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An Analysis of STEM Students' Ability to Interrelate Derivative, Integral, Power Series, and Function Concepts

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Abstract

Power series is a concept that requires knowledge of extensive calculus sub-conceptual knowledge that includes rate of change and antiderivative knowledge and the pedagogical efforts to measure conceptual understanding of STEM students' is recent ([1]-[9].) If and only if (Iff) is one of the pedagogical techniques introduced in [10] to analyze calculus questions and educators are encouraged to use this technique to structure questions. In this work, we utilize iff methodology introduced in [10] and analyze empirical data collected at a university located on the Northeastern side of the United States. The research received Institutional Review Board Approval (IRB) to collect written and interview data from 17 engineering students on two math questions. Written and video recorded information is collected and analyzed qualitatively and quantitatively to understand engineering students' ability to solve the two questions that are related to function, derivative, integral, and power series concepts. The written results indicated that 30% of the participants' ability to form a bridge between derivative, integral, power series and function concepts by answering the two math questions prior to the oral interview and this rate is increased to 45% based on the post interview responses. These results indicated a need for covering calculus questions that support the iff methodology in calculus education of STEM majors for improving their conceptual understanding.

1. Introduction

Power series expansion of functions has an important place in STEM research, education, and applications in real life, therefore it is important to understand STEM students' misconceptions to improve their comprehension of power series. Multiple calculus concepts such as derivative, integral, power series, and exponential function are used as a part of two mathematics research questions for collecting empirical data from university students in this research.

The research data is collected from 17 undergraduate STEM students at a mid-sized university located at the Northeastern side of the United States. The research received Institutional Review Board (IRB) approval, and the participants are compensated for their written and video recorded oral interview responses to the research questions. The requirement for research participation was completion of the second course of a three-course calculus sequence consisting of 12 credits. The participants were majoring in engineering with most of the participants being industrial or mechanical engineers. The gender of the research participants is not collected due to its irrelevance to the scope of the research. The research team consisted of a professor and three undergraduate engineering students. Research participants who decided to take part in the research answered a set of calculus related math questions as a part of a questionnaire. The subjects who completed the written questionnaire are called to participate the video recorded oral interviews to follow up on their written responses. These video-recorded interviews are then analyzed along with the written responses of the participants. The mathematical logic statement "iff (i.e., if and only if)" introduced in [10] is used to analyze the collected data by focusing on the following concepts related to the power series expansion of the exponential function:

- Integrals
- Differentiation
- Series approximation and index terms
- Meaning of infinity in series expansion of functions

In this work we report on the comprehension levels of the research participants on the series concepts and the associated sub-concepts listed above. Next section covers the details of the pedagogical meaning of the *iff* methodology and the analysis of the two math questions. A comprehensive analysis of the qualitative and quantitative information collected for both questions is conducted in the third section. The last section is reserved to report the conclusion of research with the main weakness of the participants observed to be their weakness on responding to the sub-concept questions for deriving the correct solution.

2. Empirical Data Analysis

If and only if (*iff*) is one of the most frequently used mathematical logical statements to prove that the two statements are equivalent and showing one of the logical statements would mean the other logical statement holds. The pedagogical insight of *iff* is similar to its mathematical meaning [10]:

> Comprehensive conceptual understanding requires a successful mental construction of the associated sub-concept solutions that can be observed by an individual's ability to design successful solutions that require application of reverse processes on the same topic; A reverse process applied on a topic in this case is the process that requires the reversion of the original treatment of the topic in a reversed manner. For instance, a person's mental ability to solve a power series question that requires derivative sub-conceptual knowledge indicates the person's ability to mentally construct a solution to solve the power series question that can be considered as the if part of iff pedagogical methodology. Applying the antiderivative to the same power series requires participant's ability to mentally construct a solution with a reverse process of derivative by calculating anti-derivative of the same power series' functions. Another example is the use of function g within a composition function f(g) and calculating its inverse function to retain f.

