

Incorporating Hybrid Virtual Simulators and Physical Tools for Angle Measurement in High School Geometry

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Abstract

Understanding geometric angles is crucial for students, as angles are the basis for more advanced mathematical concepts and real-world applications. Existing work aims to teach angles via hands-on learning and by making real-world connections. However, there is relatively little software or engaging lessons in this area. This paper describes a classroom module that teaches angles and angle addition to freshman and sophomore mathematics learners using a combination of Geogebra software; an angle addition simulator tool built using HTML5 Canvas and JavaScript; and a protractor to make abstract concepts tangible. This approach aims to enhance students' geometric understanding, critical thinking, and problem-solving skills, while integrating traditional and digital tools to engage Gen Z learners, ultimately preparing them for academic and professional success in fields like architecture, engineering, and navigation. The module was evaluated in a class of 25 students using pre- and post-tests and an opinion survey. Students found the approach engaging and effective, noting that the combination of hands-on tools like protractors and interactive software such as GeoGebra helped demystify complex concepts. Survey results indicated increased confidence in understanding geometric angles and improved problem-solving skills, with many students appreciating the real-world connections made during the lessons.

Keywords

Angles, Angle Addition, Angle Simulator, Geogebra, Geometry, Protractor, Angle Measurement

Background

Geometry is one of the most challenging areas of mathematics for students [1]. However, there is continued concern about students' low performance in geometry and angle measurement, which are crucial for workplace skills and STEM fields.

As an illustration, Figure 1 displays a pie chart of 2023 district test scores focusing on the Mathematics TEKS relative to angles and angle measurements for a class of 15 on-level Geometry students, excluding special groups such as gifted/talented, special ed, SPeD Students and emergent bilingual students [2].

Although some studies suggest that inadequate teacher preparation might contribute to students' underperformance in these areas, research on teacher knowledge related to geometry and measurement is almost nonexistent [3]. Additionally, students often become easily frustrated

when answers are not straightforward, and they tend to use little to no critical thinking skills, which further hinders their ability to grasp geometric concepts effectively [4]. Teaching and learning geometry is challenging, with studies showing that students' attitudes and willingness to engage with the subject significantly impact their success [5].

Gaining insight into characteristics of Gen Z learners can benefit mathematics teachers in establishing classroom norms and designing instruction that helps students achieve high standards. Gen Z students are constantly engaged with digital media and prefer interactive and technology-enhanced learning environments. They thrive in settings that offer immediate feedback and opportunities for collaboration, reflecting their social and connected nature.

Although Gen Z is the first generation born into a digital world, they are often not technologically adept. They frequently face challenges using technology for educational purposes and require training on how to effectively use the Internet, despite having had lifelong access to it [6].

