

Design, Manufacturing, Finite Element Analyses and Correlation with Experimental Work of Carbon Fiber – Epoxy Structures

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Hands-on Teaching of Carbon Fiber Structures Using Simulation, Manufacturing, and Experimentation of Carbon Fiber – Epoxy Structures

1. Introduction

The focus of this work is on teaching different topics about carbon fiber-epoxy structures or carbon fiber reinforced polymer (CFRP) in an engaging way. Activities during the lectures range from predicting mechanical properties that can be used in a Finite Element Analysis (FEA) to manufacturing the structures in the lab, and experimental work to be correlated with the FEA results.

The lectures are presented in the following sequence:

- a. Classification of composite materials including fiber and particle reinforced composites, and single and multilayer composites.
- b. Mechanical properties - review of stress-strain relationships of orthotropic lamina, elastic constants of an orthotropic material, stiffness and compliance matrices.
- c. Finite element analyses of carbon fiber – epoxy structures. Analyses include deflection and stresses of a cantilever plate, three-point deflection of a plate, and natural frequencies and modes of vibration.
- d. Manufacturing processes, including a hands-on session to prepare structures to later carry out experimental work.
- e. Experimental tests that can correlate to the finite element analyses mentioned above. In addition, damping properties are also determined.

Hands-on laboratories including finite element analyses and experimental tests are highly encouraged by ABET [1] and are commonly performed by R&D departments in the industry to develop new products. In the past there have been other ASEE works related to the topics presented here [2],[3].

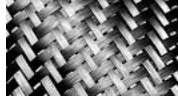
2. Classification of composite materials

A composite material is produced combining two different constituent materials with the purpose of creating a material that will have some advantages over readily available materials. There are several types of composite materials.

- Single layer
 - Continuous fiber reinforced composites
 - Unidirectional reinforcement



Bidirectional reinforcement



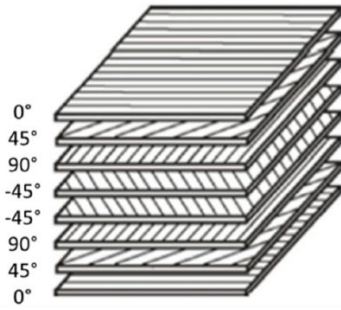
Discontinuous fiber reinforced composites

Random orientation

Preferred orientation

- Multilayered

Laminates (this is the material that will be used in labs)



Hybrids

- Particle reinforced composites

Random orientation

Preferred orientation

3. Mechanical properties of laminae

After the classification of composite materials is presented the next step to model composite structures was to define mechanical properties of a lamina. For this purpose, the stress-strain relationships of isotropic materials, orthotropic materials and transversely isotropic materials are presented.

Isotropic materials have the same properties (E , G , and ν) in all directions, these properties are related by $G = E/2(1 + \nu)$ where E is the Young's modulus, G is the Shear modulus and ν is the Poisson's ratio. Knowing two properties is enough as the third can be determined from the other two. Matrices in equations (1-2) are the compliance matrices, which are the inverse of the stiffness matrices. It must be obvious that the fibers are aligned with direction "3".

The stress-strain relationships of orthotropic materials are [4]

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{23} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \quad (1)$$

The stress-strain relationships of transversely isotropic materials, such as composite lamina are [4].

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} 1/E_p & -\nu_p/E_p & -\nu_{tp}/E_t & 0 & 0 & 0 \\ -\nu_p/E_p & 1/E_p & -\nu_{tp}/E_t & 0 & 0 & 0 \\ -\nu_{pt}/E_p & -\nu_{pt}/E_p & 1/E_t & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_p & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_t & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_t \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}, \quad (2)$$

where $E_1 = E_2 = E_p$, $\nu_{31} = \nu_{32} = \nu_{tp}$, $\nu_{13} = \nu_{23} = \nu_{pt}$,
 $G_{13} = G_{23} = G_{pt}$, $G_p = E_p/2(1 + \nu_p)$, $\nu_{tp}/E_t = \nu_{pt}/E_p$ 1,2,3 are the directions
 p, t are the longitudinal (in-plane) direction and transverse direction respectively.

