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GIFTS: Integration of Real-World Case Studies into a First-Year Engineering Mathematics Course

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GIFTS: Integration of Real-World Case Studies into a First Year Engineering Mathematics Course

Introduction

To help instructors enhance a first-year engineering math course modeled after the Wright State University approach, this paper introduces the integration of real-world scenario-based case studies. These case studies are designed to make mathematical concepts more engaging and relatable by placing students in authentic historical and current engineering contexts. The case studies also provide students with an introduction to various engineering disciplines, such as civil, biomedical, aerospace, mechanical, electrical and environmental engineering, while fostering critical thinking and problem-solving skills.

This approach dedicates one hour per week of class time to case studies, where students work in small teams to complete the case studies, with a total of ten projects over the semester, each tied to a specific math topic. This paper offers insights into how these case studies were implemented, providing a practical framework for instructors interested in adopting or adapting this method to enrich their own courses.

About the Course

This first-year university course is designed for students pursuing or aspiring to enter engineering programs. It provides a comprehensive foundation in mathematical concepts while highlighting their practical applications in engineering. Meeting five hours per week, the course balances theoretical instruction, guided practice, and hands-on learning. The mathematics and pedagogical strategies in the course are modeled after the Wright State University (WSU) model for engineering mathematics. The WSU approach to engineering mathematics uses an application-oriented, hands-on approach which focuses on only the math topics used in core engineering courses and is taught by engineering faculty [1].

Traditionally the course structure at our university includes lectures, studio and lab. The lecture is one hour per week and introduces mathematical concepts as well as describing their relevance to engineering problems. The studio time is two hours per week and offers collaborative problem-solving sessions where students practice applying mathematical concepts to engineering scenarios. The lab sessions are two hours per week. The labs provide hands-on experience with experiments and technologies, such as circuits, oscilloscopes, and coding, to reinforce mathematical principles through applications.

The mathematical topics included in the course have great breadth. The curriculum starts with algebra and progresses to advanced topics, including lines, quadratics, trigonometry, vectors, sinusoids, systems of equations, derivatives, integrals, and differential equations. Mathematical topics are always related to engineering related word problems. And in addition, lab data is often processed with MATLAB. Students are gaining an understanding of the interconnection between classroom theory, laboratory measurement and numerical representation of their engineering results [2].

Innovative Component:

In Fall of 2024, one of the professors teaching 3 sections of this course chose to use one of the five weekly hours to dedicate to solving real-world case studies inspired by current and historical events. These case studies challenged students to use the week's mathematical topic in practical real-world engineering problems, fostering critical thinking and problem-solving skills. Each real-world case study was paired with the mathematical topic of the week.

Objectives

The objectives of the WSU engineering math have been to increase retention in engineering. Unfortunately, at WSU 42% of students who started in engineering were unable to complete the calculus sequence [2]. This is common in engineering programs around the nation. The introduction of the engineering math course for freshmen allowed students early access to engineering applications and a path to take some engineering courses that would otherwise have calculus prerequisites, such as statics and circuits. Unfortunately, at our university, the course is not currently considered as a prerequisite to early engineering courses. Its primary purpose then is to increase retention by exposing students early to engineering problems and applications of math. This increases motivation as students enter the calculus sequence because of the awareness they gain around engineering applications of the math that they will encounter.

The new case study component of the course expands on the previous engineering-based discussions and problems by adding real-life and current-event based case studies, also tied into the mathematical topics. This addition is designed to enhance the real-world relevance of mathematical concepts by incorporating current events and relatable topics into the curriculum. The objectives are to expand student context around mathematics, introduce a multitude of engineering fields, engage with the world and current events, and develop problem-solving skills, as elaborated below.

1. Contextualize Mathematics: Help students connect mathematical problems and concepts to real-world scenarios, demonstrating their relevance and impact in everyday life and global events.
2. Introduce Diverse Engineering Fields: Expose students to a variety of engineering disciplines, broadening their understanding of the field and its multifaceted nature. Case studies included topics from the disciplines of aerospace, biomedical, chemical, civil, electrical, environmental and mechanical engineering.
3. Engage with Current Events: Use timely and relatable case studies to make engineering challenges more engaging and to illustrate the evolving role of engineering in addressing societal and technological issues.
4. Develop Problem-Solving Skills: Foster critical thinking and practical problem-solving by encouraging students to apply mathematical concepts to interdisciplinary challenges.