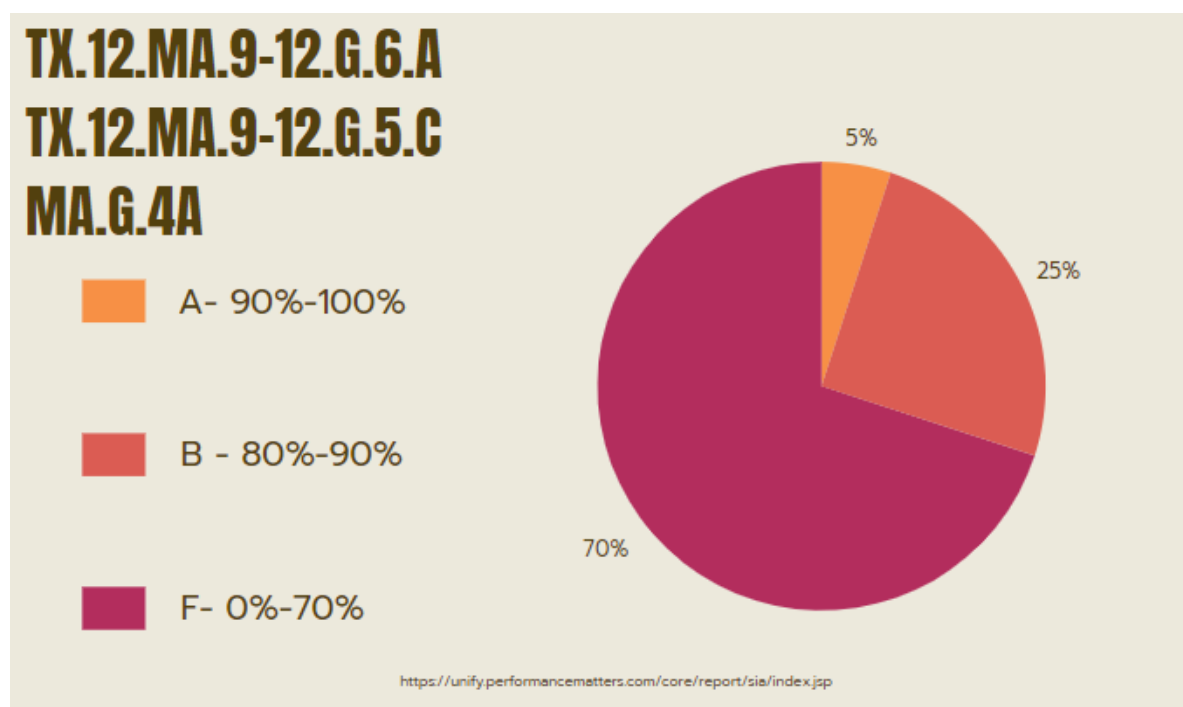


Figure 1: Grade distributions of 15 on-level geometry students.

Existing Work

Existing lesson plans on angles were reviewed.

Geometry Tools: Angles & Reflections [7]. This lesson plan introduces students to common geometry tools such as protractors and Miras, learning to create and measure angles and reflections. The lesson includes a historical overview and modern applications of these tools, practice problems, and a worksheet to reinforce concepts. Students discuss their methods and solutions, gaining an understanding of how angles and geometric tools are essential in engineering and everyday designs. This lesson sets the stage for an activity where students design and test mini-golf courses based on geometric principles. Ancient engineers used geometry tools to construct precise structures, a practice that continues in modern engineering with advanced tools for creating accurate designs. These tools are vital for measuring and designing infrastructure, products, and systems, enabling engineers to achieve precision and consistency. Through this lesson, students learn to measure and draw angles, reflect points and lines, and understand the importance of geometry in engineering, preparing them to apply these skills in real-world contexts.

Strengths. The lesson is thorough and engaging, beginning with a historical overview and modern-day uses of protractors, compasses, and mirrors. This context helps students understand the relevance of these tools in both ancient and contemporary engineering. The hands-on practice with protractors and Miras, followed by a worksheet, allows students to apply their knowledge and share their methods, fostering collaborative learning. The subsequent mini-golf activity ties the lesson to a real-world application, making the concepts more tangible and demonstrating the practical use of geometry in design and problem-solving.

Weaknesses. A potential weakness of this lesson plan is the dependency on the availability of Miras, which may not be accessible to all students. This reliance could limit the full participation and learning experience for those without access to this specific tool. Additionally, the lesson assumes a certain level of prior knowledge about angles and their applications, which may not be consistent among all students. This discrepancy could lead to gaps in understanding and make it challenging for some students to fully grasp the concepts. Finally, while the historical context is valuable, it might be too lengthy for students who are more interested in hands-on activities and direct applications.

Opportunities for improvement. Incorporating modern technology like geometry software (e.g., GeoGebra), online simulations, or geometric tools can enhance engagement and visualize geometric concepts dynamically.

Polygons, Angles and Trusses, Oh My! [8]. In this lesson plan, students explore truss structures, focusing on the geometric shapes that compose them and the variations in bridge designs. Using a guided worksheet, they draw 2D and 3D polygon shapes, analyze their forms and angles before and after applying loads, and learn how engineers reinforce shapes under compression and tension, emphasizing the strength of triangles. The lesson includes a PowerPoint presentation and prepares students for activities on polygons and trusses. Civil

engineers design strong structures like bridges using creative truss designs, illustrating how geometry plays a critical role in engineering. By the lesson's end, students should be able to identify trusses, describe their shapes, measure angles, and understand the engineering use of polygons. The lesson aligns with educational standards and includes worksheets and presentations. Students need pre-knowledge of angles, and the lesson begins with an engaging introduction using slides. The lesson emphasizes how trusses distribute weight and withstand forces, leading to hands-on activities and discussions to solidify understanding.

