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BOARD # 178: Engaging Students in Mechanics of Materials Education: Simple Demos to Understand Ultimate Tensile Strength and Angle of Twist

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Engaging Students in Mechanics of Materials Education: Simple Demos to Understand Ultimate Tensile Strength and Angle of Twist

This paper presents three innovative, low-cost demonstrations aimed at enhancing student engagement and understanding in introductory mechanics of materials courses. The first demonstration introduces a novel method to illustrate Ultimate Tensile Strength without the need for specialized equipment. By using a high-precision force gauge and copper wires of varying thicknesses, students measure and predict the breaking force required for each wire. The second and third demonstrations focus on the concept of the angle of twist, particularly relevant in components such as gears and shafts. Through hands-on experimentation with geared shafts, students can visualize and grasp the fundamental principles behind this key concept, fostering a deeper appreciation of its significance in mechanical engineering applications. The three demonstrations not only deepen students' understanding of material behavior under loading conditions but also promote active learning and critical thinking in the field of mechanics of materials.

Keywords: Mechanics of Materials, Ultimate Tensile Strength, Angle of Twist, Geared Shafts, Hands-on Demonstration

Introduction

Mechanics of Materials is a sophomore-level course that typically follows introductory courses in statics and physics. It is a core requirement for mechanical engineering and civil engineering majors and is often required for students in related disciplines such as aerospace engineering, biomedical engineering, and industrial engineering. Depending on the institution, the course may have an associated laboratory co-requisite where students conduct tensile tests, torsion tests, and beam deflection experiments to reinforce theoretical concepts. Key topics include stress and strain analysis, axial loading, torsion, shear and bending stresses, deflection of beams, combined loading, stress transformation, and failure criteria.

Two of the key fundamental concepts that students are introduced to in this course are Ultimate Tensile Strength (UTS) and the angle of twist, which play crucial roles in understanding how materials respond to forces. UTS, for instance, represents the maximum stress a material can endure before it breaks, providing valuable information about the material's strength and reliability under tension. Similarly, the angle of twist describes the rotational displacement that occurs when a torque is applied, shedding light on a material's flexibility and its ability to withstand twisting forces without failing. Together, these concepts reveal essential insights into material behavior under stress, which is critical for engineering applications where strength, and deformation tolerance are paramount.

However, for students new to mechanics of materials, grasping these ideas can be challenging, particularly if they rely solely on theoretical explanations and textbook examples. Without practical, hands-on experiences, these ideas can feel abstract and disconnected from real-world applications.

Hands-on experiments help bridge this gap by allowing students to observe and measure these behaviors firsthand, making the material properties more tangible. Unfortunately, many traditional demonstrations of UTS and angle of twist require specialized equipment, such as tensile testing machines and torque measurement devices, which can be costly and are often limited to advanced labs. This lack of accessible equipment can restrict students' opportunities to directly engage with these core principles early in their education, potentially impacting their confidence and comprehension.

To address this challenge, innovative, low-cost demonstrations provide a practical solution that makes the material more tangible without requiring high-end tools. By utilizing simple yet effective setups, such as high-precision force gauges and wires of varying thickness, students can explore Ultimate Tensile Strength (UTS). Additionally, using a torque wrench and an arrangement of gears mounted on shafts enables students to measure the angle of twist. These hands-on experiments foster an engaging, accessible learning environment that is directly applicable to their coursework.

Literature Review

Mechanics of materials courses are widely recognized as challenging for both instructors and students due to the highly analytical and theoretical nature of the content. According to Wang et al. [1], this difficulty arises from the complex concepts involved and the disconnect between theoretical material behavior and students' practical experiences.

In response to these challenges, several studies have highlighted the positive impact of hands-on demonstrations and experiments on student engagement and comprehension. Crone [2] describes how tabletop demonstrations and classroom experiments were incorporated into an introductory mechanics of materials course. These teaching tools received highly positive feedback from students, with many commenting on their educational value. One student remarked that the demonstrations were particularly helpful because they allowed students to "see what's happening," enhancing their understanding of abstract concepts.

