

## **WIP: Efficacy of Connecting Engineering and Calculus through AI Problem Generation**

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## 1. Introduction

Calculus courses have long served as gatekeepers to STEM fields, presenting significant challenges to students and contributing to high rates of attrition in engineering programs [1], [2]. Despite being foundational, these courses often fail to connect abstract mathematical concepts to their practical applications in engineering, leaving students disengaged and unprepared for real-world problem-solving [3]. This disconnect has been identified as a barrier to retention, with many students citing calculus as a primary reason for abandoning STEM majors [4].

To address this issue of disconnect between calculus courses and engineering applications, education researchers have turned to artificial intelligence in education (AIED) to bridge the gap between theoretical knowledge and its application. Tools like ProGenie, an AI-driven problem-generation platform, aim to contextualize calculus problems within authentic engineering scenarios, thereby enhancing student engagement and comprehension. Preliminary findings from this study suggest that AI-generated problems *can* align with diverse cognitive processes and knowledge dimensions, though early implementations have tended to generate problems that generally require students to apply procedural knowledge. We address this outcome as a potential shortcoming to current AI tools and we discuss implications for improving AI tools for first year calculus education and next steps for evaluation.

### *1.1. Investigating Why Calculus Causes Attrition*

Calculus related courses have long been regarded as gatekeeping to STEM degrees, including engineering technology. On average, 20% of students in calculus courses face an outcome of a 'D', 'F', or withdrawal (DFW); DFW outcomes in calculus are as high as 40% at some institutions [5]. Students have come to describe calculus as a type of “weed out” course because these courses negatively impact persistence and challenge student’s confidence and interest in STEM, often revealing inadequacies in high school preparation or readiness for college transition [2]. Even when adequately prepared, students argue that “it’s better to bail than to fail” to avoid the risk of poor performance. As such, students who switch out of STEM majors frequently cite negative experiences in calculus classes as a leading reason [6].

Researchers have found that instructors who provide meaningful connections between calculus topics and applications are most valued by all students, including those who persist in STEM (36%) and those who do not (24%) [2]. These findings align with calls for more robust applications of calculus during the landmark 1987 National Consortium to Revise Calculus for the New Century [7]. One of the referenced reasons has been the lack of innovation and meaningful problems found in stagnant Calculus textbooks [7]. This literature suggests an opportunity to improve persistence through calculus by improving the connection between mathematics and their real-world applications.

## 1.2. Revisiting Bloom's Taxonomy for Modern Learning

Effective teaching and learning are guided through well-structured objectives. Learning objectives outline the composition of lessons and practice problems by establishing the content of focus, specific student activities, and forms of assessment [8]. Similarly, they have been discussed in best practices texts for how they contribute to learning [9], [10].

Bloom's Taxonomy has frequently been referenced in developing learning objectives [5], [11]. A revision to Bloom's Taxonomy includes two dimensions: Knowledge Typology and Cognitive Process [12]. These dimensions help organize learning objectives based on their subject (noun: knowledge typology) and the activity (verb: cognitive process). Moreover, the subject of knowledge can be categorized into four developmental subjects [12], where later types refer to deeper knowledge; we present these four types in Table 1. Cognitive Processes can be broken down into six developmental activities [12], where higher numbers refer to more complex processes; these six processes are given in Table 2. The combination of these knowledge types and cognitive processes can be modeled into a taxonomy table, illustrated through Figure 1.

**Table 1. Description of the Four Knowledge Typologies.**

Knowledge Typology	Description
(A) Factual	Terminology and discrete facts that serve as the foundation of understanding
(B) Conceptual	How these elements of a subject relate to each other, which enables learners to meaningfully construct understanding
(C) Procedural	how to perform specific tasks, including methods of inquiry, algorithms, and techniques
(D) Metacognitive	Understanding around one's own cognition, including learning strategies and problem-solving strengths and weaknesses

**Table 2. Description of the Six Cognitive Processes.**

Cognitive Process	Description	Example
(1) Remembering	Retrieving of knowledge	Defining a derivative
(2) Understanding	Constructing meaning	Summarizing the relationship between a function and its derivative
(3) Applying	Conducting a procedure	Taking the derivative of a function
(4) Analyzing	Organizing material into parts and determining their relationships	Differentiating integrals from their approximation methods
(5) Evaluating	Making judgments using criteria or experience	Performing a sanity check whether a solution makes sense
(6) Creating	Generating or planning something novel	Creating a function that reflects an observed behavior

