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MSM Framework: Augmented Reality Models of 3D Vectors

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Abstract

This study investigates the potential of Augmented Reality (AR) as a pedagogical tool for the enhancement of mathematical understanding and conceptualization in physics, primarily within the context of engineering. The interactive and immersive properties of AR close the gap between representations of mathematical and physical phenomena, enabling students to grasp the issues in challenging physics topics such as vector addition, vector resolution, and the definition of coordinate systems in projectile motion, inclined planes, and magnetic fields around a wire.

Utilizing the Mathematics Sense-Making (MSM) framework, the study investigates student reasoning and engagement through think-aloud interviews and directed tutorials. The MARVLS AR apps use the camera for a phone or tablet to digitally overlay an AR representation of a physical object visually on the Merge© cube. Users are then able to modify the orientation of the AR model in response to the user rotating or translating the cube. The findings of the study suggest that AR improved students' spatial reasoning, facilitated the development of shifts between mathematical and physical reasoning, and decreased cognitive load.

The AR system developed and evaluated in this paper can be implemented by curriculum and educational designers at any level, from K-12 to university to professional career training in any STEM field.

Introduction

Students often face challenges with learning abstract concepts and spatial visualization, particularly when engaging with new 3D content in physics and engineering [1-3]. These disciplines rely heavily on foundational knowledge from mathematics and physics, where many students encounter gaps in understanding, particularly in complex, 3D topics such as vector operations and defining coordinate systems. This paper focuses on the introductory concept of vectors - a crucial building block for grasping 3D physics and engineering principles.

Vectors can be represented in multiple ways, including conceptual diagrams, symbols, and mathematical equations. While these representations are valuable, they often fall short of providing the intuitive, hands-on dynamic experience necessary for students to fully grasp the concepts. Without the ability to interact with and observe the real-time consequences of these interactions, students may struggle to develop a deep understanding. Luckily, AR has been found to be an effective learning tool in science education, particularly in aiding the understanding of vectors in physics, math, and engineering [4-12].

This research study aims to improve students' learning outcomes in STEM, particularly those who struggle with spatial and mathematical reasoning. Developing a deeper understanding of spatial transformations will serve students in their coursework, retention, and completion of STEM degrees. This foundational knowledge will eventually support students in the long run to develop expertise and have successful careers in STEM.

As part of a larger project aimed at understanding the effectiveness of augmented reality curriculum in learning abstract and 3D physics and engineering concepts, we are utilizing the Mathematical Sense-Making (MSM) framework to analyze student reasoning processes. This approach also enables us to refine both the lessons and the app to better address the research questions.

MARVLS Augmented Reality App and Lessons

The MARVLS smartphone Apps use augmented reality (AR) and Merge© cubes to allow students to hold the diagrammatic representations of abstract science concepts, then rotate and move these representations around in space. These interactions allow students to investigate the connections between 2D and 3D models in STEM and the mathematical formulas underlying these models [13-14]. The MARVLS apps use the camera for a phone or tablet to digitally overlay an AR representation of a physical object visually on the Merge© cube. Users are then able to modify the orientation of the AR model in response to the user rotating or translating the cube. By doing this, users are able to manipulate the AR model as if it were a physical object. As such in this paper we consider the digital representations as physical objects.

Each AR model includes an add representation button that directly associates labels with aspects of the model. For example, in this paper, the student presses buttons that represent variables in the vector equation that highlight different vectors in the augmented reality model.

The Lessons and AR Scenes include:

- *Vector Addition*: Through manipulation of vector magnitudes, the MARVLS AR Scene enables students to visualize the addition of vectors using the tip-to-tail method in a 3D coordinate system.
- *Vector Resolution/Components*: Students interact with 2D and 3D vector arrows and visualize how components of a vector (e.g. A_x, A_y, and A_z) combine to form the original vector. Additionally, students can highlight each component and explore corresponding equations and their physical (e.g., horizontal and vertical) implications.
- *Projectile Motion*: By rotating the Merge© cube, students adjust the coordinate systems to match a trajectory of a ball's motion with the front/direct view. This allows students to comprehend how selecting certain axes (e.g., aligning the x-axis with the horizontal and y-axis with vertical) simplifies calculations of a mathematical equation (e.g. $y=y_0+v_0t+1/2gt^2$).
- *Box on an Incline*: The goal of this AR Scene is for students to identify the difference in drawing a coordinate system for a flat versus an inclined surface. The AR Scene illustrates coordinate axes parallel and perpendicular to the incline. Viewing the AR Scene, students can

potentially draw the free body diagram of the box (including gravitational and normal forces) relative to the orientation of the coordinate system and resolve forces into components.