In the case when participants were able to observe that the derivative of the exponential function's series expansion is same as the series of the exponential function (Q1 stated in the next section), they were expected to apply this knowledge in the second question (Q2 stated in the next section) for which they needed to show that the integral of the exponential function's series expansion is same as the series expansion of the exponential function. In this application, the *if* of *iff* relies on a student's realization of the unchanging nature of the concept (i.e. power series) after the derivative is applied. If the student realizes that the power series is the exponential function, then the student can realize that the derivative of the exponential function is the exponential function itself. This can also be tested by the student through term-by-term calculations of the power series and if the student can calculate these terms, then the result will be a success if the student realizes that the infinite number of terms help with the proof of equality. The *only if* part of *iff* relies on Q2 and the integral of the exponential function to be the same as the exponential function with the addition of a constant term. Some of the students were not able to answer Q2 through calculations and could not realize this fact during the interviews. Only after the interviews are concluded, the interviewer mentioned to them that the power series in the question is the exponential function they realized automatically that the statement is true. Hence, *iff* analysis can be particularly helpful in determining sub-conceptual misunderstandings that take place in a concept and analyzing students' ability to form conceptual bridge between a variety of concepts.

2.1 Power Series & Derivative Knowledge

The responses of the research participants to the following research question and the corresponding *iff* analysis are covered in this section to analyze their basic derivative knowledge applied to one of the most basic Maclaurin series.

Q1. *Is it a true statement to say*

$$\frac{d}{dx}\sum_{n=0}^{\infty}\frac{x^n}{n!}=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

Please explain your answer.

Research participants' conceptual understanding of this research question was analyzed by considering the following:

- Relating the derivative of the exponential function's Maclaurin series to the series expansion of the exponential function.
- Recognizing the change in the index term of the power series.
- Determining the derivative of polynomial functions or exponential function.
- Recognizing the changes in the summation index.

Only 35 % of the participants' answers to the derivative research question were correct with good justification. This percentage increased to 50% during oral interviews. The rest of this subsection is devoted to the responses to Q1 and the details of students' misconceptions.
One of the participants who answered the research question mathematically correct with justification based on the exponential function is shown in Figure 1 below. This is one of the rare observations of a participant to determine a solution to both research problems. In such a case the participant was still required to calculate derivative and anti-derivative of the functions associated with the power series expansion of the exponential function.

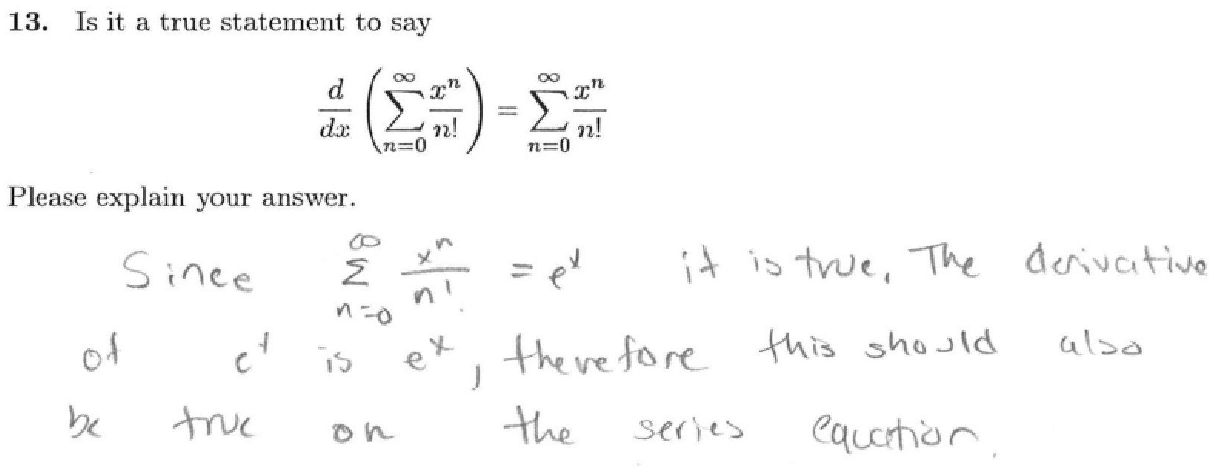


Figure 1. Response of RP 10 to the derivative of series question

PR 3 with the response displayed in Figure 2 below correctly answered the research question by deriving the result by both calculating the derivatives of the series terms and the exponential function itself.

