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Teaching the Effects of Lamina Stacking Sequence (LSS) in an Introductory Composite Materials Class

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Abstract

Composite materials have many advantages such as high specific strength, high corrosion resistance, high fatigue life under cyclic loads and enhanced flexibilities during fabrications. Hence, composite materials are currently widely used in many engineering applications. The authors, who are professors at a comprehensive university, offer an introductory level Composite Materials course for the Mechanical Engineering (ME) and Mechanical Engineering Technology (MET) students. This 5-credit course, with 4 standard classroom lectures and 1 lab session per week, is fast-paced in a quarter system. Within a 10-week time-frame, the authors cover several topics such as a basic introduction, composite constituents, manufacturing processes, micro-mechanics, ply-mechanics, macro-mechanics and failure theories.

Fiber reinforced composite laminates are prepared by stacking a single sheet of continuous fibers in different orientations to get the desired strength and stiffness. The Lamina Stacking Sequence (LSS) represents how the layers are stacked together in a composite laminate. For example, the $[0/60/-60]_s$ laminate represents a symmetric composite plate that consists of 6 layers of fibers, which are stacked at 0, 60, -60, -60, 60 and 0 degree orientations. Therefore, the $[0/60/-60]_s$, $[0/-60/60]_s$, $[60/-60/0]_s$, $[-60/60/0]_s$, [60/0/-60]s, and $[-60/0/60]_s$ represent the same laminate but with a different LSS. The concepts of LSS and its overall effects on structural behavior are only lightly covered in many textbooks. Therefore, the instructors have developed an end-of-quarter project where students were required to work on a composite laminate, defined as $[0/60/-60]_s$, to investigate its structural behavior for different LSS configurations. First, students were required to use MATLAB to analytically compute the in-plane stiffness matrix [A], bending stiffness matrix [D], in-plane moduli (E_x and E_y) and bending moduli (E_{bx} and E_{by}) for different LSS. For each LSS, students analyzed structural response, such as deformations, strains and stresses when the laminate was subjected to in-plane stretching and bending loads. Second, they were required to use the Finite Element Analysis (FEA) software ANSYS Composite Prep-Post (ACP) to re-compute the structural properties and responses that were previously analyzed by MATLAB. Students are expected to see a close agreement between the results obtained by MATLAB and ACP, which will strengthen their overall conceptual understanding. Then, they fabricated several composite laminates, for the previously mentioned LSS, using a 3D Printer. Finally, students conducted several experiments to see how the LSS can affect the structural response when these laminates are subjected to standard tensile and bending loads.

This paper outlines methodologies to teach the intended learning outcomes. This includes computational tools (MATLAB and ANSYS) and experimental techniques (mechanical testing of 3D printed samples) to gain valuable knowledge on how the LSS could affect structural response of different symmetric composite laminates. An anonymous survey was conducted at the end of the course that demonstrated strong student understanding of the core concepts taught and are included in this article.

Introduction: Composite Materials Course Structure

Composite Materials is an elective course taken by mostly seniors who are part of the university's Mechanical Engineering (ME) and Mechanical Engineering Technology (MET) programs. As we are in a quarter system, teaching the intended course objectives in a fast-paced 10 week time frame requires thoughtful planning and meaningful learning experiences. Most of the course contents covered in this class originate from the textbook "Introduction to Composite Materials Design" written by Ever. J. Barbero [1]. Other supplemental course materials are from textbooks written by Jones, Gibson and Kaw [2-4].

The course starts with basic introduction of composites where students learn the definitions of fiber, matrix, lamina, laminate, hybrid, local axes, global axes and others. Then, students become familiar with different standard symbols (such as E_f, E_m, V_f, E_1, E_2, F_{1t}, F_{1c}, F_{2t}, F_{2c}, G_{12}, Q, S, T, and others) that are listed in any standard textbooks. Then, the discussion moves on to composite constituents where students acquire in-depth knowledge about different types of inorganics, organics and natural fibers. Additionally, students learn different types of resin (matrix), which can be thermoplastic or thermoset. They learn to identify or list the key functions of fiber and matrix to fabricate the composite laminates. Students also become familiar with the various acronyms used for glass and carbon fibers. Before diving into mechanics, students learn different types of composite manufacturing processes, which include Hand Layup, Prepreg Layup, Vacuum Bagging, Resin Infusion Molding (RTM), Vacuum Assist Resin Transfer Molding (VARTM), Pultrusion and Filament Winding. By watching a variety of YouTube videos, as needed, students can strengthen their conceptual understanding of manufacturing processes. As part of micro-mechanics, students learn how to estimate or predict several mechanical properties (such as E_1, E_2, F_{1t}, F_{2t}, G_{12}, ν_{12} and others) of unidirectional (UD) lamina by applying the concept of mechanics, not from the experimental data. Students understand that these estimated mechanical properties are along the local directions (designated by 1 and 2), where 1 represents the fiber direction and 2 represents normal to the fiber direction. Also discussed in class is how to estimate other properties such as coefficient of thermal expansion (α_1 and α_2) and coefficient of moisture expansion (β_1 and β_2). Students learn how these estimated properties would change with the fiber volume fraction (V_f). The class discussion then moves to ply-mechanics, where students learn the concept of plane stress and the relationship between stresses and strains through forming the stiffness [Q] and compliance [S] matrices. They also learn the applications of transformation matrix [T] to determine the stresses and strains along the local or global direction. It is noteworthy to let students know that the concept of ply-mechanics is also applicable to the unidirectional (UD) lamina. The estimated [Q] and [S] represent the mechanical properties along the local directions. Hence, students can use the [T] matrix to recompute the stiffness and compliance matrices along the global direction. After several class lectures on ply-mechanics, the class explores many important concepts of macro-mechanics, which are applicable to a composite laminate, consists of multiple laminae where the fiber orientation can be the same or different. In this chapter, students learn how to compute different stiffness matrices, such as [A = In-Plane Stiffness], [B = Coupling Matrix] and [D = Bending Stiffness]. Students then use these matrices to determine the stresses when the laminate is subjected to in-plane stretching and/or bending loads. Students also learn different types of laminates, such as symmetric, anti-symmetric, balanced, cross-ply and others. They become highly aware of knowing when the [B] matrix is required, and when it is not. In this section, they also learn how to compute the in-plane moduli (E_X and E_y) and bending moduli (E_{bx} and E_{by}) for symmetric balanced laminate. The final lecture portion covers the different failure theories applicable to composite materials. Students learn how to compute the First Ply Failure (FPF) load using the concept of macro-mechanics that they had learned before. They also learn how to compute stresses on a composite laminate subjected to thermal and/or moisture loads. Finally, students learn how to determine the in-plane strength and moduli using different Carpet Plots.