		Increasing Complexity in Cognitive Process					
		1	2	3	4	5	6
Increasing Depth of Knowledge	D						
	C						
	B						
	A						

**Figure 1. Annotated Taxonomy Table [12].**

Effective learning objectives for practice problems and activities should incorporate varied knowledge typologies and cognitive processes to reflect the challenges students will face in their careers. Engineering problems in the real world are often ill-structured and complex with multiple solution pathways, making it essential to design course problems that align with this complexity [13]. Practicing with problems that are diverse along these dimensions provides students with rigorous experiences that prepare them for addressing real-world challenges. Storytelling can also serve as a powerful framework for addressing these challenges, as it piques curiosity and engages students by contextualizing abstract problems within relatable, real-world narratives [14], [15]. By embedding discipline-specific problems into meaningful stories, students can connect more deeply with the material, enhancing their critical thinking and problem-solving skills [16], [17].

### *1.3. Exploring the Role of AI in Education*

The integration of AIED has emerged as a critical field of study, broadly categorized into two major areas: how AI can enhance teaching and learning, and how individuals are taught about AI itself. Our review focuses on the former. While the potential of AI in education has been met with both caution and praise, the discourse remains divided, and the future of AIED is unknown and unpredictable. Gray and Kucirkova [18] highlight the transformative potential of AI, particularly in supporting personalized and adaptive learning environments. However, Selwyn [19] warns of how speculative education researchers may become about much AIED research. While AI tools have begun to demonstrate their abilities to function as digital assistants [20], personalized tutors [20], [21], and tailored educational content generators [22], [23], rigorous assessment and evaluation is needed [19]. As such, researchers have begun to publish frameworks [22] and practical strategies for integrating AI into education, emphasizing ethical considerations [24]. Challenges particularly persist in embedding AIED into broader policy discussions and ensuring that ethical implications are sufficiently addressed [25].

One area of research in AIED explores the use of AI and large language models (LLMs) to generate contextualized math problems. Although automatic problem generation has been studied since the mid-1960s [26], the accessibility and sophistication of modern AI models have significantly enhanced the personalization, generation speed, and robustness of these problems. Recent efforts, such as the use of OpenAI's ChatGPT to generate problems in real-time within classroom settings, have demonstrated the potential of these tools to adapt dynamically to learners' needs [27]. This approach is gaining traction, particularly in K–12 education, where personalized arithmetic problems are being used to establish meaningful context for students

[28], [29]. While these tools have been emerging, a formal tool designed for engineering education and the challenges first-year students face in calculus has yet to be publicly released.

## 2. Project Approach

Calculus courses often experience high attrition rates, partially because students struggle to see real-world relevance—especially in engineering contexts. To address this challenge, we explore whether an AI tool called ProGenie can generate *academically meaningful* calculus problems that highlight authentic engineering applications, thereby potentially enhancing student engagement. We aim to evaluate the *subject* (knowledge typology) and *activity* (cognitive process) embedded in these AI-generated problems, focusing on what we consider “latent objectives”—those learning goals that remain implicit in traditional problem statements. **By identifying these latent objectives, we can determine if AI-generated items truly align with calculus course requirements.** Specifically, we pose the question: *What are the latent objectives of personalized, engineering-contextualized calculus problems generated by AI?* Through this lens, we seek to ascertain how effectively ProGenie addresses long-standing issues of relevance and rigor in calculus, with the aim of curbing attrition rates among first-year engineering students.

