Engineering Educators Bringing the World Together
2025 ASEE Annual Conference & Exposition
Palais des congrès de Montréal, Montréal, QC • June 22–25, 2025
ASEE
Paper ID #47597

Effects of Spatial Ability on the Mastery of Statics Concepts

Dr. Maxine Fontaine, Stevens Institute of Technology (School of Engineering and Science)

Maxine Fontaine is a Teaching Associate Professor in Mechanical Engineering at Stevens Institute of Technology. She received her Ph.D. in 2010 from Aalborg University in Aalborg, Denmark. Maxine has a background in the biomechanics of human movement, and she currently teaches several undergraduate courses in engineering mechanics. Her research interests are focused on improving engineering pedagogy and increasing diversity in engineering.

Chaitanya Krishna Vallabh, Stevens Institute of Technology (School of Engineering and Science)

Effects of Spatial Ability on the Mastery of Statics Concepts

Abstract

Solving problems goes beyond simply “doing the math”. Before applying formulas and/or solving equations, most engineering problems require that a diagram or visual representation of the problem be developed first, e.g. a free-body diagram or vector addition triangle. The ability to generate an appropriate representation of the problem is often referred to as representational competence, which is heavily influenced by spatial visualization skills (SVS).

Spatial visualization skills refer to the ability to mentally manipulate and transform 3D objects and other spatial information. The importance of strong SVS in engineering is well established, but the mechanisms behind this link are not well understood. We aim to investigate whether certain concepts in engineering mechanics can be identified as requiring strong SVS for mastery, or put differently, whether certain errors or misconceptions in solving statics problems can be attributed to poor spatial ability. This could inform how we teach statics concepts and help guide us in providing supplemental instruction and developing targeted hands-on activities and labs to improve conceptual understanding (and perhaps build spatial skills) among low visualizers.

In a prior study, we investigated the level of mastery of fundamental statics concepts, specifically force vectors and moments of forces in the first exam, among low, medium, and high visualizers. Preliminary results indicated that the majority of students, regardless of spatial ability, were able to solve both exam problems completely accurately. We also observed, however, that low visualizers master concepts at a lower rate than high visualizers. This suggests that low SVS does not prevent a student from mastering a concept but perhaps makes it more difficult to do so.

In this study, we expand on our previous study by focusing on the following two research questions:

- RQ1. Do students with lower SVS simply need more time to grasp the concept?
- RQ2. What other skills / concepts in statics require strong visualization skills?

We identify and categorize errors on subsequent exam problems (focusing on rigid body equilibrium and truss analysis) in the second exam to address both research questions. By comparing performance on these first and second exams, we can track progress of students who were unable to demonstrate mastery of early statics concepts initially. By comparing performance on the second exam problems among low, medium, and high visualizers, we can make some observations on the role of SVS in mastery of rigid body equilibrium and truss analysis.

We hypothesize that students with low SVS have more difficulty with conceptual understanding and therefore resort to ineffective learning strategies such as memorization of example problems. This superficial comprehension is revealed by the type of errors that are made when confronted with a new problem that is too dissimilar from the pool of example problems, they have become familiar with.

Introduction

Free-body diagrams (FBD) are very significant for solving statics problems, their importance cannot be overstated. Similarly, vector addition triangles, schematics, visual representations are vital for understanding and solving a statics problem successfully. Spatial visualization skills (SVS) are often necessary for developing high levels of competence in accurately representing a problem or drawing a accurate FBD [1]. Without these SVS skills, students might purely rely on their memory or procedural mastery of certain topics. As stated in the literature SVS skills are inherently correlated with the understanding of fundamental concepts such as FBDs, and moments [2]. SVS are linked to student success in engineering courses, showing significant improvements in learning and retention [1], [3]. Additionally, SVS can contribute to a high rate of success in STEM fields, supported by strong skills in mathematics and physics [4].

Evidence strongly suggests that instructors should integrate spatial ability training and testing in foundational courses [1], [5]. In their work, Sorby et al [1] found that there is a significant correlation between spatial skills and understanding/ representing FBDs necessary to solve the problem. Further, authors in [5] demonstrated that visualization of the problems provided more in-depth understanding of the concepts rather than solely relying on equations and algebraic manipulations. Students can also learn to develop and apply these spatial skills while studying statics [6].

In our previous study [7], we examined the mastery levels and their correlation for solving a certain set of problems, namely, force vectors and moments of forces. We found that regardless of the SVS mastery level, students were able to solve the problems accurately. However, we inferred that the low visualizers master concepts at a lower rate, hence suggesting that SVS mastery might help students grasp the concepts faster. For this study, students are divided into three groups based on SVS: low, medium, and high.

To investigate our conclusions from our previous work further, we examined two research questions in this study:

- RQ1. Do students with lower SVS simply need more time to grasp the concept?
- RQ2. What other skills / concepts in statics require strong visualization skills?

Similar to the moment problem in our previous work, as shown in Figure 1, we chose one problem from the second exam for our analysis (shown in Fig. 2). The second exam was conducted four weeks after the first exam, giving students additional time to understand the concepts, specifically, moments of force and force components. Our analysis showed that the correctness of the moment calculations did not improve from exam one to exam two, rather they showed a decline in the percentage of correct answers in all three SVS groups, this could be attributed to the geometry of the problem as shown in Fig. 2.

However, it was promising to find that a couple of the common errors found in exam one was not seen in exam two, specifically, including zero-moment forces and assigning ***i***, ***j*** components to moments. Geometry errors were also found to be prevalent, leading to miscalculation of angles and force components. Some of the errors included mis-identifying the support reactions, i.e., two reaction forces at the rocker support instead of one and drawing incorrect forces on the FBDs. The geometry errors, support reaction and FBD errors can be attributed to low SVS skills. We hypothesized that the students with high SVS scores should be able to visualize the supports, understand the geometry better compared to low SVS score groups. Our observation from this study confirmed our hypothesis. Subsequent sections detail the methods, analysis and the results of our study. Other types of relevant errors to RQ2 are also addressed in this study, with an additional analysis on the efficacy of solution in correlation to the student's SVS score is also conducted and reported.