Students solve a wide variety of mathematical problems belong to micro-mechanics, ply-mechanics, and macro-mechanics using MATLAB or Python, which is one of the prerequisites for this class. This class also requires 1 lab session per week, which is 2 hours long. The lab activities are completed as teams, where each team has about 4 to 5 students. Hence, each team is allotted about 40 to 45 minutes to finish the lab exercises. Over the quarter, students participate in the following hands-on lab experiments:

Lab # **1**: Students are introduced to different composite constituents, supplies and tools for sample preparations/fabrications as well as lab safety protocols.
Lab # **2**: Students fabricate composite samples by the "Hand Layup" method using unidirectional (UD) glass fiber. Then, each team uses different fiber orientations, such as $[0]_8$ or $[0_2/90_2]_s$, or $[0/45/-45/90]_s$, to fabricate the composite laminates.
Lab # **3**: Students fabricate composite samples by using? applying? the "Vacuum Bag" molding method. This lab is similar to Lab # 2, except students use carbon fiber.
Lab # **4**: Students perform a mechanical test of glass fiber samples with different fiber orientations and then compare the mechanical properties with other teams.
Lab# **5**: Students perform a mechanical test of carbon fiber samples with different fiber orientations and compare the mechanical properties with other teams, similar to Lab # 4(?). In addition, students compare the mechanical properties of composite samples prepared by glass and carbon fibers.
Lab # **6**: Students learn different aspects of quasi-isotropic laminate followed by mechanical test of quasi-isotropic laminates.
Lab # **7**: Students solve several macro-mechanics problems using MATLAB.
Lab # **8**: Students solve several macro-mechanics problems using Finite Element Software, ANSYS.
Lab # **9**: Students are required to prepare a research presentation related to an aspect of composite materials. Most of the time, students select topics from CompositeWorlds [5] based on their interest. Finally, they provide an oral presentation during the lab hour.

Literature Review:
There are quite a few pedagogical papers on teaching aspects of composite materials for the undergraduate or graduate students. For example, Hossain and Durfee [6] provided basic concepts on composite materials through successive laboratory exercises in an introductory materials science course. The learning process started with a basic understanding of composite constituents such as matrix and fiber, their types, properties and manufacturing processes. After acquiring the necessary theoretical knowledge, students performed a series of experiments dealing with several composite samples demonstrating different concepts, such as anisotropy and laminate strength that depends on the number of plies, fiber orientation and the types of fiber. This paper outlined the details of these laboratory experiments and discussed the educational outcomes obtained in material science curriculum.

William and Janeiro [7] developed an elective composite materials course that involved some laboratory exercises providing the students a hands-on experience in the layup and testing of carbon-epoxy specimens. In that article, the authors shared lessons learned in making and testing these specimens that can be incorporated to either in a composite materials course or to supplement an introductory materials science course. They also shared details regarding material sources, low-cost tools, and cutting plans that can minimize waste in the creation of the specimens. This paper outlined different concepts including strength and stiffness variations with orientation angle, progressive failure, and thermal property mismatches leading to residual stresses and/or specimen warping.

Imran et al. [8] developed an engineering technology course where students designed AI (Artificial Intelligence) tools for analyzing mechanical properties of composite materials. The course content included (a) a review of general mechanics and failure theories, (b) composite materials overview, design, modeling, analysis, and applications, (c) Machine Learning fundamentals, and (d) design of AI tools for the analysis of composite materials. The course was designed for undergraduate engineering students to teach fundamental mechanics of fibers and composite materials, modeling stress and strains of fiber reinforced composites, and the applications of composites to design structures. The authors also included Classical Laminate Theory (CLT) and special types of laminates, but nothing specifically addressing the effects of LSS.

P. Bishay [9] developed a lab manual for undergraduate mechanical engineering course on composite materials. The lab was intended for basic composite manufacturing using wet and prepreg layups. The lab manual provided a general overview of the lamination process that students performed. The manual reviews the general structure of composite laminates made of matrix that contains continuous fibers in a specific orientation. The effects of sequencing of the laminate plies is not presented in this lab manual.

Barton and Miller [10] from the United States Naval Academy presented two elective courses involving composites in the engineering curriculum. One course focused heavily on hands-on aspects of material processes, with a section dedicated to composite fabrication, including hand layups and prepreg processes. A second elective course was dedicated to composite mechanics based on micro- and macro-mechanics, failure theories, and classical lamination theory. Students used theoretical hand-calculation and computational tools (such as IDEAS) to design and analyze various laminate structures under different loading conditions.

The LSS and fiber orientations may affect the structural response of any composite laminates subjected to in-plane stretching and bending loads. However, this concept is often lightly touched on by standard textbooks available to students and educators. Plus, this topic is missing in many educational research papers that have been recently published.

Lamina Stacking Sequence: Conceptual Understanding

The Lamina Stacking Sequence (LSS) represents how the laminae (or layers) are stacked together in a composite laminate. For example, the $[0/60/-60]_s$ represents a quasi-isotropic symmetric laminate, where 6 plies are stacked together at 0, +60 and -60 degree orientations. Hence, the $[0/60/-60]_s$, $[0/-60/60]_s$, $[60/-60/0]_s$, $[-60/60/0]_s$, $[60/0/-60]_s$, and $[-60/0/60]_s$ represent the same laminate but with different LSS. The concept of LSS is pictorially shown in **Figure 1**.

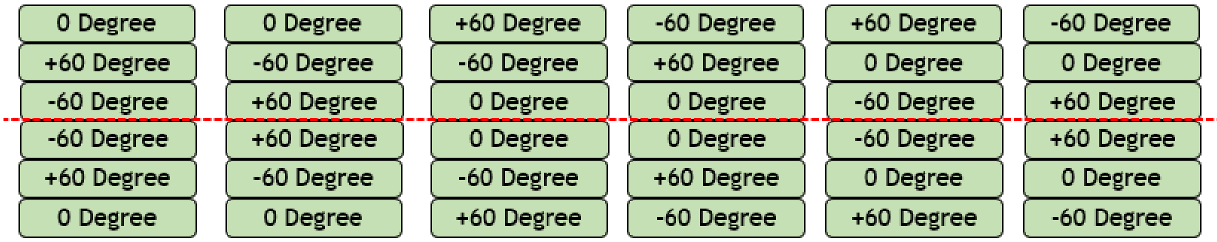


Figure 1: Different LSS for the Symmetric Laminate Defined by $[0/60/-60]_s$. The Red Line Represents the Mid-Plane.

To understand this concept further, a project is assigned (in week 8) to each lab team to predict the structural response of a composite laminate for various LSS configurations. First, students investigated the effect of LSS on the laminate in-plane stiffness [A] and bending stiffness [D]. Then, they computed the in-plane moduli (E_x and E_y) and bending moduli (E_{bx} and E_{by}). Next, the laminate was subjected to an in-plane stretching load (N) and bending moment (M). Finally, students computed the stress distribution at several locations along the laminae thickness direction.