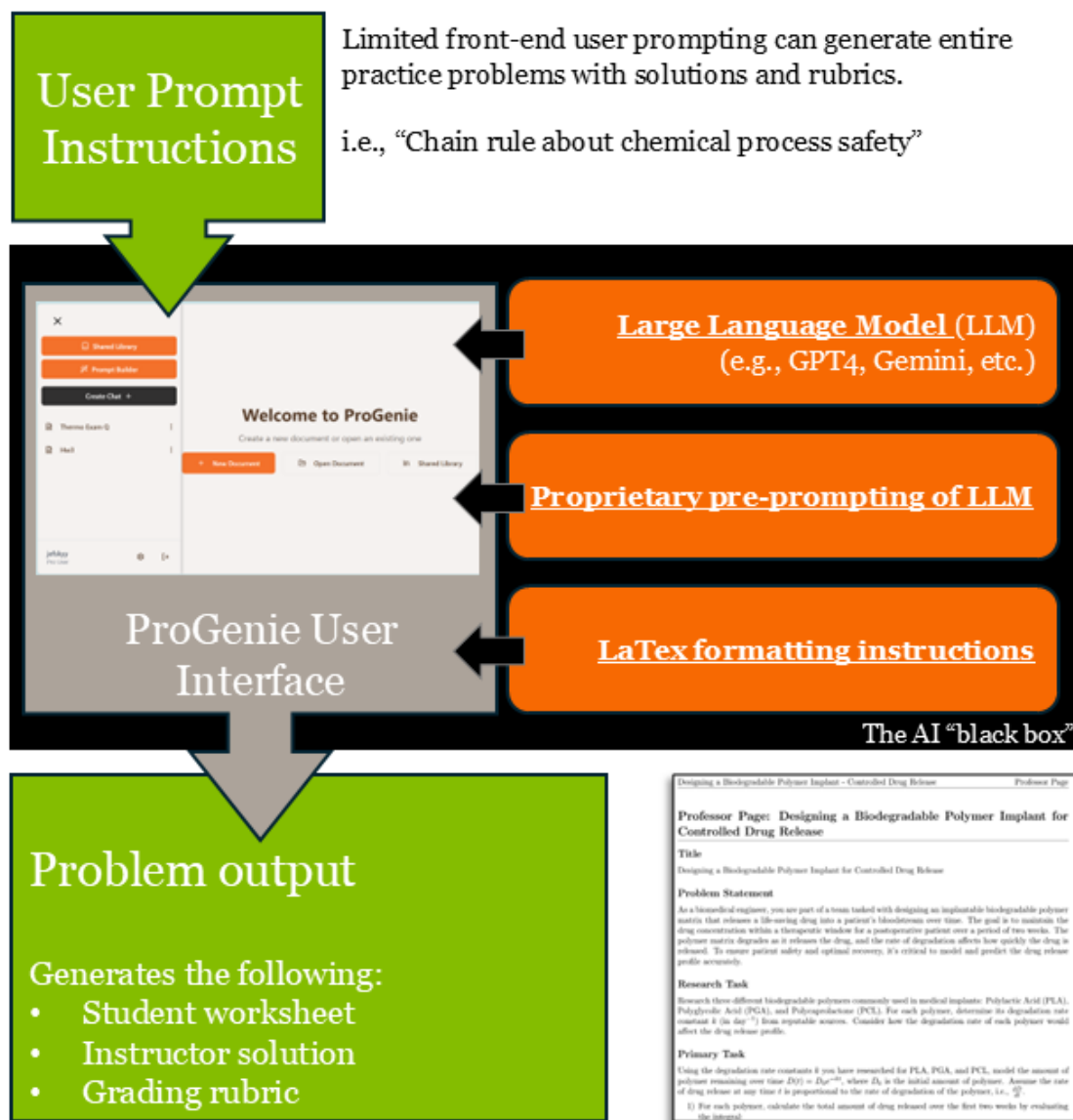
### 2.1. Generating Calculus Problems with ProGenie

Our analysis focuses on problems generated by an AI Problem Generation tool, ProGenie. ProGenie is an in-house tool, developed at [institution blinded for review], which has not yet been officially released for public use. ProGenie enables faculty to produce engineering story-based problem sets aligned with specific engineering disciplines to provide student autonomy in problem selection. For example, an instructor can quickly generate exponential decay problems wrapped in interesting stories related to biomedical, electrical, civil, and manufacturing engineering. Students can engage with the engineering discipline of their choosing or the problem scenario that piques their interest.

We share the general structure of ProGenie as a system in Figure 2. Users can request unique problems with very limited input, such as “Chain rule about chemical process safety.” ProGenie will produce a full student worksheet, instructor solution, and grading rubric within the “black box” of AI (Figure 2). ProGenie invites users to provide as little or as much detail about a particular problem scenario as they like in the problem generation process. We designed ProGenie to process commands through modular application programming interfaces (APIs) and can easily switch between large language models (LLMs) such as OpenAI’s GPT-4 to Google’s Gemini. In the background, we have configured ProGenie to interpret commands with the goal of producing structured, solvable calculus problems in the context of user-defined engineering scenarios using proprietary prompting. However, we have also configured ProGenie to format its responses using LaTeX and created a backend to automatically generate the final output in PDF-formatted documents for ease of use.