Strengths. The lesson plan provides a comprehensive and engaging approach to understanding the geometric principles behind truss designs and their applications in civil engineering. Students are able to explore 2D and 3D polygon shapes and analyze their structural integrity under different load conditions. The plan also effectively connects theoretical knowledge with real-world engineering practices, enabling students to appreciate the importance of trusses in bridge construction and other infrastructure. Additionally, the lesson aligns with various educational standards and provides detailed instructions, vocabulary, and assessment tools to ensure thorough comprehension and practical application of the concepts.

Weaknesses. A potential weakness of this lesson plan is its reliance on students' prior knowledge of interior and exterior angles, which may not be uniformly understood by all participants. Without a strong foundational understanding, some students might struggle to fully grasp the more complex concepts of truss stability and load distribution. Additionally, the lesson plan requires several materials and resources, such as rulers, protractors, and hot glue, which might not be readily available in all classroom settings, potentially limiting its accessibility. Furthermore, the plan's heavy dependence on visual and hands-on learning may not cater to all learning styles, particularly those who benefit from more traditional, lecture-based instruction or textual learning aids.

Opportunities for improvement. Plan follow-up lessons or activities that revisit and expand on the concepts taught. Use think-pair-share activities during the discussion to ensure all students are actively involved and thinking critically about the content.

Close Enough? Angles & Accuracy of Measurement in Navigation [9]. In this lesson plan, the importance of accuracy and precision in measurements, particularly in navigation and engineering, is emphasized. For navigation, the accuracy of angle measurements is crucial; even small errors in angle can lead to significant deviations in position over long distances. Engineers also rely heavily on precise measurements, ensuring spacecraft trajectories, medical instruments' safety, and structural integrity of buildings. The lesson introduces students to trigonometric concepts like sine, cosine, and tangent, essential for calculating distances and angles accurately. Through hands-on activities like locating an "Invisible Island" using protractors and string, students experience firsthand how slight variations in angle measurement can affect accuracy in real-world applications. This practical exercise helps reinforce the understanding of accuracy

versus precision and the practical applications of trigonometry in various fields of study and everyday life.

Strengths. The lesson effectively illustrates the critical concepts of accuracy and precision through real-world examples and practical activities. By using the analogy of dart throwing and practical applications in navigation and engineering, students grasp the difference between accuracy (closeness to the target) and precision (consistency of measurements). The hands-on activity of locating an "Invisible Island" encourages active learning and application of trigonometric principles, reinforcing understanding through practical experience. Students engage in collaborative learning, using tools like protractors and string to measure angles and distances, which enhances their spatial reasoning and mathematical skills. This approach not only aligns with educational standards but also fosters critical thinking by prompting students to analyze measurement errors and their implications in different scenarios.

Weaknesses. While the activity provides a practical understanding of accuracy and precision, it may pose challenges related to equipment and environmental conditions. Issues such as protractors with limited angle divisions or uneven terrain can affect the accuracy of measurements, potentially leading to inconsistencies in student results. Moreover, the need for a large, open space for outdoor activities might not always be feasible, limiting the accessibility of the lesson. In some cases, the complexity of trigonometric calculations and the coordination required between team members may overwhelm younger or less experienced students, reducing their engagement or understanding. Additionally, the focus on practical application may overshadow deeper theoretical exploration of trigonometric principles, potentially leaving some students with gaps in understanding the underlying mathematical concepts.

Opportunities for improvement. Provide digital protractors and measuring tools to improve the precision of angle and distance measurements. Include real-world examples and applications of navigation and measurement to show the relevance and importance of these skills.