3.1 Determining mechanical properties of laminae from fiber and matrix properties

Fibers are transversely isotropic materials with excellent mechanical properties in the direction in which they are aligned, the longitudinal direction. However, their mechanical properties in the transverse directions are not as good. In contrast, the mechanical properties of the matrices such as epoxy are usually isotropic. Thus, each layer, just like fibers, has orthotropic mechanical properties and the mechanical properties in the direction of the fibers are much better than in the transverse directions, which are equal. Several models have been developed to determine the values of the mechanical properties of a layer, some of the most widely used are a) the rule of mixtures and b) the Chamis model presented below

The rule of mixtures (ROM)

This model assumes fibers to be aligned along direction 1. Thus, the plane of isotropy is 2-3. [5],[6].

$$E_1 = V^f E_1^f + V^m E^m, \nu_{12} = V^f \nu_{11}^f + V^m \nu^m, E_2 = \frac{E_{22}^f E^m}{E^m V^f + E_2^f V^m}, \text{ and } G_{12} = \frac{G_{12}^f G^m}{G^m V^f + G_{12}^f V^m} \quad (3)$$

The Chamis model

This is a semi-empirical model that improves the ROM [10].

$$E_1 = V^f E_1^f + V^m E^m, \nu_{12} = V^f \nu_{12}^f + V^m \nu^m, E_{22} = \frac{E^m}{1 - \sqrt{V^f} (1 - E^m/E_{22}^f)},$$

$$G_{12} = \frac{G^m}{1 - \sqrt{V^f} (1 - G^m/G_{12}^f)}, \text{ and } G_{23} = \frac{G^m}{1 - \sqrt{V^f} (1 - G^m/G_{23}^f)}, \nu_{23} = (E_{22}/G_{23}) - 1 \quad (4)$$

In the equations above V^f is the percentage volume of the fiber and V^m is the percentage volume of the matrix (epoxy); and 1 is equal to p and 2 and 3 are equal to t . Thus 2 and 3 are interchangeable. Thus, $\nu_{12} = \nu_{13}$, $E_{22} = E_{33}$, $G_{12} = G_{13}$.

For both models - ROM and Chamis model - the density of the composite is defined as

$$\rho = V^f \rho^f + V^m \rho^m \quad \text{where } \rho \text{ is density, } f \text{ is fiber and } m \text{ is matrix} \quad (5)$$

Several values can be found in the literature for the mechanical properties and density of carbon fibers and epoxy resin. The values used and thus recommended in this work are

Table 1. Mechanical properties [11] and density of carbon fiber and epoxy resin

Carbon fiber		Epoxy resin	
$E1^f$	232 GPa	E^m	3.45 GPa
$E2^f$	15 GPa		
$\nu12^f$	0.279	ν^m	0.35
$\nu23^f$	0.49		
$G12^f$	24 GPa	G^m	1.28
ρ^f	1760 kg/m ³	ρ^m	1540kg/m ³

3.2 Determining failure of composite materials

Failure index I_F and strength ratio R [7]

Failure is predicted when either

$$I_F = \frac{\text{stress}}{\text{strength}} \geq 1 \quad \text{or} \quad \text{when } R = \frac{1}{I_F} = \frac{\text{strength}}{\text{stress}} \geq 1 \quad (6)$$

Strength Values

Table 2 shows the strength properties of the material were used considering $V^f=0.55$ and the material properties given in Table 1. The letter F is used to denote the strength value for a unidirectional lamina. Use these values only if strengths are unknown, tensile/compression tests can be used to easily determine F_{1t} and F_{1c} . Tests/analyses suggested here are meant to have one these failure modes. The failure index I_F must be determined for each layer.