For this study, we analyzed nine problems that we generated using OpenAI’s o1 model. We have included an example problem as Appendix 1 for reference.

We share the work we generated with ProGenie as a case-study for other AI generation platforms. We recognize switching the LLM and altering the prompt configuration will significantly affect the output; hence, we share our findings as ‘lessons learned.’ In doing so we seek to contribute to the collective best practice for using AI as a tool in education. In our case, we seek to share the efficacy of how AI generated problems can connect calculus to engineering applications with a breadth and depth that would be difficult for most instructors to generate on their own.



**Figure 2. Flowchart of how ProGenie Functions.**

## 2.2. Implementing Provisional Coding for Problem Sets

As we sought to understand the latent objectives of AI-generated engineering-calculus problems, we adapted the descriptions of knowledge typologies (Appendix 2) and cognitive processes

(Appendix 3) shared by Anderson et al. [5] to be relevant toward calculus. Originally, these descriptions were written towards generic education, referencing broad examples in history, government, geology, etc. Our adaptation enables us to relate the AI generated calculus problems to calculus-related descriptions for knowledge typologies and cognitive processes.

We treat these adapted descriptions (Appendix 2 & 3) as provisional codebooks [30], [31], which enables us to qualitatively analyze the AI-generated calculus problems. Each time we observed an instance of a knowledge typology, we co-coded the section for a cognitive process, and vice-versa, as each activity must have an associated subject [5]. Some problem statements included multiple objectives, so we allowed multiple provisional codes per problem. To ensure coding reliability, one researcher conducted the initial coding of all problems, while a second researcher independently reviewed and verified the work. Any discrepancies were discussed until a consensus was reached.

### 3. Results & Discussion

When we analyzed the nine generated problems, we plotted them on the Taxonomy Table (Table 3) based upon the identified knowledge dimension and cognitive complexity. We color code the cells based on the frequency of problems per typology-process, visualizing a heat map of most generated problems. This table acts as a data visualization as part of our qualitative analysis of the generated problems.

**Table 3: Taxonomy Table Frequency Heat Map of Problems per Objective Category.**

		Cognitive Process					
		1. Remember	2. Understand	3. Apply	4. Analyze	5. Evaluate	6. Create
Knowledge Typology	D. Metacognitive Knowledge				4		6
	C. Procedural Knowledge			1, 2, 3, 5, 7, 9	3, 4, 8		
	B. Conceptual Knowledge		1, 5	8	8		
	A. Factual Knowledge						

The three categories of objectives with the highest frequency were ‘understand conceptual knowledge’ (B-2), ‘apply procedural knowledge’ (C-3), and ‘analyze procedural knowledge’ (C-4). We coded Problem 5, regarding optimizing packaging design for efficiency, as B-2 because of the research task built into this problem:

*Your task is to research three types of material commonly used in packaging. Consider: 1. Cardboard with a thickness that provides moderate durability 2. Biodegradable plastic offering environmental benefits 3. Recyclable aluminum known for its sturdiness. Compare their costs, environmental impact, and suitability for beverage storage and calculate how choices impact material consumption.*

This research task asks students to relate how suitability and cost relate to material consumption, which reflects a need for conceptual knowledge (B). This activity also asks students to make a comparison of the materials, which aligns well with the *understand* cognitive process (2).



Next, we coded Problem 2, regarding transoceanic fiber optic cables, as C-3:

- a) *Using the attenuation coefficient for the selected cable, derive an expression for the rate at which the signal power loss (in decibels) increases with distance. Compute this rate.*
- b) *Determine the maximum allowable distance between amplifiers if the signal power cannot drop by more than 20 dB between amplifiers to maintain signal integrity.*
- c) *Based on your findings, calculate the number of amplifiers required for the 6,000 km cable.*

This problem gives students an equation and ask them to derive it with respect to distance. To perform this derivation, students need procedural knowledge (C) of how to apply the power rule. Enacting this knowledge reflects the *apply* (3) cognitive process.

