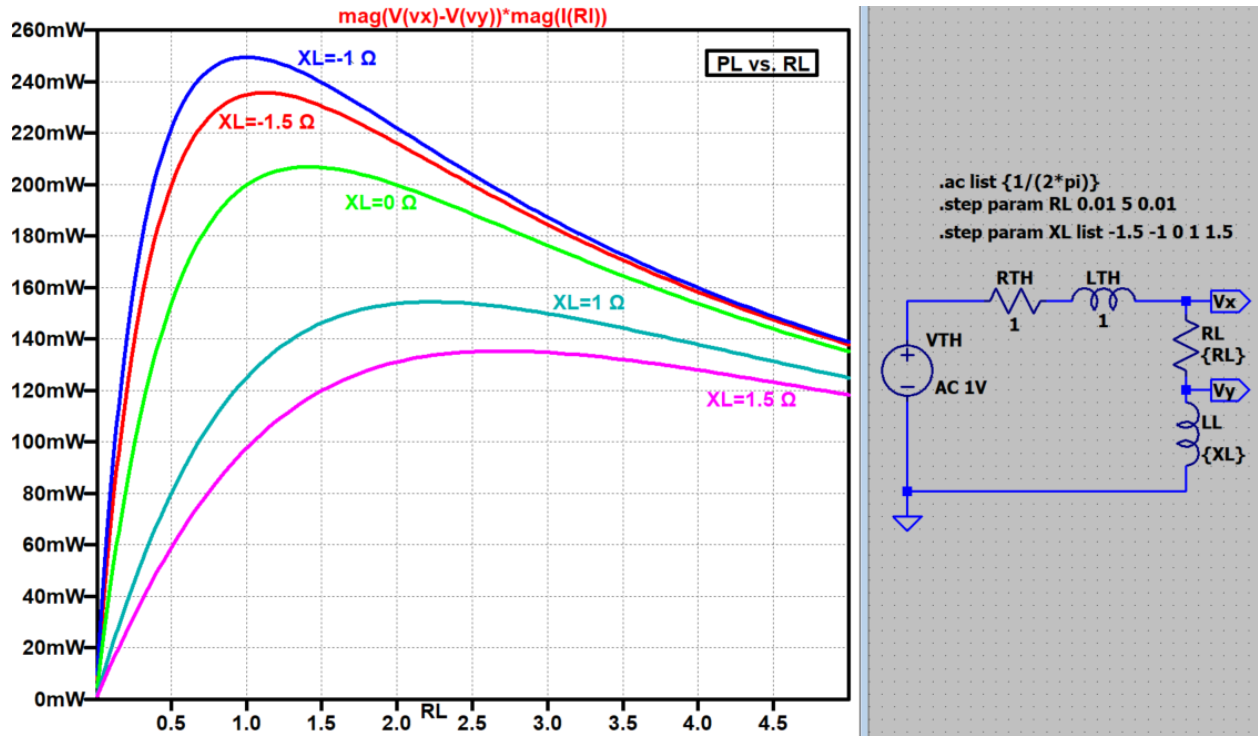


Figure 6: Load power as a function of load resistance ($V_{TH} = 1 \text{ V}$, $R_{TH} = 1 \Omega$, $X_{TH} = 1 \Omega$)

C. Load reactance is zero, i.e., the load is purely resistive and adjustable

For this load configuration, (14) and (15) can be used directly with X_L set to zero, resulting in (16) and (17), respectively.

$$P_{L,max} = \frac{V_{TH}^2}{2 \left[R_{TH} + \sqrt{R_{TH}^2 + X_{TH}^2} \right]} \quad (16)$$

$$R_L|_{\max_power} = \sqrt{R_{TH}^2 + X_{TH}^2} \quad (17)$$

This verifies the well-known maximum power transfer theorem for a purely resistive load, i.e. the load resistance needs to be equal to the magnitude of the Thevenin impedance. Using $V_{TH} = 1 \text{ V}$, $R_{TH} = 1 \Omega$, and $X_{TH} = 1 \Omega$ in (16) and (17) results in a maximum load power of 207.1 mW occurring at a load resistance of 1.414 Ω . These two values are consistent with the plot shown in Fig. 6.

D. Load impedance angle is fixed but the load impedance magnitude is adjustable

For this configuration, the load impedance is expressed in both rectangular and polar forms in (18) and their interrelationships in (19).

$$\overline{Z}_L = Z_L \angle \theta_L = R_L + jX_L \quad (18)$$

$$Z_L = \sqrt{R_L^2 + X_L^2}; \theta_L = \tan^{-1} \left(\frac{X_L}{R_L} \right); R_L = Z_L \cos \theta_L; X_L = Z_L \sin \theta_L \quad (19)$$

Using (18) and (19), the load power expression of (6) is rewritten in terms of load impedance magnitude and angle in (20).

$$P_L = \left[\frac{V_{TH}^2}{(R_{TH} + Z_L \cos \theta_L)^2 + (X_{TH} + Z_L \sin \theta_L)^2} \right] (Z_L \cos \theta_L) \quad (20)$$