Table 2. Strengths of the composite material [7]

Property	[MPa]
F_{1t}	1550
F_{1c}	1090
$F_{2t}= F_{3t}$	59
$F_{2c}= F_{2t}$	207
F_4	128
F_6	75

The subscripts associated with σ come from the Second order stress tensor [7]

$$[\sigma] = \begin{bmatrix} \sigma_1 & \tau_{12} & \tau_{13} \\ \tau_{12} & \sigma_2 & \tau_{23} \\ \tau_{13} & \tau_{23} & \sigma_3 \end{bmatrix} = \begin{bmatrix} \sigma_1 & \sigma_6 & \sigma_5 \\ \sigma_6 & \sigma_2 & \sigma_4 \\ \sigma_5 & \sigma_4 & \sigma_3 \end{bmatrix} \quad (7)$$

Subscripts for off-diagonal terms are equal to 9-i-j

Failure Criteria

Several failure criteria for composite materials have been developed such as [7]

- Hashin Failure Criterion (2D)
- Puck Failure Criterion (2D)
- Maximum Strain Criterion (3D)
- Maximum Stress Criterion (3D)
- Tsai-Wu Criterion (3D)

In this work the maximum stress criterion that defines the failure index is used. Similar equations

can be found in the literature for other theories of failure as shown in [7].

$$I_F = \max \left\{ \begin{array}{l} \frac{\sigma_1}{F_{1t}} \text{ if } \sigma_1 \geq 0 \text{ or } \frac{\sigma_1}{F_{1c}} \text{ if } \sigma_1 < 0 \\ \frac{\sigma_2}{F_{2t}} \text{ if } \sigma_2 \geq 0 \text{ or } \frac{\sigma_2}{F_{2c}} \text{ if } \sigma_2 < 0 \\ \frac{\sigma_3}{F_{3t}} \text{ if } \sigma_3 \geq 0 \text{ or } \frac{\sigma_3}{F_{3c}} \text{ if } \sigma_3 < 0 \\ \frac{abs(\sigma_4)}{F_4} \\ \frac{abs(\sigma_5)}{F_5} \\ \frac{abs(\sigma_6)}{F_6} \end{array} \right. , \quad (8)$$

where t indicates tension and c compression

4. Finite element analyses of carbon fiber – epoxy structures

The finite element analyses proposed in this work modelling carbon fiber – epoxy plates are

- natural frequencies and modes of vibration
- deflection and stresses of a cantilever plate,
- three-point deflection of a plate

In this work the finite element code Abaqus was used, but it must be clear that other codes can be used for the same purpose.

The common procedure to set up the proposed analyses in Abaqus using composite shells has the following steps:

a) **Define geometry.** Geometry is defined by a surface (without thickness) in these labs as either shell elements are used. Part – Create Shape Shell – Type Planar

b) **Define material.**

Materials – General – Density, enter value of the composite's density

then in the same window Mechanical - Elasticity – Elastic

Here a choice must be made. Composites can be modelled defining “Type” as either “Engineering Constants” or “Lamina.”

If “Engineering Constants” is selected, the following mechanical properties will be needed

$E1, E2, E3, \nu_{12}, \nu_{13}, \nu_{23}, G_{12}, G_{13}$ and G_{23} .

If “Lamina” is selected, the following mechanical properties will be needed

$E1, E2, \nu_{12}, G_{12}, G_{13}$ and G_{23} .

c) **Define section.**

Sections – Category Shell and Type Composite.

Give material name, thickness, orientation angle, and ply name for each layer. Select Symmetry if needed.

Assign (main menu) - section (pick plate model)

d) **Create instance**

Assembly – Create

Seed (main menu) – Instance

Mesh (main menu) – Element Type - S4R or switch to quadratic to assign an S8R element

4.1 Natural frequencies and modes of vibration

Natural frequencies and modes of vibration are determined in Abaqus [4] running a Frequency analysis. This type of analysis is usually one of the first to be carried out during the design of new products subjected to dynamic loads. Natural frequencies are usually to be avoided during operation with dynamic loads as the amplitude of oscillation could increase without boundaries which may cause failure of the structure.