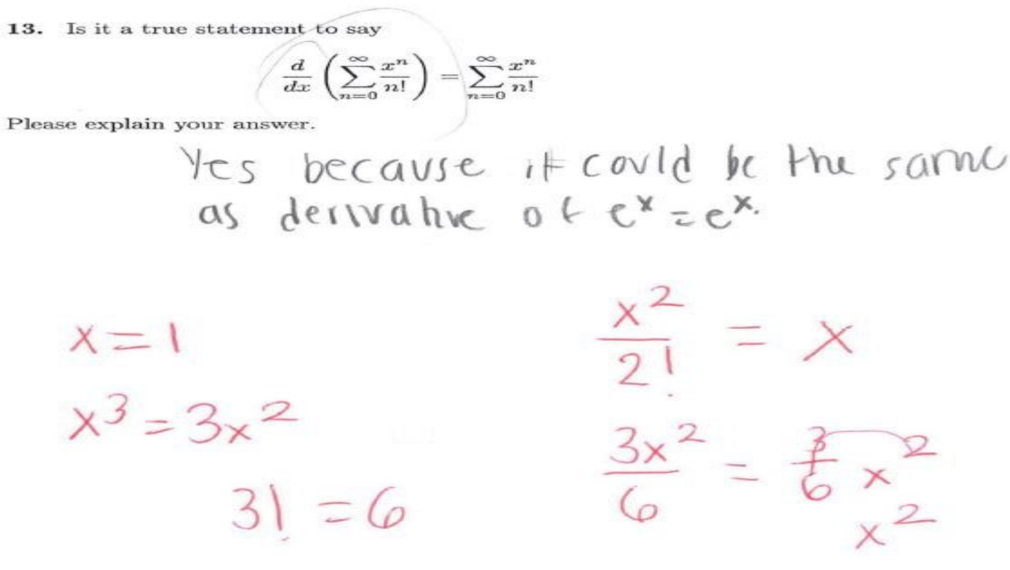
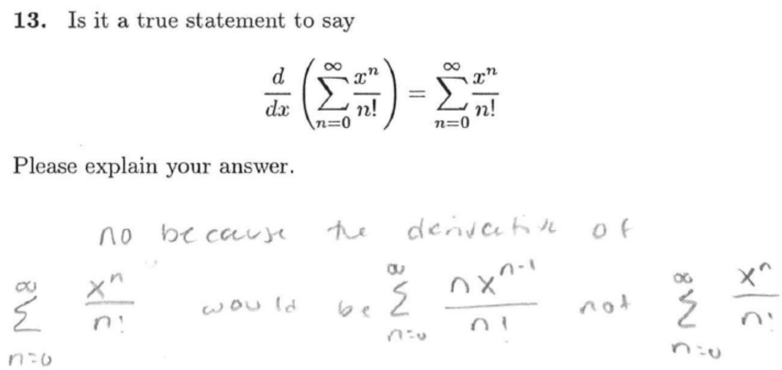


Figure 2. Derivative calculations of RP 3 to the derivative of series question.

One of the participants, RP 5, had the correct idea to respond to the research question while the summation index was the main reason for confusion during the response. This participant successfully calculated the derivative of the power series and had confusion on the connection between the index term n-1 and n. This participant reasoned the difference in the index for the equality to not hold true.



13. Is it a true statement to say

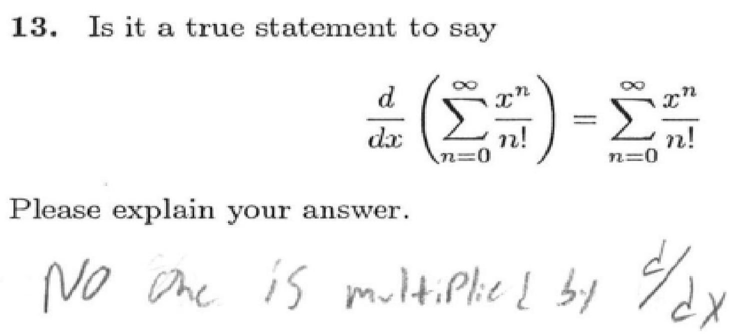
$$\frac{d}{dx}\left(\sum_{n=0}^{\infty}\frac{x^n}{n!}\right)=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

Please explain your answer.

no because the derivative of $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ would be $\sum_{n=0}^{\infty}\frac{nx^{n-1}}{n!}$ not $\sum_{n=0}^{\infty}\frac{x^n}{n!}$

Figure 3. Response of RP 5 with emphasis on the index of the series after calculating the derivative of the series of the exponential function.