Sullivan et al. [3] further support the use of experiential learning, demonstrating that combining theoretical and mathematical approaches with hands-on physical experiments can significantly improve students' understanding and retention of engineering concepts. In their study, students who participated in experiments using a beam testing system (BTS) were able to verify their analytical predictions, leading to a deeper understanding of the material. The impact on performance was found to be positive and the students who engaged in experiential learning expressed higher satisfaction with the course compared to those who only performed analysis, indicating the importance of active learning in enhancing the student experience.

Montoya [4] emphasizes the challenge of teaching fundamental concepts like stress and strain in mechanics of materials, as students often struggle to relate these theoretical concepts to their everyday experiences with objects. The study suggests that the use of visualization, experimentation, and discussion in the classroom can help bridge this gap and improve student understanding of complex material behaviors.

These studies underscore the importance of incorporating hands-on demonstrations and active learning strategies into the mechanics of materials curriculum. By combining theoretical instruction with practical experimentation, educators can enhance student comprehension, engagement, and satisfaction in this challenging subject. This paper builds on these findings by introducing three innovative, low-cost demonstrations aimed at deepening students' understanding of key concepts such as Ultimate Tensile Strength (UTS) and the angle of twist. These demonstrations not only offer students a hands-on learning experience but also promote active learning and critical thinking, further supporting the positive outcomes associated with experiential learning in mechanics of materials.

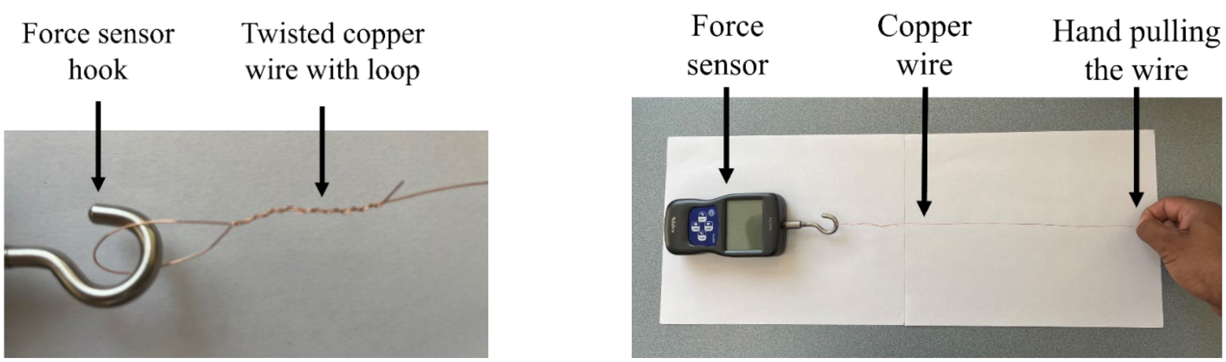
Demonstration #1

In this demonstration, a force gauge and four copper wires are used to illustrate the concept of UTS. The wires, with diameters of 0.010", 0.012", 0.016", and 0.020", provide a range of thicknesses for comparing breaking forces. Details on the properties, specifications, and sources of the force gauge and wires are available in Table 4 (See Appendix A).

The demonstration took approximately 30 minutes to complete. The initial introduction lasted about 5 minutes, followed by 15 minutes for students to perform analytical calculations. The final 10 minutes were dedicated to gathering experimental data and engaging in a discussion. Demo 1 took place on the second day of class, effectively generating excitement among students. They were particularly impressed by how closely their experimental results aligned with their predicted values. This immediate validation fostered enthusiasm and a sense of accomplishment, solidifying their understanding of ultimate tensile strength (UTS) early in the course.