- *Magnetic Field Around a Wire*: The AR Scene shows the magnetic field arrows surrounding a current carrying wire and relate their 2D and 3D representations. Students can explore the concept of the right-hand rule to have a deeper understanding of abstract mathematical laws (e.g. Biot-Savart).

Mathematical Sense-Making (MSM) Framework

Gifford and Finkelstein [15] developed the mathematical sense-making framework based on Vygotsky's [16] ideas of mediated cognition and the cognitive process strand of scientific sense-making identified by Odden and Russ [17]. Their MSM framework is a collection of modes of sense-making as shown in Figure 1. These modes define the tool and the object that a student uses to understand the object. The triangles in the figure are a representation of mediated cognition, where the student uses a tool to interact with an object or interacts with the object directly and this is an extension of the work of Vygotsky [16].

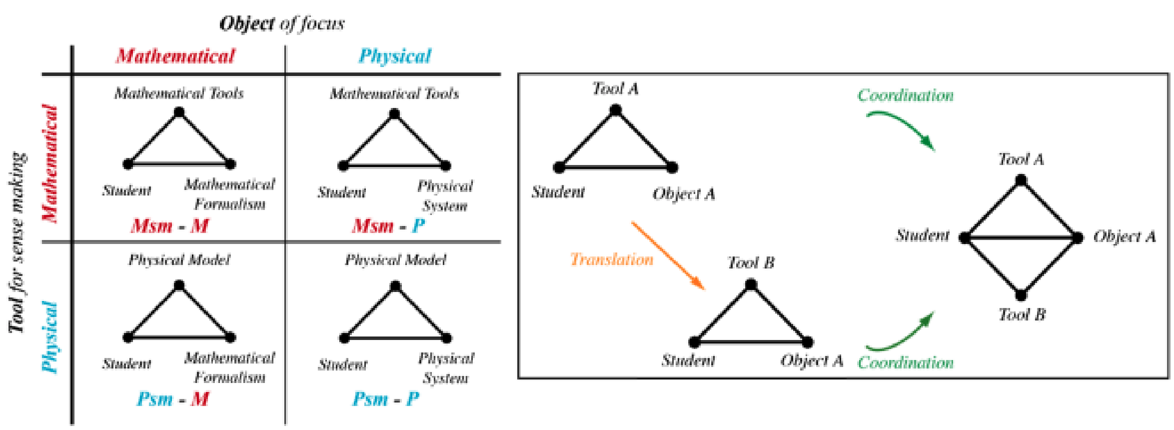


Figure 1. Left: the four modes of the MSM framework, defined by the tool and the object of sense-making. Right: a sense-making diagram, showing the processes of translation and coordination that lead to a coordinated reasoning structure.[15]

The four modes in the framework include Msm-M, Msm-P, Psm-M, and Psm-P. Specifically,

- Msm-M: a mathematical tool mediates the interaction with a mathematical object
- Msm-P: a mathematical tool mediates the interaction with a physical object
- Psm-M: a physical tool mediates the interaction with a mathematical object
- Psm-P: a physical tool mediates the interaction with a physical object

The MSM framework provides for added complexity to combine these modes into molecules using translation, chaining and coordination. Translation is a change of the tool or the object, chaining is combining tools sequentially, and coordination is combining the modes together.

Methods

Seven students participated in an IRB approved study that included a guided tutorial combined with a think-aloud interview. The interviews were video recorded and captured students' on-the-fly reasoning processes while they interacted with the tutorial and AR models.

The first author guided the students through the lessons and answered any questions. The first few activities in the lesson were an introduction to the type of questions they'd be answering along with practice for how to maneuver in the App and use the Merge© cube. Once students were ready to start, they were introduced to lessons that included vector addition, vector components, and the mathematical equations used to define vectors. Then, students were asked to consider the vectors and define a coordinate system for three AR Scenes of 3D physics concepts. Additional details about the lessons are provided in the previous section [18].