Students completed this task by MATLAB first. Then, they repeated these analyses by using the FEA software ACP. The ANSYS results were then compared to the corresponding MATLAB solutions. Finally, students performed some experiments where the 3D printed samples were tested for various LSS configurations. The details of these works are presented in the following sections.

Student Learning: Theoretical Background

The concept of reduced stiffness matrix [Q] was introduced to students in the chapter called ply-mechanics, which is used to define the stress-strain relationship of an orthotropic material as $\{\sigma\}_i = [Q]_{ij} \times \{\varepsilon\}_j$. This [Q] matrix is defined along the local directions 1 and 2, where 1 represents along the fiber, and 2 represents normal to the fiber, as shown in **Figure 2(a)**. A composite laminate is usually made of multiple plies (laminae) with different fiber orientations. For example, **Figure 2** represents a composite laminate consisting of 6 plies, where the fiber orientations can be different. Students need to understand that the [Q] is the same for each ply, as it is always defined along the local directions (i.e., 1 and 2). Once the [Q] matrix is known, then the transformed reduced stiffness matrix $[Q_b]$ can be determined based on the fiber orientation (θ). Here, students first need to compute the transformation matrix, [T] based on fiber orientation (θ). Then, they can determine the $[Q_b]$ matrix as $[Q_b] = [T^{-1}]\,[Q]\,[T^{-1}]^T$. However, the $[Q_b]$ matrix may not be the same for all plies depending on the fiber orientations.

After ply-mechanics, students are introduced to macro-mechanics, where a composite laminate could be subjected to distributed stretching load [N] and moment [M]. Hence, the laminate will offer in-plane extension or stretching $\{\varepsilon^o\}$ and curvature $\{\kappa\}$. The constitutive equation relating the loads [N] and [M] with resulting $\{\varepsilon^o\}$ and $\{\kappa\}$ is as follows:

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \tag{1}$$

Here, the [A] matrix is called an in-plane stiffness matrix, as it directly relates the in-plane extension or stretching $\{\varepsilon^o\}$ to in-plane stretching loads $\{N\}$. The [D] matrix is called bending stiffness matrix, as it directly relates curvature {κ} to bending moments {M}. The [B] matrix is called a bending-extension coupling matrix, as it relates the in-plane strains to bending moments and curvatures to in-plane stretching loads. The mathematical equations needed to compute the [A], [B] and [D] matrices are as follows. In these equations, *k* represents the ply numbers as shown in **Figure 2**.

$$A_{ij} = \sum_{k=1}^{n} Q_{b,ij} t_k \tag{2}$$

$$B_{ij} = -\sum_{k=1}^{n} Q_{b,ij} t_k z_{b,k} \tag{3}$$

$$D_{ij} = \sum_{k=1}^{n} Q_{b,ij} \left(t_k z_{b,k}^2 + \frac{t_k^3}{12}\right) \tag{4}$$

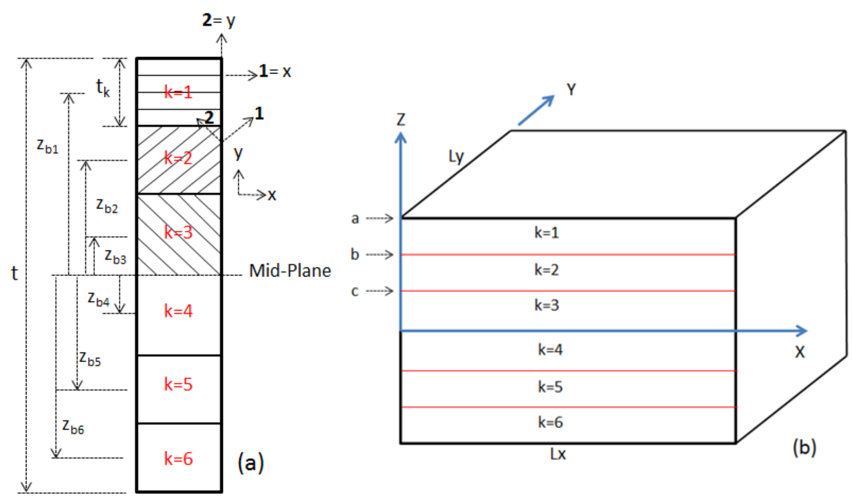


Figure 2: Lamina Stacking in a Composite Laminate.

For symmetric laminates, there is no coupling effect. Hence, the [B] matrix becomes zero. Therefore, the in-plane extensions {ε^o} and curvatures {κ} could be determined separately by [A] and [D] matrices using the following equations:

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \left[A_{ij}\right]^{-1} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \text{ and } \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \left[D_{ij}\right]^{-1} \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} \tag{5}$$

However, the [B] matrix is not equal to zero for any non-symmetric laminate. Therefore, students need to use the generalized Equation (1) to compute the in-plane extensions {ε^o} and curvatures {κ}. Once the {ε^o} and {κ} are known, then the total strain {ε} at any desired location (z) along the thickness direction can be determined. Finally, we can use the total strain (ε) to compute the global stresses using the [Q_b] matrix as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{b,11} & Q_{b,12} & 0 \\ Q_{b,12} & Q_{b,22} & 0 \\ 0 & 0 & Q_{b,66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}; where \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} - z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{6}$$

Once the global stresses are known, as defined above, the local stresses (usually, defined by σ_1, σ_2 and σ_6) can be determined by using the transformation matrix, [T].

In this project, students determined the [A] and [D] matrices first. Then, they were asked to deermine a set of equivalent laminate moduli for balanced symmetric laminates. The equivalent *in-plane* moduli along the x- and y- directions are defined as

$$E_x = \frac{A_{11}A_{22}-A_{12}^2}{tA_{22}} \approx \frac{1}{t\alpha_{11}} \text{ and } E_y = \frac{A_{11}A_{22}-A_{12}^2}{tA_{11}} \approx \frac{1}{t\alpha_{22}} \tag{7}$$

Here, the A_{ij} represents the components of the in-plane stiffness matrix, and the *t* represents the overall thickness of the laminate. The matrix [α] represents the inverse of [A] matrix. When the laminate is subjected to in-plane load [N], then the induced stresses depend on in-plane laminate moduli E_x and E_y. Similarly, the equivalent *bending* moduli in the x- and y- directions are defined as

$$E_{bx} = \frac{12(D_{11}D_{22}-D_{12}^2)}{t^3D_{22}} \approx \frac{12}{t^3\delta_{11}} \text{ and } E_{by} = \frac{12(D_{11}D_{22}-D_{12}^2)}{t^3D_{11}} \approx \frac{12}{t^3\delta_{22}} \tag{8}$$

Here, D_{ij} represents different coefficients of the bending stiffness matrix. The matrix [δ] represents the inverse of [D] matrix. When the laminate is subjected to bending load [M], then the induced stresses depend on bending moduli E_{bx} and E_{by}. Students learned all these math equations before diving into the project using MATLAB and ACP.