We coded Problem 4, regarding a drug delivery capsule, as C-4:

*Using the degradation rate constants you identified in your research, for each polymer:*

1. *Model the rate at which the medication is released into the bloodstream over time.*
2. *Determine the time duration during which the drug concentration remains within the therapeutic window (between  $C_{min} = 5\text{mg/L}$  and  $C_{max} = 20\text{mg/L}$ ).*

This problem gives students an equation of a capsule mass as a function of time. While the first task of this problem reflects C-3, the second step reflects C-4. Students must organize the multiple steps required to answer this question, which reflects the *analyze* (4) cognitive process. To perform these steps, students must have requisite procedural knowledge (C) as well.

### *3.1. Examining C-3: A Potential Shortcoming of AI Generated Problems*

A potential shortcoming of AI-generated problems is the excess of problems with objectives related to applying procedural knowledge (C-3). As discussed, students should see problems with a diversity of objectives to help prepare them for the complexity of real-world problems [32]. The over-representation of C-3 type problems may hinder students from getting this diversity. Problems that require factual knowledge (A) may not be best suited for engineering-contextualized problems. For example, students may not need a personalized word-problem to practice their factual knowledge of mathematical limits. However, there is a poignant lack of problems that have students practice evaluating (5) and creating (6) or exercising their metacognitive knowledge (D). Of the nine problems we generated, we only coded Problem 6 as D-6:

*In designing a power transmission from a power plant to a city across a river, you aim to minimize the length of the transmission line while considering the rivers constraints. The task is to calculate the angle  $\theta$ , relative to the riverbank, at which the line should be initiated to minimize material costs and ensure a consistent power supply... To solve this problem, calculate the theoretical optimal path using trigonometry, and justify your choice based on practical material research. The key steps involve optimizing the path length using trigonometric identities and constraints.*

In Problem 6, students are asked to create (6) an equation from the context of the problem, which requires metacognitive knowledge on how to set up and solve this type of problem.

The abundance of C-3 type problems may also be an artifact of calculus learning. Calculus teaching has long been recognized for its procedural manner. Recognizing the relationship



between conceptual and procedural knowledge (Hetcher et al., 2022), education researchers have sought alternative approaches to gaining robust conceptual and procedural knowledge of calculus [33], [34], [35].

This reliance on procedure is parallel to a critique on Bloom's taxonomy. Bloom's Taxonomy assumes that lower-order knowledge typologies (i.e., facts and concepts) must precede higher-order typologies (i.e., procedures and metacognition) [5]. However, as Case and Marshall [36] point out, this developmental progression often fails to capture the reality of procedural tasks, such as in calculus problem-solving, where students may memorize and apply problem-solving algorithms without truly understanding the underlying concepts. For example, students can know how to take a derivative without really having the conceptual knowledge needed to understand what that means.

We recognize the abundance of C-3 type AI-generated problems as a flag for concern that ProGenie may be perpetuating the gap in calculus learning that does not balance conceptual and procedural knowledge. Our future steps, and our recommendations for others seeking to integrate engineering and technology into calculus learning, are to continue to stay cognizant of and mitigate potential biases being built into AI tools. Our learning from analyzing the problems generated by ProGenie have led us to realize that our future work should include taxonomy references within the prompt configuration to facilitate problem generation alignment with specific learning objectives, beyond 'applying procedural knowledge' (C-3). Additionally, we believe, based on what we have learned in creating ProGenie and our broader work with LLMs, is that we can improve our internal prompting strategy to explicitly address these shortcomings.

### *3.2. Limitations*

When answering our research question regarding the latent objectives to AI-generated calculus problems, we only analyzed nine problems. It is possible that through additional analysis additional objective types and processes may emerge. We also only used OpenAI's ChatGPT models for problem generation. While the LLM used may impact the generated problems, analysis comparing models is out of scope for this work. Others who use AI-generated problems should expect some differences if using different LLMs or GPT models.