Two Augmented Reality Apps were used with the lesson. These include MARVLS: Physics I Mechanics and MARVLS AR for Physics 2. These Apps are available on the Appstore and the Google Play Store [19-20]. The menus used in the App that allow students to choose component values and to view the vector equation and press buttons are shown in Figure 2.

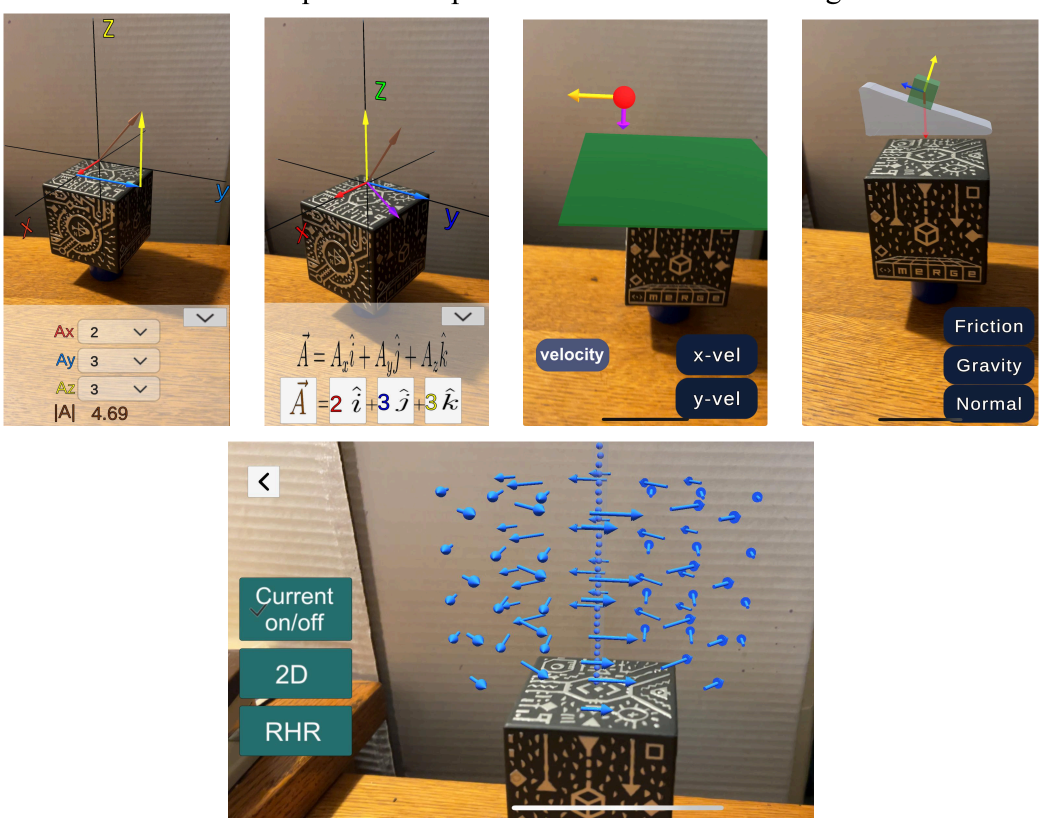


Figure 2. Examples of the AR Scenes as viewed through the smartphone screen for the vector addition and vector component Scenes. The AR Scenes for the 3D physics concepts of projectile motion, box on an incline, and the magnetic field of a current-carrying wire are shown.

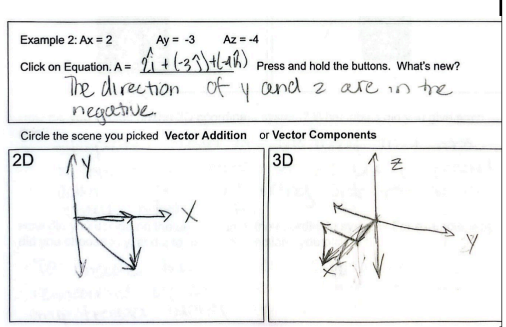
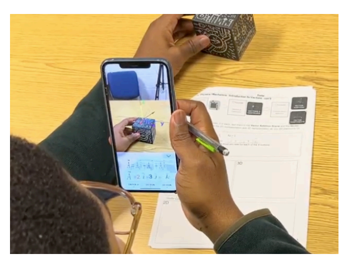
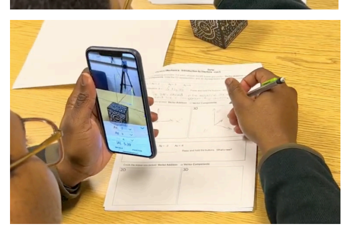
Results and Discussion

The data collected during the interviews include the student work and the video and audio of students as they worked through the lessons. Video was recorded over the shoulder of students as they viewed the AR models on their phones, manipulated the Merge© cube, answered questions, and completed drawings.