Through this innovative approach, students gain a deeper appreciation of mathematics in engineering and are inspired to explore the many ways engineering influences and improves the world around them.

Incorporating case studies into a freshman engineering math curriculum provides students with an opportunity to apply mathematical concepts to real-world engineering problems. These case studies are designed to continue to bridge the gap between theoretical math and practical engineering, allowing students to experience the challenges and rewards of engineering problem-solving in a team environment. By engaging students with scenarios rooted in historical or contemporary engineering events, such as the construction of the Central Pacific Railroad, the forces in a hip joint during recovery, or the optimization of satellite internet signals, educators can demonstrate the relevance of mathematics in engineering and foster a deeper understanding of the subject.

Practical Implementation Details

The implementation of these case studies is straightforward and adaptable to various classroom settings. Each case study is designed to fit within a single 40- to 50-minute class period, making it feasible to integrate them into weekly class time. Note that each case study was introduced with an introductory presentation with background information and short videos on the topic. Students work in small teams, typically of 2-3 members, which promotes collaboration and communication skills. The problems are constructed to align with the math topics being covered in the course, such as systems of equations, trigonometry, integrals, derivatives, and differential equations. This alignment ensures that students not only practice mathematical techniques but also see their direct application in solving engineering problems.

The materials include a clear problem statement, historical or technical background, step-by-step instructions, and expected deliverables. For example, in the case-study on resource optimization using derivatives, students learn about distributing water to remote communities, apply mathematical concepts to optimize delivery, and reflect on the societal and environmental impacts of their solutions. Similarly, the case study on electrification involves exploring integrals in utility distribution for population-dense areas. These activities emphasize both technical proficiency and broader considerations such as resource constraints and community impact. Table 1 shows a list of mathematical concepts and the associated case studies chosen. Each instructor can develop and customize case studies based on their expertise, interests, and relevant current or historical events. These are simply one set of examples.

Table 1. Case studies used in the course

Case Study Topic	**Mathematical Concept**	**Goal/Deliverable Description**
Central Pacific Railroad	Trigonometry	Calculate maximum elevation angles for train crossings
Bo Jackson's Hip Recovery	Vectors and Forces	Design a device applying forces to support hip recovery
Guided Missile Interception	Systems of Equations	Solve two linear equations to model missile interception trajectories.
Waste to Biofuels	Systems of Equations	Combine fuels using equations to optimize energy output.
Starlink Satellite Internet in Africa	Sinusoidal Addition	Analyze and combine sinusoidal waves for optimal signal and consider social implications of implementation.
Drought and Water Resource Optimization	Derivatives	Optimize resource distribution for remote communities.
Kindness Spread Modeling	Integrals	Model kindness spread using integrals and cascading effects.

Electrification of New York City	Integrals	Assess utility distribution across a city over time.
Subwoofers and Sound Waves	Differential Equations	Find transient and steady-state solutions for audio systems.

For institutions considering adopting such case studies, starting with one or two examples tailored to their syllabus can be a manageable first step, with the flexibility to expand as faculty and students become more familiar with the approach. The inclusion of case studies not only enhances the learning experience but also inspires students to view engineering as a holistic discipline that combines technical skills with societal impact.

Assessment

At the end of the semester, students participated in a survey that included questions to assess their opinions of the case studies; results are provided in Appendix D. An open-ended question asked students to indicate their favorite case study. All of the case studies had at least 1 vote, but the most common were the Central Pacific Railroad, Bo Jackson's Hip Recovery, and Drought and Water Resource Optimization. Each of these case studies is included in Appendices A-C.

Conclusion

The integration of real-world case studies into the engineering math class has proven to be an effective way to bridge the gap between theoretical concepts and practical applications. By engaging students in authentic engineering challenges across diverse disciplines, the case studies enhance motivation, foster critical thinking, and provide a meaningful introduction to the field of engineering. This approach not only helps students develop their mathematical skills but also equips them with the ability to approach complex problems with a broader, real-world perspective, preparing them for future academic and professional success.

References

[1] N. Klingbeil, "The Wright State Model for Engineering Mathematics Education:," [Online]. Available: https://www.oswego.edu/celt/sites/www.oswego.edu.celt/files/nwk_bio_abstract.pdf. [Accessed 16 02 2025].