Methodology

For this study, a lesson plan was designed to integrate three different tools: Geogebra (a math platform available online at <https://www.geogebra.org/>), a custom-built online angle simulator, and physical protractors. GeoGebra is a dynamic mathematics software that combines geometry, algebra, statistics, and calculus in a single, easy-to-use package as shown in Figure 4. It allows users to create and manipulate mathematical constructions and models visually and interactively. GeoGebra is widely used in education to help students and teachers visualize and explore mathematical concepts, enhancing understanding through interactive learning. It supports various platforms, including desktop, web, and mobile devices, making it accessible for both classroom and individual use [11].

Combining Geogebra with the angle simulator and a protractor allowed us to address common challenges in geometry education, such as misconceptions and difficulty transitioning from algebraic concepts, by fostering spatial awareness and critical thinking skills [10]. The goal is to make abstract geometric principles more tangible and relevant, preparing students for practical applications in fields such as architecture, engineering, and navigation. Ultimately, this approach aims to build a solid foundation in geometry, equipping students with essential problem-solving skills for both academic and professional success [10].

This lesson plan was implemented in a high school Geometry course. The lesson consisted of five activities. Each activity consisted of two separate days of instructional time, with each day being about an hour. Activities 1, 2, and 3, along with assigned homework, took place on Day 1. Activities 4 and 5, along with a final report, took place on Day 2.

1. Introduction to Geogebra
2. Creating angles using Geogebra
3. Adding angles using Geogebra
4. Interactive angle addition in Geogebra & Angle Simulator
5. Real-World Applications using Protractors

Activity 1: Introduction to Geogebra

For this activity, the purpose of angles and why Geogebra is being used was discussed. Students used Geogebra software to create points, lines, segments, and angles as shown in Figure 1.

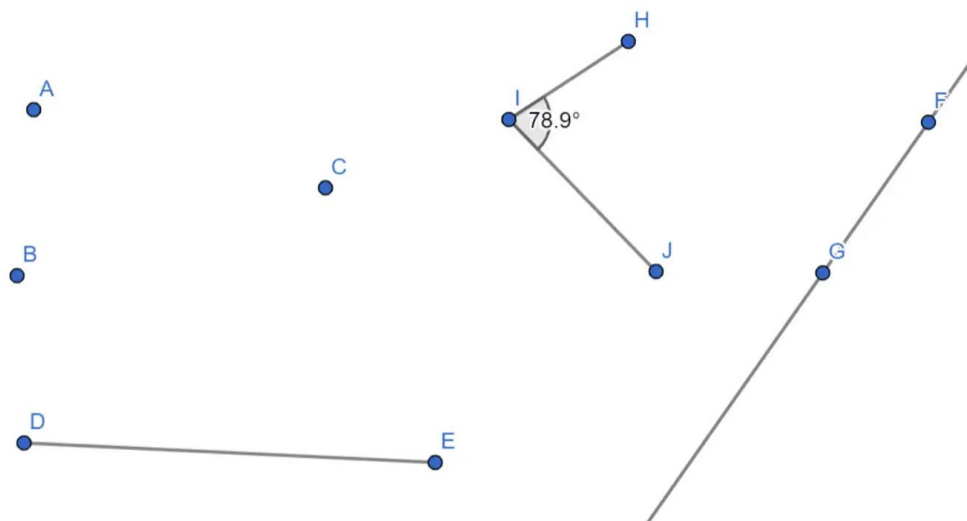


Figure 1: Points, lines, segments, angles created in Geogebra.

Students saved their constructions in Geogebra files directly to their school laptops.

Activity 2: Creating Angles using Geogebra

This activity began with a class discussion about the definition and properties of an angle. The teacher demonstrated how to create a 45° angle. The students then created their own angles, including 45° , 90° , 180° , 270° , 360° angles, as shown in Figure 2.