Methods

All second-year engineering students at Stevens Institute of Technology are required to take Statics & Introduction to Engineering Mechanics. Math prerequisites for the course include two semesters of calculus. Force vectors and moment of a force are covered in the first two weeks of the semester, and students are tested on these concepts in the first exam (around week 4 or 5). Rigid body equilibrium, trusses, and frames are introduced afterward and tested in the second exam (around week 8 or 9). The problem on moments of forces from Exam 1 and the rigid body equilibrium problem from Exam 2 in Fall 2022 were selected for this study. It should be noted that the sample size for Exam 2 (n=129) was slightly lower than that for Exam 1 (n=136), since several students took a make-up exam 2, which was excluded from this study.

In problem 2 of Exam 1, students were asked to determine the moment of each force about a given point, then find the magnitude and direction of the resultant moment (see Figure 1). Using the scalar approach to calculating 2D moments, this requires that students are able to:

1. Identify which force components do and do not cause a moment about the given point.
    a. Ability to recognize vertical forces that cause moment.
    b. Ability to recognize horizontal forces that cause moment.
2. Find the moment arm distance for each force component.
3. Determine the direction of each moment of force.
4. Add to find the resultant moment.

Problem 2.

Three coplanar forces are acting on the structure as shown, where $F_1 = 200$ N, $F_2 = 150$ N, and $F_3 = 275$ N. For this system solve for the following:

a. Determine the moment of each of the three forces about point *A*.

b. Determine the resultant moment about point *A*.

c. Determine the moment of each of the three forces about point *B*.

d. Determine the resultant moment about point *B*.

Be sure to include the correct sign with your answers. Note that counterclockwise moments are positive.

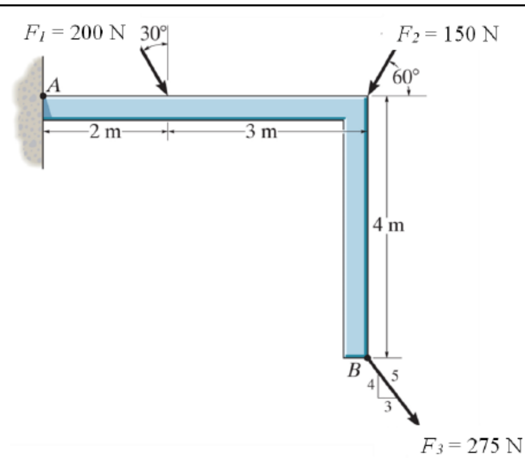


Figure 1. Exam 1 problem on moments of forces.

The errors identified for each of the above skills are summarized in Table 1 below. Minor calculation errors were not included in the analysis.

Table 1. Errors identified for Exam 1 problem on moments of forces.

Fundamental Skill	**Errors Identified**
Resolve force vector	F - Did not resolve forces at all
Recognize which force components cause moments	O - Omitted one or more force components contributing to resultant moment (e.g. calculated moments due to vertical force components only) Z - Included one or more force components that should be zero moment
Recognize that moment arm is perpendicular to force	D - Incorrect moment arm distance, or did not identify moment arms at all
Determine the direction of the moment of force	S - Incorrect sign on one or more moments of forces
Add moments	C - Assigned i- and j- directions to one or more moments

In problem 1 of Exam 2, students were asked to draw the appropriate free-body diagram, then use equilibrium equations to determine the support reactions (see Figure 2). This problem requires students to be able to:

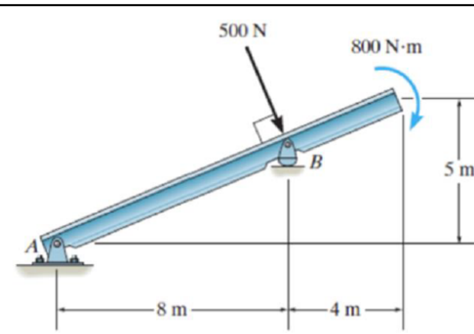
1. Identify the appropriate reactions at the supports to draw a proper free-body diagram.
2. Write an appropriate moment equilibrium equation (presumably about point *A*) and solve.
    a. Determine the moment about *A* due to the applied force acting at *B*.
    b. Determine moment about *A* due to the reaction at the roller *B*.
    c. Consider the applied moment acting on the beam.
    d. Set the sum of the moments to zero.
3. Write the appropriate force equilibrium equations and solve.

Problem 1.

The following structure is supported by a pin at *A* and a rocker at *B*.

a. Draw the free-body diagram.

b. Determine the reactions at the supports. Be sure to clearly indicate the direction of each force in your final answers, e.g. $P = 50$ lb (←) .



Problem 1.

The following structure is supported by a pin at *A* and a rocker at *B*.

a. Draw the free-body diagram.

b. Determine the reactions at the supports. Be sure to clearly indicate the direction of each force in your final answers, e.g. $P = 50$ lb (←) .

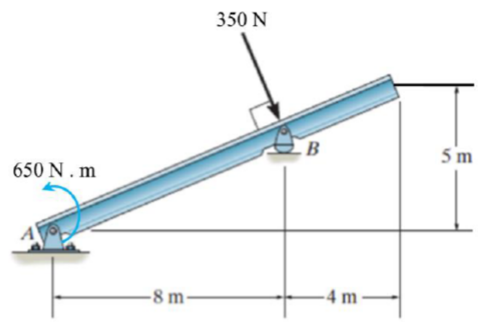


Figure 2. Exam 2 problem on rigid body equilibrium (versions A and B).

Focusing only on the skills tested in the Exam 1 problem, the errors identified for steps 2a and 2b are summarized in Table 2 below. Minor calculation errors were not included in the analysis.

Table 2. Errors identified for Exam 2 problem on rigid body equilibrium (steps 2a and 2b only).