[2] N. Klingbeil, R. Mercer, K. Rattan, M. Raymer and D. Reynolds, "Redefining Engineering Mathematics Education at Wright State University," in *American Society of Engineering Education (ASEE) Annual Conference & Exposition*, Chicago, Illinois, 2006.

Appendix A Central Pacific Railroad Case Study

Building the First Transcontinental Railroad: Engineering the Path through the Santa Cruz Mountains

Historical Context:

Theodore Judah and Charles Crocker, along with three other investors (Leland Stanford, Collis Huntington, and Mark Hopkins), embarked on the monumental task of building the Central Pacific Railroad, part of the first transcontinental railroad in the United States. One of the greatest challenges was crossing the Sierra Nevada and Santa Cruz Mountains. As engineers, they had to consider the steepness of the terrain and determine the safest routes while keeping the grade (slope) within limits that trains could handle.

Scenario:

You are a team of engineers in 1861, tasked with designing the railroad path through the Santa Cruz Mountains. Theodore Judah, the chief engineer, has instructed you to calculate the maximum allowable slope for the rail lines based on the steep terrain. The railroad must rise to meet higher elevations, but the angle of ascent must not exceed a maximum safe angle for train travel.

Topographical Details:

The Santa Cruz Mountains are part of California's Coastal Range, characterized by rugged terrain, steep inclines, and numerous ridgelines. The highest point in the range is Loma Prieta, which rises to 3,790 feet. However, the proposed railroad route avoids the highest peaks and instead traverses a more accessible pass, with a summit elevation of approximately 1,500 feet. The terrain between your starting point and the summit varies, featuring a combination of gradual slopes and steeper sections.

Terrain Profile (Approximate):

- **Starting Point**: 300 feet above sea level (base of the mountain)
- **First Ridgeline**: 800 feet above sea level (after 2 miles)
- **Second Ridgeline**: 1,200 feet above sea level (after an additional 1.5 miles)
- **Final Summit**: 1,500 feet above sea level (after another 1 mile)
- **Total Distance to Summit**: 4.5 miles (approx. 23,760 feet)

The engineers of the day must find a way to construct the track along this uneven terrain while keeping the grade within safe limits for train operation.

Task:

Using trigonometric principles, determine the necessary length of track to safely climb from the starting point to the summit. You will also calculate the horizontal and vertical distances for specific sections of the track, ensuring that the elevation angle stays within safe limits.

Given Information:

1. **Max Safe Elevation Angle for Trains**: The maximum grade for a railroad should not exceed 2.2% (equivalent to a 1.26-degree angle).
2. **Mountainous Terrain**: You will be traversing ridgelines with varying elevations.
3. **Elevation Data**: Starting elevation is 300 feet above sea level, and the summit you must reach is 1,500 feet above sea level.

Questions/Steps:

4. **Calculate the Track Length Required**: Given the total elevation difference of 1,200 feet and the maximum allowable grade of 2.2%, use trigonometry to calculate the minimum total track length required to reach the summit.
5. **Verify the Maximum Slope at Each Ridgeline**:

- Calculate the vertical and horizontal distances between each ridgeline (from 300 to 800 feet, 800 to 1,200 feet, and 1,200 to 1,500 feet).
- Ensure that the grade (slope) between ridgelines remains within safe limits.

For example, calculate the angle of ascent between the base and the first ridgeline, and verify that the angle does not exceed the safe limit of 1.26 degrees.

6. **Designing the Track Segments**:

- Propose track segments that ascend gradually, potentially curving around steeper ridgelines or following natural valleys to reduce the steepness of the incline.
- Determine the total track length required if you follow a more gradual path that avoids direct steep ascents. How does this compare to the straight-line distance?

7. **Reflection on Challenges**: Compare your approach to the real-world challenges faced by Theodore Judah and his engineering team. How did topography, technology, and mathematical considerations influence their decisions? Reflect on how modern engineering techniques would allow for more precise control over the terrain.

Extension Activity:

If time allows consider alternative paths through the mountains. For instance, analyze whether cutting through the mountain with tunnels or creating switchbacks would allow for a shorter or safer route. How do these alternative designs affect the total track length and grade?