Students reported how the App supported their interaction with complex physics by verbalizing their thoughts. In addition, the video highlighted moments of cognitive struggle or breakthrough. Analyses of the recorded videos showed that active learning was supported by the AR environment, as students dynamically tested and refined their understanding. For example, on tasks involving vector resolution or the direction of the magnetic field, the interactivity of the App allowed immediate feedback, aiding in the identification and correction of misconceptions. Students started drawings, viewed the AR image while rotating the Merge© cube into different orientations, pressing buttons to highlight the vectors, and made corrections to their drawings. Each of the students participated with the AR Apps and lessons in this way, referring back to the App and the AR Scene several times for each drawing.

Table 1 includes samples of student work, snapshots of students viewing the AR Scenes with their smartphones, and the MSM mode chosen to best represent the sense-making mode students used during the lesson. The drawings completed by the students were selected as the students spent most of their time on these drawings and spent the most time using the App to complete the drawings.

Table 1. Data collected from the think-aloud interviews including student work and pictures of students using their smartphone to view the augmented reality models.

Vector Activity	Student work	Interview	MSM mode
Vectors Addition & Vector Components			Msm-P & Psm-M

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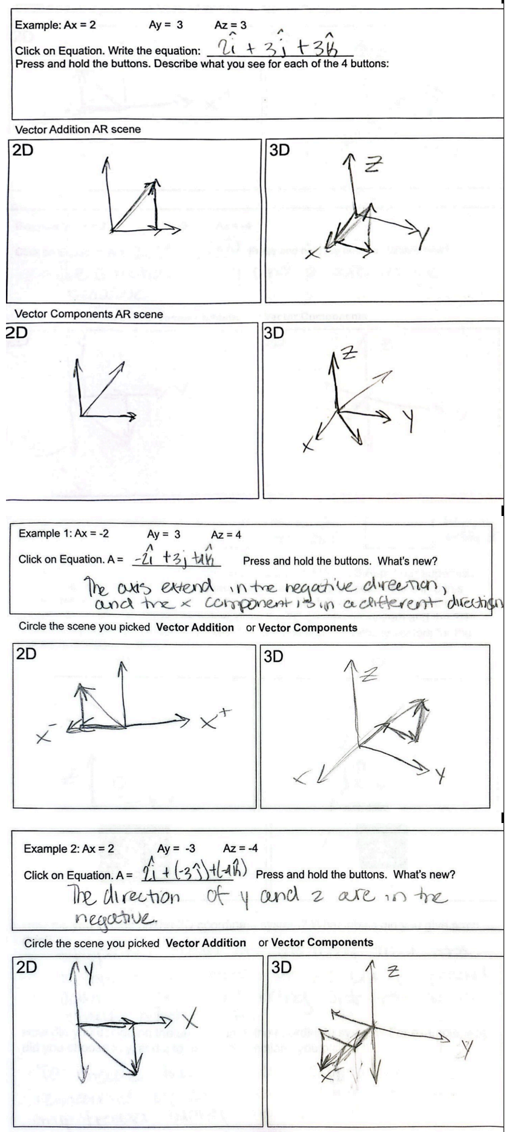
<td>
Example: Ax = 2 Ay = 3 Az = 3
Click on Equation. Write the equation: $2\hat{i} + 3\hat{j} + 3\hat{k}$
Press and hold the buttons. Describe what you see for each of the 4 buttons:
Vector Addition AR scene
2D 3D
Vector Components AR scene
2D 3D
Example 1: Ax = -2 Ay = 3 Az = 4
Click on Equation. A = $-2\hat{i} + 3\hat{j} + 4\hat{k}$ Press and hold the buttons. What's new?
The axis extend in the negative direction, and the x component is in a different direction
Circle the scene you picked **Vector Addition** or **Vector Components**
2D 3D
Example 2: Ax = 2 Ay = -3 Az = -4
Click on Equation. A = $2\hat{i} + (-3\hat{j}) + (-4\hat{k})$ Press and hold the buttons. What's new?
The direction of y and z are in the negative.
Circle the scene you picked **Vector Addition** or **Vector Components**
2D 3D
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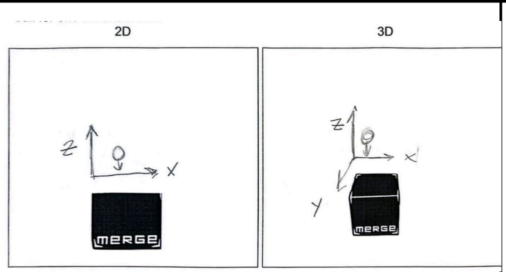
<td>
2D 3D
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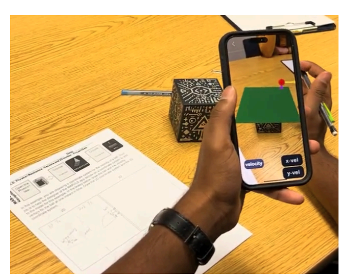
<td></td>
<td>Psm-P</td>
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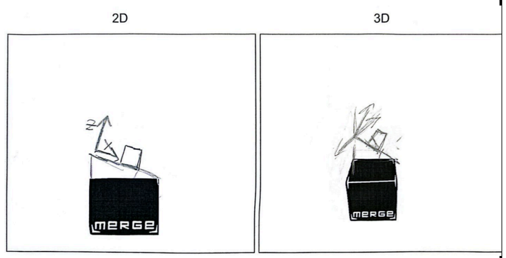
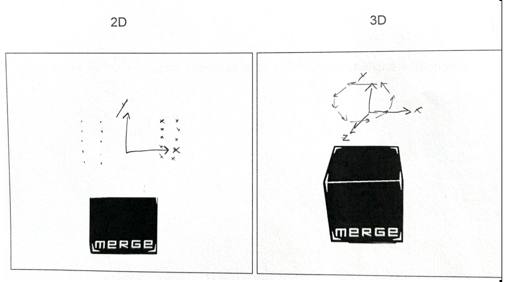
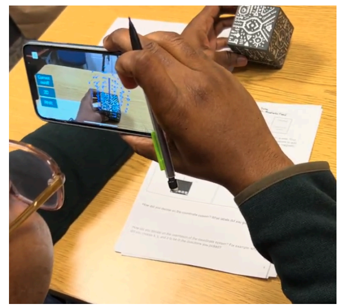
Inclined Plane			Psm-P
B field of a wire			Psm-P