Experimental Procedure

To begin, the instructor secures a 0.020” wire to the force gauge by twisting it approximately 15-20 times, as shown in Figure 1(a). Once secured, the force gauge is held in place with one hand and the wire is pulled with the other hand until it breaks, as depicted in Figure 1(b).



(a) (b)

Figure 1: (a) Securing the wire to the force gauge, (b) Wire pull setup.

The force gauge is set to peak mode, which records the maximum force required to break the wire, allowing students to measure and observe the breaking force with accuracy. The wire snaps at a recorded breaking force of 12.8 lbf. This initial break serves as a reference point, allowing students to observe the force required to reach the material’s UTS. With this known breaking force, students are then challenged to predict the breaking forces for three additional wires with diameters of 0.010", 0.012", and 0.016".

To guide their predictions, students first calculate the UTS of the copper material using Equation 1, based on the breaking force and cross-sectional area of the 0.020" wire:

$$\text{UTS} = \frac{\text{Breaking Force}}{\text{Cross}-\text{sectional Area}} = \frac{12.8\ \text{lbf}}{\pi * (0.020/2)^2} = 40{,}744\ \text{psi} \qquad \text{Equation 1}$$

With the UTS established, students use Equation 2 to predict the breaking force for each remaining wire, by multiplying the UTS calculated in Equation 1 by the new cross-sectional area:

$$\text{Breaking Force} = \text{UTS} * \text{Cross} - \text{sectional Area} \qquad \text{Equation 2}$$

Table 1 presents both the predicted and experimentally observed breaking forces for each wire diameter, illustrating how well the calculated UTS predicts real-world behavior. These results are from one random run, and every single run conducted showed experimental values within ±10% of the predicted values, which is within the range of expected uncertainty. Although no formal statistical analysis was conducted to assess repeatability, the consistency across multiple runs suggests that the observed variability falls within acceptable bounds for practical applications.

Table 1: Experimental breaking force and calculated predicted breaking force.

Wire Diameter (inch)	Predicted Breaking Force Calculation (lbf)	Predicted Breaking Force (lbf)	Experimental Breaking Force (lbf)
0.010	$40{,}744 \times \pi \left(\frac{0.010}{2}\right)^2$	3.2	3
0.012	$40{,}744 \times \pi \left(\frac{0.012}{2}\right)^2$	4.6	5
0.016	$40{,}744 \times \pi \left(\frac{0.016}{2}\right)^2$	8.2	7.7

Discussion

The results demonstrate that the predicted and experimental breaking forces align closely within the range of allowable uncertainty, effectively validating the calculated UTS for the material. This correlation reinforces students' grasp of the principles governing material strength and behavior under axial loads. By bridging theoretical calculations with tangible outcomes, this hands-on approach provides a robust learning experience, allowing students to observe firsthand the predictive accuracy of UTS and deepen their understanding of real-world applications in mechanics of materials. Furthermore, this experiment presents an excellent opportunity for not only understanding material behavior but also for engaging in critical discussions about measurement uncertainty and error propagation. This dual focus enhances the overall learning experience, providing students with a comprehensive understanding of both the complexities of materials science and the intricacies of accurate measurement in experimental settings.

Demonstration #2

In this demonstration students explore the angle of twist under torsional stress using a torque wrench and a gear-mounted shaft setup (Figure 2). The setup allows students to visually grasp how rotational deformation occurs when torque is applied to one end of a shaft. Detailed specifications of the equipment are provided in

Table 5 (See Appendix A).

For demonstrations #2 and #3, the class of 24 students was divided into six groups of four. Each group took turns making their measurements. A total of two hours of lab time was allocated for students to complete their measurements and perform calculations. Following the lab work, the instructor conducted a 45-minute lecture during the next class session to compare the groups' measurements, review the solutions, and facilitate a discussion.