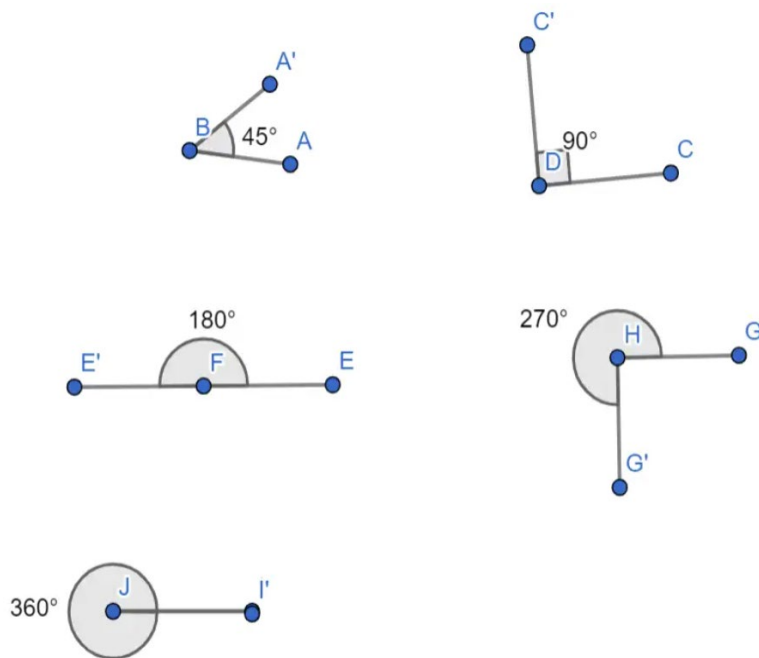


Figure 2: Examples of five specific angles created in Geogebra.

Activity 3: Adding Angles using Geogebra

The teacher led a class discussion about angle addition. The teacher then demonstrated adding angles in Geogebra as shown in Figure 3, leading the students to produce their own angle additions.

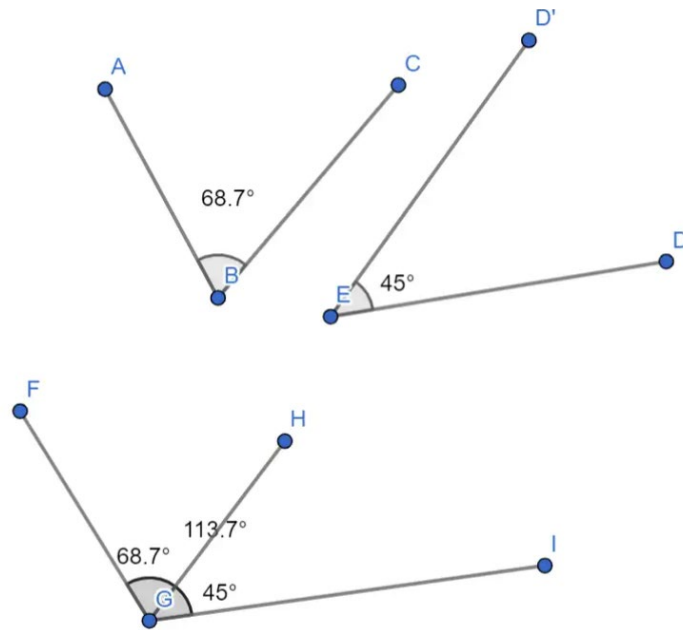


Figure 3: Angle Addition created in Geogebra.

Once an angle has been created, students can draw a point to manipulate the angle measure, as well as the total angle measure for the adjacent angles. Students created three sets of angle addition measurements to demonstrate their understanding.

Activity 4: Interactive Angle Addition in Geogebra and Angle Simulator

Students used a custom-built Angle Simulator web application to explore geometry concepts. The Angle Simulator uses HTML5 canvas and JavaScript to create a dynamic, visual representation of angles within a circle.

The application has these key features:

1. Interactive circle diagram with draggable points
2. Real-time angle calculations
3. Visual representation of angles with color-coded sectors
4. Display of angle measurements and equations

As shown in Figure 5, learners can interact with the diagram by dragging points B, C, and D around the circle, while point A remains fixed. As the points move, the application calculates and displays the angles in real-time. Manipulating points on the circle allows students to instantly see how angles change and relate to each other.

Students were provided links to the Interactive Angle Addition Simulator in Geogebra (Figure 4) and the Angle Simulator (Figure 5). Students explored and engaged with the simulators to

familiarize themselves with the configurations. They used the simulators to answer the odd numbered questions from the class worksheet (Figure 6).

