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WIP: Calculus for the Modern Engineer: Putting the Joy Back in Learning Advanced Mathematics

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I am a (full) Professor of Robotics and hold the titles of the Elmer Gilbert Distinguished University Professor and the Jerry and Carol Levin Professor of Engineering. I was Director of the Michigan Robotics Institute from September 2016 through June 2022, and led the transition of the institute to a full-fledged department so that we could better serve our students and the aspirations of the robotics faculty. I jointly hold sixteen patents dealing with emissions reduction in passenger vehicles through improved control system design. I am also a Fellow of the IEEE and IFAC. In addition, I received the Paper of the Year Award from the IEEE Vehicular Technology Society in 1993, the George S. Axelby Award in 2002, the Control Systems Technology Award in 2003, the Bode Prize in 2012, the IEEE Transactions on Control Systems Technology Outstanding Paper Award in 2014, the IEEE Transactions on Automation Science and Engineering, Googol Best New Application Paper Award in 2019, and the 2023 Kalman Best Paper Award from ASME. My work on bipedal locomotion has been the object of numerous plenary lectures and has been featured on CNN, ESPN, Discovery Channel, The Economist, Wired Magazine, Discover Magazine, Scientific American, and Popular Mechanics.

WIP: Calculus for the Modern Engineer: Putting the Joy Back in Learning Advanced Mathematics

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Abstract

Work in Progress: Many engineering students enter college excited about mathematics, only to have their enthusiasm diminished by a rigid, outdated calculus curriculum. The Robotics Department at Michigan has piloted a new course, CALCULUS FOR THE MODERN ENGINEER, designed to restore the joy of learning advanced mathematics. This 4-credit course integrates Differential and Integral Calculus of a single variable, vector derivatives, and Ordinary Differential Equations (ODEs) into a one-semester curriculum tailored specifically for robotics students. The pilot cohort included 24 students: one first-semester freshman with no college calculus credit, five students with credit for Calculus I, and 18 students who had completed both Calculus I and II. The recommended prerequisite is ROB 101: COMPUTATIONAL LINEAR ALGEBRA, which ensures familiarity with Julia programming.

Departing from the traditional calculus sequence codified in the 1950s, the course begins with definite integration—a concept students readily understand through sums—before progressing through limits, differentiation, antiderivatives, and ODEs. By leveraging modern computational tools such as Julia, Large Language Models (LLMs), and Wolfram Alpha Pro, the course shifts the focus from tedious hand calculations to conceptual mastery and real-world application. Three engineering projects reinforce this approach: (1) numerically integrating drone IMU acceleration data to estimate velocity and position while correcting acceleration bias, (2) optimizing motion through gradient descent and equality constraints in applications such as basketball trajectories and gymnast posture, and (3) modeling and designing controllers for a planar BallBot using state-variable models and Laplace-transform-based feedback control.

Student evaluations indicate strong engagement: 85% of students reported an increased interest in calculus, and the course received an overall excellence rating of 4.8/5. Written feedback highlights the effectiveness of integrating programming and real-world applications, making calculus a more intuitive and empowering tool for engineering problem-solving. While direct comparisons with traditional calculus courses are not yet available, ROB 201 is designed to condense three semesters of calculus into a single semester, emphasizing practical applications that prepare students for elective coursework in Numerical Methods, Optimization, and Feedback Control.

1 Motivation for Reform and Engineering Education Perspective: Why Calculus Needed a Complete Overhaul in 2024

The calculus curriculum taught at most universities today remains rooted in a structure developed during the 1950s. Designed during the Sputnik era, this traditional sequence was intended to meet the educational needs of its time, emphasizing manual problem-solving techniques and abstract theoretical concepts. However, the landscape of engineering has evolved dramatically since then. Fields such as robotics, artificial intelligence, and data science now require not only a deep theoretical understanding but also the ability to engage confidently with computational tools and solve real-world problems. Despite these changes, the core structure of calculus courses—typically covering differential and integral calculus along with ordinary differential equations—remains largely unchanged, creating a widening gap between classroom learning and the dynamic problems modern engineers face.

To address this gap, the Robotics Department at the University of Michigan launched an experimental course, Calculus for the Modern Engineer. This pilot course explores how mathematics can be better aligned with the computational and practical needs of students entering fields like robotics and engineering. The goal of this article is to report on a new curriculum that integrates computation and real-world applications into calculus education while simultaneously making it a more engaging subject to learn.