Student Learning: Exposure to MATLAB and ACP

The elasticity of a material can be defined as its ability to resist deformation under an applied load. The LSS can change the magnitude of the laminate stiffness; hence this was analyzed first. Students used MATLAB to investigate the effects of LSS on the resulting in-plane [A] and bending [D] stiffness matrices of a symmetric quasi-isotropic laminate defined by $[0/60/-60]_s$. From the [A] and [D] matrices, the in-plane and bending moduli were determined using the Equations (7) and (8). Then, the class studied how the in-plane and bending moduli affect the induced stresses at different locations along the thickness direction by computing the stresses at points *a*, *b* and *c*, as shown in **Figure 2(b)**.

The MATLAB simulation used the material properties of AS4/3501-6 unidirectional carbon fiber composite with E_1 = 126 GPa, E_2 = 11 GPa, G_{12} = 6.6 GPa, ν_{12} = 0.28, and ply thickness (t_k) = 2 mm. The laminate in-plane moduli (E_x and E_y) and bending moduli ($E^b{}_x$ and $E^b{}_y$) are shown in **Table 1** for different LSS, where the fiber orientations were kept the same for all laminates.

Table 1: Comparison of In-plane and Bending Moduli for Different LSS.

LSS	E_x (MPa)	E_y (MPa)	E_{bx} (MPa)	E_{by} (MPa)
$[0/60/-60]_s$	*5.11 x*10^4	5.11 x10^4	*9.26 x*10^4	2.52 x 10^4
$[0/-60/60]_s$	*5.11 x*10^4	5.11 x10^4	*9.26 x* 10^4	2.52 x 10^4
$[60/0/-60]_s$	*5.11 x*10^4	5.11 x10^4	*4.24 x* 10^4	3.17 x 10^4
$[-60/0/60]_s$	*5.11 x*10^4	5.11 x10^4	*4.24 x* 10^4	3.17 x 10^4
$[60/-60/0]_s$	*5.11 x*10^4	5.11 x10^4	*1.77 x* 10^4	5.08 x 10^4
$[-60/60/0]_s$	*5.11 x*10^4	5.11 x10^4	*1.77 x* 10^4	5.08 x 10^4

The in-plane moduli (E_x and E_y) were the same for different LSS, as shown in **Table 1**. The in-plane stiffness resists the extension (or compression) when the laminate is subjected to in-plane stretching loads. This analogy is shown in **Figure 3(a)** and **Figure 3(b)**, where two springs of unequal stiffness are connected in parallel with a different stacking sequence, but subjected to the same in-plane stretching load, N. Both configurations should offer the same in-plane stiffness. Hence, the deformation of each spring (or ply or lamina) would be the same regardless of LSS.

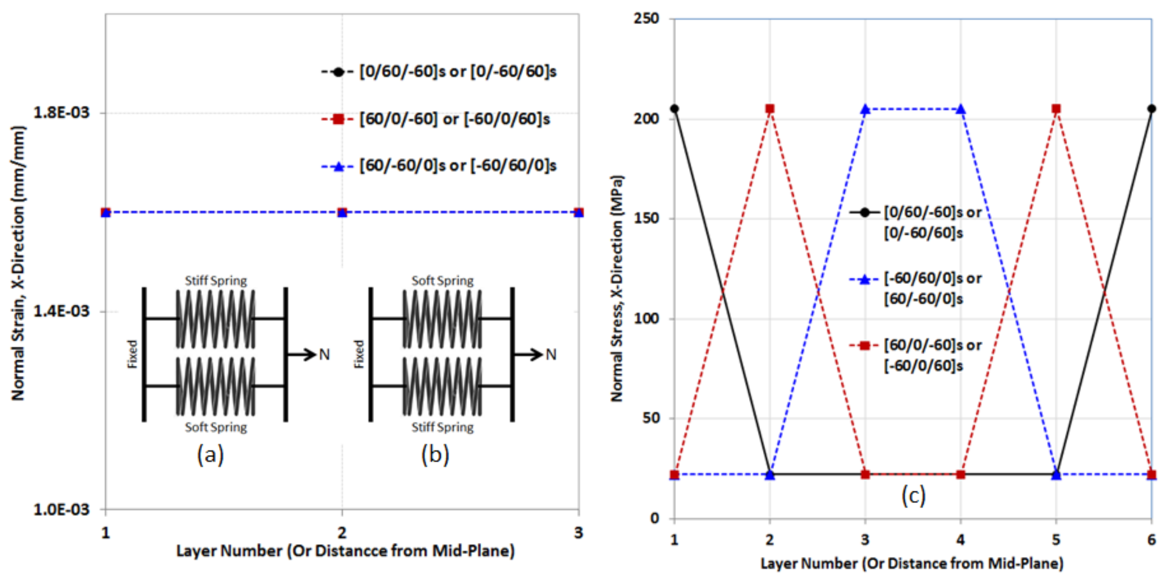


Figure 3: Normal Strain and Normal Stress of $[0/60/-60]_s$ Laminate for Different LSS, Subjected to In-plane Stretching Load (N_x).

After determining the E_x and E_y, we applied an in-plane distributed stretching load (N_x = 1,000 N/mm) along the global x-direction. As the laminate offers the same in-plane stiffness, the normal strain along the x-direction (ε_x), also called mid-plane strain or stretching, for all plies was found to be the same for different LSS. This is also shown in **Figure 3**. The strain magnitude and sign will be the same for all laminae (or plies) located above or below from the mid-plane.

However, stress induced in each lamina depends on the corresponding lamina moduli and strain. This can easily be explained by the well-known Hooke's Law, which is $\sigma = E \times \varepsilon$. Students noticed a few important takeaways from this analysis, which are listed below.

- If the laminate is subjected to an in-plane stretching load along the global x-direction, then the [0] degree lamina will always take the maximum load, compared to the other fiber orientations. Hence, the normal stress along the x- direction (σ_x) will always be the maximum for the [0] degree lamina for all possible LSS configurations, as shown in **Figure 3(c)**.
- The maximum (or peak) stress will also be the same for different LSS, as also shown in **Figure 3(c)**.
- The [60] and [-60] plies will also take the same amount of load. Hence, the stress σ_x will also be the same for the [60] and [-60] plies, as also shown in **Figure 3(c)**.

Composite laminates offer curvature when they are subjected to a bending moment. In this case, one side of the laminate will be in tension while the other side will be in compression. Hence, there will be an optimal LSS that will offer the maximum resistance to deformation. This analogy is shown in **Figure 4(a)** and **Figure 4(b)**, where two springs of unequal stiffness are connected in parallel with a different stacking sequence, but subjected to the same bending load, M.