Adding Adjacent Angles

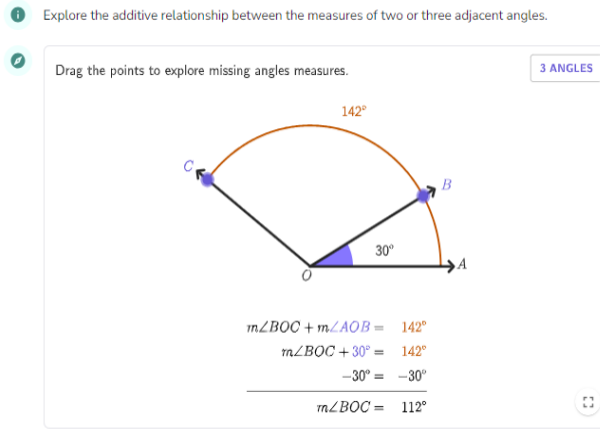


Figure 4: Angle Addition Simulator in Geogebra

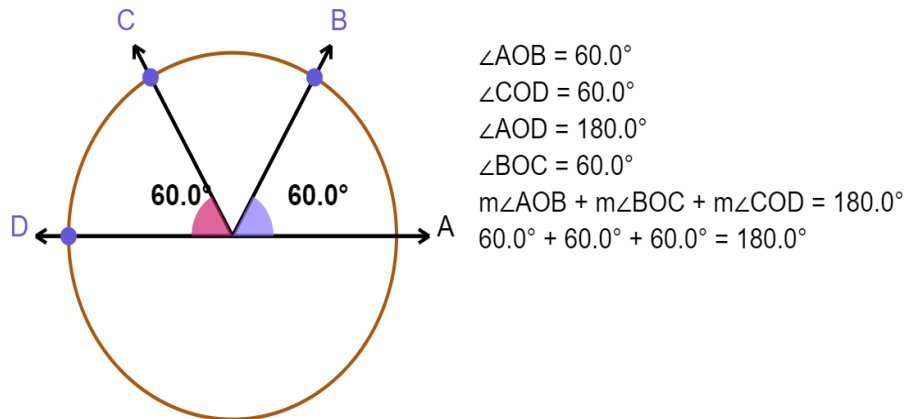
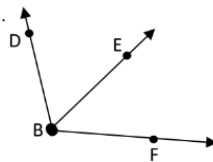


Figure 5: Angle Simulator

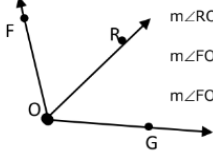
Geometry L
Practice: ANGLE ADDITION

Name: _____
 Date: _____ Per: _____

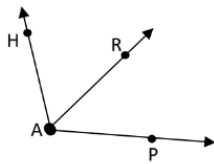
Lesson
Find the missing angle measure.

1.  $m\angle DBE = 15^\circ$
 $m\angle EBF = 18^\circ$
 $m\angle DBF = \underline{\hspace{2cm}}$

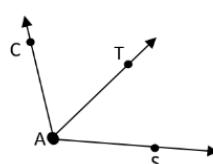
Geometry set-up _____
 Equation _____

2.  $m\angle ROG = 21^\circ$
 $m\angle FOG = 100^\circ$
 $m\angle FOR = \underline{\hspace{2cm}}$

Geometry set-up _____
 Equation _____

3.  $m\angle RAP = 67^\circ$ $m\angle HAR = 42^\circ$
 $m\angle HAP = \underline{\hspace{2cm}}$

Geometry set-up _____
 Equation _____

4.  $m\angle CAT = x^\circ$ $m\angle CAS = (2x+4)^\circ$ $m\angle SAT = 64^\circ$
 $x = \underline{\hspace{2cm}}$ $m\angle CAT = \underline{\hspace{2cm}}$ $m\angle CAS = \underline{\hspace{2cm}}$

Geometry set-up _____
 Equation _____

Figure 6: Angle Addition Worksheet.

Activity 5: Real-World Applications using Protractors

A protractor is a semicircular or circular tool used measure and draw angles; Figure 7 shows a semicircular protractor. Typically made of transparent plastic or metal, it has degree markings from 0° to 180° (or 0° to 360° for a circular protractor) [12]. Protractors are essential in fields requiring precise angle measurements, such as mathematics, engineering, and architecture.