The following steps must be added

e) **Create frequency step**

Steps – Create – Procedure Type: Linear Perturbation – Frequency, Define the number of eigenvalues (natural frequencies) requested.

f) **Define Boundary Conditions**

BCs – Continue – Select edge(s) -Pinned or Encastre

g) **Create and submit the job**

Jobs – Continue – OK

Jobs - Submit

h) **Post-process**

Jobs – Results

Click on icon - Plot contours on deformed shape icon

Click on arrows to change the mode of vibration. Frequencies appear on display below modes.

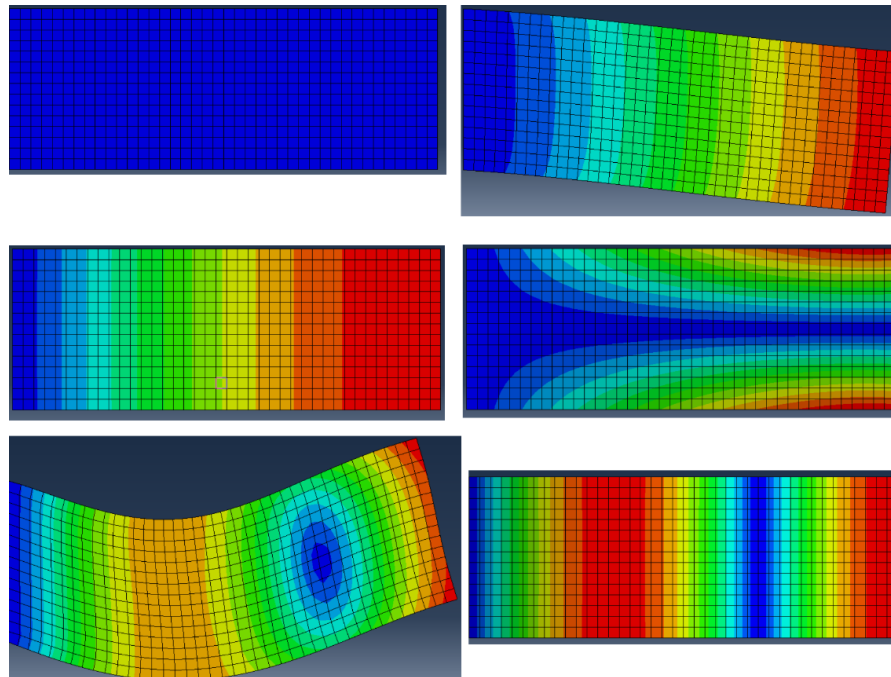


Figure 1. Example of composite plate model and modes of vibration

Figure 1 shows the model (blue) followed by the first five modes of vibration. Frequencies and modes will vary, depending on boundary conditions, mechanical properties, density, number and orientation of layers. This allows the lecturer to easily run different labs every semester, changing some or all the specifications above.

4.2 Deflection and stresses of a cantilever plate

Typically, one of the most important FEA of a new product is to determine the level of stress to avoid failure and the maximum displacement of the structure to avoid contact with other parts. In this case, we can apply a load on the edge, center or any other part of the plate, but if correlation with experimental work is required within an academic demonstration, we may choose a location easy to load and identify.

The following steps must be added to the common procedure presented in 4 to complete the analysis.

e) Create step

Steps – Create – Procedure Type: General – Static, General

f) Define Boundary Conditions (See Figure 2)

BCs – Continue – Select edge(s) -Pinned or Encastre

g) Create Loads (See Figure 2)

Create – Shell edge load – Select Edge, - Traction Transverse - Define magnitude of the load

h) Create and submit the job

Jobs – Continue – OK Jobs - Submit

i) Post-process (See Figure 3)

Jobs – Results

Click on icon - Plot contours on deformed shape icon

Tools - Query – Probe values – Field output variable for Probe MSTRS (Maximum stress theory failure measure at integration points) – Section points – Plies – Select ply. Results give values of I_F . Repeat for each layer. Field Output can also be changed to determine displacements.