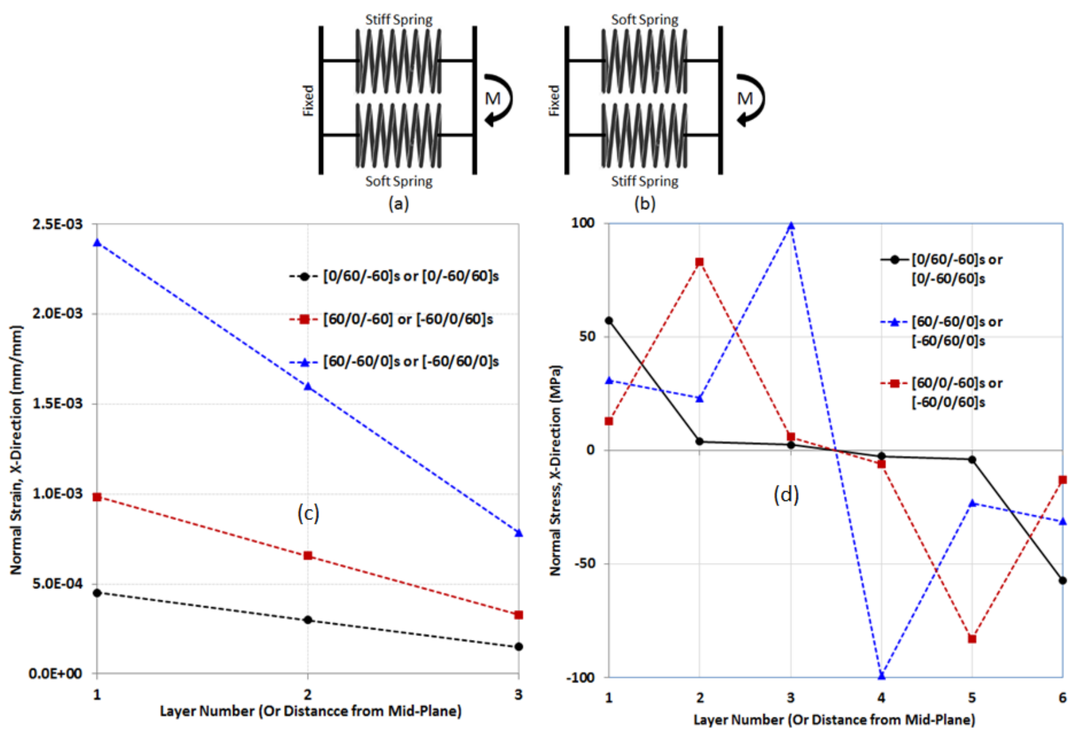


Figure 4: Normal Strain and Normal Stress of $[0/60/-60]_s$ Laminate for Different LSS, Subjected to Distributed Bending Load $[M_x]$.

The spring configuration (a) will offer higher bending stiffness compared to the configuration (b), as the stiffer spring is placed at the top. Hence, the bending moduli about x-axis (E_{bx}) were found to be the maximum for the $[0/60/60]_s$ laminate, followed by $[60/0/-60]_s$ and $[60/-60/0]_s$ laminates, as shown in **Table 1**. In other words, the (E_{bx}) was found to be the maximum when the [0] degree lamina was located at the top. And the (E_{bx}) was found to be the minimum when the [0] degree lamina was located at the center.

Then, students were asked to think about the $[0/60/-60]_s$ vs. $[0/-60/60]_s$ laminates. For these two laminates, the bending moduli (E_{bx}) and (E_{by}) were found to be the same, as expected. In other words, the [60-degree] fiber orientation either at clockwise (+) or counterclockwise (-) does not affect the E_{bx} and E_{by}. A similar analogy was explained for the [60/0/-60] vs. [-60/0/60] and [60/-60/0] vs. [-60/60/0] laminates.

After determining the bending moduli, the students applied a distributed moment load (M_x = -1,000 N.mm/mm) to the laminates to analyze the strain and stress distributions. Students also noticed a few major takeaways from this analysis, which are listed below.

- As shown in Figure 4(c), under bending load, the normal strain along the x-direction (ε_x) was found to be the lowest for the $[0/60/-60]_s$ laminate, as the [0] lamina acts as the strongest spring compared to other fiber orientations.
- Under bending load, the maximum strain (ε_x) occurred for the top-most layer regardless of the LSS configuration. This is due to the maximum effective moment acting at the top-most layer, which is also shown in **Figure 4(c)**. The strain magnitude will be the same but with opposite signs for the layers located at the same distance above or below from the *mid-plane*.
- As mentioned before, stress induced in each lamina depends on the corresponding lamina moduli and strain. As we applied bending moment about the x-direction, the [0] degree lamina takes the maximum load, and subsequently offered the highest stress compared to the other fiber orientations. This is shown in **Figure 4(d)** for different LSS configurations.
- The **Figure 4(d)** also reveals that the magnitude of peak (or maximum) stress is different for different LSS. The peak stress was found to be the lowest for the $[0/60/-60]_s$ laminate followed by $[60/0/-60]_s$ and $[60/-60/0]_s$ laminates. Therefore, students realized that if the laminate is subjected to a bending load about x-direction, then the top-most layer must have [0] degree fiber orientation to reduce the stress.

After the MATLAB analyses, students repeated these numerical simulations using the FEA software ACP. The FEA model represented the $[0/60/-60]_s$ laminate with a side dimension of (1m x 1m = 1,000 mm x 1,000 mm). The thickness (t_k) of each ply was 2 mm, as used in the MATLAB. We used the material properties of AS4/3501-6 unidirectional carbon fiber composite that are mentioned before. The left and bottom sides of the model were attached to a roller-boundary conditions. The right side of the model was subjected to an in-plane stretching load, N. Students applied different magnitude for N, such as N = 100 kN or N = 1,000 kN. There were two teaching objectives here, as listed below.

- In the context of macro-mechanics, the symbol N_x represents the *distributed* in-plane stretching load along the x-direction. The unit of N_x can be (N/mm) or (lb/inch). Similarly, the symbol M_x represents a *distributed* moment load acting on a plane that is perpendicular to the x-axis. The unit of M_x can be (N.mm/mm) or (lb.inch/inch). So, if we applied N = 100 kN to our ACP model, then it actually represents N_x = (100 x 1000 N)/1000 mm = 100 N/mm. Similarly, the applied load N = 1,000 kN to the ACP model actually represents the N_x =1000 N/mm. Our students had some difficulties to conceive the physical meaning of N_x and M_x that they used in MATLAB codes. Hence, we provided additional efforts to facilitate this concept to our students.

- The second learning objective was to expose students to the concept of linear simulation. For example, if the FEA model offers the maximum deformation of 0.158 mm for N_x = 100 N/mm, then it must offer the maximum deformation of 1.58 mm for N_x = 1,000 N/mm. Students realized that all the output will vary linearly with the applied load. In other words, students got opportunities to understand the *Theory of Superposition* to differentiate the linear vs. nonlinear analysis.