Appendix B Bo Jackson’s Hip Recovery Case Study

Bo Jackson's hip injury occurred during a 1991 NFL game when a tackle caused his hip to dislocate. The injury disrupted blood flow to the femoral head, leading to a condition called avascular necrosis, where the bone tissue in the hip joint began to die. This resulted in chronic pain and joint damage, eventually requiring a hip replacement.

Bo Jackson's Hip Injury and Force Recovery Design

Introduction: Bo Jackson, a legendary two-sport athlete, suffered a severe hip injury during a football game in 1991. His injury was caused when he was tackled from behind while running, causing his hip to dislocate. This resulted in a significant force being applied to his hip joint, which eventually led to avascular necrosis—a lack of blood flow to the bone. As engineers, your task is to understand the forces involved in the hip joint and design a device that could support and apply corrective pressure to aid in his recovery.

Brief Overview of the Hip Joint: The hip is a ball-and-socket joint, where the head of the femur fits into the acetabulum of the pelvis. It can withstand large forces during movement but can be vulnerable to trauma from abnormal forces or angles, as in Bo Jackson’s case.

Task 1: Resolving Forces in the Hip Joint (15 minutes)

Bo Jackson's body weight and the force from the impact applied to his hip can be modeled using vectors. Assume the following:

- Bo's body weight was 1000 N (downward force).
- The tackling force from behind caused an additional 800 N at a 30-degree angle to the horizontal.
- Frictional force from the ground is negligible.

Step 1: Resolve these forces into x and y components. Use vector addition to calculate the total force acting on Bo’s hip joint at the moment of the injury.

- **Body Weight** (Vertical Force, Fy)
  - Magnitude = 1000 N
  - Direction: Straight downward (negative y-direction)
- **Tackling Force** (Applied Force, F)
  - Magnitude = 800 N
  - Direction: 30 degrees from the horizontal

Questions for the students:

1. What are the x and y components of the tackling force?
2. What is the total force acting on the hip joint in the x and y directions?
3. What is the magnitude and direction of the resulting force?

Task 2: Designing a Recovery Support Device for Bo Jackson's Hip (20 minutes)

Now that you've calculated the forces during the traumatic event, let's focus on the aftermath of Bo Jackson's injury. The dislocation led to **avascular necrosis**, a condition where blood flow to the bone is compromised, causing pain and deterioration of the bone tissue. Your task now is to design a device that can help correct this problem, either by applying forces that relieve pain or by promoting blood flow to the hip joint.

New Information:

- **Medical Insight**: Blood flow to the head of the femur was significantly reduced due to damage to the surrounding blood vessels during the injury. Restoring this blood flow is essential to prevent further degeneration of the bone.
- **Design Requirements**: The device should apply gentle but effective pressure to areas around the hip joint, encouraging blood circulation. It must also help alleviate hip pain by redistributing weight and reducing stress on the injured area.

Device Parameters:

- The device should not exceed 5 kg in weight.
- It should fit comfortably around the waist and upper thigh and apply adjustable pressure to the hip joint.
- The design should consider both support for the hip joint and stimulation of blood flow.

Your Design Task:

1. Design a device that applies forces to the hip joint that can aid in restoring blood flow and easing pain.
2. Determine the direction and magnitude of the forces your device will apply to improve circulation to the hip area.
3. Consider how the device will redistribute Bo's body weight to alleviate pressure on the injured area.

Deliverables:

- A vector diagram showing the forces your device would apply to the hip joint to stimulate blood flow or reduce pain.
- A minimum of 3 sentences describing how your device works and how it will help Bo Jackson recover from his injury.

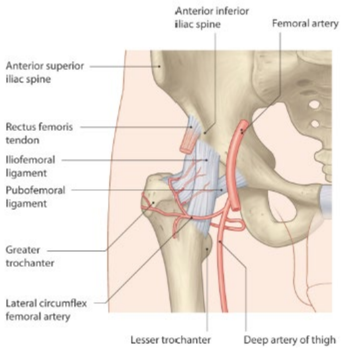

Image from: https://www.orthobullets.com/recon/12769/hip-anatomy

Appendix C Drought and Water Resource Optimization Case Study

Water Delivery Case Study

1. Background

In remote or underserved communities, access to essential resources like clean water is often limited due to high transportation costs, challenging terrain, and fluctuating demand based on population density. Engineers working in these environments must design resource distribution systems that are both cost-effective and responsive to community needs.

