2026 ASEE ANNUAL CONFERENCE June 21–24, 2026 | Charlotte Convention Center, NC

Host Sponsor UNC Charlotte

Paper ID #49990

Vectors to Diagrams to Component Equations: A Step-by-Step Process to Draw and Use the Relative Velocity Diagram

Dr. Amir H. Danesh-Yazdi, Rose-Hulman Institute of Technology

Dr. Danesh-Yazdi is Associate Professor of Mechanical Engineering at Rose-Hulman Institute of Technology. His teaching and research interests include vibrations, heat transfer, and fluid mechanics. He is the co-author of two textbooks: Mechanical Vibrations: A State-Space Perspective (with Yi Wu) and Heat Conduction, 4th Edition (with Latif Jiji).

Vectors to Diagrams to Component Equations: A Step-by-Step Process to Draw and Use the Relative Velocity Diagram

Abstract

Students are taught a rigorous process in their Statics and Dynamics courses on how to draw proper kinetic and free-body diagrams to subsequently use to simplify Newton's and Euler's 2nd Law. However, when it comes to a topic many struggle with such as relative motion, most introductory physics, fluid mechanics, and dynamics texts do not provide detailed enough guidance for students to follow to obtain a correct and useful diagrammatic representation of this important concept. In this paper, a step-by-step process for drawing the relative velocity diagram for two particles is introduced and is then used to properly resolve the associated vector relation into its components. Three very different examples are provided to show the application of this diagrammatic approach. The value of incorporating the relative velocity diagram when solving comprehensive problems that involve relative motion is assessed through the evaluation of the performance of eight cohorts on two different assessments and their feedback in evaluations. A comparison of these cohorts suggests that integrating this step-by-step process into the lecture notes, as opposed to providing it through supplemental course material, may help improve student performance on exams.

Introduction and Motivation

In introductory courses such as Statics and Dynamics, there is general consensus among instructors that correct system selection and proper kinetic and free-body diagram development is essential because they provide an opportunity for the student to (i) visualize how the system is being affected by its interactions with the surroundings and to (ii) use the diagrams to simplify and solve the applicable equations. There is, however, less agreement on emphasizing other types of diagrams that may help students better understand and solve the problems they encounter, as outlined in a previous work [1]. Another such example is the relative velocity diagram (RVD).

All motion is defined *relative* to a reference frame and yet many students struggle with problems that involve *relative motion*, which is the term used to describe how two objects move with respect to one another from the perspective of two reference frames. Engineering students initially encounter this topic in introductory physics, but outside of occasionally appearing in courses such as Vibrations, Fluid Mechanics, and Turbomachinery, the most rigorous treatment of relative motion occurs in Dynamics. In virtually every popular dynamics textbook [2, 3, 4, 5, 6, 7, 8], relative motion is introduced as a subtopic of particle kinematics, where the relative velocity vector relation for particles A and B is presented as

$$\vec{V}_{\mathrm{A}} = \vec{V}_{\mathrm{A/B}} + \vec{V}_{\mathrm{B}}, \tag{1}$$

where $\vec{V}_{\mathrm{A}}$ and $\vec{V}_{\mathrm{B}}$ are the absolute (or relative to a stationary observer) velocities of particles A and B, respectively, and $\vec{V}_{\mathrm{A/B}}$ is the velocity of particle A *relative to* B. The RVD is simply the graphical representation of (1). Relative motion is revisited in each of these texts for the case of two points separated by a fixed distance $|\vec{r}_{\mathrm{A/B}}|$ on a rigid body experiencing rotation $\vec{\omega} = \omega\hat{k}$, for which (1) (and the RVD, as will be presented in this paper) still applies with $\vec{V}_{\mathrm{A/B}} = \vec{\omega} \times \vec{r}_{\mathrm{A/B}}$. In each text, this discussion is followed by the more complex case of a changing distance between points A and B over time.

The *diagrammatic* treatment of relative velocity varies in the seven dynamics textbooks reviewed by the author. Meriam and Kraige [2] present a step-by-step process on how to create the "velocity triangle" through examples, which is by far the most extensive of the texts reviewed. Their approach to drawing the RVD, however, is very different than the one that will be discussed in this paper as it utilizes the law of sines and cosines to establish the vector magnitudes in order. Beer and Johnston [3] also mention velocity triangles in their particle kinematics and general planar motion examples, but there is no process outlined on how these diagrams are constructed. In all but one of the remaining dynamics texts that were reviewed [4, 5, 6, 7], the authors show the RVD for one example involving particle kinematics, but make no mention of how to construct the diagram nor do they show it for any other example. Outside of the introductory definition of the relative velocity, Tongue and Kawano [8] do not show the RVD in any of their examples. There is very little difference in how three popular introductory physics texts [9, 10, 11] treat this topic: only the basic relative motion of particles is discussed and the RVD in the examples are shown without mentioning how they have been created. Similarly, there is virtually no difference in how relative velocity is treated in four popular fluid mechanics texts [12, 13, 14, 15]. Relative velocity is discussed without an RVD in the derivation of the Reynolds Transport Theorem for a moving or deforming control volume, but more significantly, the "velocity kinematic diagram" (read: RVD) makes several appearances in the examples of the turbomachinery chapter of each text, yet again without outlining a procedure on how to create them.

A review of papers in ASEE's PEER repository does not reveal a significant amount of prior publication on relative motion. Tang and Grigg [16] attributed student struggle with relative motion to the suboptimal selection of practice problems and insufficient supportive information and addressed this learning gap by utilizing the 4-Component Instructional Design framework. They encouraged a graphical representation of velocity and acceleration vectors, but not in the form of the RVD. Flori et al. [17] considered different methods for teaching relative velocity in their sophomore-level engineering dynamics course and indicated their preference for scalar methods for teaching planar kinematics, but since only the abstract of their paper is available, it is difficult to draw further conclusions from their work. Hassanzadeh Gorakhki et al. [18] studied the effect of flipping lectures on relative velocity and instantaneous centers of zero velocity on student performance. While they do not highlight how they go about presenting relative velocity, they mention employing a student activity involving Mindstorms Lego systems to manipulate and evaluate rigid-body motion systems. A graphical approach using Working Model [19] and a pseudographical technique [20] used to analyze the kinematics of linkages mention relative velocity, but do not expand on how the RVD is obtained.

The lack of a step-by-step process to generating this visual tool in textbooks and the literature coupled with the author's experience in observing student struggles with relative motion in the context of larger problems has motivated the exploration of a reasonably straightforward approach to drawing and subsequently utilizing the relative velocity diagram to correctly resolve the vector relation in (1) into components. This approach is highlighted in the next section.