Fundamental Skill	Errors Identified
Moment: Recognize which force components cause moments	F* – Incorrect resolution of force vector O - Omitted one or more force components contributing to resultant moment (e.g. calculated moments due to vertical force components only)
Moment: Recognize that moment arm is perpendicular to force	D - Incorrect moment arm distance, or did not identify moment arms at all
Moment: Determine the direction of the moment of force	S – Incorrect sign on one or more moments of forces

Spatial visualization skills were evaluated using the Purdue Spatial Visualization Test: Rotations (PSVT:R), which is a timed, standardized assessment of mental rotation abilities [8].The PSVT:R, developed by Bodner and Guay, is a prevalent test employed to evaluate spatial ability in students within the engineering education research field. This assessment is administered to all first-year engineering students during the initial week of the Introduction to Engineering Design course to gauge their spatial skills. Typically, a passing threshold of 60% or 70% is established to identify students with lower spatial abilities. At Stevens, thresholds of 60% and 80% are utilized to categorize students into groups demonstrating low, medium, and high spatial abilities.

Results and Discussion

Students were divided into three groups, based on their PSVT:R score. Low visualizers were defined by scores below 60%, the medium visualizers scored between 60% to 79%, and the high visualizers achieved scores of 80% or higher. In this cohort (n=129), the percentage of students with low, medium, and high SVS was 24%, 30%, and 46%, respectively, as seen in Figure 3.

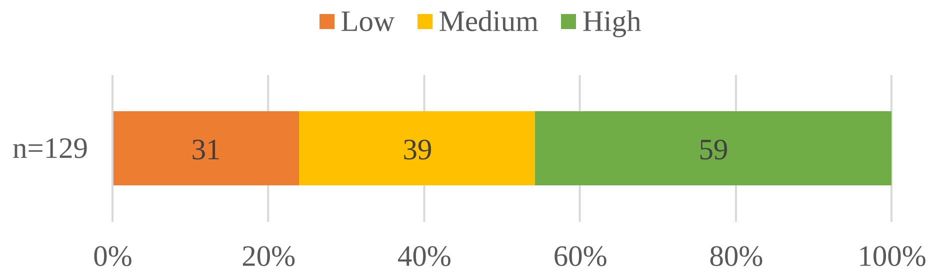


Figure 3. Distribution of students with low SVS, medium SVS, and high SVS.

Longitudinal comparison of overall mastery

To address RQ1, we compared the overall performance on calculating moments among students on the Exam 1 problem and the Exam 2 problem. On the Exam 2 problem, only the moment equation portion was considered to provide a more direct comparison between the two problems. The FBD and proper use of the applied moment were not considered.

Mastery of the moment of forces concept was defined as calculating the moment completely correctly. As seen in Figure 4 below, student performance worsened from exam 1 to exam 2, across all SVS groups. For the exam 1 problem, a much larger percentage of students were able to demonstrate mastery of the concept, i.e. 48%, 46%, and 79% of low, medium, and high visualizers, respectively. In exam 2, only 16%, 31%, and 39% of low, medium, and high visualizers, respectively, were able to demonstrate complete mastery of the concept.

The lower mastery rates may be attributed to a higher difficulty in the exam 2 problem rather than an actual regression of skills. We observed that geometry of the problem in exam 2 (angled beam) likely contributed to this effect, which will be discussed in a later section.

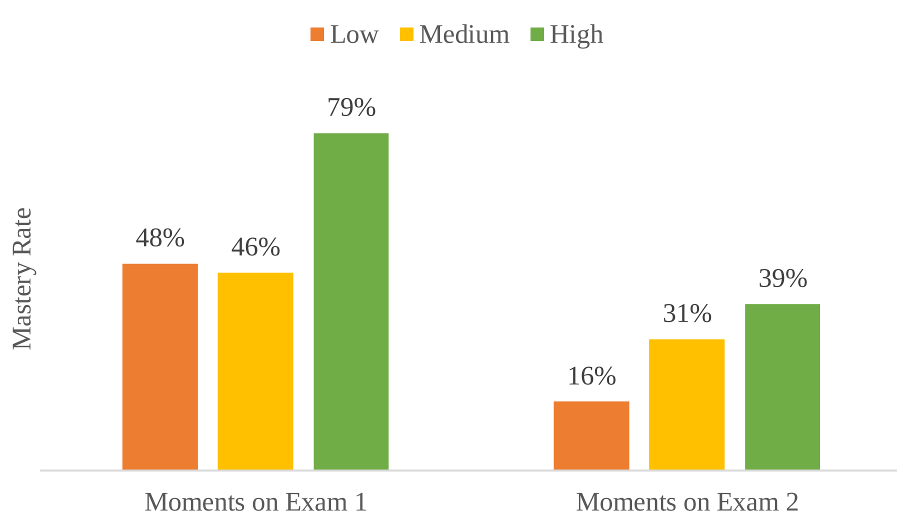


Figure 4. Comparison of student mastery of the moment of force concept on Exam 1 vs. Exam 2 among low, medium, and high SVS groups.

Longitudinal comparison of error types and prevalence

Focusing again on only the moment equation portion of the exam 2 problem, we determined the types and prevalence of errors made among the three SVS groups. A comparison of error types and prevalence between the exam 1 problem and exam 2 problem can be made using Figures 5 and 6.

Note that the error categories are not identical, as some were not relevant to both problems. In exam 1, the error "F" refers to the case where students did not resolve forces at all, whereas in exam 2, the error "F*" represents one or more incorrect force components in the moment equation.

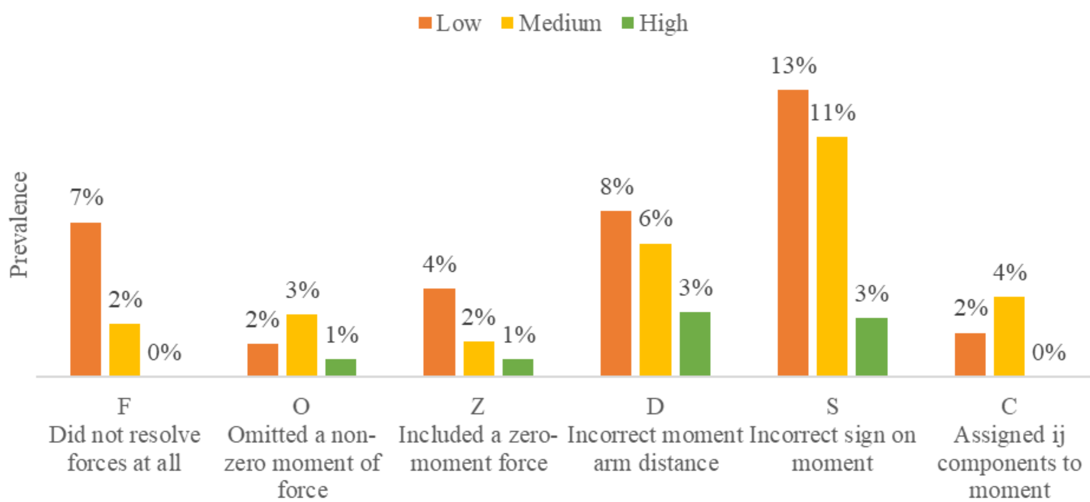


Figure 5. Error prevalence metrics for Exam 1 problem among low, medium, and high SVS groups.