The authors completed the lessons, watched the interviews, and categorized the modes in the videos independently and met to discuss the choice of sense-making modes for each of the lessons. The evolution and construction of the modes is illustrated in the figures below. These modes are based on the interviews with the students and represent how they interacted with the lessons and the AR Scenes. Each of the modes start out with a PSM-P mode where the tool is the AR model and the object is the concept of interest.

The sense-making modes for the vector activities are shown in Figure 3. The diagram starts out in the Psm-P mode with the AR vector model as the tool and the concept of the Vector as the object. There are two translations where the tool changes. The first translation to the 2D and 3D drawings as a new tool does not change the mode. The second translation is to the vector equation as a new tool and a new mode of Msm-P. Three chaining steps occur with changes of the tool to the vector components (Msm-M), then the highlighting component buttons (Msm-M), then the vector equation (Msm-P) as the final tool to understand the vector object. This sense-making molecule is joined with the sense-making mode that we started with, which is the chained 2D and 3D drawings to the AR model (Psm-P) to understand the vector object. Studying the chaining process to get to the vector equation as a tool, we determined that the sense-making mode was a coordinated M and P tool as described by Gifford and Finklestein as [M&P]sm-P [15]. The [M&Psm]-P mode was chosen as students navigated between the equation and pressing buttons and the AR model several times and seemed to use the equation as a tool sometimes and the AR model as a tool to understand the equation.