The FEA model with meshing and the associated boundary conditions is shown in **Figure 5**. Then, the numerical simulations were repeated for different LSS, subjected to N_x = 100 N/mm and N_x = 1,000 N/mm. These results are shown in **Tables 2** and **3**. The contour plots of σ_x are shown in **Figures 6 and 7**. The key learning outcomes are as follows:

- Students compared the FEA results with the corresponding MATLAB solutions. For example, **Figure 3** represents the MATLAB stress outputs for the $[0/60/\text{-}60]_s$ laminate for N_x = 1,000 N/mm. These outputs are then compared with the corresponding FEA results as shown in **Table 3** and **Figure 7**. Students witnessed that a strong agreement was found between MATLAB and FEA.

- Students learned the concept of linear analysis. All the results such as deformation and normal stress were changed linearly with the applied load for the FEA models. For example, if the applied load was 10 times higher, then all the deformations and stresses were also increased by 10 times.

- In both MATLAB and FEA, students learned that the LSS does not change the in-plane stiffness. Hence, the total deformation was found to be the same for different LSS subjected to stretching load, N_x. As the N_x represents the in-plane load along the global x-axis, the [0] degree lamina carries the maximum load. Hence, the [0] degree lamina offered the maximum normal stress, σ_x regardless of LSS. These results are shown in **Tables 2** and **3**.

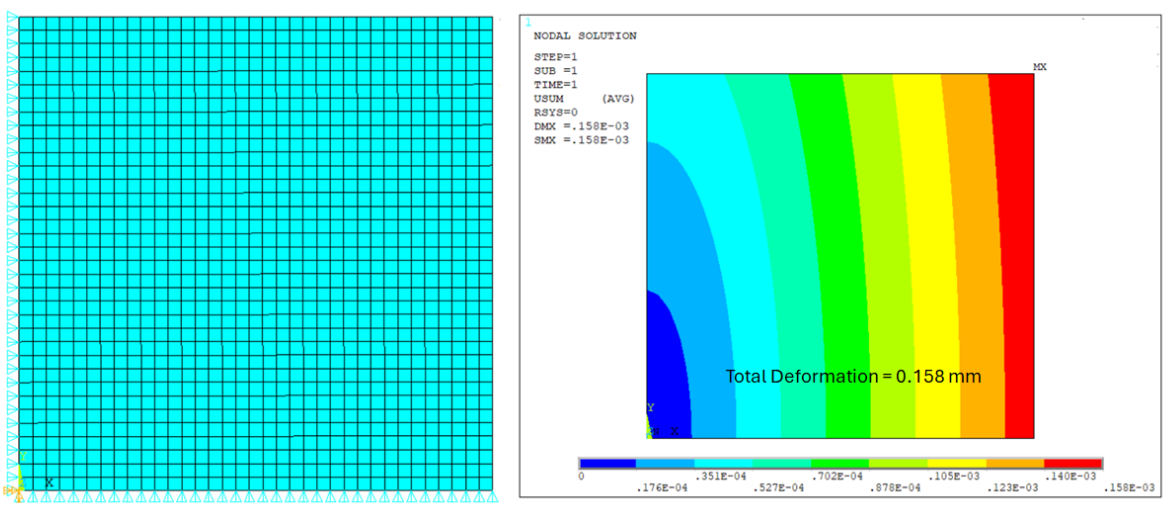


Figure 5: FEA Model with Meshing and the Total Deformation for Loading N_x = 100 N/mm.

Table 2: Comparison of Total Deformation and σ_x for Different LSS, for N_x = 100 N/mm.

LSS	Total Def	σ_x at Ply 1	σ_x at Ply 2	σ_x at Ply 3
$[0/60/-60]_s$	0.158 mm	20.6 MPa, at $\theta = 0^o$	2.2 MPa, at $\theta = 60^o$	2.2 MPa, at $\theta = 60^o$
$[60/-60/0]_s$	0.158 mm	2.2 MPa, at $\theta = 60^o$	2.2 MPa, at $\theta = -60^o$	20.6 MPa, at $\theta = 0^o$
$[-60/0/60]_s$	0.158 mm	2.2 MPa, at $\theta = -60^o$	20.6 MPa, at $\theta = 0^o$	2.2 MPa, at $\theta = 60^o$

Table 3: Comparison of Total Deformation and σ_x for Different LSS, for N_x = 1,000 N/mm.

LSS	Total Def	σ_x at Ply 1	σ_x at Ply 2	σ_x at Ply 3
$[0/60/-60]_s$	1.58 mm	206 MPa, at $\theta = 0^o$	22 MPa, at $\theta = 60^o$	22 MPa, at $\theta = 60^o$
$[60/-60/0]_s$	1.58 mm	22 MPa, at $\theta = 60^o$	22 MPa, at $\theta = -60^o$	206 MPa, at $\theta = 0^o$
$[-60/0/60]_s$	1.58 mm	22 MPa, at $\theta = -60^o$	206 MPa, at $\theta = 0^o$	22 MPa, at $\theta = 60^o$

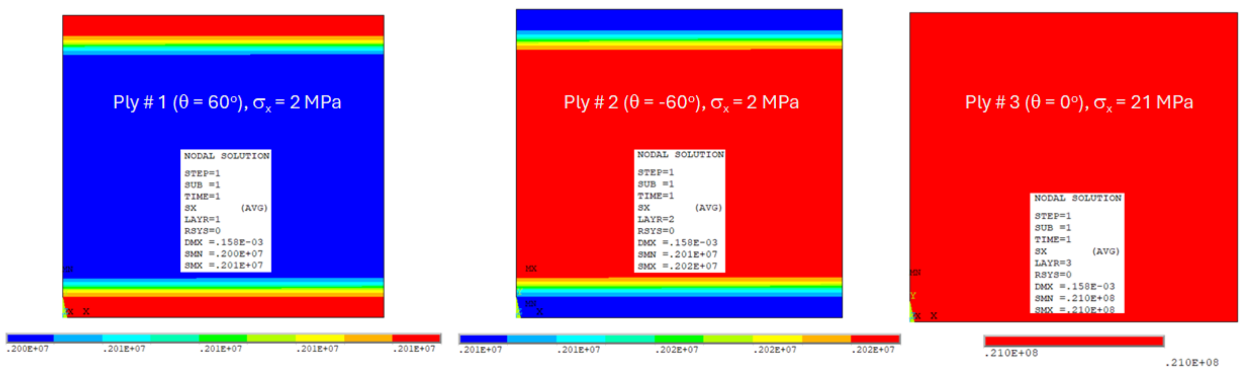


Figure 6: FEA Model, Normal Stress (σ_x) at Different Plies of $[60/-60/0]_s$ Laminate for N_x = 100 N/mm.