A Simple Approach to Drawing a Correct Relative Velocity Diagram

Limiting the discussion to the one- or two-dimensional relative motion between particles (or points of a fixed length on a rigid body), the RVD will take one of two forms: three co-linear vectors (1D) or a triangle of vectors (2D), as shown in Figure 1.

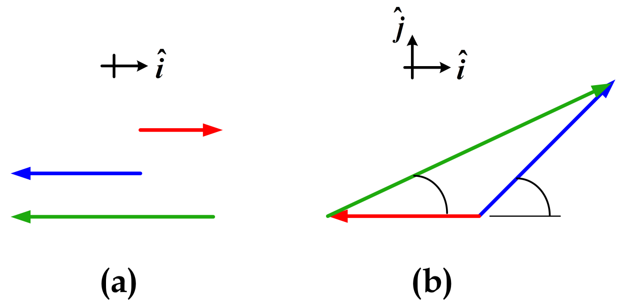

Figure 1 Sample (a) 1D and (b) 2D relative velocity diagrams

The series of steps that follow are presented to help any student develop the RVD, particularly for those who are newly introduced to this topic:

Step 1: Velocities with known direction For two particles A and B, there will always be three velocities involved in a relative velocity diagram: V_A, V_B, and $V_{\text{A/B}}$. The two vectors whose *direction* is known should be identified in this step.

Step 2: Relative velocity vector relationship The vector relation between the three velocities (1) should be solved for the vector whose direction is unknown. If, for example, the direction of the relative velocity is not known, (1) would be recast as

$$\vec{V}_{\text{A/B}} = \vec{V}_\text{A} - \vec{V}_\text{B} = \vec{V}_\text{A} + \left(-\vec{V}_\text{B}\right). \tag{2}$$

Step 3: Graphical implementation The equation established in *Step 2* should be graphically implemented through vector addition or subtraction. If, for example, the two vectors with known directions are being subtracted from one another, the graphical implementation would require that the subtracting vector be flipped and added to obtain the relative velocity, as stated in (2).

Step 4: Finalizing the RVD If any of the vectors with known directions have been flipped because of a vector subtraction, they should be switched back to their original form in *Step 1*. Otherwise, the RVD can be finalized by adding a coordinate system to the diagram.

Step 5: Using the RVD The vector relation established in *Step 2* can be resolved in terms of its components and according to the coordinate system from *Step 4*.

In the next section, three examples are provided to demonstrate the application of the relative velocity diagram as a necessary part of solving more comprehensive problems.

Examples

Example 1: Open system with moving bounds A rocket flies vertically upward with (absolute) velocity $V_{\mathrm{r}}(t)$. It ejects propellant of density ρ_{p} with an unknown (absolute) velocity $V_{\mathrm{p}}(t)$ and a mass flow rate $\dot{m}_{\mathrm{p}}(t)$ from a nozzle with an exit cross-sectional area A_{n}, as shown in Figure 2. The rocket with the fuel it contains has an initial mass $m_{\mathrm{sys,i}}$. Neglect the drag force. Determine an expression for

(a) the mass of the system as a function of time, $m_{\mathrm{sys}}(t)$, and

(b) the absolute velocity of the rocket as a function of time, $V_{\mathrm{r}}(t)$.

Simplify each expression assuming that the velocity of the propellant relative to the rocket remains constant, *i.e.*, $V_{\mathrm{p/r}}(t) = V_{\mathrm{p/r,0}}$:

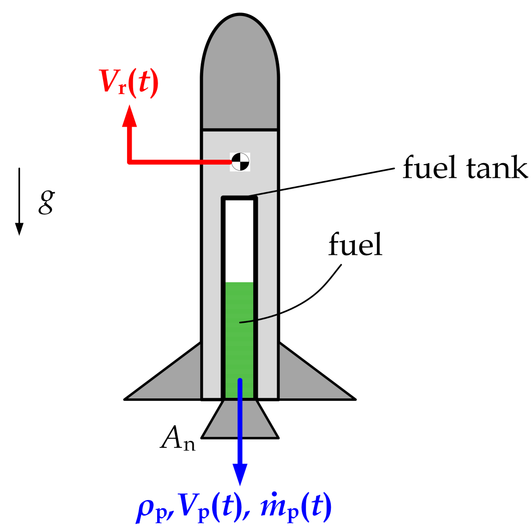

Figure 2 Example 1 - Rocket ejecting propellant to fly upwards

Example 1 Solution The system is the rocket and the propellant inside.

(a) The mass storage and interaction diagrams for this system are shown in Figure 3(a). Conservation of mass yields

$$\frac{\mathrm{d}m_{\mathrm{sys}}}{\mathrm{d}t} = \underbrace{\cancel{\sum_{\mathrm{in}} \dot{m}}}_{\text{no inflow}} - \sum_{\mathrm{out}} \dot{m}, \text{ or}$$

$$\frac{\mathrm{d}m_{\mathrm{sys}}}{\mathrm{d}t} = -\dot{m}_{\mathrm{p}}(t). \tag{3}$$

The propellant mass flow rate out of the rocket is defined as

$$\dot{m}_{\mathrm{p}}(t) = \int_A \rho_{\mathrm{p}} \left(\vec{V}_{\mathrm{p/r}}(t) \cdot \hat{n} \right) \, \mathrm{d}A, \tag{4}$$

where $\hat{n} = -\hat{j}$ is the unit normal associated with the cross-sectional area. Assuming the propellant density and velocity remain constant along the cross-section, the propellant mass flow rate reduces to

$$\dot{m}_{\mathrm{p}}(t) = \rho_{\mathrm{p}} V_{\mathrm{p/r}}(t) A_{\mathrm{n}}. \tag{5}$$

Substituting (5) into (3) and integrating over time results in

$$m_{\mathrm{sys}}(t) = m_{\mathrm{sys,i}} - \rho_{\mathrm{p}} A_{\mathrm{n}} \int_0^t V_{\mathrm{p/r}}(t) \, \mathrm{d}t. \tag{6}$$