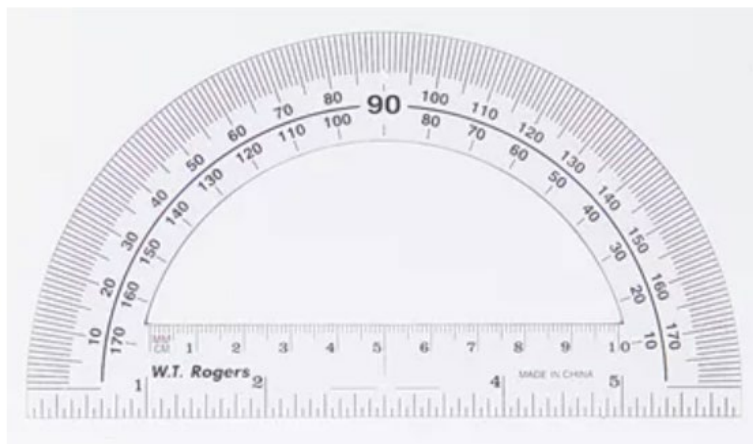


Figure 7: Protractor: a tool used in geometry to measure and draw angles.

The teacher led a class discussion about how angle addition is in various fields, such as architecture, engineering, and art. Students worked in groups of 3 or 4 at stations supplied with cardboard cutouts of various shapes (Figure 8). Groups selected at least five items from the station and used the protractor to record the angle measures. No right angles or straight angles were to be recorded.



Figure 8: Cardboard cutouts and colored shapes.

Students also used protractors to calculate how wide a Bobcat (Figure 9) needs to open its mouth to eat certain sizes of food. Figure 10 displays various fruits and Figure 11 displays various animals.



Figure 9: Google Image of Bobcat with 60 degree angle constructed in Geogebra.



Figure 10: Various fruits.



Figure 11: Various animals.

For the final report, each student found a real-world image and imported it into Geogebra. They created five angles within the image and displayed the angle measurements (Figure 12).

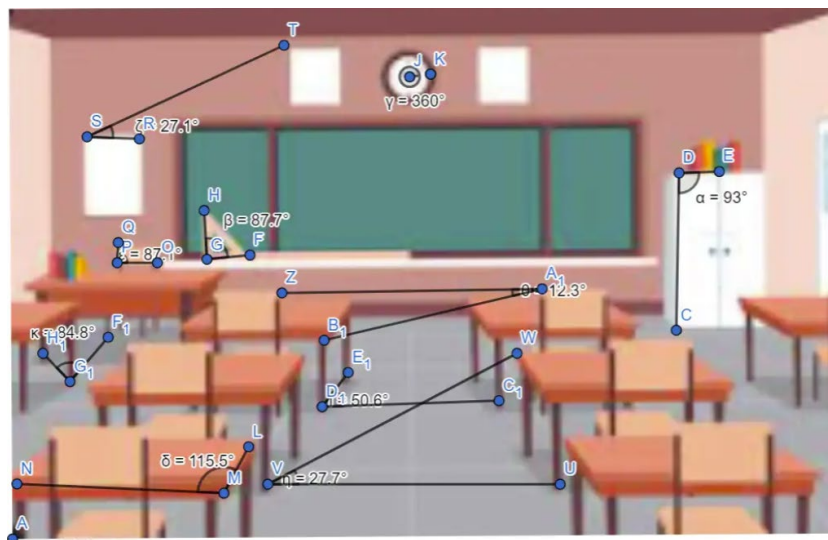


Figure 12: Google Image uploaded to Geogebra with angle measurements.

Evaluation

To evaluate the lesson plan, a pre-test and post-test were used. Sample questions included:

1. Use a protractor to measure these angles (as drawn on a worksheet):
 - Angle XUY
 - Angle YUZ
 - Angle XUZ
2. Add these angles:
 - $45^\circ + 30^\circ$
 - $90^\circ + 90^\circ$
3. Why is the sum of the angles in a triangle always 180° ?
4. If two angles add up to 90° , and one angle is 45° , what is the other angle?
5. What is a common mistake people make when measuring angles?