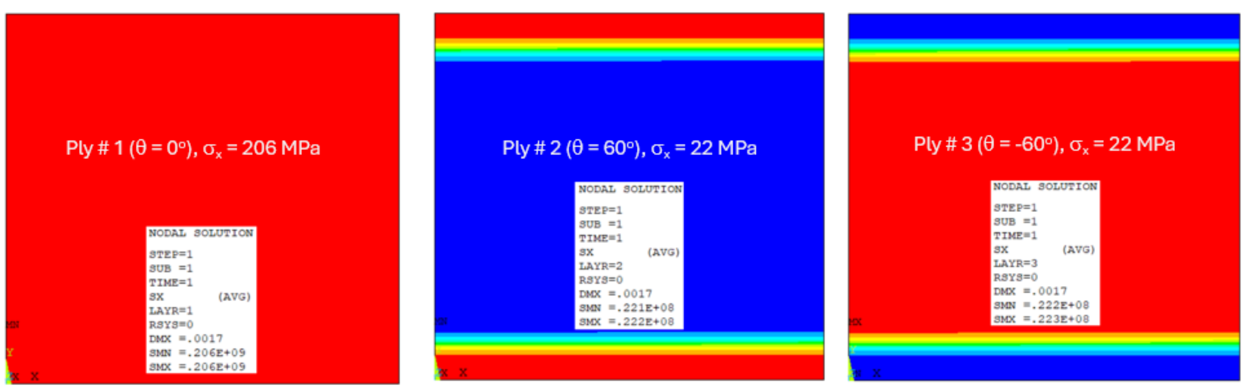
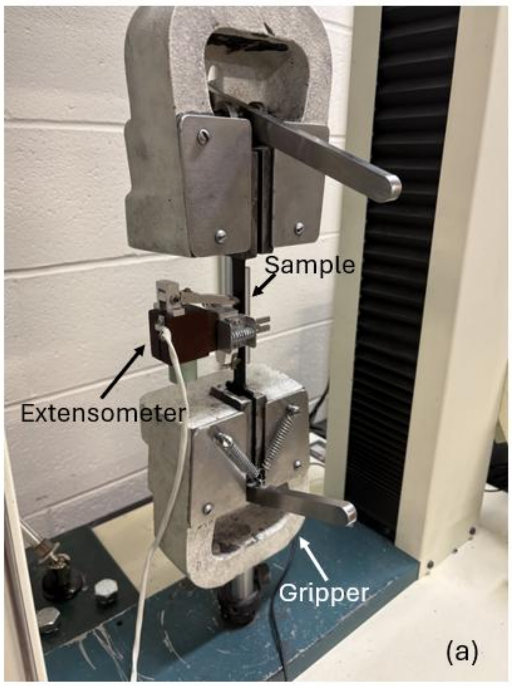


Figure 7: FEA Model, Normal Stress (σ_x) at Different Plies of $[60/-60/0]_s$ Laminate for N_x = 1,000 N/mm.

Student Learning: Exposure to Experiments

In this project, students fabricated some 3D printed samples with the PETG (Polyethylene Terephthalate Glycol) filament. They produced 4 types of samples defined by $[0]_6$, $[0/60/-60]_s$, $[60/-60/0]_s$ and $[60/0/-60]_s$. The $[0]_6$ sample represents 6 plies, where filaments are oriented along [0] degree for all plies. The $[0/60/-60]_s$ sample also represents the 6 plies, where filaments are oriented along [0], [60] and [-60] degrees. Similar analogy for the $[60/-60/0]_s$ and $[60/0/-60]_s$ samples.

The experiments posed several challenges for students to overcome. First, the samples must be thick enough so that a typical tensile tester jaw (or gripper) can grip the samples without slippage. In that case, the number of plies during the sample fabrication needed to increase; however, that increases the overall cost. Second, the samples should be placed properly so that they are aligned along the loading direction. Any misalignment will provide inaccurate data. A typical tensile test apparatus that students used is shown in **Figure 8 (a)**.



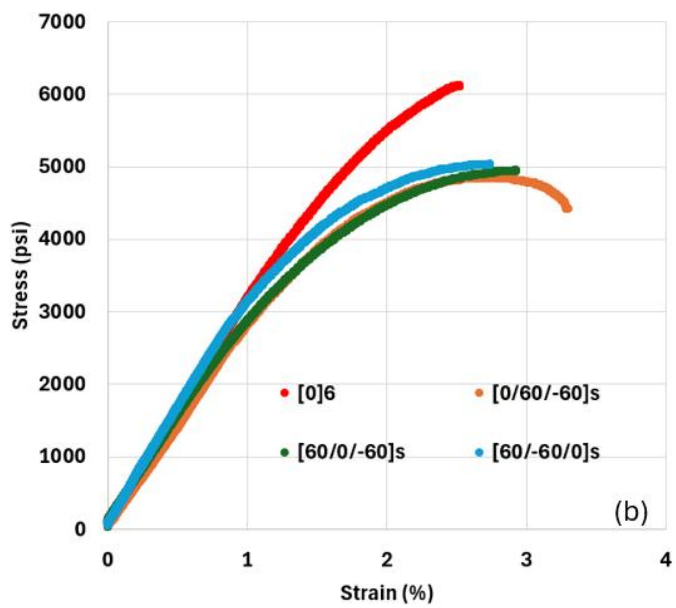


Figure 8: Experimental Response of 3D Printed Samples with Different LSS.

Due to limited resources, the students made 3 samples of each type that are mentioned above. Then, students performed the tensile test using a Tinius Olsen Universal Tensile Machine, which is available at our Materials Lab. The average results from these samples are shown in **Figure 8(b)**. Students learned that the $[0]_6$ sample took relatively more load, compared to the other samples, as all the plies are oriented along the [0] degree. As the LSS does not impact the in-plane stiffness [A] and the associated in-plane moduli (E_x and E_y), the experimental response of other samples, such as $[0/60/-60]_s$, $[60/-60/0]_s$ and $[60/0/-60]_s$ were found to be almost the same. These experimental responses were found to be intuitively correct based on the theoretical knowledge that students learned in class.

Students also faced some challenges conducting the 3-point bending experiments. As mentioned before, the fabricated 3D printed samples using the PETG only used 6 plies. The PETG filament is very flexible, and the samples were relatively thin. Hence, the bending experiment did not go well as expected. During the bending experiment, all the samples flexed excessively, and students were not able break them. To resolve this problem, the future plan is to fabricate the 3D printed samples with PETG-Carbon filament. This filament is made of PETG mixed with short carbon fibers, resulting in a material that is stronger and stiffer than standard PETG under bending load. Another future plan is to increase the number of plies. For example, we can use 12 plies, instead of 6 plies, where the samples can be defined by $[0]_{12}$, $[0_2/60_2/-60_2]_s$, $[60_2/-60_2/0_2]_s$ and $[60_2/0_2/-60_2]_s$. As the samples become relatively thick and stiffer, they will be easier to break under bending load without difficulties. These refined experiments will help students visualize the effects of LSS under bending load.