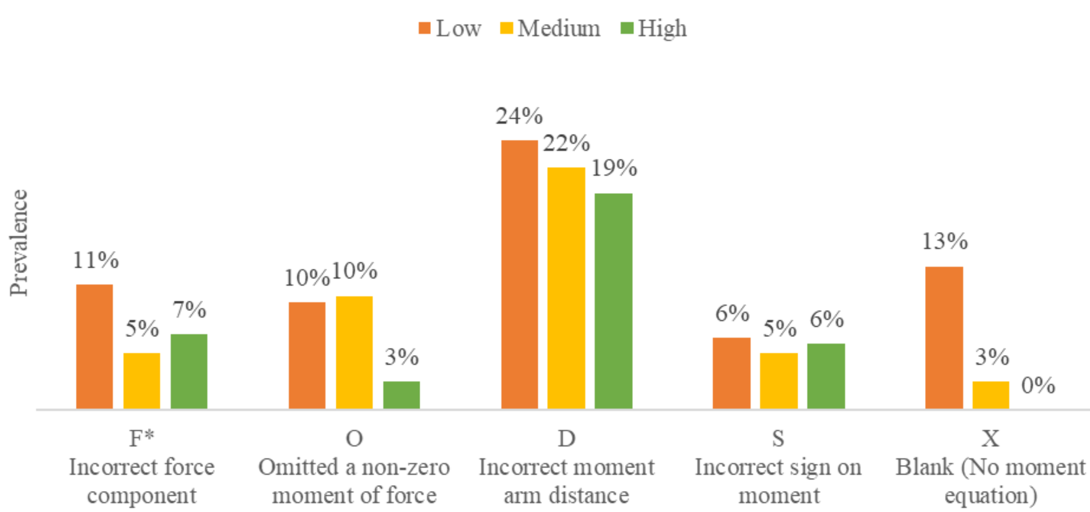


Figure 6. Error prevalence metrics for Exam 2 problem among low, medium, and high SVS groups.

From Figs. 5 and 6 we can observe that the students made fewer sign errors when calculating moments and more importantly, errors related to zero-moment forces (Z) and assigning components to moments (C) are no longer observed. This is promising as it appears that the students have improved their basic understanding of moments.

However, errors involving omission of a force contributing to moment (O) and incorrect moment arm distances (D) were more prevalent in exam 2. Students often neglected to include the moment due to the horizontal force component. For the moment-arm distance errors, this included students swapping the appropriate distances (e.g. using a vertical distance for a vertical force component) but we also observed that students struggled to find proper moment arm distances due to the geometry of the problem. If it was clear that students conceptually understood which distance was needed as the moment arm, but calculated the distance incorrectly, the error could be tagged as "G" for geometry rather than "D". It was often difficult, however, to discern the difference and definitively attribute to a geometry error. Due to this difficulty, metrics on geometry errors were not separated explicitly, but would contribute to errors "F*" and "D".

A starker observation between exam 1 and exam 2 is the rise in the error prevalence of error "D", in exam 1 8%, 6%, and 3% errors were found in low, medium, and high SVS groups respectively, where as 24%, 22%, and 19% were reported in exam 2. This decline in the accuracy of incorrect moment arm distance could be attributed to the geometry of the problem, however, based on our hypothesis this error should be low in high SVS groups. This error highlights the lack of understanding in moments and geometric errors. The next section delves deeper into the observed geometric errors for this problem.

Geometry errors

Many students struggled with the geometric aspects of the problem. This contributed to the inability to calculate the distances and/or force components properly, even if students may have conceptually understood what was needed to determine the moment appropriately.

Poor skills in trigonometry, angle properties, and similar triangles led to the following creative solutions for calculating force components and moment arm distances:

a. Assumed orientation of force vector was the same as a beam, such that $F_x = \frac{12}{13}F$ and $F_y = \frac{5}{13}F$
b. Assigned some angle value, e.g. 30°, 45° or 60° for the force and/or beam.
c. Assigned 3-4-5 triangle for the force (possibly due to provided dimensions)
d. Attempted to incorporate the provided 90° angle to find force components
e. Used angle properties incorrectly (e.g. corresponding, alternate, and/or interior angles)
f. Calculated distance *AB* as sqrt (8^2+5^2), possibly due to inability to use similar triangles

Examples of the above errors are illustrated in Figure 7 below. Additional examples are included as an appendix.