A key challenge the course seeks to address is how calculus is traditionally taught. The focus on rote memorization and mechanical application of formulas often stifles curiosity, skill acquisition, and deep understanding. Students are rarely shown how calculus can be a creative, powerful tool for solving real-world engineering problems. Instead, they are burdened by repetitive manual computations that fail to connect with the complex, technology-driven tasks they will face in their careers. Moreover, outdated assessment methods, such as high-stakes exams, reinforce this disconnect, often producing median scores below 40% and fostering frustration rather than confidence.

This experimental course aims to reframe calculus education to emphasize not just theoretical principles but their practical applications. Traditional timed exams have been mostly replaced with three major projects that focus on solving real-world engineering challenges using the Julia programming language. This shift in assessment methods reflects the evolving needs of modern engineers, who increasingly rely on computational tools and iterative algorithms rather than closed-form solutions. The course allows students to engage with calculus concepts by directly applying them to projects, such as numerically integrating drone data, optimizing robotic movement, and designing feedback control systems.

By introducing computational tools early and focusing on real-world applications, the pilot course strives to rekindle students' enthusiasm for calculus. Julia helps students visualize and manipulate complex mathematical models, fostering a deeper, more intuitive understanding of calculus concepts. Rather than being seen as a barrier, calculus becomes a dynamic enabler of engineering solutions.

This article reports on the progress and outcomes of this pilot course, detailing its structure, the rationale behind its development, and the practical experiences of both students and instructors. While the experiment is ongoing, early feedback suggests that this reimagined approach offers a more relevant and engaging way to teach calculus to the engineers of tomorrow.

2 Contextualizing Reform in Mathematics Education

Here we give a snapshot of prior and current reform efforts in calculus education, providing context for how Calculus for the Modern Engineer aligns with and advances these initiatives.

2.1 Calculus in Context: Historical Precedent for Applied and Computational Focus

Calculus in Context, developed by the Five College Consortium[1] in 1995, restructured the traditional calculus sequence to emphasize real-world applications and computational methods[1]. Key reforms included replacing hand computations with algorithms, de-emphasizing special-case topics, and fostering collaborative and experimental learning environments. The curriculum prioritized broadly applicable concepts, such as differential equations and dynamical systems, while encouraging iterative problem-solving with computational tools.

Despite its innovative approach, challenges such as unpolished materials and articulation problems between Calculus II and III hindered widespread adoption[2]. However, the program demonstrated improved conceptual understanding of limits and derivatives, highlighting its potential for discovery-based learning.

2.2 Calculus for Economists: Domain-Specific Adaptation of Calculus

`Calculus for Economists`[3], published in September 2024, integrates foundational topics from Calculus I, Calculus II, and Ordinary Differential Equations (ODEs) into a comprehensive curriculum tailored for economics students. The course explores calculus principles, differential calculus applications, and advanced techniques in relation to economic modeling and policy formulation[4]. This curriculum bridges mathematics and economics, reinforcing the relevance of calculus in economic decision-making.

2.3 Other Collaborative Efforts to Innovate Calculus Education: Expanding Engagement and Accessibility

SimCalc introduces calculus concepts of change and variation—such as rates, accumulation, and limits—to younger students through interactive software and dynamic visualizations[5].

[1]Smith College, Amherst College, Hampshire College, Mount Holyoke College, and the University of Massachusetts Amherst.

Aimed at democratizing access to mathematics, it fosters engagement with real-world motion scenarios and promotes both qualitative and quantitative reasoning.

The *Learning Lab Calculus Report* underscores the role of active learning and inclusivity in addressing STEM retention, particularly for underrepresented populations[6]. Recommendations include mentorship programs, collaborative environments, and adaptive teaching practices.

Project NExT, sponsored by the MAA[7], focuses on faculty development, promoting teaching innovations and addressing equity in mathematics education. By fostering collaboration among early-career mathematicians, it supports curricular reforms and best practices in core courses like calculus.

2.4 Efforts Toward Interactive Calculus Texts or the Inclusion of Programming: Leveraging Technology for Learning

Interactive textbooks and programming tools offer transformative approaches to teaching calculus. `Active Calculus` and `Calculus With Julia` encourage exploration through computational experiments and applications[8,9]. Despite their strengths, these resources retain the traditional calculus sequence, limiting their potential to revolutionize the curriculum.