One of the participants' confusions was based on the differential operator's application to the power series. Participant 8 with the response displayed in Figure 4 used the term "multiplied" for the differential operator applied to the power series.



13. Is it a true statement to say

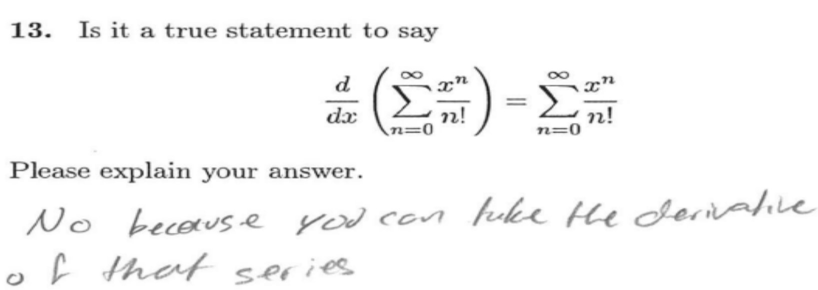
$$\frac{d}{dx}\left(\sum_{n=0}^{\infty}\frac{x^n}{n!}\right)=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

Please explain your answer.

NO one is multiplied by d/dx

Figure 4. Response of RP 8 with confusion on differential operator's application to the series.

Another participant's confusion was based on the differentiation of the series. RP 12 stated that "you can take the derivative of that series" and this is the reason for the derivative of the series to be not equal to itself.



13. Is it a true statement to say

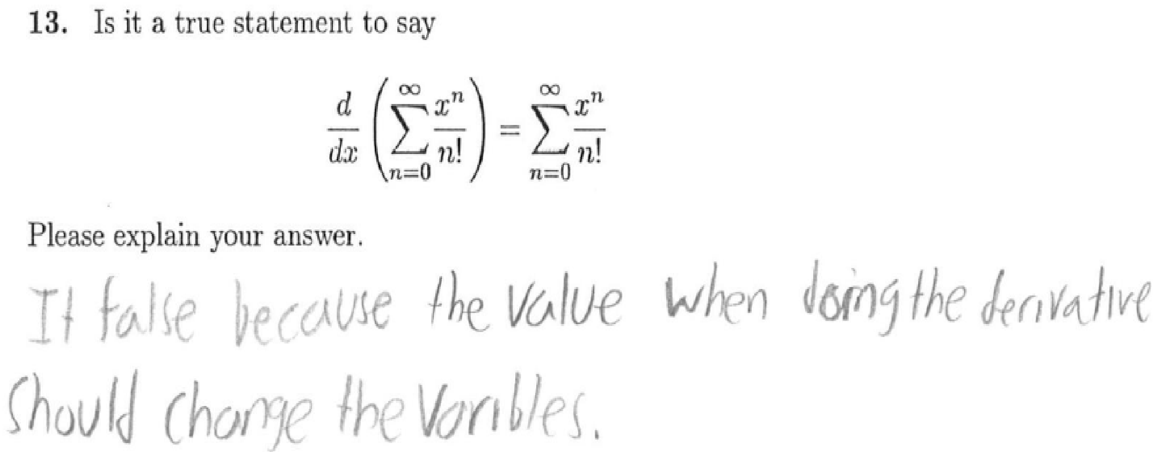
$$\frac{d}{dx}\left(\sum_{n=0}^{\infty}\frac{x^n}{n!}\right)=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

Please explain your answer.

No because you can take the derivative of that series

Figure 5. Confusion of RP 12 was based on differentiating the power series.

Participant 14 indicated that the appearance of the series after taking the derivative doesn't change therefore it is not possible for the derivative of the series to be equal to the series itself. This justification was neither depending on the derivative of the exponential function nor the actual term-by-term calculations of the series.



13. Is it a true statement to say

$$\frac{d}{dx}\left(\sum_{n=0}^{\infty}\frac{x^n}{n!}\right)=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

Please explain your answer.

It false because the value when doing the derivative should change the Varibles.

Figure 6. RP 14 claimed that the variable cannot change while differentiating power series.

In conclusion, the main reasons for 65% of the participants' wrong responses were centered around the following concepts:

- Application of the differential operator to a power series
- Change in the index of the series
- Challenges faced regarding taking the derivative of basic polynomial functions.

The participants were asked to go through the calculations step by step during the video recorded interviews. 15% of all the participants changed their answers and derived the correct one during the interview.