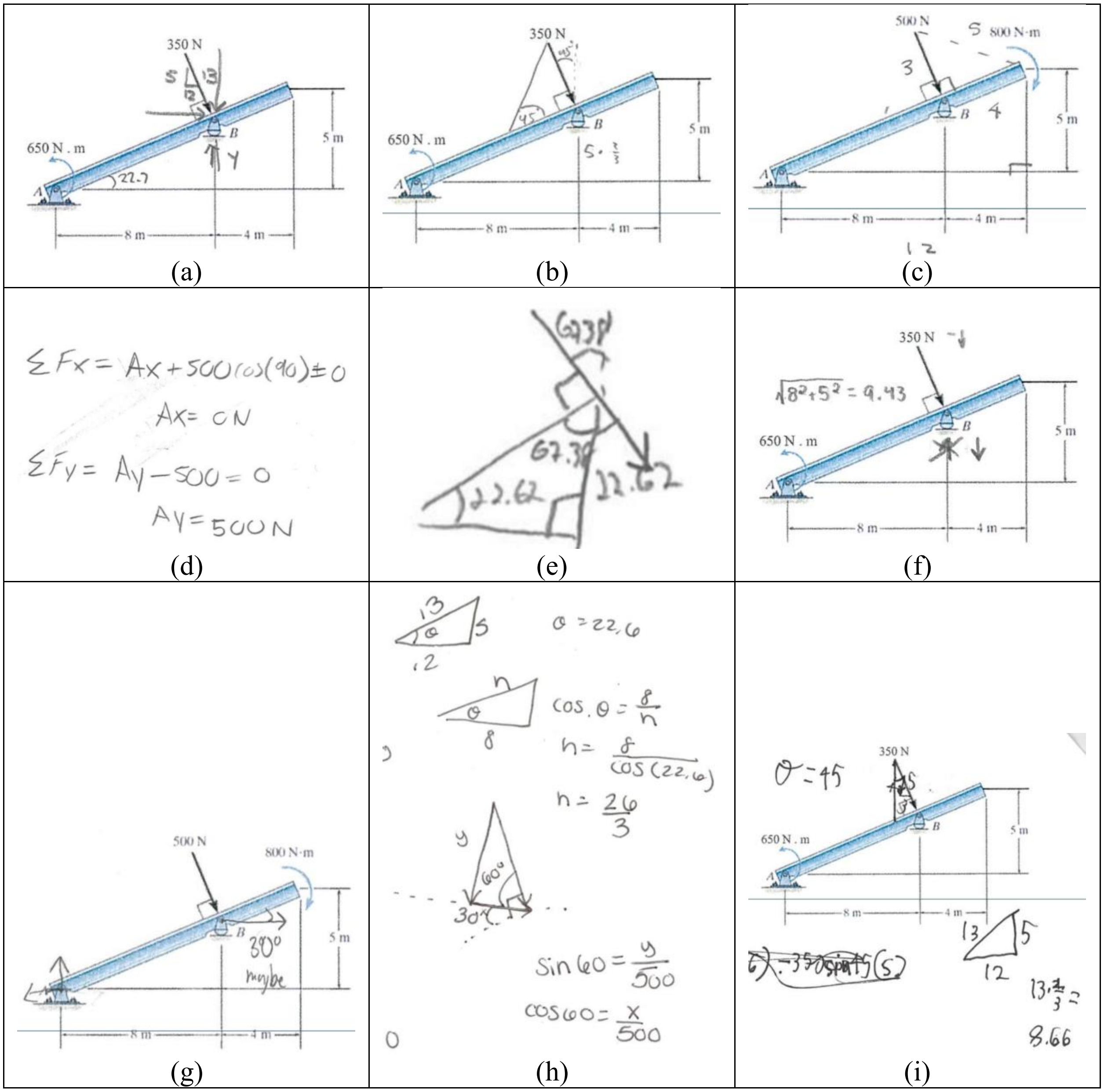


Figure 7. Examples of student errors related to geometry

Although more prevalent among low and medium visualizers, geometry errors were also observed among high visualizers. While high visualizers were generally able to use trigonometry properly to calculate angles and/or lengths, they were sometimes unable to apply this information to solve the problem correctly. This revealed an inability to connect all the given information together in a coherent manner.

Overall, there were several inconsistencies observed with determining angles, distances, and force components. It was surprising to find that many students did not know how to handle the right angle illustrated in the problem, leading to some students assigning an arbitrary angle for the force vector and others to simply using sin 90° and cos 90° as force components.

Additional statics concepts and skills

In addition to the moment of force, the exam 2 problem tested two other skills:

- Drawing a proper free-body diagram (FBD)
- Understanding the effect of a free moment

The required FBD for the problem was relatively straightforward, and most students drew it correctly. The reaction at roller *B* proved to be problematic for some, as shown in Figure 8. Students either assumed two reactions ($\boldsymbol{B_x}$ and $\boldsymbol{B_y}$) or a reaction force colinear with the applied force, in the opposite direction.

The observed error prevalence indicates probable improper visualization of the rocker support and more fundamental error of having four unknowns with only three available equations. The latter does not fall under the purview of SVS, however, shows the lack of fundamental procedural mastery as well.

High SVS group made less errors compared to the other groups in this aspect, further strengthening the notion that SVS skills are strongly correlated to the understanding and drawing the FBDs.

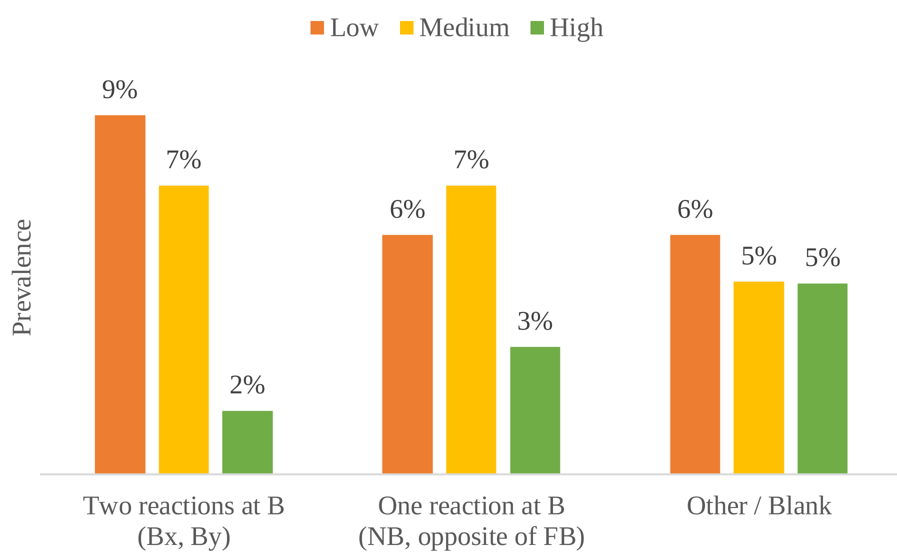


Figure 8. Error prevalence metrics for the free-body diagram in the Exam 2 problem among low, medium, and high SVS groups.