Other efforts include the integration of computational tools, such as MATLAB for engineering applications[10] and inferential thinking in data science[11]. Tools like Mathematica and MATLAB have also been used to enhance the teaching of multivariable calculus and its applications in geometry and physics[12,13].

2.5 Comparison and Contrast with Calculus for the Modern Engineer: A New Paradigm for Engineering Mathematics

Calculus for the Modern Engineer aligns with the educational philosophies of *Calculus in Context*[1] and *Calculus for Economists*[3], which emphasize real-world applications and streamlined curricula. Distinctively, this new course reorders topics to build student skills progressively and equips them with a mathematical toolkit tailored to modern engineering challenges. Its integration of theory and application supports both conceptual understanding and practical problem solving, particularly for advanced topics in robotics. Most results in the textbook are proved in optional sections, adding depth for interested students.

3 Curriculum Design: Content of the Open-source Textbook

The custom textbook composed for Calculus for the Modern Engineer provides a comprehensive resource for integrating pre-calculus principles, single-variable calculus, and advanced topics like ordinary differential equations (ODEs) and Laplace transforms into a unified curriculum. Starting with pre-calculus fundamentals, the text emphasizes the conceptual and computational aspects of calculus, presenting topics such as definite integration, differentiation, and antiderivatives in a sequence that builds intuition and practical problem-solving skills. Computational tools like Julia and SymPy are used throughout to reinforce understanding and apply calculus concepts to engineering case studies, including optimization, feedback control, and robotic systems.

Advanced chapters focus on improper integrals, ODEs, and Laplace transforms, connecting theoretical ideas to engineering applications such as proportional-derivative feedback controllers. By balancing rigorous mathematical principles with computational tools and practical examples, the textbook aims to make calculus both engaging and directly relevant to engineering practice. A detailed outline is available in Appendix A. The open-source textbook is available on GitHub[14].

4 Applying Calculus Through Computational Engineering Challenges: Three Pillar Projects

The structure of CALCULUS FOR THE MODERN ENGINEER revolves around three major Julia-based projects that integrate core mathematical concepts with practical engineering applications. These projects progressively develop students' computational and theoretical skills, emphasizing real-world challenges in robotics and engineering. Eight homework assignments supplement the projects, with each consisting of a written component for traditional problem-solving and a Jupyter notebook component for computational exercises.

The first project introduces numerical integration through drone IMU acceleration data. Students estimate velocity and position using clean data with the Trapezoidal Rule applied to

$$v(t) = v(t_0) + \int_{t_0}^{t} a(\tau)\, d\tau, \quad p(t) = p(t_0) + \int_{t_0}^{t} v(\tau)\, d\tau.$$

They then address acceleration bias with a correction step inspired by Kalman Filtering, tuning gains to improve accuracy. This foundational project prepares students for later work on ODEs.

The second project explores optimization through gradient descent and equality constraints. Beginning with a basketball free-throw problem, students calculate the initial speed and angle using both analytical and gradient-based approaches. They advance to more complex tasks, such as optimizing the landing posture of a simple gymnast model and simulating a diver on a 10-meter platform. Tools like JuMP guide students in solving these increasingly sophisticated problems.

The final project focuses on modeling and feedback control of a planar BallBot. Students derive equations of motion using Lagrange's Method and develop state-variable models and transfer functions. By designing and testing controllers for the BallBot's linear and nonlinear models, students gain hands-on experience with feedback control, supported throughout by Julia-based tools.

5 Evaluating Engagement and Learning Outcomes through Student Assessments from the Pilot Term

Student evaluations provided valuable insights into the pilot offering of CALCULUS FOR THE MODERN ENGINEER. Midterm feedback, based on 22 out of 24 responses, highlighted strong engagement and satisfaction, with an overall course rating of **4.29/5** and **4.50/5** for advancing subject understanding. **85% of students** reported increased interest in calculus, crediting the course's integration of Julia programming and real-world applications for making the material more engaging. Comments emphasized how programming assignments illustrated practical uses of calculus and helped reinforce mathematical concepts.