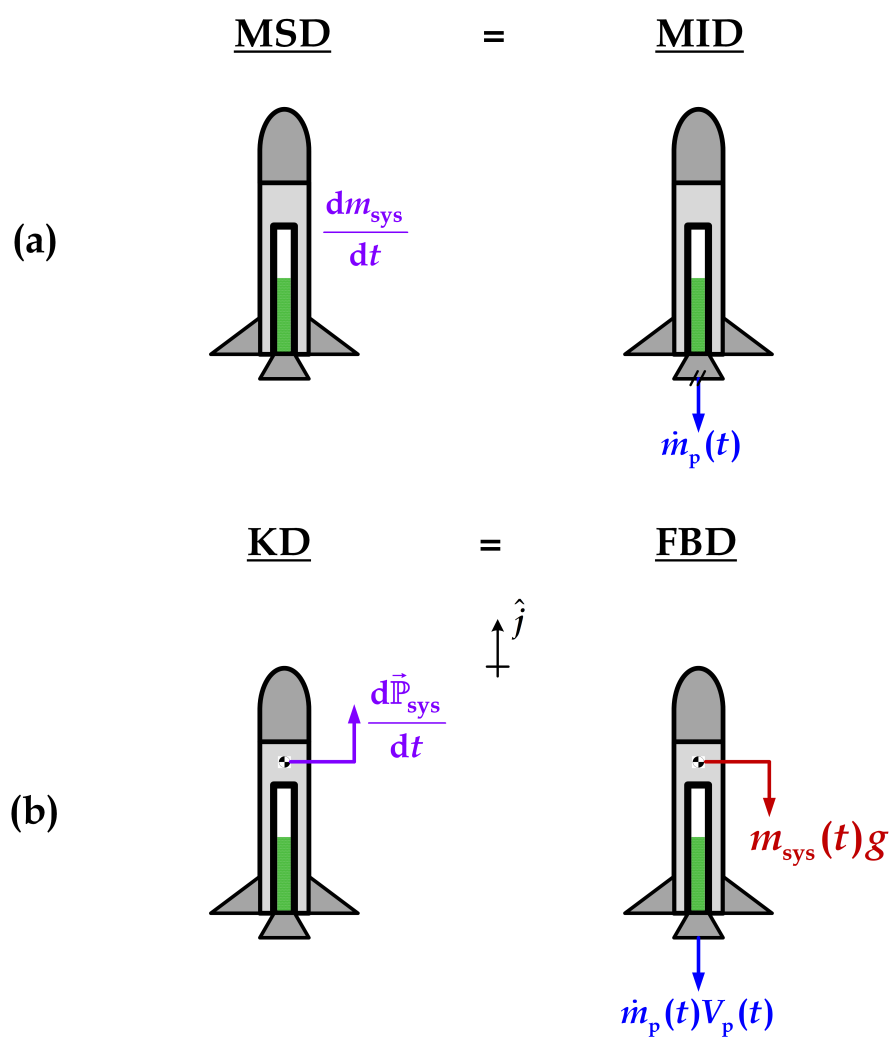

Figure 3 Example 1 (a) mass storage (MSD) and interaction (MID) diagram pairs and (b) kinetic (KD) and free-body diagram (FBD) pairs of the rocket and the propellant inside.

For a constant relative velocity,

$$m_{\text{sys}}(t) = m_{\text{sys,i}} - \rho_{\text{p}} V_{\text{p/r,0}} A_{\text{n}} t. \tag{7}$$

(b) The kinetic and free-body diagrams for this system are shown in Figure 3(b). Conservation of linear momentum (Newton's 2nd Law generalized for open systems) results in

$$\frac{\mathrm{d}\vec{\mathbb{P}}_{\text{sys}}}{\mathrm{d}t} = \sum_{\text{ext}} \vec{F} + \underbrace{\cancel{\sum_{\text{in}} \dot{m}\vec{V}}}_{\text{no inflow}} - \sum_{\text{out}} \dot{m}\vec{V}.$$

Resolving the vector relation in the $\hat{j}$ coordinate and using the definition of linear momentum leads to

$$m_{\text{sys}}(t)\frac{\mathrm{d}V_{\text{r}}}{\mathrm{d}t} + V_{\text{r}}(t)\frac{\mathrm{d}m_{\text{sys}}}{\mathrm{d}t} = -m_{\text{sys}}(t)g - \dot{m}_{\text{p}}(t)\left(-V_{\text{p}}(t)\right). \tag{8}$$

Substituting (3) and (5) into (8) and solving for the acceleration yields

$$\frac{\mathrm{d}V_{\text{r}}}{\mathrm{d}t} = \frac{\rho_{\text{p}} V_{\text{p/r}}(t) A_{\text{n}}}{m_{\text{sys}}(t)}\left(V_{\text{p}}(t) + V_{\text{r}}(t)\right) - g. \tag{9}$$

Equation (9) can further be simplified by expressing the absolute velocities of the propellant and the rocket in terms of the propellant velocity relative to the rocket. As shown in *Step 1* of Figure 4, the direction of the two absolute velocities is known. In *Step 2*, the vector relation between the absolute and relative velocities is expressed in terms of the velocity whose direction is unknown, which is $V_{\text{p/r}}(t)$ in this example:

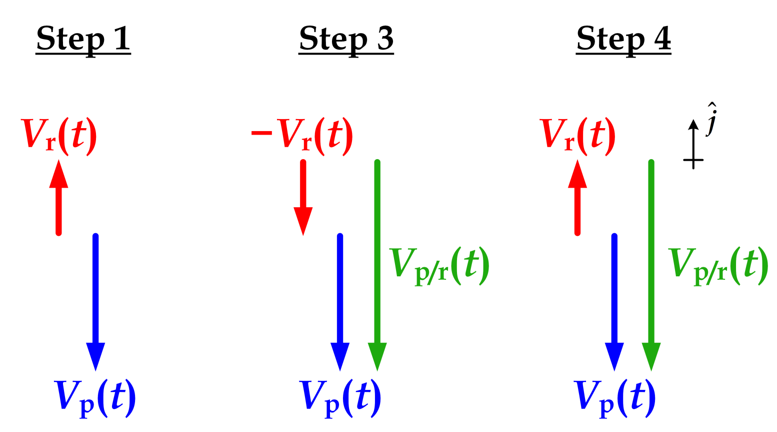

Figure 4 Steps 1, 3, and 4 of the process to correctly determine the relative velocity diagram for Example 1.

$$\vec{V}_{\text{p/r}}(t) = \vec{V}_{\text{p}}(t) - \vec{V}_{\text{r}}(t) = \vec{V}_{\text{p}}(t) + \left(-\vec{V}_{\text{r}}(t)\right). \tag{10}$$

