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The Power of Movement: Exploring Gestures as Tools for Engineering Students Conceptualizing Statistics

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The Power of Movement: Exploring Gestures as Tools for Engineering Students Conceptualizing Statistics

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Abstract

This paper presents a multiple-case study examining first-year engineering students' conceptual understanding and associated gestures for concepts of central tendency including median, mean, and mode, which are critical concepts in statistics and engineering education. Statistics education is fundamental to STEM careers and relevant to peoples' everyday lives including personal choices and workplace success across professions, however, people tend to struggle with interpreting, communicating, and applying statistics in professional settings and their daily lives. The embodied learning literature presents evidence that gestures are particularly useful in promoting the learning and application of STEM concepts. Importantly, a few studies have documented the benefits from both instructors' and students' gestures in learning statistics, but these studies do not focus on the fundamental statistical concepts of median, mean, and mode or how engineering students conceptualize these concepts. Additionally, education in the post-pandemic age is increasingly leveraging the accessibility, reach, and flexibility that online and asynchronous instructional methods offer in higher education. There is sparse research on how students leverage their body movement and actions when reasoning about statistical concepts and even less on how to apply these within online and asynchronous settings. Thus, the challenge lies in bringing the benefits of effective embodied design practices used for in-person educational settings to remote and asynchronous learning. This study used video recordings of semi-structured interviews that focus on conceptual understanding of median, mean and mode. Phenomenography was used to describe how engineering students think and gesture about fundamental statistical concepts. Students' spontaneous gestures were classified using McNeil's gesture categories and their speech was analyzed in relation to their gesture representations. These results will inform future studies on the development of gesture-based digital video learning environments to support engineering students' learning of statistics.

Introduction

This paper examines first-year engineering students' conceptual understanding of a key concept in data analytics: central tendency. This research is important because incoming engineering students encounter challenges in adapting to college-level coursework due to poor foundational knowledge [1]. Introductory courses are designed to help build this type of foundational knowledge and support student development. In considering what type of instructional aids might make learning foundational knowledge less arduous, recent research points to representational gestures—those gestures which serve to represent an object or concept as useful in supporting STEM learning, including mathematics [2], [3]. Representational gestures are instrumental in facilitating conceptual learning in STEM because these gestures depict semantic knowledge, either directly or metaphorically, via hand placement, shape, and trajectory [4]. Capitalizing on this, this study examined the types of representational gestures engineering students produce to understand how they conceptualize various measures of central tendency.

This study represents a critical precursory step in understanding the underlying imagery and metaphors students use to make sense of abstract statistical concepts. This step is critical for the design of digital learning environments that incorporate gestures to facilitate the conceptual learning of foundational statistics concepts. By examining gestures that students produce, our ultimate goal is to inform the creation of embodied and gesture-based pedagogies for teaching statistics. Central tendency is a critical foundational statistical concept covered in introductory courses that is used to make engineering decisions such as deciding which process is most efficient, or whether a particular component is appropriate for the design. Therefore, our research questions are:

RQ1: What representational gestures do first-year engineering students produce after learning about central tendency?

RQ2: To what extent do the spontaneous gestures align with conceptual understanding of central tendency?

Literature Review

Representational Gestures

Representational gestures are bodily simulations of previous actions and perceptions experienced in the world [4], [5]. As we interact with our environment, we build perceptual and sensorimotor traces that we activate later to make sense of abstract ideas [6], [7]. The connection between concepts and sensorimotor experiences often happens intuitively. For example, students intuitively connect the concept of addition with the physical action of putting objects together [8] or the concept of fraction with splitting objects [9], [10]. However, much less is known about how students, and particularly engineering students, intuitively connect their sensorimotor experiences to more abstract concepts such as statistics.

What Can Spontaneous Gestures Reveal About Students' Knowledge?

Research in cognitive science has long found that gesture and speech co-occur and together represent an individual's conceptual understanding [11]. This research also shows that students' gestures serve as an index of their knowledge [12]-[14]. For example, students gesture more frequently when they find a task difficult [15], and also when they have greater task knowledge [16]. Furthermore, mismatches between students' speech and gesture often indicate that their old and more rudimentary knowledge state is transitioning into a new, and (hopefully) more advanced, knowledge state [13], [17], [18]. Because of this, students' spontaneous gestures can predict students' readiness to learn from mathematical lessons [13], [19].