Final evaluations, with 20 out of 24 responses, yielded even stronger ratings, including an **overall excellence score of 4.8/5**, **4.7/5** for advancing understanding, and **4.6/5** for conceptual clarity. Students valued the balanced approach of theoretical learning and computational practice, with many praising the textbook and its long-term utility. Key strengths cited included the structured material, the real-world focus of projects, and the ability to visualize concepts through Julia programming. One student remarked, *"The programming assignments provided context where math concepts could be applied and will help me with future classes and the real world."* Another noted, *"The homework formats helped me understand the material and see how it applies to real engineering problems."*

Constructive feedback highlighted areas for refinement. Some students found written homework less valuable than programming tasks, while others requested more challenging computational exercises and shorter, more frequent lectures to sustain engagement. Suggestions included adding quizzes or group activities to promote in-class participation and expanding programming challenges to push more advanced learners.

The overwhelmingly positive feedback underscores the course's success in bridging theory and practice while pointing to refinements that will further enhance engagement. These insights will shape the next iteration, affirming the course's role as a scalable model for modernizing calculus education.

6 Discussion on Course Structure and Computational Focus

Developing CALCULUS FOR THE MODERN ENGINEER within a newly established Robotics Department provided the freedom to break from traditional norms and reimagine the mathematics curriculum. This autonomy allowed for integrating essential topics like numerical integration, optimization, dynamical systems, and feedback control, equipping students with skills directly applicable to robotics and engineering. The course condenses three semesters of traditional calculus into one semester, freeing up time for advanced mathematical topics while maintaining rigorous math requirements for Robotics majors.

A central innovation of the course is its integration of computation using Julia. Students use Julia to visualize and solve complex mathematical problems, such as optimizing multivariable functions, deriving differential equations for robots, and designing feedback controllers. To ensure that students are well-rounded, the course strikes a balance between computational fluency and manual problem-solving. While students gain extensive experience with computational tools, they also engage in enough hand computation to succeed in traditional courses. This dual focus prepares students not only for academic success, but also for real-world engineering challenges.

Unlike other computational calculus initiatives, Calculus for the Modern Engineer fundamentally reorders the curriculum, starting with definite integration. This intuitive topic, tied to sums and areas, provides a strong foundation before abstract concepts like single-sided limits, continuity treated correctly, and differentiation. The course employs project-based learning, with robotics case studies demonstrating the practical relevance of calculus, making it an empowering and accessible tool for engineering students.

7 Reflections on Impact and Next Steps

Calculus for the Modern Engineer seeks to align calculus education with the computational and practical needs of contemporary engineering. By rethinking the traditional structure and emphasizing project-based learning, computation, and real-world applications, this course offers a blueprint for modernizing the mathematics curriculum for engineers. The successful pilot has shown that integrating computational tools like Julia with rigorous mathematical principles not only deepens student understanding but also enhances their ability to apply calculus meaningfully in engineering contexts. Student evaluations indicate strong engagement, with 85% reporting increased interest in calculus and the course receiving an overall excellence rating of 4.8/5. Feedback highlights the effectiveness of programming assignments and real-world case studies in reinforcing conceptual understanding.

To build on this success, a GitHub repository with a common open-source license has been launched, providing access to the textbook source files[14]; soon to follow: written homework sets, and Jupyter notebooks for programming assignments and projects. This initiative aims to broaden the course's reach and invite contributions from educators and the global learning community. By making the materials freely available, the course can adapt to the specific needs of diverse student populations while fostering a spirit of collaboration and innovation in mathematics education.

Future iterations of the course will be refined through student feedback, focusing on elements such as additional coding challenges for advanced learners. This iterative approach ensures the course evolves alongside the demands of both students and the engineering profession. The interdisciplinary appeal of the curriculum is evident, with the pilot semester drawing interest from the Ross School of Business, the School of Kinesiology, and other engineering departments. This underscores the curriculum's versatility beyond robotics, highlighting its potential to become a model for teaching applied mathematics across a broad range of disciplines.

Challenges to Adoption: Broader enrollment in Calculus for the Modern

Engineer is currently limited by prerequisite structures and institutional hesitancy to recognize the course as equivalent to the traditional calculus sequence. Administrative and curricular constraints present additional barriers to widespread adoption. These challenges, as well as their implications for expanding the course, are discussed in Appendix B. A longitudinal study comparing the impact of this experimental course to traditional calculus courses would provide deeper insights into its effectiveness in preparing engineering students for the workforce, thereby easing adoption.