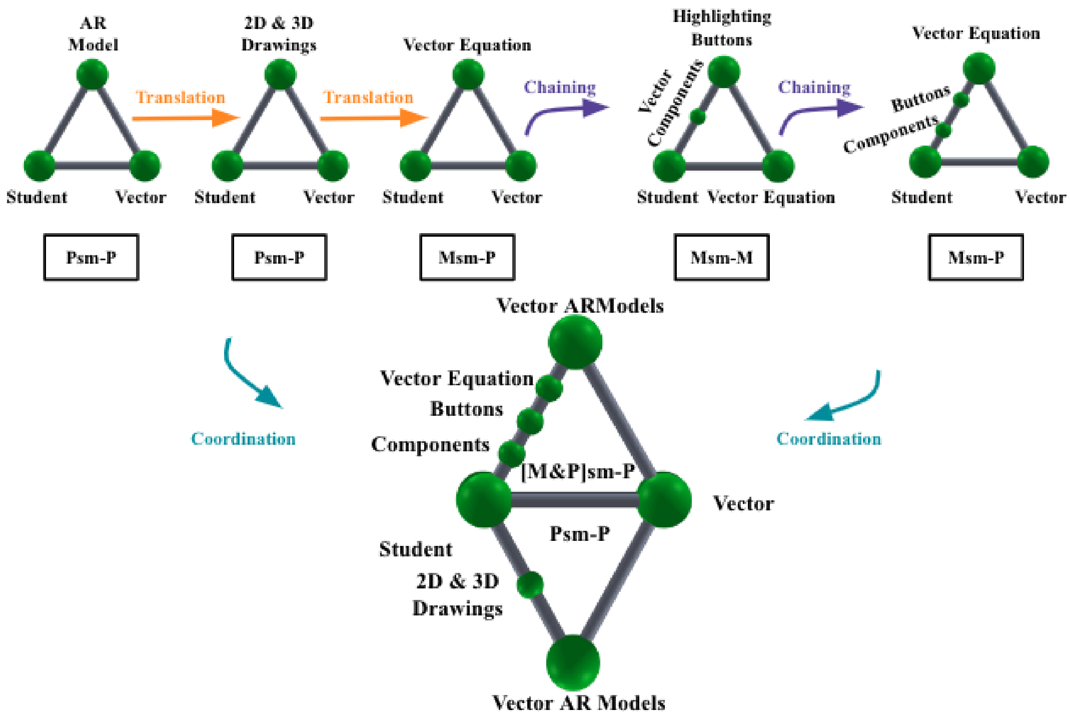


Figure 3. Vector sense-making modes, translations, chaining and coordination.

The sense-making modes for the projectile activity are shown in Figure 4. Each of the modes in this sense-making example is a Psm-P mode. The diagram starts out with the AR projectile model as the tool and the concept of the projectiles as the object. There is one translation where the tool changes. This translation to the 2D and 3D drawings as a new tool does not change the mode. Two chaining steps occur with changes of the tool to the coordinate system (Psm-P) and the 2D and 3D drawings (Psm-P) as the final tool to understand projectile motion in 3D. This sense-making molecule is joined with the sense-making mode that we started with which is the AR model (Psm-P) to understand the projectile object.

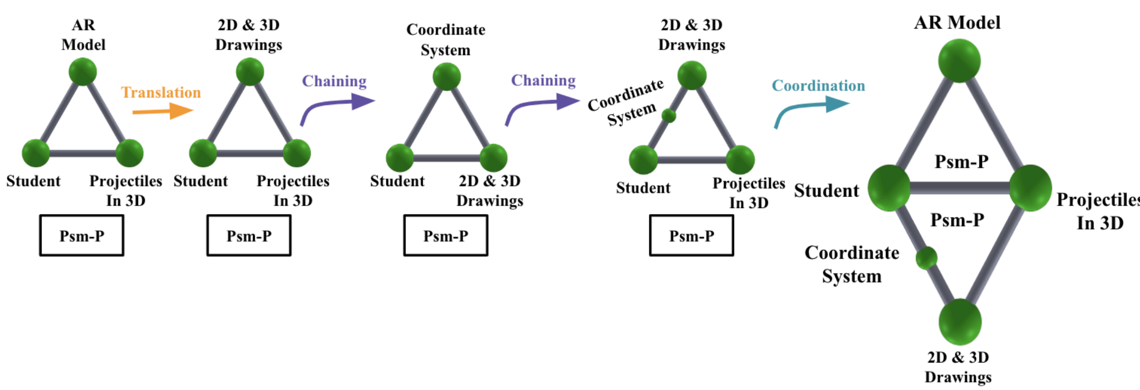


FIgure 4. Projectile sense-making modes, translations, chaining and coordination.