2.2 Power Series & Integral Knowledge

The responses of the research participants to the following research question and the corresponding *iff* analysis are displayed in this section to analyze their basic integral knowledge applied to one of the most basic Maclaurin series.

Q2. *Is it a true statement to say*

$$\int\sum_{n=0}^{\infty}\frac{x^n}{n!}dx=\sum_{n=0}^{\infty}\frac{x^n}{n!}+constant$$

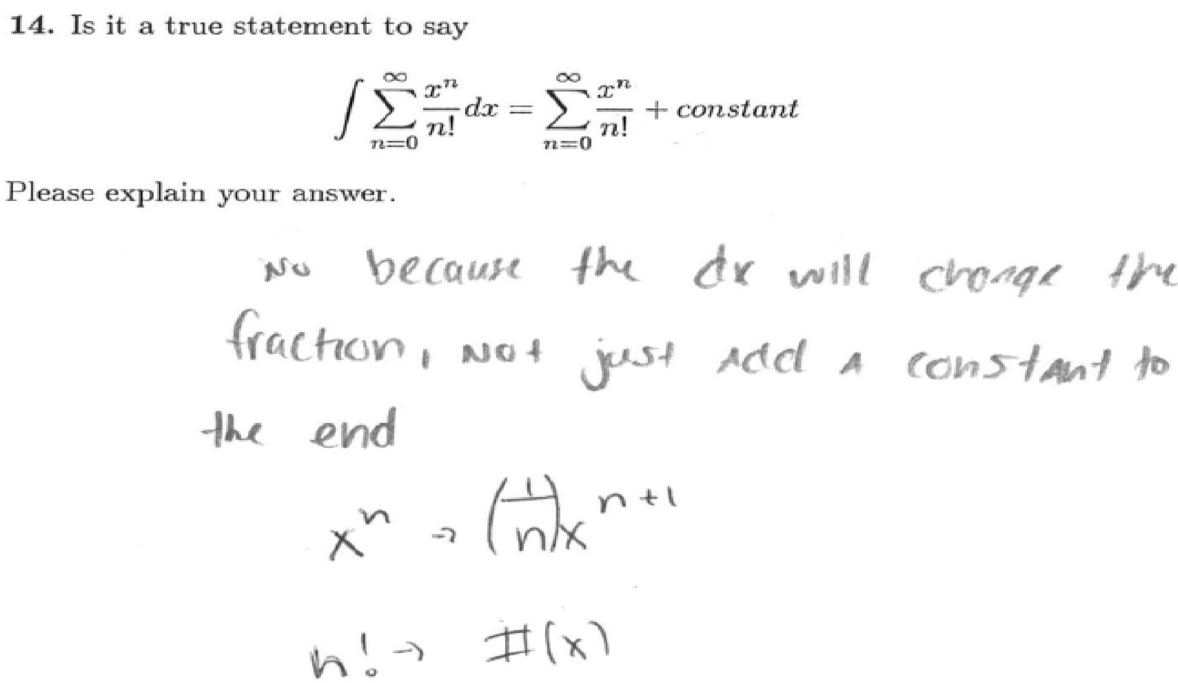
Please explain your answer.

Research participants' conceptual understanding of this research question was analyzed by considering the following:

- Ability to relate the integral of the exponential function's Maclaurin series to the series expansion of the exponential function.
- Recognizing the change in the index term of the power series.
- Determining the integral of polynomial functions or exponential functions.
- Recognizing the changes in the summation index.

Only 45 % of all the participants' answers to the derivative research question were correct with good justification. This percentage increased to 60% during the oral interviews for following up on the written responses. The rest of this subsection is devoted to the responses to Q2 and the details of students' misconceptions.

One of the responses to the research question by participant 1 displayed in Figure 7 below was focusing on the differential term of the integral; the reasoning for equality to not hold was explained by the participant as "the dx will change the fraction, not just add a constant to the end."


14. Is it a true statement to say

$$\int \sum_{n=0}^{\infty} \frac{x^n}{n!} dx = \sum_{n=0}^{\infty} \frac{x^n}{n!} + constant$$

Please explain your answer.

No because the dx will change the fraction, not just add a constant to the end

$x^n \to \left(\frac{1}{n}\right) x^{n+1}$

$n! \to \#(x)$

Figure 7. Response of participant 1 with reasoning depending on the differential term dx.