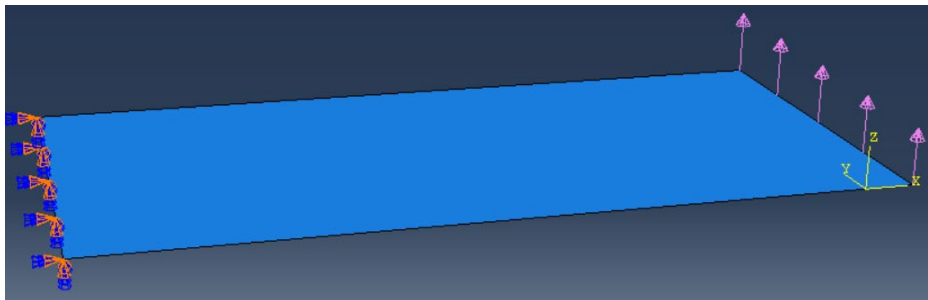


Figure 2. Boundary conditions and loads

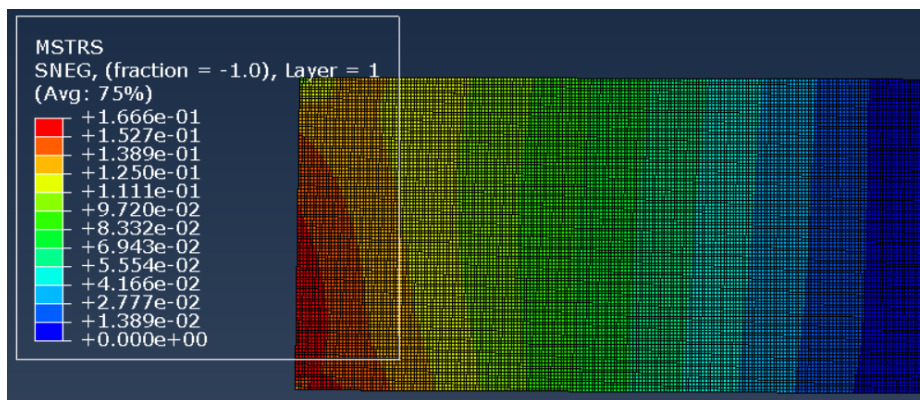


Figure 3. Stresses in layer 1 of the plate – Maximum stress Theory

4.3 Three-point deflection of a plate

e) Create Supports

Create Supports as shown in Figure 4. Rounded supports must be created and were constrained using an Encastre (clamped) boundary condition. The plate rests on these supports. Another element identical to the supports but rotated 180° will be needed to apply a displacement of 6 mm. The recommended size of the plate is a planform of 120 mm x 50 mm, with a thickness of at least 2mm (16 layers were used to achieve 2 mm in thickness in the experimental work). Supports are 100 mm apart. Element bending the composite plate is identical to the supports but upside down. Location of the elements must consider the actual thickness of the shells, which can be made visible selecting view in the main menu – Assembly Display Options – Render shell thickness.

f) Create step

Steps – Create – Procedure Type: General – Static, General

In Incrementation tab, define an initial increment of 0.15 and maximum number of increments to 10,000. These values may need to be modified depending on the finite element model.

g) Define Interaction

Module: Interaction, select icon to find contact pairs, find contact pairs. Click on an empty Property cell and define contact using a) Tangential behavior – Friction formulation: penalty – Friction coef. 0.15 or better value. B) Normal behavior – using default “Hard” Contact.

h) Define Boundary Conditions

BCs – Continue – Select supports’ surfaces below plate – Encastre

BCs – Continue – Select surface of the element above the plate – Give displacement

Define symmetry boundary conditions if they applied, paying attention to the orientation of the fibers.