The direction of the velocity of the propellant relative to the rocket is graphically determined in *Step 3* of Figure 4 with the final relative velocity diagram shown in *Step 4*. From the RVD, (10) can be resolved in the $\hat{\jmath}$-direction as follows:

$$\left[-V_{\mathrm{p/r}}(t)\right] = \left[-V_{\mathrm{p}}(t)\right] - \left[+V_{\mathrm{r}}(t)\right], \text{ or}$$

$$V_{\mathrm{p/r}}(t) = V_{\mathrm{p}}(t) + V_{\mathrm{r}}(t). \tag{11}$$

Substituting (11) into (9) and simplifying results in

$$\frac{\mathrm{d}V_{\mathrm{r}}}{\mathrm{d}t} = \frac{\rho_{\mathrm{p}}\left[V_{\mathrm{p/r}}(t)\right]^2 A_{\mathrm{n}}}{m_{\mathrm{sys}}(t)} - g. \tag{12}$$

For a constant relative velocity,

$$\frac{\mathrm{d}V_{\mathrm{r}}}{\mathrm{d}t} = \frac{\rho_{\mathrm{p}} V_{\mathrm{p/r,0}}^2 A_{\mathrm{n}}}{m_{\mathrm{sys,i}} - \rho_{\mathrm{p}} V_{\mathrm{p/r,0}} A_{\mathrm{n}} t} - g. \tag{13}$$

Assuming the rocket is initially at rest, integrating (13) over time leads to

$$V_{\mathrm{r}}(t) = V_{\mathrm{p/r,0}} \ln\left[\frac{m_{\mathrm{sys,i}}}{m_{\mathrm{sys,i}} - \rho_{\mathrm{p}} V_{\mathrm{p/r,0}} A_{\mathrm{n}} t}\right] - gt. \tag{14}$$

In summary,

$$\boxed{\begin{aligned} m_{\mathrm{sys}}(t) &= m_{\mathrm{sys,i}} - \rho_{\mathrm{p}} V_{\mathrm{p/r,0}} A_{\mathrm{n}} t \\ V_{\mathrm{r}}(t) &= V_{\mathrm{p/r,0}} \ln\left[\frac{m_{\mathrm{sys,i}}}{m_{\mathrm{sys,i}} - \rho_{\mathrm{p}} V_{\mathrm{p/r,0}} A_{\mathrm{n}} t}\right] - gt \end{aligned}}$$

This example demonstrates the usefulness of the RVD in dealing with open systems with moving boundaries where the velocity of the fluid relative to the boundary is of importance both within the definition of the mass flow rate term and especially in the conservation of linear momentum. If the magnitude of the relative velocity had been incorrectly obtained in (11), the absolute velocities of the propellant and rocket would have remained in (12), resulting in a problem that would not have been solvable without further information. Next, a problem involving the multiple moving elements will be considered.

EXAMPLE 2: CLOSED SYSTEM WITH MULTIPLE MOVING ELEMENTS On a dare, a golfer is challenged to hit a ball from the top of a moving car as shown in Figure 5. If the car moves at a constant velocity V_{C} to the left, what initial velocity with respect to the car, $V_{\mathrm{B,i/C}}$, should the golfer impart on the ball to make the shot? Neglect air drag.

EXAMPLE 2 SOLUTION The system is the golf ball. The proper kinetic and free-body diagram pair is shown in Figure 6. Since the weight of the golf ball is the only force acting on it, this is, in essence, a projectile motion problem.

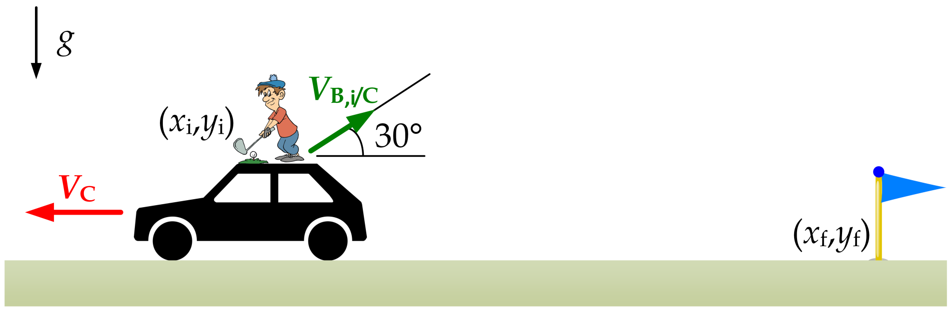

Figure 5 Example 2 - Golfer on top of a moving car

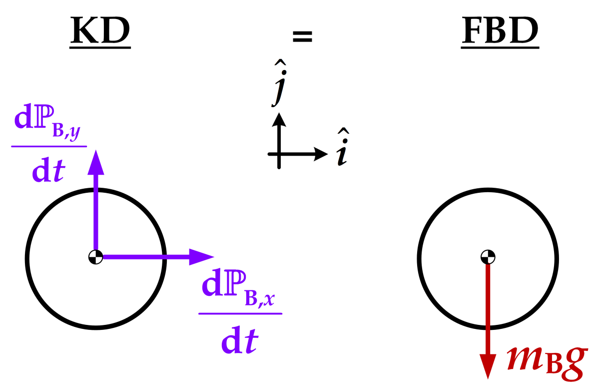

Figure 6 KD and FBD of the golf ball in Example 2

Conservation of linear momentum results in

$$\frac{d\vec{\mathbb{P}}_B}{dt} = m_B \frac{d\vec{V}_B}{dt} + \cancelto{\text{closed}}{\vec{V}_B \frac{dm_B}{dt}} = \sum_{\text{ext}} \vec{F} + \cancelto{\text{closed}}{\sum_{\text{in}} \dot{m}\vec{V} - \sum_{\text{out}} \dot{m}\vec{V}}.$$

Resolving the vector relation in the $\hat{i}$ and $\hat{j}$ coordinates leads to

$$\hat{i}: \qquad m_B \frac{dV_{B,x}}{dt} = 0, \text{ and} \tag{15}$$

$$\hat{j}: \qquad m_B \frac{dV_{B,y}}{dt} = -m_B g. \tag{16}$$

Assuming that the ball is hit with an initial (absolute) velocity $V_{B,i}$ at an angle θ from the horizon, integrating (15) and (16) with respect to time results in

$$\hat{i}: \qquad V_{B,x}(t) = V_{B,i} \cos\theta, \text{ and} \tag{17}$$

$$\hat{j}: \qquad V_{B,y}(t) = V_{B,i} \sin\theta - gt. \tag{18}$$

If the ball is initially positioned at (x_i, y_i), integrating (17) and (18) with respect to time leads to

$$\hat{i}: \qquad x(t) = x_i + (V_{B,i} \cos\theta)t, \text{ and} \tag{19}$$

$$\hat{j}: \qquad y(t) = y_i + (V_{B,i} \sin\theta)t - \frac{1}{2}gt^2. \tag{20}$$