Prior to the interview, participant 2 stated that equality doesn't hold based on a method of integration that the participant didn't know about; exponential function was used as the reason for the equality to hold during the interview.

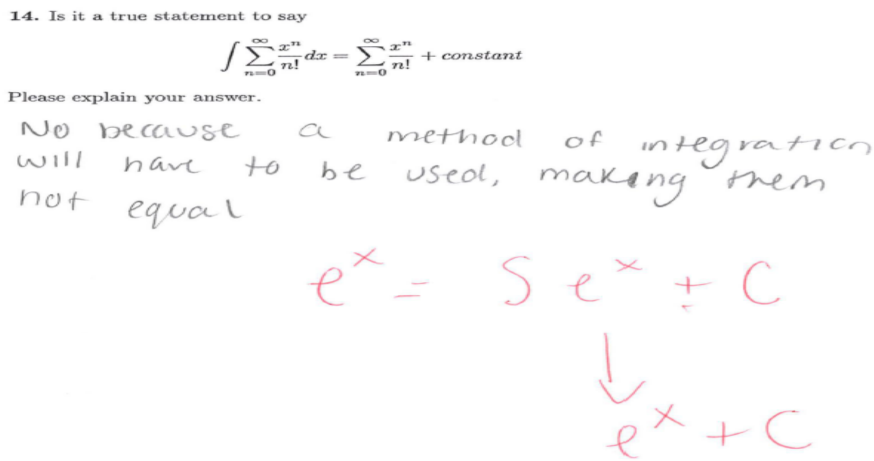


Figure 8. Responses of participant 2 during the written (black ink) and oral (red ink) interviews.

Participant 3 had the correct written response in Figure 9 to the derivative question by using the exponential function, however, couldn't calculate the integral of the cubic function.

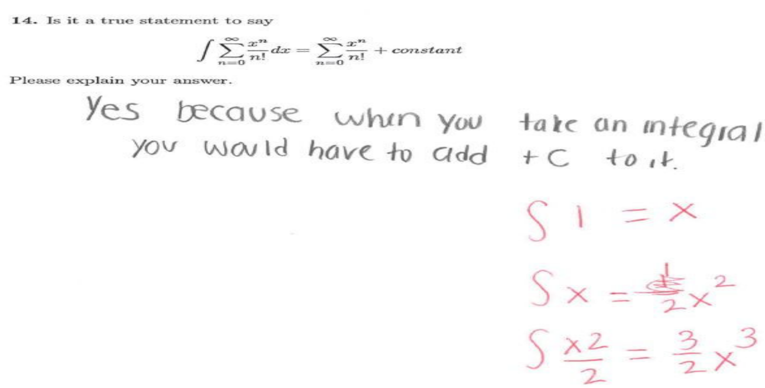


Figure 9. Response of participant 3 with during the written and oral interviews.

Participant 5 based the response to the question on the wrong integral calculation of the general polynomial term in Figure 10.

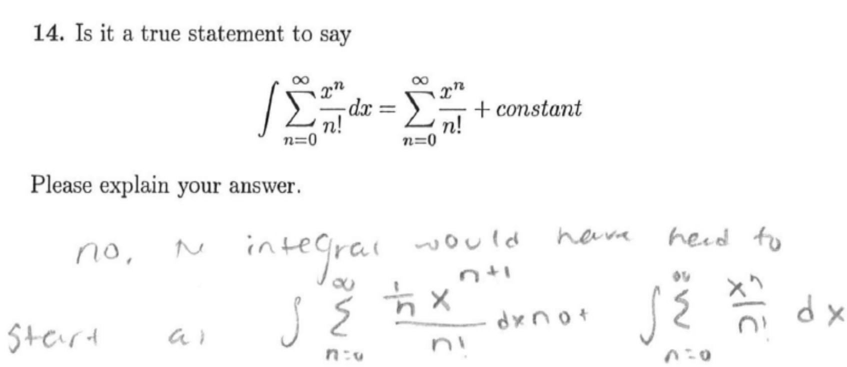


Figure 10. Response of participant 5 with a mistake made on calculating integral of the series term.

Participant 16 (with the response in Figure 11) claimed that the integral should change the series and therefore equality doesn't hold.