### *3.3. Projecting Implications for First-year Programs*

Regarding first-year programs, we still hold the belief that engineering-contextualized, AI-generated calculus problems can help improve calculus learning and STEM program retention during the first year. Tools like ProGenie provide an opportunity to enhance student curiosity and interest and readily connect calculus concepts to applications, which has been a major shortcoming in traditional calculus learning materials [37]. The findings from our study revealed that while the applications were largely feasible, ProGenie continued to reinforce the challenges associated with integrating calculus into engineering curricula. After correcting this potential shortcoming, our next step will be to evaluate first-year students' perceptions of these application-based problems and whether these problems can improve their calculus experience.

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# Professor Page: Designing an Optimal Drug Delivery Capsule

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## Title

### Designing an Optimal Drug Delivery Capsule

## Problem Statement

As a biomedical engineer at MedTech Innovations, you are part of a team developing a new drug delivery capsule designed to release a medication at a controlled rate over time. The goal is to maintain the drug concentration within the therapeutic window (between  $C_{\min}$  and  $C_{\max}$ ) in the patient's bloodstream for as long as possible to improve efficacy and patient compliance.

The capsule releases the medication as it dissolves inside the body, and the rate at which the capsule dissolves follows an exponential decay model. The mass  $m(t)$  of the capsule remaining at time  $t$  is given by:

$$m(t) = m_0 e^{-kt}$$

where:

- $m_0$  is the initial mass of the capsule (in milligrams, mg),
- $k$  is the degradation rate constant (in  $\text{h}^{-1}$ ),
- $t$  is the time (in hours, h).

## Research Task

Research three different biodegradable polymers commonly used in drug delivery systems:

1. Polylactic acid (PLA)
2. Polyglycolic acid (PGA)
3. Poly(lactic-co-glycolic acid) (PLGA)

For each polymer, find the typical degradation rate constant  $k$  (in SI units). Consider how the degradation rate affects the release rate of the medication.

## Primary Task

Using the degradation rate constants you identified, for each polymer:

1. Model the rate at which the medication is released into the bloodstream over time.
2. Determine the time duration during which the drug concentration remains within the therapeutic window (between  $C_{\min} = 5 \text{ mg/L}$  and  $C_{\max} = 20 \text{ mg/L}$ ).

Assume the following:

- The patient's blood volume is  $V = 5 \text{ L}$ .
- The medication is uniformly distributed in the bloodstream.
- The initial mass of the capsule  $m_0 = 100 \text{ mg}$ .

## Solution Steps for Professors

To solve this problem, follow these steps for each polymer:

1. **Find the degradation rate constant  $k$**  (use typical values for educational purposes):

- PLA:  $k_{\text{PLA}} = 0.02 \text{ h}^{-1}$
- PGA:  $k_{\text{PGA}} = 0.05 \text{ h}^{-1}$
- PLGA:  $k_{\text{PLGA}} = 0.03 \text{ h}^{-1}$

2. **Model the mass over time:**

$$m(t) = m_0 e^{-kt}$$

3. **Calculate the cumulative amount of medication released:**

$$M(t) = m_0 - m(t) = m_0 (1 - e^{-kt})$$

4. **Determine the drug concentration in the bloodstream:**

$$C(t) = \frac{M(t)}{V} = \frac{m_0 (1 - e^{-kt})}{V}$$

5. **Solve for time when  $C(t) = C_{\min}$ :**

$$C_{\min} = \frac{m_0 (1 - e^{-kt_{\min}})}{V} \implies e^{-kt_{\min}} = 1 - \frac{C_{\min} V}{m_0}$$

$$t_{\min} = -\frac{1}{k} \ln \left( 1 - \frac{C_{\min} V}{m_0} \right)$$

6. **Solve for time when  $C(t) = C_{\max}$ :**

$$C_{\max} = \frac{m_0 (1 - e^{-kt_{\max}})}{V} \implies e^{-kt_{\max}} = 1 - \frac{C_{\max} V}{m_0}$$

$$t_{\max} = -\frac{1}{k} \ln \left( 1 - \frac{C_{\max} V}{m_0} \right)$$

7. **Calculate numerical values:**

- Compute  $\frac{C_{\min} V}{m_0} = \frac{5 \text{ mg/L} \times 5 \text{ L}}{100 \text{ mg}} = 0.25$
- Compute  $\frac{C_{\max} V}{m_0} = \frac{20 \text{ mg/L} \times 5 \text{ L}}{100 \text{ mg}} = 1$

Note: Since  $\frac{C_{\max} V}{m_0} = 1$ ,  $t_{\max} \rightarrow \infty$ .