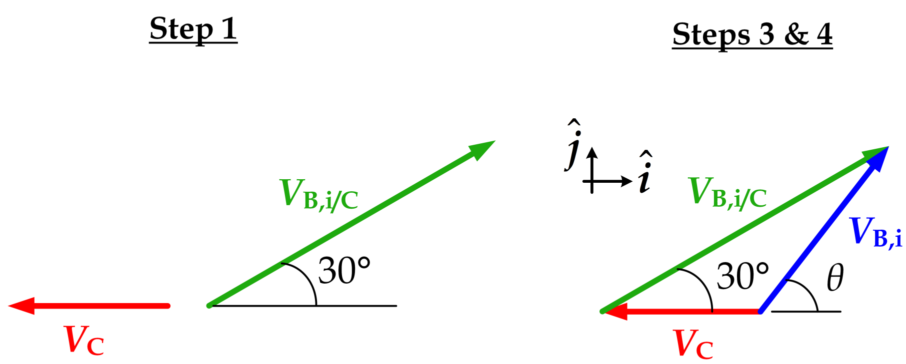

Figure 7 Steps 1 and 3 & 4 of the process to correctly determine the relative velocity diagram for Example 2.

There is no information provided about the initial (absolute) velocity of the ball, but the RVD will provide additional insight to solve this problem. The direction of the absolute velocity of the car and the relative velocity of the golf ball are known, as depicted in *Step 1* of Figure 7. In *Step 2*, the vector relation between the absolute and relative velocities is expressed in terms of the velocity whose direction is unknown, which is $V_{\mathrm{B,i}}$ in this example:

$$\vec{V}_{\mathrm{B,i}} = \vec{V}_{\mathrm{B,i/C}} + \vec{V}_{\mathrm{C}}. \tag{21}$$

The direction of the absolute velocity of the ball is subsequently determined in *Step 3* of Figure 7. This will also be the final relative velocity diagram since no vectors have to be switched back to their original form. From the RVD, (21) can be resolved in both the $\hat{\imath}$- and $\hat{\jmath}$-directions as follows:

$$\hat{\imath}: \quad +V_{\mathrm{B,i}}\cos\theta = +V_{\mathrm{B,i/C}}\cos 30^\circ + (-V_{\mathrm{C}}), \text{ and} \tag{22}$$

$$\hat{\jmath}: \quad +V_{\mathrm{B,i}}\sin\theta = +V_{\mathrm{B,i/C}}\sin 30^\circ + 0. \tag{23}$$

Substituting (22) and (23) into (19) and (20), respectively, and evaluating at the hole location $(x_{\mathrm{f}}, y_{\mathrm{f}})$ leads to

$$t_{\mathrm{f}} = \frac{\Delta x}{V_{\mathrm{B,i/C}}\cos 30^\circ - V_{\mathrm{C}}}, \text{ and} \tag{24}$$

$$\Delta y = (V_{\mathrm{B,i/C}}\sin 30^\circ)t_{\mathrm{f}} - \frac{1}{2}g t_{\mathrm{f}}^2. \tag{25}$$

where $\Delta x = x_{\mathrm{f}} - x_{\mathrm{i}}$ and $\Delta y = y_{\mathrm{f}} - y_{\mathrm{i}}$. Substituting (24) into (25) and simplifying results in the following quadratic equation for $V_{\mathrm{B,i/C}}$:

$$\left[\Delta y\cos^2 30^\circ - \Delta x\sin 30^\circ\cos 30^\circ\right]V_{\mathrm{B,i/C}}^2 + \left[\Delta x V_{\mathrm{C}}\sin 30^\circ - 2\Delta y V_{\mathrm{C}}\cos 30^\circ\right]V_{\mathrm{B,i/C}} + \left[\Delta y V_{\mathrm{C}}^2 + \frac{g}{2}(\Delta x)^2\right] = 0. \tag{26}$$

There are two solutions to (26):

$$V_{\mathrm{B,i/C}} = \frac{\left[\Delta x - 2\sqrt{3}\Delta y\right]V_{\mathrm{C}} \mp \Delta x\sqrt{V_{\mathrm{C}}^2 + 2g\left[\sqrt{3}\Delta x - 3\Delta y\right]}}{\sqrt{3}\Delta x - 3\Delta y}. \tag{27}$$

For this example, $\Delta x > 0$ and $\Delta y < 0$. Thus, the only physical solution from (27) is

$$\boxed{V_{\mathrm{B,i/C}} = \frac{\left[\Delta x - 2\sqrt{3}\Delta y\right] V_{\mathrm{C}} + \Delta x\sqrt{V_{\mathrm{C}}^2 + 2g\left[\sqrt{3}\Delta x - 3\Delta y\right]}}{\sqrt{3}\Delta x - 3\Delta y}}$$

This example shows how the RVD may be used to relate the absolute velocities that appear in the conservation of linear momentum principle (and kinematic relations), to their relative counterparts, which may be known or of interest to determine. A common student misconception in 2D relative motion problems such as this is to assume that the relative and absolute velocities point in the same direction. By explicitly drawing the RVD similar to Figure 7, it should be clear that this will only be true if the car does not move.
Next, a problem involving determining the direction of sliding friction will be considered.

EXAMPLE 3: DETERMINING THE DIRECTION OF SLIDING FRICTION A crate of mass m with an initial (absolute) velocity of $V_{\mathrm{A,i}}$ is placed on a conveyor belt that moves with a constant (absolute) velocity V_{B} as shown in Figure 8. Assume that there is friction present between the crate and the conveyor belt. Determine an expression for the time it takes for the crate to stop slipping on the conveyor belt when

(a) $V_{\mathrm{A,i}} > V_{\mathrm{B}}$, and

(b) $V_{\mathrm{A,i}} < V_{\mathrm{B}}$.

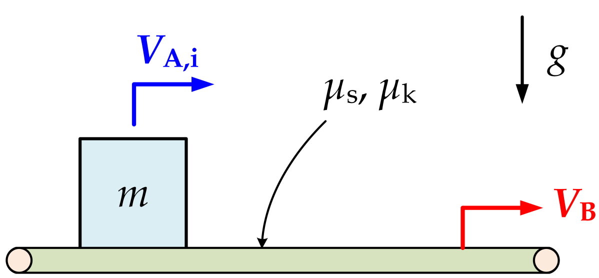

Figure 8 Example 3 - Crate sliding on a conveyor belt

EXAMPLE 3 SOLUTION The system is the crate. Recall that the sliding friction force will always oppose the direction of the *relative* velocity between the two surfaces, so it is imperative that the direction of the velocity of crate relative to the belt is identified first for each case.