Other instructors are free to explore emerging topics in engineering and mathematics, including machine learning and stochastic processes, in place of control systems. Integrating programming assignments and projects on these advanced topics will align the course with cutting-edge trends in engineering education. Expanding the scope of case studies to include applications in aerospace, mechanical engineering, and biomedical fields will further emphasize the universal relevance of this new approach to teaching calculus. For example, projects on modeling the aerodynamics of unmanned aerial vehicles, optimizing mechanical linkages in robotic arms, or analyzing physiological signals in wearable health monitors could serve as additional avenues to demonstrate the power of calculus in solving complex real-world problems. These innovations require instructors with domain expertise.

In summary, Calculus for the Modern Engineer represents a potentially transformative approach to teaching calculus, equipping students with computational proficiency and an appreciation for how mathematical theory underpins practical problem-solving. Student engagement and evaluation data affirm the effectiveness of integrating computation and project-based learning into calculus education. Michigan Robotics is committed to continually refining the curriculum to foster an educational experience that prepares students to excel in the complex and rapidly changing landscape of engineering and technology. Future work will expand the course's interdisciplinary reach and establish equivalency with traditional calculus sequences to broaden adoption.

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Appendix A Overview of the Open-Source Textbook

Lectures are 4 hours per week; Michigan has 14 weeks in the semester.

1. **Pre-calculus: Notation, Functions, and Various Algebraic Facts**

   Notes: • 2-hour lecture. • Students are expected to review this material mostly on their own. • 20-question quiz to ensure mastery. • The Approximation Principle is highlighted in HW01.

   Learning Objectives: • Recognize the utility of mathematical notation for precision and expressivity; • Understand and apply the Bisection Algorithm as an example of the Approximation Principle; • Reaffirm understanding of fundamental concepts such as functions, domains, ranges, and inverses.

   Content: Introduction, Got Calculus Dread? Additional Resources, Notation or the Language of Mathematics, The Approximation Principle, Algebraic Manipulation and Inequalities, Functions (Domains, Ranges, Inverses, and Compositions), Trigonometric and Inverse Trigonometric Functions, Powers and Roots, Exponentials and Logarithms, Euler's Formula, Hyperbolic Trigonometric Functions, Summing Symbol, Binomial Theorem, Special Functions, Shifting and Scaling, and Feedback Control. (Optional Reads: Binomial Theorem meets Euler and Proofs Associated with the Chapter.)

2. **Calculus Foundations: Proofs, Finite Sums, Limits at Infinity, and Geometric Sums**

   Notes: • 4 hours lecture + recitation. • Written and Julia HWs.

   Learning Objectives: • Understand the significance of mathematical proofs; • Master proof by induction and apply it to sums of powers of integers; • Comprehend limits and their initial uses in calculus, with emphasis on limits at infinity for rational functions and exponentials.

   Content: Introduction, Mathematical Proofs, Countable Sets, Proofs by Induction, Finite Sums, Limits at Infinity, Geometric Sums. (Optional Reads: Euler's Number, Proofs Associated with the Chapter.)

3. **Definite Integration as the Signed Area Under a Curve**

   Notes: • 5 hours lecture + recitation. • Written and Julia HWs.

   Learning Objectives: • Define and explain definite integration à la Riemann-Darboux; • Execute computations using methods like Trapezoidal Rule and Simpson's Rule; • Recognize applications of definite integrals in engineering.

   Content: The Riemann Integral, Properties of the Riemann Integral, Numerical Methods for Approximating Integrals, Applications of Definite Integral. (Optional Reads: Proofs Associated with the Chapter.)

4. **Properties of Functions: Left and Right Limits, Types of Continuity, Boundedness, and Generalizations of Max and Min**

   Notes: • 5 hours lecture + recitation. • Written and Julia HWs.

   Learning Objectives: • Analyze function behavior using one-sided limits; • Understand the nature of function continuity; • When can limits be taken inside a function; • Explore generalizations of maximum and minimum values.

   Content: Limits from the Left and Right, Continuity, Maximum and Minimum Values, The Squeeze Theorem, Piecewise Continuity, Boundedness, Generalizations of Max/Min. (Optional Reads: Intermediate Value Theorem, Proofs Associated with the Chapter.)

5. **Differentiation**

   Notes: • 5 hours lecture + recitation. • Written and Julia HWs.

   Learning Objectives: • Understand single-variable and partial derivatives; • Apply differentiation rules and software tools; • Explore real problems where derivatives are used in engineering.

Content: Derivative as Local Slope and as Linear Approximation, Differentiation Rules, Software Tools for Computing Derivatives, Use Cases of Derivatives, Partial Derivatives, Jacobians, Gradients, Hessians, Total Derivative. (Optional Reads: Proofs Associated with the Chapter.)