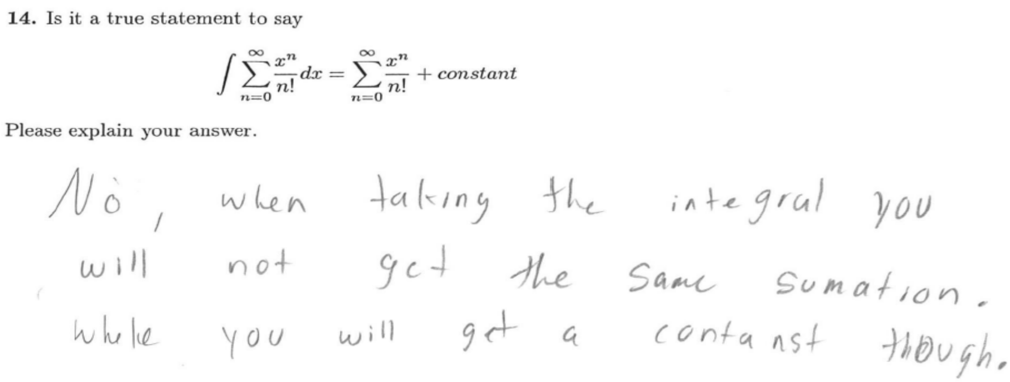


Figure 11. Response of participant 16 claiming that the series should change as a result of calculating the integral.

Participant 19 made a mistake during the calculations of the integral as displayed in Figure 12 and just responded to the question "maybe".

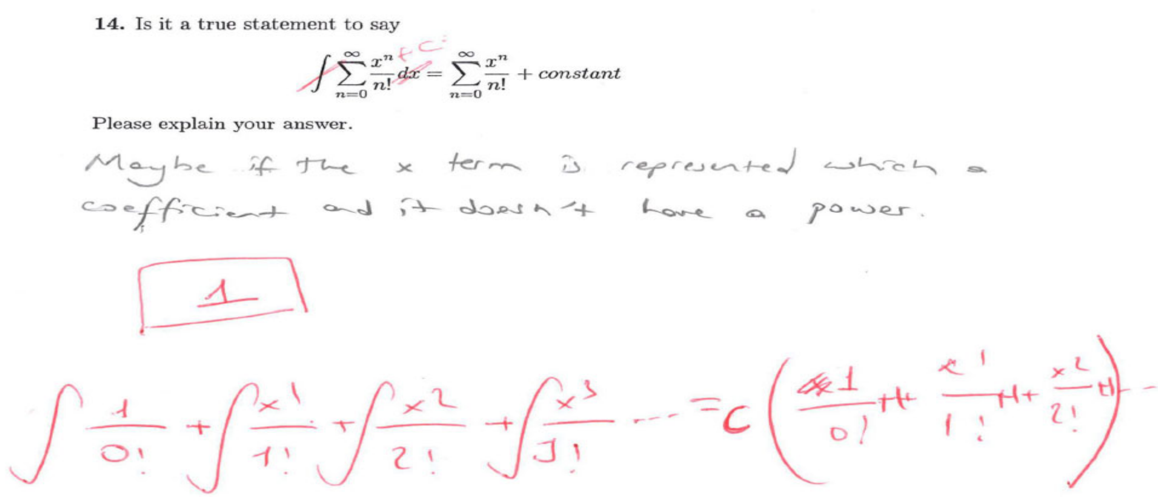


Figure 12. Doubtful response of participant 19 to the integral question by stating "maybe".

In conclusion, the main reasons for 55% of the participants' mistakes while responding to the integral research question were centered around the following:

- Application of the integral operator to the polynomial terms of the power series.
- The series cannot remain the same when its' integral is calculated.
- The constant term needs to change when the integral of a series is calculated.

The participants were asked to go through the calculations step by step during the video recorded interviews. Only 15% of all the participants changed their responses to the research question during the interview (similar to the results attained in Section 3) that increased the correct responses to

60%. Next section will contain a comprehensive analysis of the responses to both math questions and the *iff* analysis of the results along with the future work plans.

3. Analysis of Qualitative and Quantitative Responses to the Math Questions

The collected qualitative and quantitative analysis of the data is conducted by the PI. The quantitative analysis is based on the statistical categories of the responses given by using the written information provided by the research participants. The qualitative analysis relied on the specific key words used by the research participants as well as the match between their pre-interview responses and post-interview responses. The qualitative analysis relied on the students' ability to expand their written responses and explain their written responses with correct justifications.