The sense-making modes for the box on an incline activity are shown in Figure 5. Each of the modes in this sense-making example is a Psm-P mode. The diagram starts out with the AR box on an incline model as the tool and the concept of a box on an incline as the object. There is one translation where the tool changes. This translation to the 2D and 3D drawings as a new tool does not change the mode. Three chaining steps occur with changes of the tool to the direction of motion of the box (Psm-P), the coordinate system, and the 2D and 3D drawings (Psm-P) as the final tool to understand the motion of the box on an incline in 3D. This sense-making molecule is joined with the sense-making mode that we started with which is the AR model (Psm-P) to understand the box on an incline object.

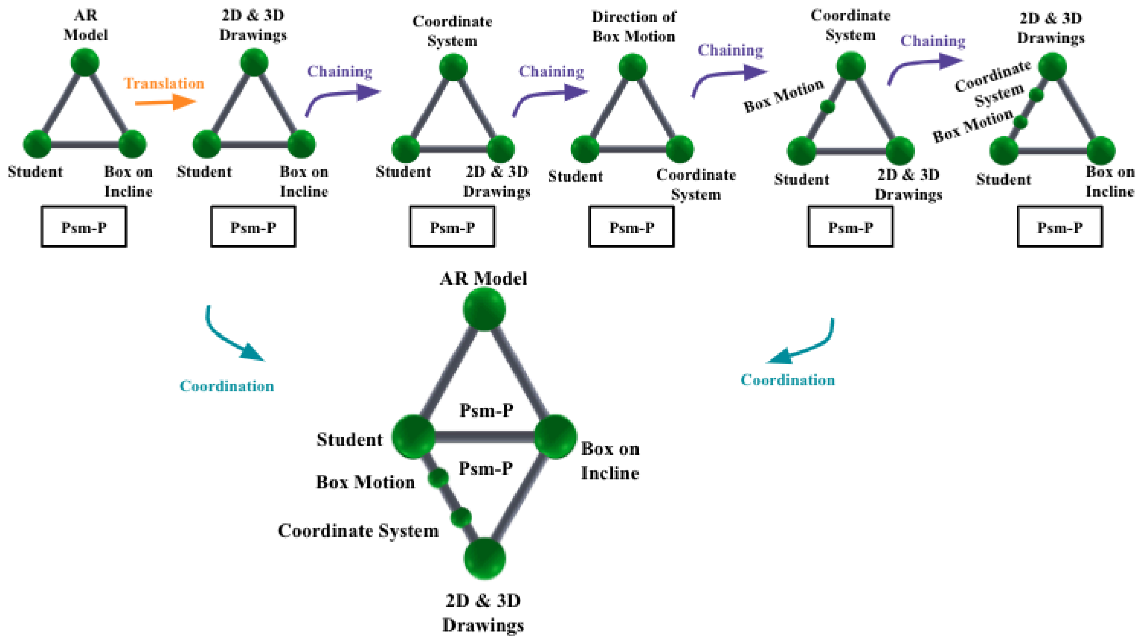


FIgure 5. Box on an incline sense-making modes, translations, chaining and coordination.

The sense-making modes for the magnetic field of a current-carrying wire activity are shown in Figure 6. Each of the modes in this sense-making example is a Psm-P mode. The diagram starts out in the Psm-P mode with the AR magnetic field model as the tool and the concept of the B field of a current-carrying wire as the object. There is one translation where the tool changes. The translation to the 2D and 3D drawings as a new tool does not change the mode. Four chaining steps occur with changes of the tool to the current direction (Psm-M), then the coordinate system (Psm-P), then the 2D image highlighting button (Psm-M), then the 2D and 3D drawings (Psm-P) as the final tool to understand the B field of a current-carrying wire object. This sense-making molecule connects to the initial sense-making mode—linking 2D and 3D drawings to the AR model (Psm-P)—to support understanding of the B field around a current-carrying wire