6. **Engineering Applications of the Derivative**

   Notes: • 6 hours lecture + recitation. • Written and Julia HWs.

   Learning Objectives: • Develop strategies for solving optimization problems; • Apply Lagrange's equations to dynamics.

   Content: Path Length, Root Finding, Unconstrained Minimization, Gradient Descent with and without Equality Constraints, Lagrange Multipliers, Dynamics à la Lagrange. (Optional Reads: More on Lagrangian Dynamics, Proofs Associated with the Chapter.)

7. **Antiderivatives and the Fundamental Theorems of Calculus**

   Notes: • 5 hours lecture + recitation. • Written and Julia HWs.

   Learning Objectives: • Understand antiderivatives and their relationship to definite integrals; • Apply key techniques for finding antiderivatives.

   Content: Introduction, Fundamental Theorems of Calculus, Antiderivatives, Techniques for Finding Antiderivatives, Software Tools for Antiderivatives. (Optional Reads: Proofs Associated with the Chapter.)

8. **Improper Integrals**

   Notes: • 2 hours lecture + recitation. • Written and Julia HWs.

   Learning Objectives: • Define and understand improper integrals; • Explore practical applications in probability theory.

   Content: Type-I Improper Integrals, Type-II Improper Integrals, Improper Integrals in Probability. (Optional Reads: Proofs Associated with the Chapter.)

9. **Ordinary Differential Equations (ODEs)**

   Notes: • 9 hours lecture + recitation. • Written and Julia HWs.

   Learning Objectives: • Learn analytical and numerical methods for solving ODEs; • Apply ODEs to model dynamic systems in robotics.

   Content: Introduction to ODEs, Higher-Order ODEs, Vector ODEs, Linear Systems of ODEs, Numerical Solutions, Matrix Exponential, Eigenvalues and Eigenvectors. (Optional Reads: Resonance in ODEs, Proofs Associated with the Chapter.)

10. **Laplace Transforms through the Lens of Feedback Control**

    Notes: • 9 hours lecture + recitation. • Written and Julia HWs.

    Learning Objectives: • Understand Laplace transforms and their application in solving ODEs; • Derive and apply transfer functions in feedback control systems.

    Content: Laplace Transform, Solving ODEs Using Laplace, Transfer Functions, Poles and Zeros, Feedback Systems, Transient Response of Systems, Proportional and Proportional-Derivative Feedback, Pre-compensators. (Optional Reads: Impulse Function, Proofs Associated with the Chapter.)

11. Balance of Time (4 hours): Additional Student-driven Examples, Special Topics.

Appendix B Challenges in Implementing Curriculum Change and other Barriers to Calculus Reform

A major current barrier to broader enrollment in this experimental course lies in the prerequisite structure of engineering electives. Students in Robotics take courses in Circuits, Dynamics, and Machine Learning, just to name a few, which are taught in other departments and have calculus prerequisites explicitly defined in terms of traditional Calculus I, II, and ODEs. Because Calculus for the Modern Engineer is not yet recognized as equivalent, students must go on a waitlist for these courses, often resulting in exclusion. Additionally, approximately 30% of our students are double majors in CS, CSE, ECE, or ME—departments that prefer to wait and assess the performance of current students before granting equivalency. This creates a classic chicken-and-egg problem: students hesitate to enroll until the course gains broader acceptance, while departments hesitate to accept it until they see a sufficient number of students succeeding in follow-on courses.

A second major barrier to calculus reform is the outsourcing of most first-year STEM courses, such as calculus and physics, to departments outside of engineering. This division discourages both mathematics and engineering faculty from pursuing radical curriculum changes, as neither group has full ownership of the content's alignment with engineering needs. Developing innovative materials, such as textbooks and projects, requires significant time—time that active research faculty often cannot allocate due to their primary research commitments. Administrative support is essential; without it, the author's sabbatical plans to develop this curriculum would not have been possible. Even with initial success, gaining widespread adoption among engineering faculty remains challenging, as structural change within academia is inherently slow and difficult.

A third concern raised by faculty is skepticism about the curriculum's breadth and depth: *"You are compressing three courses into one. Either you are leaving out important material that we need, or you are creating a course that is too difficult."* This perception presents another hurdle, requiring careful communication of the curriculum's scope, the role of computational tools, and the pedagogical approach that balances rigor with efficiency.