A simplified version of (20) is given in (21).

$$P_L = \frac{V_{TH}^2 Z_L \cos \theta_L}{R_{TH}^2 + X_{TH}^2 + Z_L^2 + 2Z_L(R_{TH} \cos \theta_L + X_{TH} \sin \theta_L)} \quad (21)$$

Next, a parameter named n is defined as the ratio of load impedance magnitude to the Thevenin impedance magnitude.

$$n \equiv \frac{Z_L}{\sqrt{R_{TH}^2 + X_{TH}^2}} \quad (22)$$

Combining (21) and (22), results in (23).

$$P_L = \frac{V_{TH}^2 \left(n \sqrt{R_{TH}^2 + X_{TH}^2} \cos \theta_L \right)}{R_{TH}^2 + X_{TH}^2 + n^2 (R_{TH}^2 + X_{TH}^2) + 2n \sqrt{R_{TH}^2 + X_{TH}^2} (R_{TH} \cos \theta_L + X_{TH} \sin \theta_L)} \quad (23)$$

A simplified version of (23) is given in (24).

$$P_L = \frac{V_{TH}^2 \cos \theta_L}{\left(n + \frac{1}{n} \right) \sqrt{R_{TH}^2 + X_{TH}^2} + 2(R_{TH} \cos \theta_L + X_{TH} \sin \theta_L)} \quad (24)$$

Again, the principle of AM-GM inequality is applicable to (24). Using (4), it can be stated that the minimum value of $(n + 1/n)$ is 2, and it occurs at $n = 1$. Using this information in (24), the maximum average load power is expressed in (25). Additionally, using the definition of n in (22), the load impedance magnitude value corresponding to maximum load power is given by (26).

$$P_{L,max} = \frac{V_{TH}^2 \cos \theta_L}{2 \left(\sqrt{R_{TH}^2 + X_{TH}^2} + R_{TH} \cos \theta_L + X_{TH} \sin \theta_L \right)} \quad (25)$$

$$Z_L|_{\max_power} = \sqrt{R_{TH}^2 + X_{TH}^2} \quad (26)$$

Using (25) and (26), Table V below shows the calculated maximum power values for a few specific load impedance angle values, with $V_{TH} = 1$ V, $R_{TH} = 1$ Ω , and $X_{TH} = 1$ Ω . It can be observed that the highest value of maximum power (250 mW) is obtained when the load impedance angle is -45° , as this specific load impedance angle results in a load impedance that is complex conjugate of the Thevenin impedance.

Table V: Load maximum power for various load impedance angle values

$V_{TH} = 1$ V, $R_{TH} = 1$ Ω , $X_{TH} = 1$ Ω , $Z_L = \sqrt{2}$ Ω	
θ_L (degree)	$P_{L,max}$ (mW)
60	89.9
45	125.0
0	207.1
-45	250.0
-60	238.5

The LTspice simulation schematic and the associated power plot, shown in Fig. 7, clearly indicates that maximum power values and the corresponding load impedance magnitude values are consistent with the values calculated in Table V.

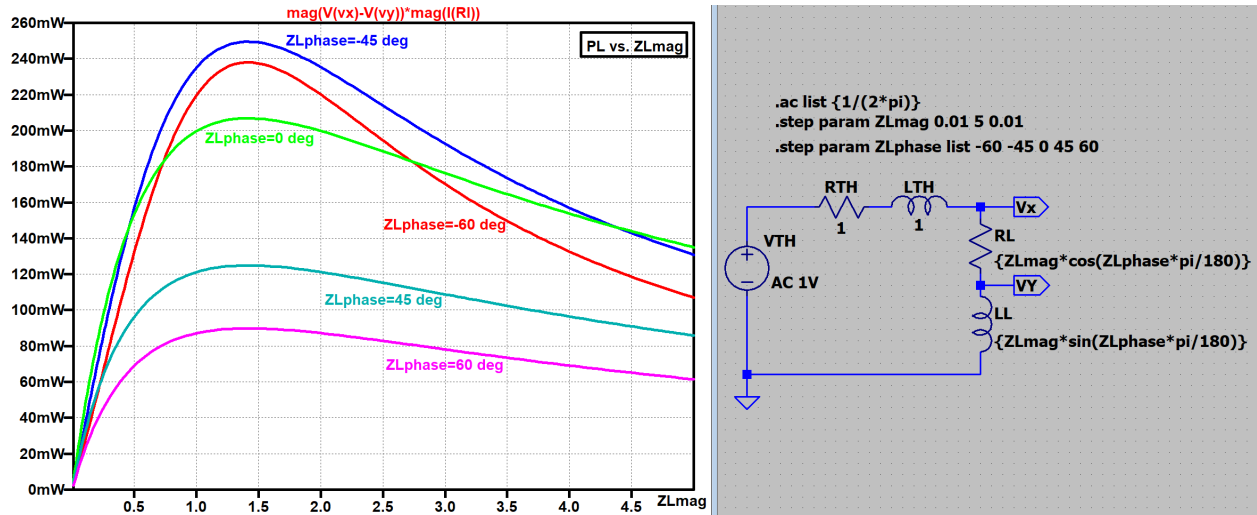


Figure 7: Load power as a function of load impedance magnitude ($V_{TH} = 1$ V, $R_{TH} = X_{TH} = 1$ Ω)