In these cases, we see how gestures act as a “window” into student thought [11]. As such, gestures’ spatial and imagistic nature allows researchers and instructors to access details about students’ internal representations that words and symbols often cannot provide. Specifically, representational gestures depict—literally or metaphorically—students’ internal simulations of actions and perceptions related to the mathematical idea they are trying to express [4]. For example, a person’s diagonal placement of hand and forearm and subsequent tilting can be used as an iconic representation of a slope alteration [4]. Alternatively, a student’s placement of both hands with palms facing outward while alternating lifting the right and left hand can be used as metaphorical representation of a mathematical relationship (i.e., “sameness” of two fractions) [9].

How Can Gestures Impact Learning and Inform Pedagogies?

Gestures not only represent thought and knowledge; they also *impact and manifest* learning. For instance, encouraging students to gesture when explaining mathematical solutions improves student learning [20] and can lead students to develop new mathematical strategies [21], [22]. In contrast, restricting students from gesturing negatively impacts mathematical thinking and problem solving compared to students whose gestures were unrestricted [23], [24].

Instructors’ gestures also impact student learning. For instance, observing instructors’ concept-relevant gestures enhances conceptual learning, transfer, and retention [25]-[27]. Furthermore, asking students to produce specific content-related gestures increases students’ learning [28]-[30]. The benefit of prompting students to gesture can lead to better student outcomes than simply observing the instructors gesturing [31], [32]. However, it is important to note that producing gestures that are not aligned with the correct conceptual understanding can be detrimental to learning [30]. As such, the ability for gesture to enhance learning relies on the use of gestures that are aligned with correct conceptual understanding and are accessible to students’ current thinking. Therefore, to determine which gestures to use in gesture-based interventions, a precursory step is to *scrutinize the spontaneous gestures that individuals produce when talking about the target concept.* Observing spontaneous gestures can help to identify sensorimotor-concept connections that can be used to inform embodied pedagogies.

Current Study

Although gestures serve many purposes (e.g., mediate communication, direct someone’s attention), *representational gestures* are uniquely instrumental in revealing students’ mental representations and conceptualization of abstract ideas [11]. Therefore, it is crucial to understand how the target population of engineering students currently conceptualizes key statistical ideas and how these conceptual understandings are manifested through their gestures. Therefore, this study focuses on representational gestures that engineering students spontaneously produce when thinking, talking, and explaining central tendency (i.e., median, mean, and mode).

Methods

Participants

Ten first-year engineering students who were enrolled in an engineering program at a large Midwestern state university volunteered for this study. All participants were enrolled in a first-year engineering design course that taught descriptive statistics (e.g., central tendency, variability), mathematical modelling (i.e., least-squares regression), and probability. The participants were introduced to central tendency in this course prior to being recruited to this study. In addition, all participants self-reported learning these statistics concepts in high school or middle school. The demographics of the participants roughly matched the course demographics (see Table 1). Pseudonyms are used throughout the paper to maintain the participants' privacy. While all participants consented to have their interview video recorded, one participant (Jim) elected to not have the images from his video shared in publications or presentations.

Table 1. Demographics

	Male	Female
Gender	6	4
	Domestic	International
Residency	5	5

Procedures

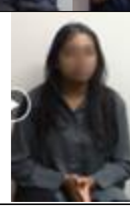
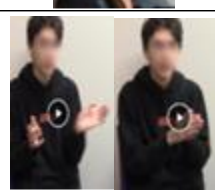
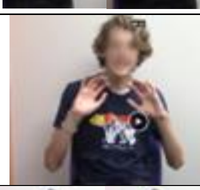
Participants completed the study in the research lab of the last author. When participants arrived, they completed a brief (approximately 30-minute) semi-structured interview that asked them to explain how they understood various statistics concepts, including median, mean, and mode. The interview prompts were designed to elicit participants' understanding of these statistical concepts, while allowing for follow-up questions to clarify participant' responses. For instance, one prompt asked, "How would you explain *mean* to a friend who was struggling with the concept?" and another asked "Can you show me what *median* represents?" Participant interviews were transcribed before analysis.