8. **Compute  $t_{\min}$  for each polymer:**

$$t_{\min} = -\frac{1}{k} \ln (1 - 0.25) = -\frac{1}{k} \ln(0.75)$$

$$t_{\min} = \frac{0.2877}{k}$$

- PLA:  $t_{\min} = \frac{0.2877}{0.02} = 14.38 \text{ h}$
- PGA:  $t_{\min} = \frac{0.2877}{0.05} = 5.75 \text{ h}$
- PLGA:  $t_{\min} = \frac{0.2877}{0.03} = 9.59 \text{ h}$

9. **Determine the optimal polymer:**

- PLA provides the longest duration above  $C_{\min}$  (14.38 h).
- PLGA is intermediate (9.59 h).
- PGA provides the shortest duration (5.75 h).

## Numerical Solution

- **PLA:**  $t_{\min} \approx 14.38$  h
- **PLGA:**  $t_{\min} \approx 9.59$  h
- **PGA:**  $t_{\min} \approx 5.75$  h

**Recommendation:** Choose **PLA** as it maintains the drug concentration within the therapeutic window for the longest duration.



# Student Page: Designing an Optimal Drug Delivery Capsule

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## Title

### Designing an Optimal Drug Delivery Capsule

## Problem Statement

As a biomedical engineer at MedTech Innovations, you are part of a team developing a new drug delivery capsule designed to release a medication at a controlled rate over time. The goal is to maintain the drug concentration within the therapeutic window (between  $C_{\min}$  and  $C_{\max}$ ) in the patient's bloodstream for as long as possible to improve efficacy and patient compliance.

The capsule releases the medication as it dissolves inside the body, and the rate at which the capsule dissolves follows an exponential decay model. The mass  $m(t)$  of the capsule remaining at time  $t$  is given by:

$$m(t) = m_0 e^{-kt}$$

where:

- $m_0$  is the initial mass of the capsule (in milligrams, mg),
- $k$  is the degradation rate constant (in  $\text{h}^{-1}$ ),
- $t$  is the time (in hours, h).

## Research Task

Research three different biodegradable polymers commonly used in drug delivery systems:

1. Polylactic acid (PLA)
2. Polyglycolic acid (PGA)
3. Poly(lactic-co-glycolic acid) (PLGA)

For each polymer, find the typical degradation rate constant  $k$  (in SI units). Consider how the degradation rate affects the release rate of the medication.

## Primary Task

Using the degradation rate constants you identified, for each polymer:

1. Model the rate at which the medication is released into the bloodstream over time.
2. Determine the time duration during which the drug concentration remains within the therapeutic window (between  $C_{\min} = 5 \text{ mg/L}$  and  $C_{\max} = 20 \text{ mg/L}$ ).

Assume the following:

- The patient's blood volume is  $V = 5 \text{ L}$ .
- The medication is uniformly distributed in the bloodstream.
- The initial mass of the capsule  $m_0 = 100 \text{ mg}$ .

**Student Work Section**

Provide detailed solutions to the tasks above, showing all your work and calculations. Clearly state which polymer you recommend and justify your choice based on your findings.

## Rubric Page: Designing an Optimal Drug Delivery Capsule

Criteria	Exemplary (4)	Proficient (3)	Developing (2)	Needs Improvement (1)
<b>Mathematical Accuracy</b>	Solutions are correct and calculations are accurate for all polymers.	Solutions are mostly correct with minor errors in calculations.	Solutions have significant errors affecting results.	Solutions are incorrect or incomplete.
<b>Clarity of Explanation</b>	Explanations are clear, logical, and detailed.	Explanations are mostly clear but may lack some detail.	Explanations are unclear or insufficient.	Explanations are missing or very unclear.
<b>Engagement with Research Task</b>	Thorough research with accurate $k$ values, properly referenced.	Adequate research with mostly accurate $k$ values.	Limited research with some inaccuracies.	Minimal or no research conducted.
<b>Justification and Analysis</b>	Decision is well-justified with strong reasoning and evidence.	Decision is justified but may lack depth.	Decision is weakly justified with minimal reasoning.	Decision is unjustified or missing.
<b>Organization and Presentation</b>	Work is well-organized, neat, and professional.	Work is organized but could be neater.	Work is somewhat disorganized or hard to follow.	Work is disorganized and difficult to read.