6. How are angles used in architecture, engineering, or navigation? Give one example.
7. Give an example of a real-life situation where you might need to add or subtract angles.

In addition, an opinion survey was used to gather students' feedback on the lesson and their learning experience. Sample questions included:

Rate your level of agreement with the following statements on a scale of 1 (Strongly Disagree) to 5 (Strongly Agree):

The GeoGebra simulator helped me better understand geometric angle concepts.

1 - 2 - 3 - 4 - 5

Using the simulator along with traditional tools like protractors enhanced my spatial awareness.

1 - 2 - 3 - 4 - 5

The simulator made abstract geometry principles feel more concrete and applicable to real life.

1 - 2 - 3 - 4 - 5

Practicing with the simulator improved my ability to add and subtract angles.

1 - 2 - 3 - 4 - 5

I can see how the skills learned could be useful in fields like architecture, engineering, or navigation.

1 - 2 - 3 - 4 - 5

Overall, integrating the GeoGebra simulator into geometry lessons was more effective than traditional instruction alone.

1 - 2 - 3 - 4 - 5

The simulator activities helped build my critical thinking and problem-solving abilities.

1 - 2 - 3 - 4 - 5

Open-Ended Feedback:

What did you like most about using the GeoGebra simulator for geometry learning?

What challenges or difficulties, if any, did you face when using the simulator?

Any other comments or suggestions for improving the use of technology like GeoGebra for geometry education?

Data Analysis

The evaluation of the simulator's effectiveness reveals a notable average score difference between pre-test and post-test results. The pre-test average score was 71.4, while the post-test average increased to 79.6, resulting in an overall improvement of 8.2 points. This increase suggests that the simulator had a positive impact on students' learning outcomes, helping them to grasp the material better after its use. However, the range of scores indicates variability in performance; for instance, some students experienced significant improvements, such as Student 3, who went from a score of 0 to 74, while others, like Student 15, saw a decline from 88 to 48. This suggests that while the simulator benefited many students, it may not have been equally effective for all.

In terms of student feedback regarding the simulator's helpfulness, the average rating on a scale from 1 to 5 was 3.32, indicating a generally moderate level of satisfaction. The most common rating was a 3, suggesting that many students found the simulator to be "okay" but not overwhelmingly beneficial. Comments from students reflected a mix of opinions; some expressed that the simulator was helpful or interesting, while others remained uncertain about its value or indicated that they struggled with its content. Notably, one student described it as "too difficult," which points to potential issues with accessibility or alignment with students' current skill levels.

Overall, while the simulator appears to have contributed positively to test score improvements for many students, the mixed responses regarding its helpfulness suggest that further enhancements might be necessary. Addressing concerns related to clarity and difficulty could help maximize its effectiveness and ensure that all students can benefit from this educational tool.

Conclusion and Future Directions

Understanding geometric angles is essential for students as it forms the foundation for advanced mathematical concepts and real-world applications. Geometry, especially involving spatial awareness, presents significant challenges for students, leading to lower performance in this area, which is crucial for STEM fields. Inadequate teacher preparation and students' frustration with non-straightforward answers further impede comprehension. While existing methods focus on teaching angles through hands-on learning and real-world connections, there is a lack of engaging software and lessons. This paper presents an approach to teach geometric angles and angle addition to freshman and sophomore math learners using GeoGebra software, an angle addition simulator tool built with HTML5 canvas and JavaScript, and a protractor to make abstract concepts more tangible. This approach aims to enhance students' geometric understanding, critical thinking, and problem-solving skills by defining angles, addressing student difficulties, and employing interactive software. It integrates traditional and digital tools to engage Gen Z learners, preparing them for academic and professional success in fields like

architecture, engineering, and navigation. The lesson will be evaluated using a pre-test, a post-test, and a survey of 25 students. Student feedback indicates that the use of GeoGebra software, the angle addition simulator, and hands-on tools like protractors significantly enhanced their engagement and comprehension of geometric concepts, making learning more interactive and effective. Overall, this study aims to demonstrate that the designed tools positively impact students' learning in angle measurement.

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