Overall analysis of the responses to the two math questions indicated a higher success rate of participants to the integral question during the oral interviews; the number of correct responses to both math questions increased by 15% during the video recorded interviews. Analysis of the written questionnaire responses indicated 30% of all participants' ability to satisfy *iff* successfully by applying the integral and derivative operators on the series correctly. This rate was increased to 45% as a result of post interview analysis of the data. These participants recognized the fact that the series doesn't change regardless of the operator applied. The common mistakes made by the participants during their responses to both math questions were based on the following concepts:

- Difficulty of applying the integral and derivative operators to a power series: Participants couldn't mentally construct how derivative and integral operators can be applied to a power series, and they had hard time with the associated calculations.
- The power series needs to change as a result of applying an operator to it: Some of the participants thought that the power series should change after an integral or derivative operator applied to it without any prior calculations. Participants' initial responses were biased due to this initial biased thinking and their initial reaction to the question's appearance.
- Impact of the constant term: Some of the participants tried to justify their responses based on the constant term existing in the equations. Neither one of the two equations were mentioned to hold by these participants because of the impact of the constant term.

4. Conclusion & Future Work

In this work we emphasized STEM students' ability to apply the derivative and integral operators on power series expansion of the exponential function. Pedagogical research in this area of interest is recent and limited [10]. A well-established understanding of a person's power series concept requires to measure the person's ability to apply derivative and integral operators on the power series; These applications are particularly important for STEM students to comprehend their use in various areas of STEM including but not limited to noise differentiation of wave lengths and observing the area between the wave-length and input information by integrating the function as a part of the Fourier analysis [10].

The data analyzed in this work is collected from 17 students at a university located on the Northeastern side of the United States and the data collection procedure followed an IRB procedure. The participating students were compensated for their written and oral interview responses to two math questions. The video recorded oral interviews were conducted for further understanding of participants' written responses by the leading researcher. The research team consisted of four investigators: three undergraduate students and a professor of engineering. The quantitative analysis of the data was based on the statistical analysis of the collected data while the qualitative data consisted of the written and the oral interview responses.

There were 7 participants responding to the derivative of the series question correctly. The main reasons for the rest of the participants' responses to the derivative question were centered around the following:

- Application of the differential operator to a power series
- Change in the index of the series
- Challenges faced regarding taking the derivative of basic polynomial functions

There were 9 participants who responded to the integral of the series question correctly. The main reasons for the remaining 55% of the participants' mistakes were centered around the following:

- Application of the integral operator to the polynomial terms of the power series.
- The series needs to change when its integral is calculated.
- The constant term needs to change when the integral of a series is calculated.

The participants were asked to go through the calculations step by step during the video recorded interviews. Only 15% of all the participants corrected their responses to both math questions during the interviews.

Given that systematic or structural way to interpret student's conceptual understanding is up to the research direction and content that can be followed, the application of iff can change depending on the research objectives. Analysis of the written questionnaire responses indicated 30% of all participants' ability to satisfy *iff* conditions successfully by applying the integral and derivative operators on the series of the exponential function. This rate was increased to 45% as a result of post interview analysis of the data; These participants recognized the fact that the series doesn't change regardless of the operator applied. The common mistakes made by the participants during their responses to both math questions included the following:

- The idea of applying the integral and derivative operators to the power series.
- The series needs to change as a result of applying an operator to it.
- Impact of the constant term on the calculations.

The outcomes of the *iff* method explained in this work can help educators to do the following:

- Structuring exam and assignment questions that can help them to observe the learners' conceptual understanding.
- Design examples to be provided to STEM students in the classroom or as additional materials that follow the logic of *iff* to explain the concepts better.

The *iff* methodology can help the pedagogical researchers to investigate the depth of learners' conceptual comprehension of the STEM topics therefore they are recommended to test this method for measuring its effectiveness in applications. The application of two conceptual processes such as derivative and anti-derivative by the learners may stimulate their mental constructions of the concepts and help them to understand concepts. Given the building blocks of concepts existing in mathematics, successful comprehension of mathematical concepts has a critical role in STEM education. As a part of the future work, the *iff* technique can be used for furthermore advancement of assignment and exam question writing as well as investigating other areas of its utilization in research. Repetition of the same mistakes could be the indicators of the ways of how STEM students' mind work for responding to mathematics-based questions [10].

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