Errors related to the applied moment are summarized in Figure 9. The most common error was multiplying the moment by a distance, signifying a lack of conceptual understanding about what a moment is. Other errors included omitting the applied moment altogether in the moment equation and/or applying the incorrect sign for the direction of the moment.

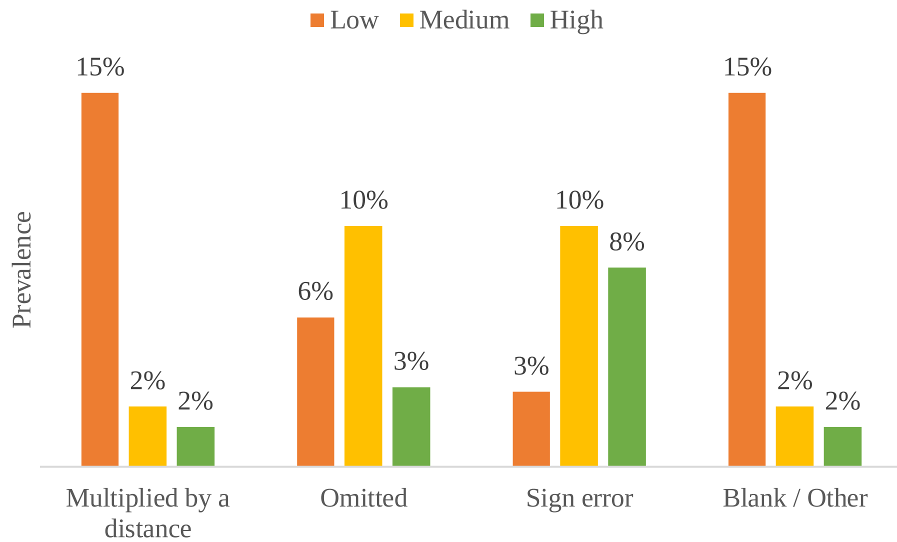


Figure 9. Error prevalence metrics for the free moment in the Exam 2 problem among low, medium, and high SVS groups.

While the exam 2 problem was not meant to explicitly test this skill, the solution method that students chose for moment of the applied force about *A* is interesting to compare. Many students recognized that the applied force was perpendicular to the beam, and the moment of the force could thus be calculated with one term (more efficient – Method A) instead of resolving the force vector and using two terms (less efficient – Method B) for the moment. Method A also led to higher accuracy rates than Method B, as shown in Figure 10.

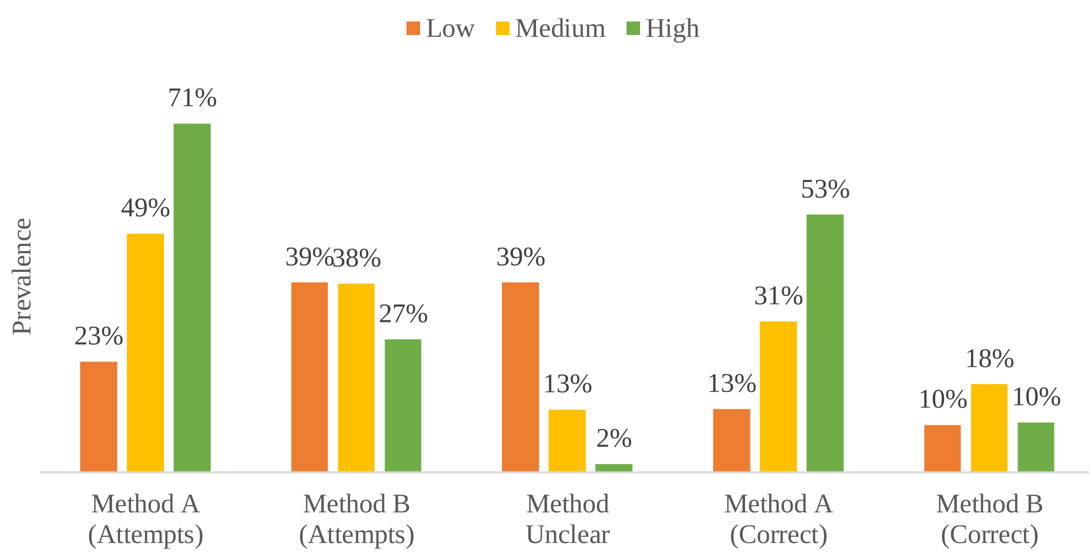


Figure 10. Comparison of prevalence and accuracy rates of various solution methods among low, medium, and high SVS groups.

The majority of the high SVS group (71%) chose Method A, leading to a high accuracy rate of 53%. Almost half of the medium SVS group (49%) chose Method A, leading to a fairly high accuracy rate of 31%. The low SVS group (23%) chose Method A, with an accuracy rate of 13%.

Recognizing that the force does not need to be split into components for this situation demonstrates that the student can flexibly apply their statics knowledge, showcasing versatility in problem solving.

Conclusions

In our previous study, we found that students with lower SVS master concepts at a lower rate than their higher SVS counterparts. By comparing problems involving a specific concept (i.e. moment of forces) from Exam 1 and Exam 2, we investigated whether mastery of this concept would improve with time among students with lower SVS (RQ1).

We observed that the capability for accurate moment calculations declined from exam 1 to exam 2, indicating that the amount of time did not aid in strengthening all the conceptual aspects of moments. As discussed, these results may have been confounded by the geometry of exam 2 problem, which further emphasizes the need for spatial skills in engineering problem solving, primarily, statics in this case. However, some of the major conceptual errors (Z – including zero-moment forces and C – assigning ij components to moment) were no longer observed in exam 2, which is a positive sign of the students learning and retention.

Geometric errors were considered under both RQ1 and RQ2, these errors led to the miscalculation of moment arms, angles, and force components. These errors showcased that the students need more practice in trigonometry and be able correctly visualize the angles and components involved in the problem. Geometry skills are a pre-requisite for solving statics problems properly. Students with low SVS struggle with geometric concepts which makes mastery of statics concepts more difficult.