(a) As shown in *Step 1* of Figure 9, the direction of the absolute velocities of the crate and belt are known. In *Step 2*, the vector relation between the absolute and relative velocities is expressed in terms of the velocity whose direction is unknown, which is $V_{\mathrm{A,i/B}}$ in this scenario:

$$\vec{V}_{\mathrm{A,i/B}} = \vec{V}_{\mathrm{A,i}} - \vec{V}_{\mathrm{B}} = \vec{V}_{\mathrm{A,i}} + \left(-\vec{V}_{\mathrm{B}}\right). \tag{28}$$

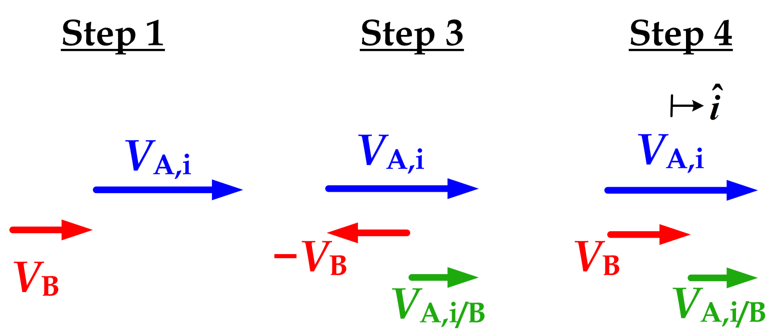

Figure 9 Steps 1, 3, and 4 of the process to correctly determine the relative velocity diagram for Part (a) of Example 3.

The direction of the velocity of the crate relative to the belt is graphically determined in *Step 3* of Figure 9 with the final relative velocity diagram shown in *Step 4*. From the RVD, it is clear that since $V_{\mathrm{A,i/B}}$ points to the right, the sliding friction the crate experiences would act to the left.
The proper kinetic and free-body diagram for this case are shown in Figure 10. From conservation of linear momentum,

$$\frac{\mathrm{d}\vec{\mathbb{P}}_{\mathrm{A}}}{\mathrm{d}t} = m\frac{\mathrm{d}\vec{V}_{\mathrm{A}}}{\mathrm{d}t} + \cancel{\vec{V}_{\mathrm{A}}\frac{\mathrm{d}m}{\mathrm{d}t}}^{\text{closed}} = \sum_{\text{ext}} \vec{F} + \cancel{\sum_{\text{in}} \dot{m}\vec{V} - \sum_{\text{out}} \dot{m}\vec{V}}^{\text{closed}}. \tag{29}$$

Resolving the vector relation (29) in the $\hat{j}$- and $\hat{\imath}$-coordinates and utilizing the definition of sliding friction leads to

$$\hat{j}: \qquad N = mg, \text{ and} \tag{30}$$

$$\hat{\imath}: \qquad m\frac{\mathrm{d}V_{\mathrm{A},x}}{\mathrm{d}t} = -\mu_{\mathrm{k}} mg. \tag{31}$$

The crate will stop sliding when its velocity matches the belt. Thus, integrating (31)

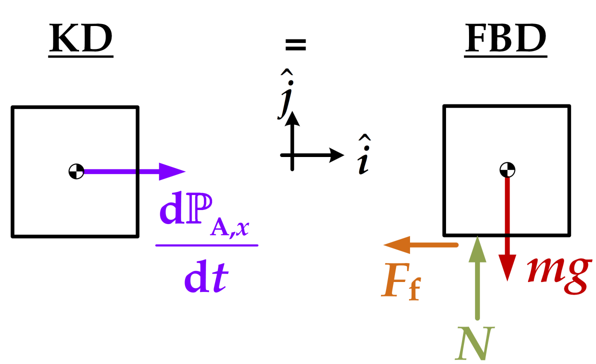

Figure 10 KD and FBD of the crate in Part (a) of Example 3

once with respect to time results in

$$\int_{V_{\mathrm{A,i}}}^{V_{\mathrm{B}}} \mathrm{d}V_{\mathrm{A},x} = -\mu_{\mathrm{k}} g \int_{0}^{t_{\mathrm{f}}} \mathrm{d}t, \text{ or}$$

$$t_{\mathrm{f}} = \frac{V_{\mathrm{A,i}} - V_{\mathrm{B}}}{\mu_{\mathrm{k}} g}. \tag{32}$$

(b) The steps to determining the direction of the velocity of the crate relative to the belt are identical to Part (a). For this case, however, the RVD from Figure 11 suggests that $V_{\mathrm{A,i/B}}$ would point to the left, which means that the sliding friction the crate experiences would act to the right.

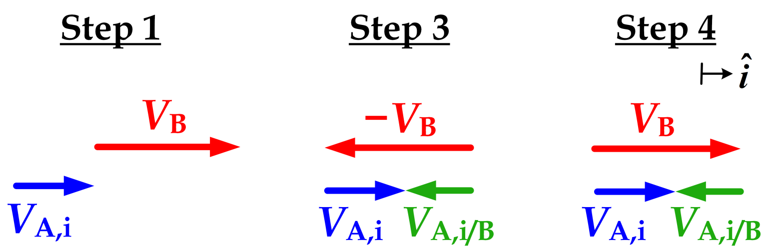

Figure 11 Steps 1, 3, and 4 of the process to correctly determine the relative velocity diagram for Part (b) of Example 3.

The proper kinetic and free-body diagram for this case are shown in Figure 12.

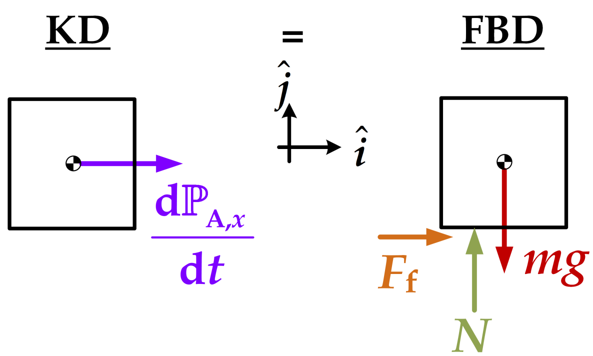

Figure 12 KD and FBD of the crate in Part (b) of Example 3

Going through a similar series of steps as in Part (a), the time at which the crate stops sliding is

$$t_{\mathrm{f}} = \frac{V_{\mathrm{B}} - V_{\mathrm{A,i}}}{\mu_{\mathrm{k}} g}. \tag{33}$$

The results from Parts (a) and (b) can be summarized as

$$\boxed{t_{\mathrm{f}} = \frac{|V_{\mathrm{A,i}} - V_{\mathrm{B}}|}{\mu_{\mathrm{k}} g}}$$

This example demonstrates the usefulness of the RVD in aiding in the determination of the direction of sliding friction. This is especially true for the case highlighted in Part (b), where, in the author's experience, nearly every student initially guesses the direction of friction incorrectly since they likely remember that "friction opposes motion" and thus, assume that friction would point in the direction opposite to that of the absolute velocity of the crate.