Phenomenographic analysis was used to analyze participants' spoken and gestured explanations [33]. The videos were coded by two of the authors and all discrepancies were resolved through discussion. Participants' spontaneous gestures were coded using McNeill's [34] gesture framework, categorizing gestures into representational (i.e., iconic and metaphoric gestures), and non-representational (i.e., deictic and beat gestures) [4]. For this paper, we focus on the representational gestures and co-occurring speech. Following the gesture coding, we examined how participants described the general concepts of median, mean, and mode. When analyzing participants' verbal responses, we documented the themes that emerged. We also examined the connection between gestures and the participants' verbal responses to draw connections between gestural representations and thematic findings.

Results and Discussion

All ten participants spontaneously gestured while explaining the meaning of the measures of central tendency (i.e., median, mean, and mode). As expected, there were some demographic differences in the size and frequency of spontaneous gesture production during the interviews. However, much of these observed differences were found in the production of non-representational gestures. For example, domestic students in our sample tended to produce larger and more frequent beat gestures—gestures that align with the rhythm of speech but do not appear to convey meaning—than international students. On the other hand, representational gestures were spontaneously produced by all participants at roughly the same frequency, except for during their explanations of mode, which will be described below. Example gestures and codes for each topic along with the co-occurring speech are shown in Table 2 and described in more detail below. While the sample size is too small to reach any definitive conclusions, the fact that representational gestures appear to be spontaneously produced by all participants is encouraging.

Table 2. Example Gestures and Verbal Explanations

Concept	Gesture		Gesture Type	Verbal Description
Mean	One hand has a finger extended, while the other hand waves over the top of the finger.		Metaphoric	The mean of the distribution is the balancing point of the distribution.
	Using two hands to group objects together, then using one hand to make a slashing gesture.		Iconic	The mean is the sum of the numbers divided by the number of observations.
Median	Both hands extended with palms touching at the center of one's body.		Metaphoric	Median is the middle most value.
	Hands far apart with palms facing each other, then bring hands together so that palms are touching in the middle of the body.		Iconic	You sort it [the data] from the biggest to the smallest and you find the one in the middle.
Mode	Fingers slightly extended on both hands. Hands move in circular motion to indicate a set.		Metaphoric	The mode, is just, like, the biggest single group of data.
	Hands extended with fingers making a pinching motion to represent height		Iconic	The number that occurs most frequently.

Central Tendency

Central tendency is a measure that describes the typical measurement. As there are many interpretations of 'typical', several different measures are needed. When asked how they would explain the meaning of central tendency, some participants referred to either the middle of a data set, or the location of the majority of the data. For example, Amy described central tendency as, "how much the data is like *grouped together* towards, not the middle of the range exactly, towards the *middle* of whatever data points." Conversely, Christina described central tendency as, "in the whole group of data, what is the *biggest amount* of that features of that data." Other participants, like Jim, described central tendency as, "numbers that describes the center of the data." Similarly, Ernie indicated that central tendency is the "idea of what the *majority* of data tells us, this is done in three different ways; mean median and mode…to get a feel for the *average* response or um average sort of value." In the next sections, we focus our analysis on how the participants gestured while describing the mean, median, and mode specifically.

Median

During the interview, all participants correctly described median as the "middle," "center," or "central" value, however, most participants described the median by beginning with a procedural description of how to calculate the median before describing that the median represents the middle number in a data set. There was a great deal of similarity in the representational gestures used when explaining the concept of the median. Eight of the participants initially began by describing the need to arrange the data in numeric order. For example, Della began her explanation of median by highlighting the need to, "arrange the terms from ascending to descending... no, smallest to biggest." While discussing the procedure, Della used both of her hands to create a number line or x-axis by spreading her hands apart (Figure 1a, 1b). She then continued by describing how the median is then located in the middle. Della then moved her hands together so that her palms touched when her hands ended up located in the middle of her body (Figure 1c). Amy produced a similar gesture as she explained the median (Figure 1d); however, as she brought her palms together, she used her index fingers to cross out the highest and lowest terms (Figure 1e) before ending with her index fingers touching at the center of her body (Figure 1f). A similar series of gestures was produced by eight of the ten participants.