Most of the FBDs in the solutions were drawn correctly, however, the majority of errors were the misidentification of the rocker support reactions or the drawing incorrect forces (direction) in the FBD. Students with low and intermediate SVS scores contributed to a higher percentage of errors compared to the high SVS students. This is most likely due to the proper visualization and understanding of the support structures and their function in equilibrium. Both the geometric and FBD errors could be attributed to low SVS skills which can be improved with more practice either in class or through worksheets.

Another interesting comparison among the three groups is the efficiency of the solution chosen by the three groups. 71% of the high SVS students chose the most efficient method (Method A) to solve the problem by directly finding the moment arm for the applied force rather than splitting it into components, compared to 23% and 49% of low and intermediate SVS groups respectively. Out of these students who chose the most efficient method, 53% (high), 31% (med), and 13% (low) correctly solved the problem. This choice of the most efficient method can be viewed as an indication that the high SVS student group visualized the problem comparatively better and chose the efficient way, highlighting the importance of developing practical visualization skills in students.

In our future work, we would like to further explore the RQ2 in a broader data set from multiple years to understand the observed correlations better. Further, with the inclusion of lab components with practical moment examples, we anticipate that the fundamental errors will diminish further.

References:

[1] S. Sorby, G. Duffy, D. Soni, and G. Panther, "An Evaluation of The Relationship between Spatial Skills and Creating a Free Body Diagram," in *2022 ASEE Annual Conference & Exposition*, 2022. Accessed: Feb. 07, 2024. [Online]. Available: https://peer.asee.org/an-evaluation-of-the-relationship-between-spatial-skills-and-creating-a-free-body-diagram.pdf

[2] O. Ha and N. Fang, "Spatial Ability in Learning Engineering Mechanics: Critical Review," *J. Prof. Issues Eng. Educ. Pract.*, vol. 142, no. 2, p. 04015014, Apr. 2016, doi: 10.1061/(ASCE)EI.1943-5541.0000266.

[3] S. Sorby, N. Veurink, and S. Streiner, "Does spatial skills instruction improve STEM outcomes? The answer is 'yes,'" *Learning and Individual Differences*, vol. 67, pp. 209–222, 2018.

[4] N. S. Foundation (US), *Preparing the next generation of stem innovators: Identifying and developing our nation's human capital*. National Science Foundation, 2010.

[5] E. Davishahl, T. Haskell, L. Singleton, and M. P. Fuentes, "Do They Need To See It To Learn It? Spatial Abilities, Representational Competence, and Conceptual Knowledge in Statics," in *ASEE Annual Conference proceedings*, 2021. Accessed: Feb. 08, 2024. [Online]. Available: https://par.nsf.gov/biblio/10297086

[6] S. D. Wood, W. H. Goodridge, B. J. Call, and T. L. Sweeten, "Preliminary analysis of spatial ability improvement within an engineering mechanics course: Statics," in *2016 ASEE Annual Conference & Exposition*, 2016. Accessed: Feb. 07, 2024. [Online]. Available: https://peer.asee.org/preliminary-analysis-of-spatial-ability-improvement-within-an-engineering-mechanics-course-statics

[7] M. Fontaine and C. K. Vallabh, "Correlating Common Errors in Statics Problem Solving with Spatial Ability," in *2024 ASEE Annual Conference & Exposition*, 2024. Accessed: Jan. 15, 2025. [Online]. Available: https://peer.asee.org/47079.pdf

[8] G. M. BODNER and R. B. GUAY, "The Purdue Visualization of Rotations Test," *Chem. Educator*, vol. 2, no. 4, pp. 1–17, Oct. 1997, doi: 10.1007/s00897970138a.

Appendix

Many students did not know how to handle the given dimensions and simply assigned a 30° or 45° angle instead of using the 5-12-13 triangle. Some examples of the geometry errors made by low, medium, and high visualizers are provided below.

Beginner/Low SVS: Student correctly calculated angles and distances but did not know how to use them properly to calculate moments of forces.

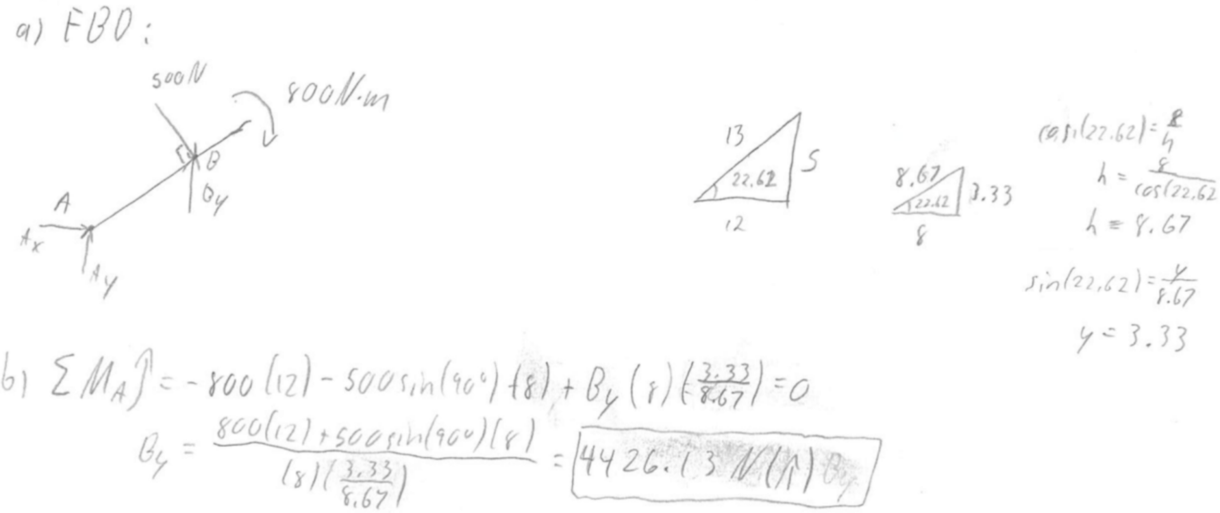


Figure A1: Representative example of a geometric error by a low SVS student

Intermediate SVS: Student calculated angle properly but was unable to use it to determine distance and force components properly.