i) Create and submit the job

Jobs – Continue – OK

Jobs - Submit

j) Post-process

Same as in Section 4.2

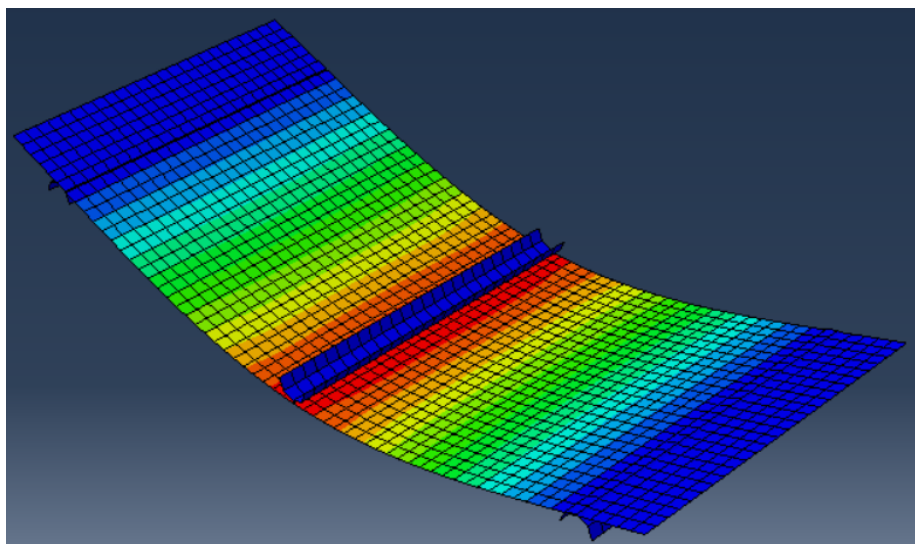


Figure 4. Three-point bending FEA

5. Manufacturing and Experimental work

Manufacturing

After different manufacturing processes were presented in class. The manufacturing of the plates for our labs followed the procedure shown in [8] Hand- Layup but using metallic plates to compress the composite plate. After that the composite plates were cut to the exact required dimension including additional material required for support.

5.1 Natural frequencies

Natural frequencies are commonly measured using a modal hammer and an accelerometer (Figure 5). Both instruments connect to a DAQ that send the signal to a computer to perform a Fast Fourier Transformation (FFT) to determine the Frequency Response Function (FRF). Peaks of the FRF give the natural frequencies of the structure. Both FFT and FRF are covered in the Numerical Methods course. Modes of vibration can be determined placing sensors (accelerometers or strain gages) at the nodal points (points without displacement). Other solutions such as laser vibrometers or high-speed cameras can be used to determine the modes of vibration.



Figure 5. Plate with accelerometer and modal (impact) hammer.

5.2 Stress, displacement and three-point bending

FEA results of Sections 4.2 and 4.3 can be compared applying loads and measuring displacements using a Universal Machine as the one shown in Figure 6. Three-point bending requires support and an element to apply the load.



Figure 6. Universal machine and composite plate on support.

Shear stress can be measured with a special sensor, but in this case only the axial strain was measured and compared to the FEA results and hand calculations. A schematic of the strain gage appears in Figure 7. The theory of the strain gage and logarithmic decrement to determine damping is also given in class.

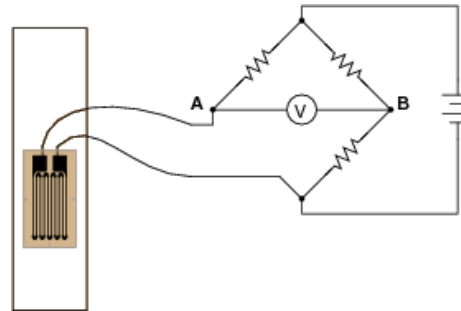


Figure 7. $\frac{1}{4}$ bridge strain gage connection

6. Assessment of the laboratories

To ensure teaching success, it is important that students present their results in writing in an organized way together with comments and observations. Conclusions will also be used to analyze if students grasped the main concepts of the laboratories. The proposed rubric to assess students' work is given in Table 3.

Table 3. Rubric of the hands-on components of the course.