Figure 1. Iconic Gestures accompanying procedural explanations for median

The two exceptions were Ben and Jim. Ben reversed the gesture sequence by beginning with his hands extended and palms together before spreading his hands apart (Figure 1g, 1h). In his explanation Ben began by incorrectly explaining that the median is the most likely outcome or value in a data set. In contrast, Jim described the median by saying, "if you look to the left of it [the median] you have 50%, and if you look to the right of it [the median] you have 50%. The median would be the one element that is right in the center." His gestures were similar to figure 1b and 1d with his hands apart. However, Jim moved this gesture from his left to his right, before ending with his palms together as in Figure 1c.

Seven of the ten participants described the median as the middle value, which is true for data sets with an odd number of observations. However, Isaac, Ernie, and Della also specifically talked about data sets with an even number of observations. For example, Isaac mentioned that the median may not be a member of the data set and describe, “if there’s two data points at the center you just average them.” Similarly, Della described how an even number of observations will have two middle numbers, and the median is halfway between them. Both descriptions were accompanied by the same palms together gesture as above, however when they produced the gesture, their hands were located slightly apart from each other rather than touching.

Mean

All participants interpreted the term *mean* to refer to the arithmetic mean, but only Jim discussed the existence of the geometric and harmonic means. This makes sense as the engineering course in which they were enrolled only taught about calculating and interpreting the arithmetic mean. Seven of the ten participants correctly described the formulas and procedure for calculating the arithmetic mean but were not able to explain what the mean represents. As the participants were describing the procedure for calculating mean, they produced iconic gestures representing a number line or x-axis. This was often done by spreading their hands out in front of them. This referred to using subsequent gestures to represent data points with various values. Interestingly, these gestures were made from each of the participants’ perspectives—with smaller numbers to the participant’s left—suggesting that these gestures were serving a cognitive purpose as participants think about the concepts, as well as a communicative purpose for researchers to observe how participants are thinking. For example, Ernie explained the concept of the mean by saying, “the mean value is just the average so you add up the values of each data, add it all up together, and you divide by the total amount of data and that will just get your mean. It will be your average value.” As he described the procedure, Ernie spread his hands, then brought them together as he described adding the values (Figures 2a, 2b). He then used his left hand to make a slashing gesture as he described dividing by the number of observations (Figure 2c). Heather, Greg, and Isaac produced similar gestures when describing the procedure for calculating mean.

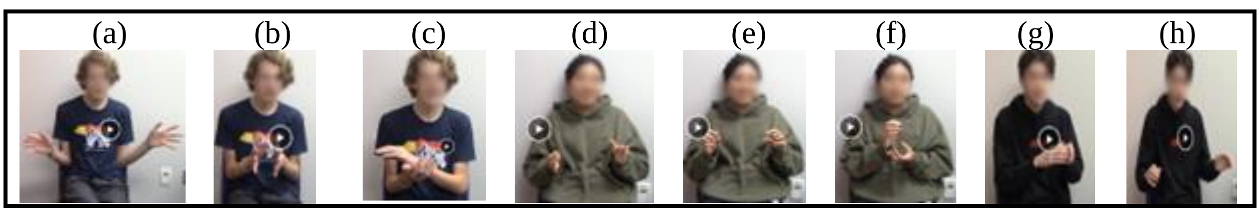


Figure 2. Representational gestures accompanying procedural explanations for mean

Christine also described the procedure for calculating mean using gestures representing procedural steps for calculating mean. However, Christine used both a chopping gesture (Figure 2d) and a pinching gesture (Figure 2e) to indicate the individual values as she described calculating the sum of the data. She then used her right fist to represent the sum, while cupping her left hand to represent dividing by the number of observations (Figure 2f). Both Christine and Della produced gestures that appeared to represent division as a fraction with the sum as the numerator of a fraction, and the number of observations as the denominator. Ben used similar gestures to represent adding values in a data set but produced a metaphorical gesture for division by starting with his hands touching before moving them in a linear motion down and away from his body, representing the process of division by separating his hands (Figures 2g, 2h).