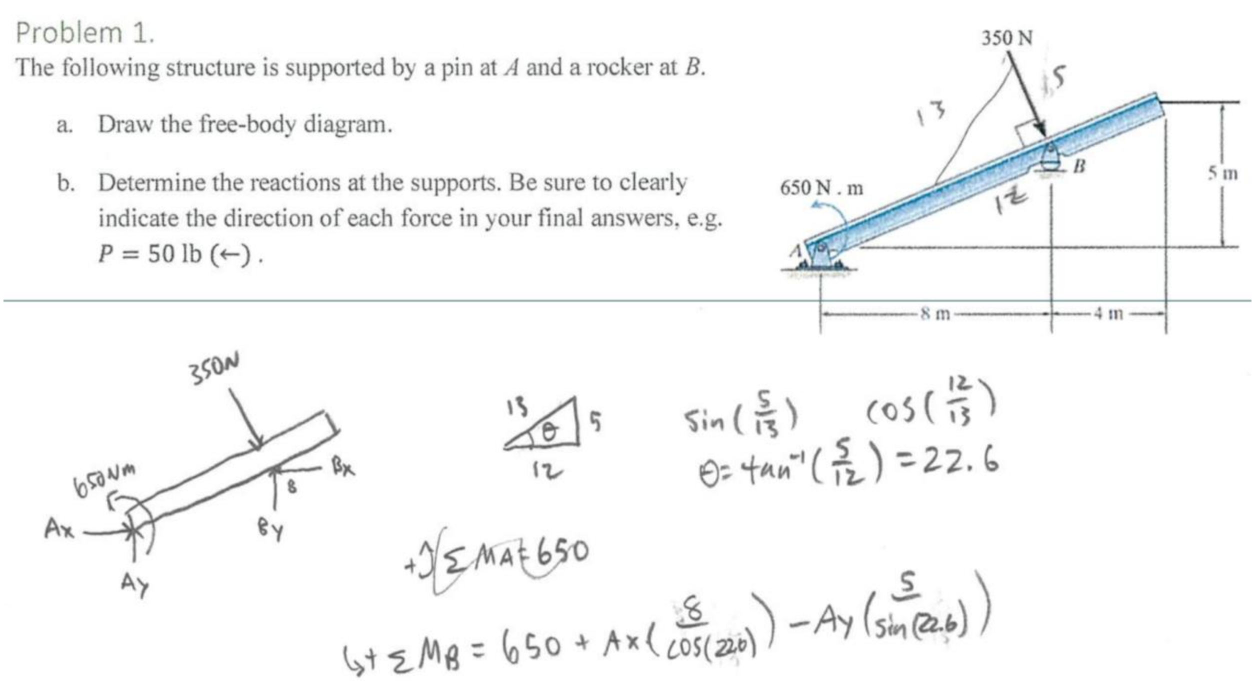


Figure A2: Representative example of a geometric error by an intermediate SVS student

Master/High SVS: Student calculated distance *AB* properly but did not use it as the moment arm distance. In addition, student assigned a 30° angle for the applied force.

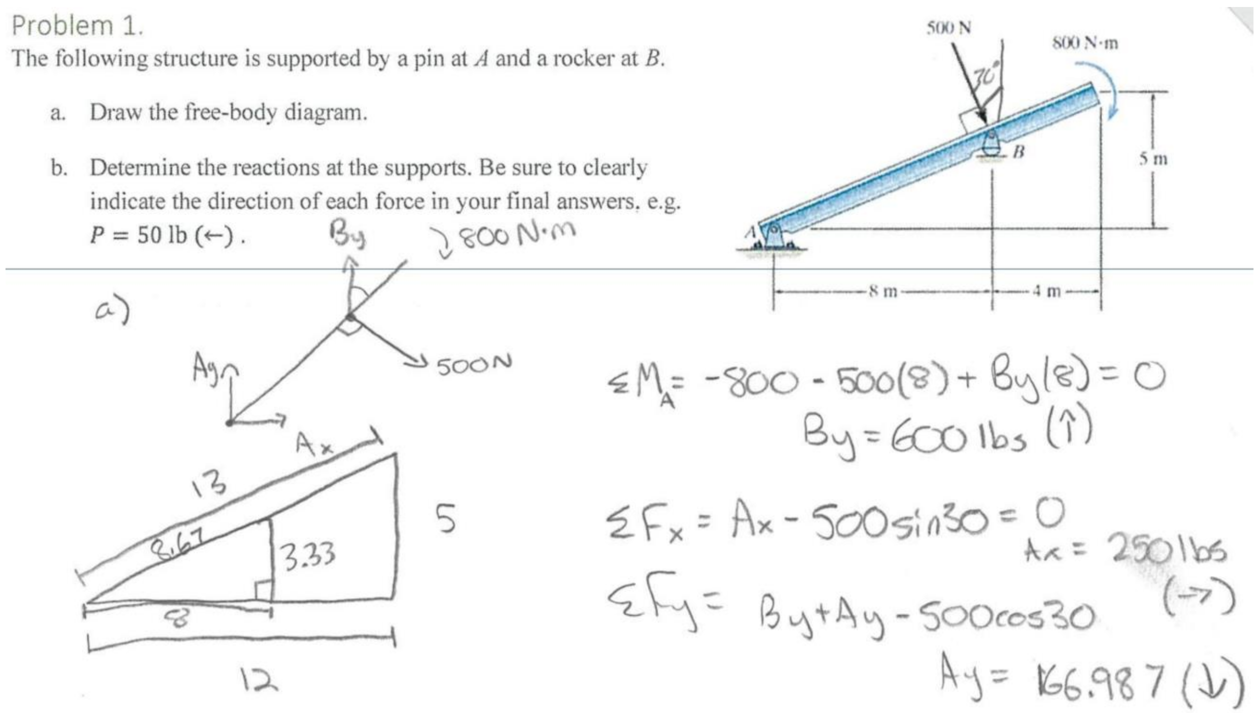


Figure A3: Representative example of a geometric error by a high SVS student