Student Assessment

In this class, we attempt to accomplish several Student Learning Outcomes (SLOs), which are listed below for the completeness of this paper. The SLOs are clear, concise statements about the knowledge, skills, values, and attitudes that students are expected to achieve after successfully completing a course.

1) Comprehend different standard terminologies (e.g., fiber, matrix/resin, lamina, laminae, laminate, hybrid, and others) related to composite materials.
2) Good understanding of composite constituents such as continuous fibers, discontinuous fibers, inorganic fibers, organic fibers, thermoset matrices, thermoplastic matrices and others.
3) Good understanding of composite manufacturing processes such as Hand Lay-up, Prepreg Lay-up, Vacuum Bag Molding, Autoclave, Compression Molding, Resin Transfer Molding, Vacuum Assist Resin Transfer Molding (VARTM), Pultrusion and Filament Winding.
4) Apply the concept of Micromechanics to predict mechanical properties including Longitudinal Modulus, Transverse Modulus, Shear Modulus, Poisson's ratio, Longitudinal Strength, and Transverse Strength.
5) Apply the concept of Ply Mechanics to develop the constitutive equations of a lamina, and forming the compliance, stiffness, and transformation matrices.
6) Apply the concept of Macro Mechanics to analyze the structural response of laminated composites subjected to in-plane loads and moments.
7) Apply the concept of micro, ply, and macro mechanics to predict failure of laminated composites subjected to in-plane loads and moments.

This pedagogical research that outlines the instructional activities is very much related with the SLO (5) and (6). At the end of the quarter, the students were asked to rate the following statements based on a 5-point scale, where 5 = Strongly Agree, 4 = Agree, 3 = Neutral, 2 = Disagree and 1 = Strongly Disagree. We had 19 students enrolled in this class during the Winter 2025 quarter. The assessment survey is shown in Figure 9. It seems evident that the entire class was very satisfied with this project.

1) *I understand the concept of Lamina Stacking Sequence (LSS). In other words, I understand how LSS affects the in-plane stiffness, [A] and bending stiffness, [D].*
   Average rating is 4.84. Most of the students "Strongly Agreed" with this question.

2) *I understand how to compute the laminate in-plane moduli (E_x and E_y) and bending moduli (E_{bx} and E_{by}) from the [A] and [D] matrices, respectively.*
   Average rating is 4.84. Most of the students "Strongly Agreed" with this question.

3) *The application of MATLAB to determine the [A], [D], E_x, E_y, E_{bx}, and E_{by} for different LSS enhanced my overall understanding of macro-mechanics of composite laminates.*
Average rating is 4.79. Most of the students "Strongly Agreed" with this question.

4) *The application of ACP (ANSYS Composite Prep-Post) to determine the [A], [D], E_x, E_y, E_{bx}, and E_{by} for different LSS enhanced my overall understanding of macro-mechanics of composite laminates.*
Average rating is 4.63. Most of the students "Strongly Agreed" with this question. Students were not required to know about the FEA software for this class. Hence, a few students had difficulties using the ACP module.

5) *I understand how the LSS would impact the structural response of a composite laminate subjected to in-plane stretching and bending loads?*
Average rating is 4.79. Most of the students "Strongly Agreed" with this question.

6) *I enjoyed the hands on experience with the 3D printed samples with different LSS. The experimental data under the tensile test was found to be intuitively correct when compared to MATLAB and ACP results.*
Average rating is 4.90. Most of the students "Strongly Agreed" with this question.

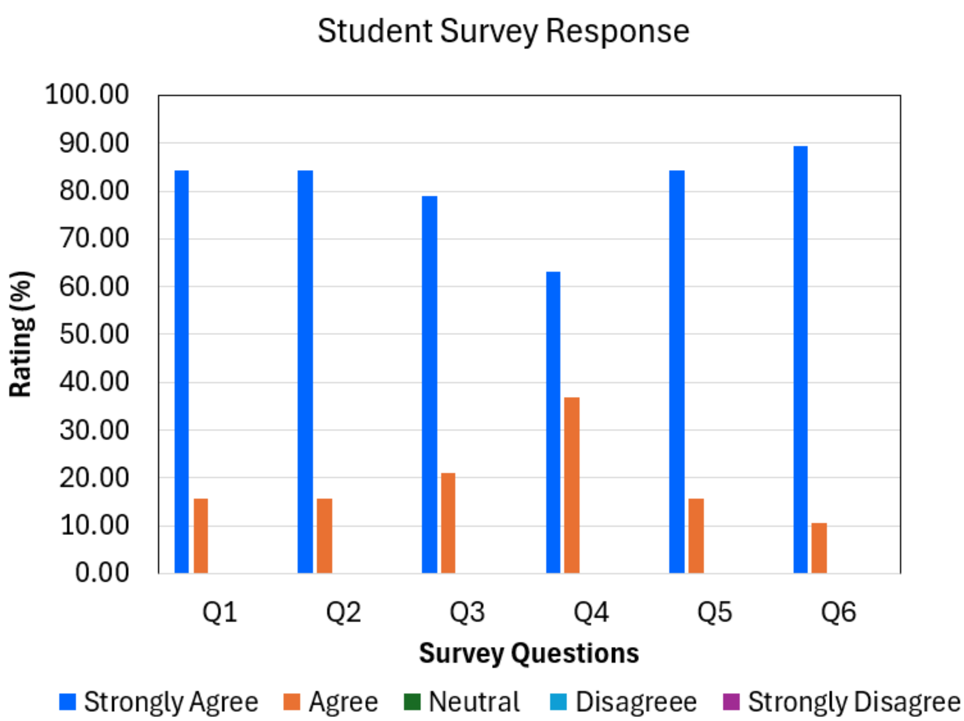


Figure 9: Student Survey to Assess the Learning Outcomes.

Conclusion

The LSS and fiber orientations affect the structural response of any composite laminates subjected to in-plane stretching and bending loads. However, this concept is lightly touched on by standard textbooks available to students and educators. Therefore, we attempted to teach the effects of LSS on the structural responses of symmetric laminates by different methodologies. These methodologies included standard lectures and then solving macro-mechanics problems using computational tools, such as MATLAB and ACP. To strengthen the overall concept of LSS, we also performed some limited experiments using 3D printed samples. Based on our overall efforts, students gained a better understanding of the general concept of LSS and its effects on the composite laminates subjected to in-plane stretching and bending loads. This is evident from the assessment data completed by our students. In the future, the authors plan to add additional experiments where the 3D printed composite samples with different LSS can be tested properly under bending load. These hands-on experiences engage students to learn several core concepts of composite materials with increased understanding.

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