In contrast to median, where the procedural and conceptual meanings overlap, these participants were less able to describe the conceptual meaning of the mean. This is best illustrated by Jim as he described, "If I had to think about what the mean would be without using numbers [the procedural explanation], that would be much harder." Later in the interview Jim added to this by saying, "If you try to understand what the mean is, like it's hard to explain what the mean is without using the word average. I feel like that it is sort of counterintuitive to how we explain mean."

Despite this conceptual challenge, three of the ten participants attempted to describe the conceptual meaning of mean using metaphorical gestures to represent the abstract concept of mean using analogies to physical objects. Both Fred and Jim described the mean as the balancing point or center of mass of a distribution. For example, Fred described the mean by saying, "the mean is the balancing point. So, if you imagine all your data is a bunch of points or, like, marbles or whatever, and you're trying to balance all these marbles then the mean is essentially the point at which you can get it to balance and not tip over." As Fred described the data as a bunch of marbles, he spread his hands wide, similar to the other participants (Figure 3a). Fred then moved his right hand and extended his index finger under the number line he drew using gestures (Figure 3b). This second gesture appears to represent the balancing point where the number line might balance on his finger. Fred repeated this gesture later in the interview when he described the mean as, "the middle point of all your numbers. Like if I actually, you know, plotted my numbers on a graph and then I looked at where is the middle of all these numbers, like the weighted middle." Similarly, Jim described mean using balancing.

> "The way I have learned mean in a distribution is like a tipping point. So, like if you decided to balance a distribution on a scale, or like one point, the mean is like the point where it balances. So, you think of mean intuitively as the center [of] gravity of the distribution, you know. So, if you place your finger on it [the mean], it [the distribution] won't tip on either side because you have equal mass to the left and equal mass to the right."

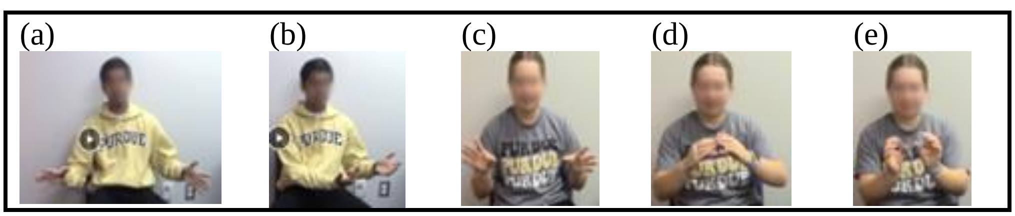


Figure 3. Example gestures accompanying conceptual explanations of mean

Amy also used representational gestures as she described her conceptual understanding of mean. In contrast to Fred and Jim, Amy explained the mean by saying, "the mean, the way I think of it is if you put them [the data points] all together into like one big bar and then you like break it down the middle or however many parts you know like you break it up into." As she described her understanding, Amy began with her arms spread to represent the data laid out on a number line, then brought her hands together to represent a solid object, before breaking it in the middle (Figures 3c-e), which seemed to represent a procedural explanation similar to Ben's. This indicates a gesture-speech mismatch where her gestures were consistent with a procedural description for how to calculate the arithmetic mean, but her verbal explanation of mean seemed to describe the conceptual meaning of median. In this case, it may be useful for Amy to be provided with a different metaphor through gesture, similar to the ones used by Fred and Jim.

Mode

Of the three measures of central tendency, mode was the measure least likely to produce spontaneous gestures when explaining. Only six of the ten participants produced spontaneous gestures when describe mode as a measure of central tendency. This is likely due to the participants simply describing the mode as the most frequently occurring value in a data set. While Ben, Della, Fred, and Jim did not gesture when explaining their understanding of mode, the other six participants produced similar gestures. Ernie described the mode as, “just whichever response was the largest overall…particular number shows up the most” or “just the biggest single group of data.” He reiterated by incorporating frequency uttering, “whichever number shows up the most often...it’s just whatever is biggest tallest on a bar graph.” Ernie engaged both hands by spreading them apart, creating a number line or x-axis (Figure 4a). Following this, Ernie used his hands to show the heights of bars reminiscent of either a bar chart or histogram (Figure 4b).