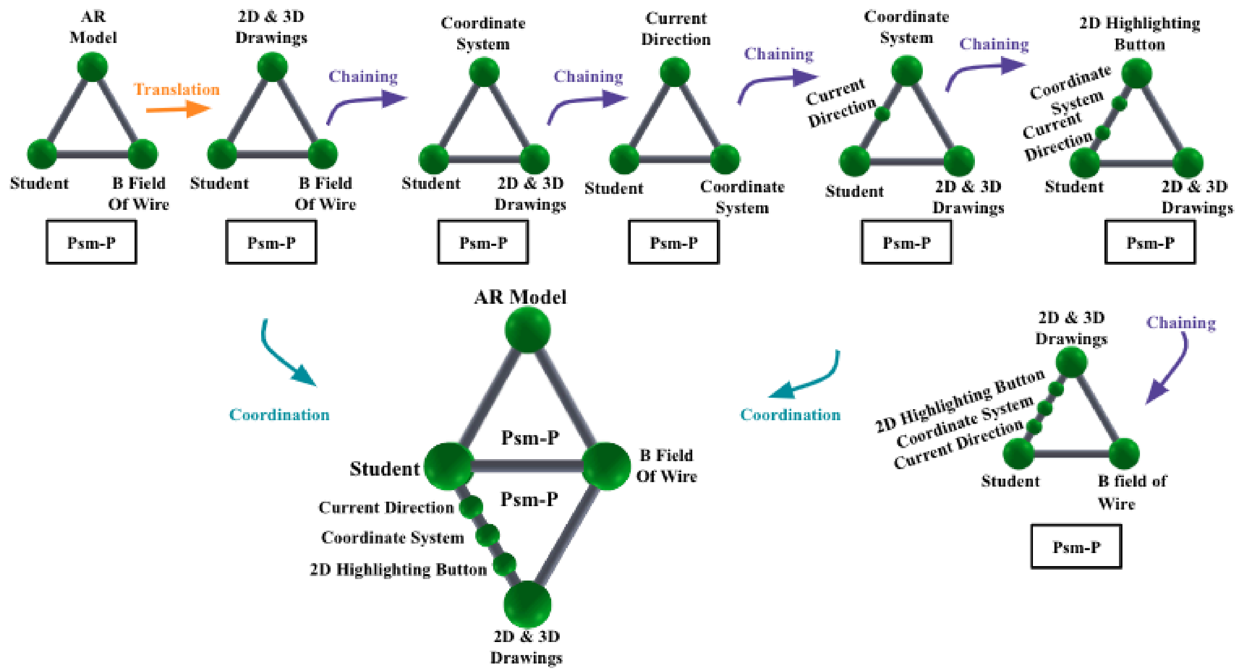


Figure 6. Magnetic field of a wire sense-making modes, translations, chaining and coordination.

Conclusions and Future Work

In this work, we studied students engaged in mathematical sense-making and the impact of AR tools at bridging the gap between abstract mathematics and physical phenomena. Utilizing the MSM analytical framework, we examined students' reasoning as they interacted with the MARVLS AR applications. The MARVLS App facilitated transitions between reasoning modes, particularly between MSM-P and PSM-P. Students effectively connected the mathematical formalisms with physical interpretations, especially in areas that required spatial reasoning, such as understanding vector addition, resolving vectors into components, aligning coordinate systems, and visualizing magnetic fields in 3D space. Very high engagement levels were observed where students spent much time interacting with features that enabled the dynamic manipulation of vectors and the visualization of motion providing an active learning experience. The AR apps foster the thinking abilities required to grasp difficult physics concepts while also deepening conceptual understanding through dynamic, interactive exploration. Future work will look to further refine the design of the app in terms of the identified challenges and further its application within wider educational settings.

Future research development involves focusing on improving the effectiveness of the AR-based learning experience identified during interviews to improve the curriculum. One key improvement pertains to smoothing the transition from 2D to 3D representations. We noticed that some students struggled when switching between 2D and 3D representations, often failing to include the third dimension. For example, students who correctly named the vertical axis as the y-axis demonstrated confusion about how to incorporate the z-axis into their reasoning. To address this, we plan to design AR examples which explicitly relate 2D and 3D visualizations, and to practice working with all possible 2D vector projections in A_{xy}, A_{xz}, and A_{yz} planes. In addition, we will refine the lesson structure to encourage consistent reference to foundational concepts and diagrams throughout the activity. This will support students to maintain a comprehensive perspective as they progress through the printed guided tutorial.

Finally, we plan to conduct collaborative AR exercises where students are engaged in group problem-solving activities to enhance peer-to-peer learning and conceptual engagement. A more robust, evidence-based framework for integrating AR into the physics and engineering curriculum can be developed based on the insights gained from this study.

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