For this case study, students will imagine they are engineers working on a project to optimize water delivery routes for a region with several small, scattered communities. By modeling the impact of population density and geographical accessibility on transportation costs, they can determine the most efficient distribution route and delivery frequency, minimizing costs while meeting demand.

2. Mathematical Practice

Problem Setup:
Suppose the cost C (in dollars) of delivering water to a community depends on:

- Population density D (people per square kilometer),
- Geographical accessibility G (a measure from 0 to 1, where 1 is fully accessible terrain),
- Resource quantity W (liters of water delivered).

The cost function is defined as:

$$C = k \cdot \frac{W}{G \cdot D}$$

where k is a constant that incorporates factors like transportation type and fuel cost.

1. **Example 1: Solving with Variables**

Find the partial derivatives of C with respect to D and G to understand how changes in population density and geographical accessibility impact the cost of water delivery.

Using the expressions for $\frac{\partial C}{\partial D}$ and $\frac{\partial C}{\partial G}$, explain how adjusting population density or improving accessibility might help reduce costs in a real-world scenario.

2. Example 2: Solving with Numbers

Assume:

a. $k=100$,

b. $W=5000$ liters,

c. Population density $D=20$ people per square kilometer,

d. Accessibility $G=0.3$.

Task:

e. Calculate the cost C with these values.

f. Then, find the impact of a 10% increase in population density and a 10% increase in accessibility on the cost.

3. Reflection

Reflection Questions for Students:

- **Engineering Context**: Which factor—population density or geographical accessibility—would be most practical to improve in a real-life situation? Why?
- **Derivatives in Decision-Making**: How did using derivatives help in understanding the cost sensitivities in water distribution? What other factors might you consider if you were working as an engineer on this project?
- **Broader Impacts**: How could knowledge of optimizing resource distribution be applied to other essentials, like food or medical supplies, in humanitarian efforts?

Appendix D. Assessment

At the conclusion of the semester, students were asked for their feedback around the case studies as part of a larger survey to evaluate the course (conducted under an approved human subjects research protocol; #DEID). 55 students completed the survey. Students were asked to rate 5 aspects of their experience with the case studies on a 7-point scale (1=strongly disagree, 7 = strongly agree) and if they liked the case studies on a 5-point scale. Results to these quantitative survey items are summarized in Table 2. The strongest benefits of the case studies appeared to be exposing the students to a range of different engineering disciplines and making the class more interesting.

Table 2. Case Study Student Experience Feedback

Question Statements	**Mean**
I remember the 'case studies' that were presented during many of the Monday lecture periods	5.3
The case studies made the class more interesting	5.6
The case studies helped me learn the related math principles	5.1
The case studies improved my understanding of engineering	5.4
The case studies exposed me to a range of topics relevant to different engineering majors	6.0
To what extent did you like the reflection parts of the case studies? [1 = strongly disliked, 3 = neutral, 5 = liked a lot]	3.0

For the reflection parts, 36% of the students disliked/strongly disliked them, 25% liked/liked a lot, and 39% were neutral. This indicates that some improvements could be made to the reflection elements.

When asked if the math problems associated with the case studies were too simple or too difficult the responses indicated that the level of difficulty was about right (2% much too simple, 13% somewhat too simple, 66% at the right level of difficulty, 18% somewhat too difficult, 2% much too difficult). The distribution of responses might mirror the diversity of math skills of the students and the math related learning goals in the course overall, where some students placed into Calculus 3 while others had placed into pre-calculus.

In terms of recommendation for future semesters, 54% responded keep the same number of case studies; 29% said keep some but fewer case studies; only 9% wanted more case studies and 9% said do not have Monday case studies.

Open ended questions invited students to indicate their favorite case study. The responses were well distributed across case studies, with votes across all case studies. Three stood out with a higher number of votes. Those case studies are the Central Pacific Railroad, Bo Jackson's Hip Recovery and Drought and Water Resource Optimization. Each of these case studies is included in Appendix A-C. Note that each case study was introduced with an introductory presentation with background information and possibly short videos on the topic.

Students were also asked to provide suggestions to improve the case studies. Among the 8 responses were suggestions for making more effective use of the TAs to help with the case studies during class, ensuring all engineering disciplines are covered in at least one case study and providing more directions.