Purely resistive load ($\theta_L = 0$)

It is interesting to note that for a purely resistive load ($\theta_L = 0$), (25) and (26) simplifies to (27) and (28), respectively.

$$P_{L,max} = \frac{V_{TH}^2}{2 \left(\sqrt{R_{TH}^2 + X_{TH}^2} + R_{TH} \right)} \quad (27)$$

$$R_L|_{\max_power} = \sqrt{R_{TH}^2 + X_{TH}^2} \quad (28)$$

As expected, (27) and (28) are identical to (16) and (17), respectively, since both sets of equations are for a purely resistive load in an ac circuit.

Load impedance and Thevenin impedance are complex conjugate of each other

For a load impedance that is complex conjugate of the Thevenin impedance, (25) can be rewritten as (29), using $\theta_L = -\theta_{TH}$.

$$P_{L,max} = \frac{V_{TH}^2 [\cos(-\theta_{TH})]}{2 \left[\sqrt{R_{TH}^2 + X_{TH}^2} + R_{TH} \cos(-\theta_{TH}) + X_{TH} \sin(-\theta_{TH}) \right]} \quad (29)$$

Replacing the Thevenin impedance angle in terms of Thevenin resistance and reactance in (29), results in (30).

$$P_{L,max} = \frac{V_{TH}^2 \left(\frac{R_{TH}}{\sqrt{R_{TH}^2 + X_{TH}^2}} \right)}{2 \left[\sqrt{R_{TH}^2 + X_{TH}^2} + R_{TH} \left(\frac{R_{TH}}{\sqrt{R_{TH}^2 + X_{TH}^2}} \right) - X_{TH} \left(\frac{X_{TH}}{\sqrt{R_{TH}^2 + X_{TH}^2}} \right) \right]} \quad (30)$$

Algebraic simplification of (30) results in (31).

$$P_{L,max} = \frac{V_{TH}^2 R_{TH}}{2[R_{TH}^2 + X_{TH}^2 + R_{TH}^2 - X_{TH}^2]} \Rightarrow P_{L,max} = \frac{V_{TH}^2}{4R_{TH}} \quad (31)$$

This is the well-known maximum power equation for a load impedance that is complex conjugate of the Thevenin impedance, derived earlier in (8).

Topic-specific Assessment of Performance Indicators for Student Outcomes and Program Criteria Student Competency Areas

Direct assessment of student outcomes and program competency areas specific to the topic of interest was conducted in fall-2025 using several instruments, including homework and in-class quizzes. These assessment data were evaluated to generate action items to be implemented during the fall-2026 offering of the *Circuit Analysis* course. This feedback process is part of the EET program's continuous improvement process, contributing to the improvement of student learning. In addition to direct assessment, the topic-specific ABET-ETAC Criterion-3 student outcomes performance indicators and Program Criteria student competency areas were assessed indirectly via student survey. Results of the indirect assessment for fall-2025 offering are shown in Table VI. It is to be noted that at the program level, a level of attainment greater than 75% combining the *Definitely* and *Substantially* responses is considered acceptable.

Table VI: Indirect assessment data for Criterion-3 student outcomes performance indices and Program criteria student competency areas

Criterion-3 Student Outcomes Performance Indicator or Program Criteria Student Competency Area		Level of Attainment?				
Did the the topic of <i>algebraic derivation of the maximum power transfer theorem for dc and ac circuits</i> improve your proficiency in		Total number of students: 28				
		Definitely (D)	Substantially (S)	Marginally (M)	Not really (N)	(D+S) [%]
1-4	mathematical analysis	12	10	5	1	79
1-5	theoretical and/or applied procedure in solving engineering problems	12	11	5	0	82
a-1	application of circuit analysis	11	12	5	0	82
a-2	computer simulation of circuits using LTspice software	15	11	2	0	93
b-1	application of mathematics to circuit operation	10	12	4	2	79

It can clearly be observed in Table VI that the first time introduction of the *algebraic derivation of the maximum power transfer theorem for dc and ac circuits* topic is well received by the students, resulting in attainment levels exceeding the program-level expectation.

Conclusion

In both associate and baccalaureate engineering technology programs, the first circuit analysis course typically avoids the use of calculus, instead it is offered using only algebra and trigonometry. This paper addressed the algebraic presentation of the maximum power transfer theorem for dc and ac circuits. For ac circuits, four different load configurations are analyzed in terms of load resistance and reactance variations. Each derivation is illustrated using calculated numbers as well as LTspice simulation. Additionally, a topic-level student outcomes assessment demonstrating the effectiveness of the proposed approach to mathematical analysis is presented. This work will aid the instructors in presenting the maximum power transfer theorem for electric circuits in a non-calculus approach to the entry level engineering technology students.

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