Component	Sophisticated	Competent	Not yet Competent
FEA Frequency	Results are correct and presented in an organized way listing all relevant parameters to set up the FEA	Results are correct, but presentation of the results lack information, making it impossible to reproduce the FEA	Results are incorrect
FEA stress and deflection			
FEA Three-point bending			
Experimental Frequency	Results are correct and presented in an organized way listing all relevant steps to set up the test and differences with the FEA results are as expected	Results are correct but the presentation of the results can be improved. Conclusions of the comparison with the FEA are not accurate.	Results are incorrect
Experimental stress and deflection			
Experimental Three-point bending			

Written reports will be used to evaluate students, as well as the quality outcome of their practical

work. One report is required for each laboratory. In most cases, a lab report template was given, but I think it is a good idea to ask student to write reports with the following content:

- Introduction 15%
- Procedure 25%
- Results 30%
- Conclusions 30%

The following ABET outcomes are taking into account to evaluate students:

1. an ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics.
 6. an ability to develop and conduct appropriate experimentation, analyze and interpret data, and use engineering judgment to draw conclusions.
- Our target is that 80% of students must get a grade of 80% or higher in the overall assessment of an outcome. In this case the percentage is higher than 90%.

7. Students' comments

Students show more enthusiasm in all hands-on activities than in theoretical lectures. Students were interested in the result of the manufacturing of the CFRP plates. After completion of the curing, students made a visual inspection of the plates and were able to identify manufacturing problems and identify plates with good quality.

8. Conclusions

Results show that students enjoy working with hands-on activities such as labs using a commercial finite element method code, manufacturing processes and taking part in experimental work. Average grades in this part of the course are typically 90%, while the average grades in exams may be around 70%.

Most of the activities and theory in this work are not repeated in other mandatory courses in the Mechanical Engineering program.

Experimental work is also a good time to review standards, and testing equipment such as strain gages, accelerometer, impact hammer, universal machine, data acquisition system and theory such as FRF and FFT. All these activities complement all other knowledge in the Finite Element Method course and other courses such as Dynamics, Engineering Materials, Machine Design and Numerical Methods.

There isn't time to check the internet, emails or to get distracted during manufacturing, experimental work and finite element analysis. So, these hands-on activities are clearly more engaging than theoretical lectures.

References

- [1] <https://www.abet.org/accreditation/accreditation-criteria/criteria-for-accrediting->

engineering-programs-2024-2025/#GC3

- [2] J. Tito-Izquierdo, A. Gomez-Rivas, W. Feng, G. Pincus, “An Experiment Based Structural Dynamics Course For Engineering Technology Students,” in *2006 Annual Conference & Exposition, 2006, Chicago, IL, USA, June 18-21, 2006*.
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- [4] ABAQUS (2017) Analysis User’s Manual. Dassault Systemes Simulia, Inc.
- [5] W. Voigt, “Über die Beziehung zwischen den beiden Elastizitätskonstanten Isotroper Körper” *Annalen der Physik*, vol. 274, 12, pp. 573-587, 1889.
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Appendix – Reading Materials. List suggested by Tony Abbey – NAFEMS Course “Composite FEA”

1. E. J. Barbero, *Introduction to Composite Materials Design*, CRC Press. 1999. ISBN 1-56032-701-4.
2. E. J. Barbero, *Finite Element Analysis of Composite Materials*, CRC Press. 2008. ISBN 1-4200-5433-3.
3. R. Jones, *Mechanics of Composite Materials*, McGraw-Hill 1975. ISBN 0-07-032790.
4. Z. Gurdal *et al*, *Design and Optimization of Laminated Composite Materials*, Wiley. 1999. ISBN 0-471-25276-X.
5. S. Swanson, *Introduction to Design and Analysis with advanced composite materials*, Prentice-Hall. 1997. ISBN 0-02-418554-X.
6. A. Baker *et al.*, *Composite Materials for Aircraft Structures*, AIAA 2004. ISBN 1-56347-540-5.
7. J.N.Reddy., *Mechanics of laminated Composite Plates and Shells*, CRC Press. 2004 ISBN 0-8493-1592-1.
8. C. Kassapoglou, *Design and Analysis of Composite Structures*, Wiley 2010 ISBN 978-0-470-97263-2.
9. M. Niu. *Composite Airframe Structures*, Hong Kong Conmilit Press 5th Ed. 2008. ISBN 962-7128-06-6.

My personal suggestion is to read [7] and 1.