Amy and Greg produced a similar sequence of gestures (Figures 4c, 4d), however Amy was more explicit how this set of gestures connected to her understanding of mode, when she said, “I think of it [mode] kind of like a bar chart almost right like you can see how many times each number is represented, so in my head when I’m like looking at it or if I knock out how many times each point is I kind of think of it like higher.” Greg similarly said, “I feel like mode would be the highest frequency of the number that’s appearing so if like from if you have a data set and if seven is the number which appears more most frequently then that would be the mode.”

Figure 4. Representational gestures for explanations of mode

Christine demonstrated a different version of this gesture sequence by instead using her thumb and index fingers on both hands to represent the height of bars (Figure 4e), which allowed her to compare directly the heights of each bar. Isaac demonstrated another variation of this gesture sequence by bending his right hand so that his fingers pointed up, then raised this hand vertically to represent the height of bars (Figure 4f). Finally, Heather explicitly drew both the y-axis (Figure 4g) and x-axis (Figure 4h) before using her fingers to represent the number of objects in each bar (Figure 4i).

Conclusion

Representational gestures uniquely reveal students' knowledge, impact learning, and inform pedagogies [38]-[41]. To inform pedagogies effectively, a precursory step is to identify intuitive sensorimotor connections that students make with the target concept. To this end, the present multi-case study scrutinizes engineering students' representational gestures when talking about a fundamental statistical concept (i.e., measures of central tendency such as median, mean, and mode).

We found that engineering students excelled at describing the procedural knowledge for calculating central tendency measures accurately. All participants produced spontaneous representational gestures with somewhat similar frequency when talking about median and mean, and with less frequency when talking about mode. Strikingly, the series of gestures produced for these concepts were largely similar, tending to depict visual representations such as number lines and bar charts. The participants' ability to describe how to calculate the median and mode aligned with their conceptual understanding of the median and mode, likely because the procedural method for calculating these measures aligns with the conceptual meaning of these measures. Participants' representational gestures for median involved organizing arrays and locations in the middle of this array, whereas representational gestures describing mode depicted the height of bars, finding the high point in space. This result is promising for the development of gesture-based digital video learning environments, as the similarity of spontaneous gesturing suggests a small number of productive gestures that could be used by instructors and students when learning these concepts.

In contrast, while the participants were able to describe the procedure for calculating the arithmetic mean, many participants struggled to explain the concept of mean and what this measure of central tendency conceptually represents. Most participants produced gestures depicting a number line to represent the data, used a gathering gesture to depict finding the sum, then depicted division using either a slashing gesture or fraction. While they were able to correctly describe the procedure for calculating arithmetic mean, these participants struggled to describe the conceptual meaning of mean. Conversely, the two participants who described mean conceptually produced representational gestures depicting a balancing center-point or a central location in space. In doing so, the participants evoked the metaphor that numbers have mass, and by balancing the data set or finding the center of mass, they understood the mean as a "weighted middle." This gesture may represent a potentially productive gesture for teaching the idea of arithmetic mean to support conceptual understanding as well as procedural fluency.

Our results show that participants spontaneously gestured when they think and talk about statistics. These results provide insight to the potential for gesture to support conceptual understanding of mean, median, and mode. The results from this study provide insight on the similarities and differences of the mental imagery, as depicted by gesture, used when thinking about these statistical concepts. Researchers and educators can use these results to inform their pedagogies when teaching data analytics involving measures of central tendency.

The use of gestures has been shown to have cognitive benefits and support learning [35]-[37]**.** In the classroom, instructors may find educational benefits by explicitly using gestures that represent the conceptual understanding of abstract concepts, and by encouraging students to gesture as they think and talk about these concepts. Engineering educators can also attend to students' gestures to get a glimpse of students' conceptual and procedural understanding of these concepts. Specifically, instructors can use the additional information gestures provide to verify whether students' responses are aligned with target ideas. In turn, gestures can provide feedback to instructors and inform pedagogical strategies to support students' conceptual learning and address potential misconceptions. Drawing from these results, future work can investigate how different representational gestures support student procedural and conceptual understanding of engineering concepts.

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