Methodology and Assessment

At the author's current institution, Rose-Hulman Institute of Technology, which is a small private institution located in the Midwest, the Mechanical Engineering curriculum was redesigned three decades ago to add a foundational course called Conservation and Accounting Principles. Students first encounter relative motion in the context of the comprehensive problems in this course, which has been described in detail previously [1].

The author has been teaching Conservation and Accounting Principles regularly for nearly a decade. While there has historically been a strong emphasis in defining systems and drawing interaction diagrams by all the instructors in the course, there has not been as much focus on depicting the relation between velocity vectors through the RVD. In recent years, the author has gradually developed and introduced the process for drawing the RVD to their classes similar to what has been outlined in this paper. A comparison of the performance of eight cohorts of students on a "rocket-type" problem, where mass enters or exits a system to propel or stop it similar to Example 1, along with how the RVD was presented to the students is provided in Table 1. The "rocket-type" problem was selected for assessment comparison for three reasons: (i) an incorrect simplification of the relative velocity vector relation to component form would leave the student unable to solve the problem, making it a vital step in the solution process, (ii) with one exception, it was assessed on two different exams during the term for each cohort, and (iii) it was the only type of problem involving relative velocity that was consistently tested.

Prior to comparing the performance of the cohorts, it is important to provide a general description of the class composition. Cohorts *A*,*C*, *E*, and *H* took the course in-person and in its primary offering during the academic year. Each cohort consisted of mostly sophomore Mechanical Engineering majors taking the course for the first time along with less than a handful of students from similar majors. The difference in the mean grade point average (GPA) between any combination of these cohorts was statistically insignificant[a]. The story is somewhat more complex for the other groups considered. The off-sequence offering of the course *typically* consists of a combination of freshmen who are ahead and sophomores (and possibly juniors) who are either repeating the course or taking it for the first time because they were unable to complete the prerequisite courses on the first try. This often leads to a bimodal distribution of grades. For reference, roughly one quarter of cohorts *B* and *D* and two-fifths of *G* consisted of freshmen who were ahead. Cohort *F*, however, did not include any freshmen. Every cohort but *B* and *G* were taught fully in-person. For cohort *B*, the first three weeks of the course were taught in a synchronous, fully online format, while the last seven weeks were completely in-person. Cohort *G* was taught fully online and asynchronously.

[a]No p value exceeded $\alpha = 0.05$ in a two-tailed, unpaired t-test between every combination of GPAs.

Table 1 Mean student score percentages (standard deviations in parentheses) on the "rocket-type" problem on two different assessments for eight cohorts of Conservation and Accounting Principles. The superscript 'm' indicates the main offering of the course during the academic year. Assessment questions that were very similar have been highlighted by the same number of asterisks. The p value is highlighted in red for cohorts whose mean assessment scores were statistically significantly different ($p < \alpha = 0.05$) from one another.

Cohort	Mode	N	Assessment 1	Assessment 2	p value	RVD presentation
A	In-person$^{\mathrm{m}}$	24	79.0 (16.4)	63.3 (10.7)	0.00039	No RVD
B	Hybrid	16	N/A	62.0 (15.6)**	N/A	No RVD
C	In-person$^{\mathrm{m}}$	37	76.8 (15.7)	72.3 (21.7)*	0.135	Supplemental example
D	In-person	22	72.5 (20.0)	68.8 (21.2)*	0.412	Beta RVD
E	In-person$^{\mathrm{m}}$	24	75.8 (18.1)	71.8 (20.7)*	0.397	Beta RVD
F	In-person	29	73.0 (15.1)	62.3 (15.2)*	0.00152	Final RVD in supplement
G	Online	26	67.8 (18.0)	67.5 (14.4)**	0.933	Final RVD in lecture
H	In-person$^{\mathrm{m}}$	18	81.6 (19.1)	82.8 (16.1)*	0.749	Final RVD in lecture

As mentioned earlier, the development of the steps to drawing the RVD was a gradual process. Cohorts *A* and *B* were only taught relative velocity through direct manipulation of the vector relation (1) without any diagram requirements. Subsequently, the author incorporated more explanations about relative velocity into the supplemental muddiest points documents that they prepared for use outside of the class environment, first through an additional example (cohort *C*), then with a "beta" (less formalized) version of the process needed to draw a correct RVD (cohorts *D* and *E*), followed by the series of steps outlined in this work earlier (cohort *F*). These steps were ultimately integrated directly into the class notes with examples in an identical manner for both cohorts *G* and *H*.

In Table 1, the first assessment is the "rocket-type" problem from a midterm exam conducted about halfway through the term, while the second assessment is from a problem on the final exam. Notably, there is a drop in the mean between assessments for every cohort but *H*. The difference between the assessment means is statistically significant for cohorts *A* and *F*[b]. For cohort *A*, this

[b]The p values in the table are calculated based on a two-tailed, paired *t*-test, which is used to determine whether the mean of the *difference* in the scores of the two assessments from the same students in each cohort is *statistically* significantly different ($p < \alpha = 0.05$) from zero. This test assumes that these differences are approximately normally distributed. The Shapiro-Wilk test [21] of each cohort's data set *difference* verifies that the null hypothesis of normally distributed differences cannot be rejected for $p < \alpha = 0.05$, *i.e.*, the differences are normally distributed.

was due to the second assessment question being significantly more challenging than the first (and more difficult than any of the other problems whose mean is listed in Table 1). The students in cohort *F* performed very poorly on the final exam overall, so their low mean score on this problem was possibly for another reason of which the author is unaware. Cohorts *C*, *D*, *E*, *F*, and *H* were all tasked with solving very similar problems for the second assessment. Outside of cohort *F*, the mean scores are not significantly different from one another for the cohorts who were *not* provided the RVD in the lecture packet, indicating that perhaps not all students read and gain information from supplemental documents as they are expected to. However, the performance of cohort *H* on both assessments clearly stands out compared to their counterparts, especially on the final exam where there is a statistically significant difference in nearly every case (maximum p value is 0.06[c]). Admittedly, further data is needed with future cohorts to see whether these results are repeatable, but a comparison between cohorts *B* and *G* on the same final exam problem suggests that the RVD process outlined in this paper hopefully provides a step towards gaining a better understanding of relative velocity, even for struggling students.