Table 1: Rubric for Designing an Optimal Drug Delivery Capsule

**Appendix 2. Adaptation of Knowledge Typology from [5] to be Relevant to Calculus.**

<b>Knowledge Typology</b>	<b>Examples</b>
<b>A. Factual Knowledge- The basic elements students must know to be acquainted with a discipline or solve problems in it</b>	
A-a. Knowledge of terminology	Definitions of derivatives, integrals, limits, and continuity
A-b. Knowledge of specific details and elements	Common formulas such as the power rule, trigonometric identities, and area under a curve
<b>B. Conceptual Knowledge- The interrelationships among the basic elements within a larger structure that enable them to function together</b>	
B-a. Knowledge of classifications and categories	Types of functions (e.g., polynomial, exponential, trigonometric), types of limits (finite, infinite, indeterminate)
B-b. Knowledge of principles and generalizations	Fundamental Theorem of Calculus, rules for continuity and differentiability
B-c. Knowledge of theories, models, and structures	Theory of limits, relationship between differentiation and integration, optimization models
<b>C. Procedural Knowledge- How to do something, methods of inquiry, and criteria for using skills, algorithms, techniques, and methods</b>	
C-a. Knowledge of subject-specific skills and algorithms	Skills for computing derivatives and integrals, applying the chain rule, solving related rates problems
C-b. Knowledge of subject-specific techniques and methods	Techniques such as substitution in integration, partial fraction decomposition, implicit differentiation
C-c. Knowledge of criteria for determining when to use appropriate procedures	Criteria for choosing between integration by parts or substitution, deciding when to apply L'Hôpital's rule
<b>D. Metacognitive Knowledge- Knowledge of cognition in general as well as awareness and knowledge of one's own cognition</b>	
D-a. Strategic knowledge	Strategies for setting up and solving optimization problems, using heuristics to analyze a graph
D-b. Knowledge about cognitive tasks, including appropriate contextual and conditional knowledge	Understanding the type of calculus problem (e.g., initial value problem versus optimization) and the methods required for each
D-c. Self-knowledge	Awareness of personal strengths in solving derivatives but challenges with integration techniques

### Appendix 3. Adaptation of Cognitive Processes from [5] to be relevant to Calculus.

Cognitive Processes		Examples
1. Remember	Retrieve relevant knowledge from long-term memory.	
	1.1 Recognizing	Recognize the standard formulas for derivatives and integrals
	1.2 Recalling	Recall the definition of a limit or the Fundamental Theorem of Calculus
2. Understand	Construct meaning from instructional messages, including oral, written, and graphic communication.	
	2.1 Interpreting	Interpret graph of a function to identify critical points
	2.2 Exemplifying	Give examples of functions that are continuous but not differentiable
	2.3 Classifying	Classify functions as polynomial, exponential, or trigonometric
	2.4 Summarizing	Summarize the steps for solving a related rates problem
	2.5 Inferring	Infer the behavior of a function as $x \rightarrow \infty$
	2.6 Comparing	Compare the derivatives of linear and quadratic functions
	2.7 Explaining	Explain why the derivative of a constant is zero
3. Apply	Conduct or use a procedure in a given situation.	
	3.1 Executing	Compute the derivative of a polynomial function using the power rule
	3.2 Implementing	Apply the chain rule to differentiate a composite function
4. Analyze	Break material into constituent parts and determine how parts relate to one another and to an overall structure or purpose.	
	4.1 Differentiating	Distinguish between when to use the product rule versus the quotient rule
	4.2 Organizing	Organize the steps needed to solve a complex optimization problem
	4.3 Attributing	Attribute the shape of a graph to the behavior of its first and second derivatives
5. Evaluate	Make judgments based on criteria and standards.	
	5.1 Checking	Verify that the solution to a differential equation satisfies the original equation
	5.2 Critiquing	Judge the validity of a proposed method to approximate the area under a curve
6. Create	Put elements together to form a coherent or functional whole; reorganize elements into a new pattern or structure.	
	6.1 Generating	Generate a real-world scenario modeled by a given differential equation
	6.2 Planning	Plan the steps needed to solve a multivariable calculus problem
	6.3 Producing	Construct a new function that meets specific criteria, such as a maximum at a given point