The author received no actionable comments on evaluations about relative velocity from any of the cohorts except *H*. One of the two students who provided feedback from this cohort mentioned struggling with recognizing which arrow to flip (performing vector subtraction by adding the negative of the vector) when drawing the RVD. Another suggested bringing in a physical model to show the difference between absolute and relative velocity. Both of these comments hint at students struggling with vector math, which is prerequisite knowledge for the course. It has been the author's experience that when students are asked to define a vector, they reply by highlighting that it has a magnitude and direction, but when it comes to applying it in the context of a larger problem, some have a difficult time dealing with it differently than a scalar.

Conclusions and Future Work

In this work, a step-by-step process to drawing the relative velocity diagram for two particles and using it to correctly resolve the associated vector relation into its components has been presented and its utility is demonstrated through three examples. The approach to drawing the RVD involves identifying two velocity vectors whose direction is known, expressing the general vector relation (1) in terms of the unknown vector, graphically implementing the resulting relation, defining a coordinate system, and finally using the RVD to resolve the previously obtained vector relation into its components. As the three examples in the paper show, this consistent approach not only provides a useful visual tool for students to see how the absolute and relative velocity vectors are related to one another, but it aids in solving a wide variety of problems.

While student performance on assessments suggests that the outlined approach to drawing the RVD is a step in the right direction, there is plenty of work to be done. Since this process was only recently integrated into the course lecture notes, it would be valuable to collect more data

[c]The p values for this comparison are calculated based on a Welch's (two-tailed, two-sample, unequal variance) *t*-test, which is used to determine whether cohort *H* and any other cohort have Assessment 2 mean scores that are *statistically* significantly different ($p < \alpha = 0.05$) from one another. Similar to the paired case, Welch's *t*-test requires problem scores to be normally distributed. The Shapiro-Wilk test of each cohort's Assessment 2 results verifies that the null hypothesis of normally distributed scores cannot be rejected assuming a significance level of 0.05.

from primary offering and off-sequence cohorts of Conservation and Accounting Principles to see whether this approach is truly useful in improving student understanding of relative motion.

References

[1] A. H. Danesh-Yazdi, "Storage and Interaction Diagrams: Extending the Diagrammatic Framework of Kinetic and Free-Body Diagrams to other Conservation and Accounting Principles," in *2024 ASEE Annual Conference & Exposition*, (Portland, Oregon), ASEE Conferences, June 2024.

[2] J. L. Meriam, L. G. Kraige, and J. N. Bolton, *Engineering Mechanics: Dynamics*. Hoboken, NJ: Wiley, 9th ed., 2018.

[3] F. P. Beer, E. R. Johnston, D. F. Mazurek, P. J. Cornwell, and B. P. Self, *Vector Mechanics for Engineers: Statics and Dynamics*. New York, NY: McGraw-Hill Higher Education, 2024 Release ed., 2024.

[4] R. C. Hibbeler, *Engineering Mechanics: Statics & Dynamics*. Hoboken, NJ: Pearson, 14th ed., 2020.

[5] A. Bedford and W. Fowler, *Engineering Mechanics: Statics and Dynamics*. Hoboken, NJ: Pearson, 6th ed., 2023.

[6] M. E. Plesha, G. L. Gray, F. Costanzo, and R. J. Witt, *Engineering Mechanics: Dynamics*. New York, NY: McGraw-Hill Higher Education, 3th ed., 2023.

[7] A. Pytel and J. Kiusalaas, *Engineering Mechanics: Dynamics*. Boston, MA: Cengage Learning, 4th ed., 2016.

[8] B. H. Tongue and D. T. Kawano, *Engineering Mechanics: Dynamics*. Wiley, 1st ed., 2020.

[9] J. Walker, D. Halliday, and R. Resnick, *Fundamentals of Physics*. Hoboken, NJ: Wiley, 10th ed., 2014.

[10] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*. Hoboken, NJ: Pearson, 15th ed., 2019.

[11] D. C. Giancoli, *Physics: Principles with Applications*. Hoboken, NJ: Pearson, 7th ed., 2021.

[12] P. M. Gerhart, A. L. Gerhart, and J. I. Hochstein, *Munson, Young and Okiishi's Fundamentals of Fluid Mechanics*. Hoboken, NJ: John Wiley & Sons Inc, 8th ed., 2016.

[13] Y. A. Çengel and J. M. Cimbala, *Fluid Mechanics: Fundamentals and Applications*. New York, NY: McGraw-Hill Education, 4th ed., 2018.

[14] R. C. Hibbeler, *Fluid Mechanics*. New York, NY: Pearson, 2nd ed., 2018.

[15] F. M. White and H. Xue, *Fluid Mechanics*. New York, NY: McGraw-Hill Education, 9th ed., 2021.

[16] Y. Tang and S. J. Grigg, "Enhancing Relative Motion Mastery through Strategic Instructional Design," in *2025 ASEE Annual Conference & Exposition*, (Montreal, Quebec, Canada), ASEE Conferences, June 2025.

[17] R. E. Flori, D. B. Oglesby, and N. Hubing, "Observations on Teaching Relative Velocity in Engineering Dynamics," in *2005 ASEE Midwest Section Conference*, (University of Arkansas, Fayetteville, AR), ASEE Conferences, September 2005.

[18] M. Hassanzadeh, D. W. Baker, and S. F. Pilkington, "Evaluating the Effect of Flipped Classroom on Students' Learning in Dynamics," in *2019 ASEE Annual Conference & Exposition*, (Tampa, Florida), ASEE Conferences, June 2019.

[19] E. Vavrek, "Incorporating Working Model into the Lab of an Applied Kinematics Course," in *2002 ASEE Annual Conference*, (Montreal, Canada), ASEE Conferences, June 2002.

[20] P. Boyle, "Kinematics of the 3d Spatial Four Bar Linkage: Pseudographics - A Computational Method," in *2002 ASEE Annual Conference*, (Montreal, Canada), ASEE Conferences, June 2002.

[21] S. S. Shapiro and M. B. Wilk, "An Analysis of Variance Test for Normality (Complete Samples)," *Biometrika*, vol. 52, no. 3/4, pp. 591–611, 1965.