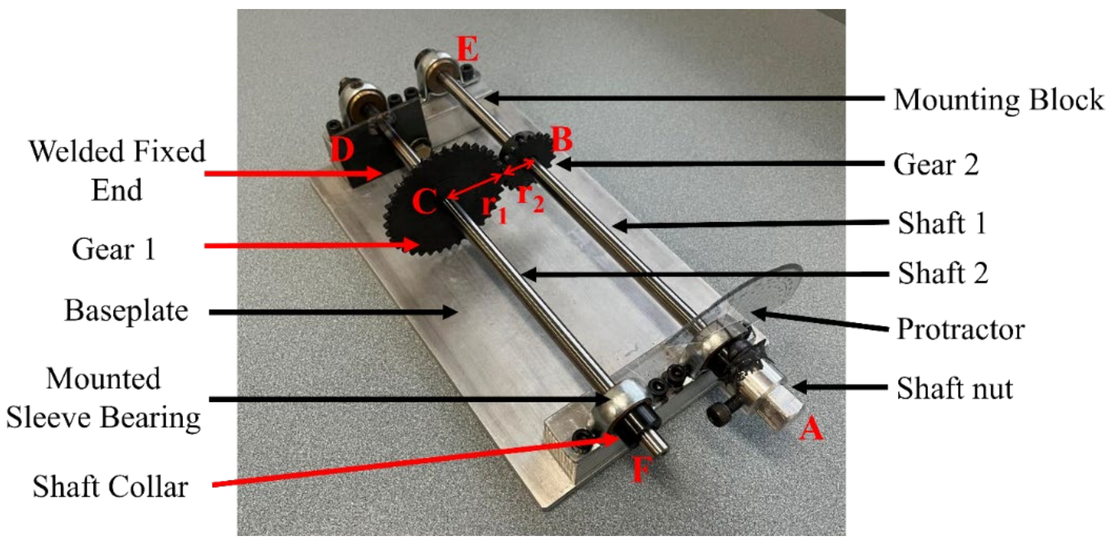


Figure 2: Demonstration #2 setup.

Experimental Procedure

Students start by measuring essential parameters—shaft diameters, lengths, and gear radii—and, after instructor verification, recording these values in Table 2 (See Appendix A). Using these measurements and a 45 lb-in torque applied at Point A, they calculate the torque and torsional stresses in each shaft segment (AB, BE, CD, and CF), noting variations based on each segment's length, diameter, and material properties. They then calculate the theoretical angle of twist and compare it to experimental results, observing how torque induces a progressive twist along the shaft system. All values are documented in Table 2 (See Appendix A). The experimental values varied within ±10% across multiple runs conducted by different students, indicating acceptable repeatability despite the lack of formal statistical analysis. Finally, students assess the accuracy of their calculations, considering factors that may cause real-world deviations.

Addressing Common Student Challenges in Measurements and Calculations

When students start taking measurements and making calculations, they often have questions or encounter challenges in the following areas:

Clarifying the Length of Shaft Segments: Students may ask if they need to measure the shaft segment (e.g., AB) to the precise location where the torque is applied, such as at the nut, rather than just the length between visible points on the shaft. It's important to emphasize that measuring to the exact point of torque application ensures accurate angle of twist calculations.

Determining the Radius for Torque Calculations: Some students may be unsure whether to measure the radius to the end of the gear teeth or to the actual point of contact between gears. Remind them that the effective radius is the measured pitch circle diameter, as this is the radius used in torque transmission calculations.

Understanding Torque and Stress in Free Ends (BE and CF): Students sometimes question why torque and stress are zero in sections BE and CF. This can be a useful opportunity to explain that since these ends are free, there's no applied torque or reaction torque, making the stress in these sections negligible.

Using Gear Radii vs. Shaft Radii for Gear Ratios: Students may ask whether to use the shaft or the gear radius when calculating gear ratios. Clarify that only the radius of the gears should be used for calculating gear ratios, as the gears themselves determine the torque and speed changes between the input and output.

Calculating Stress in the Shaft: When calculating the torsional stress, some students might mistakenly use the radius of the gears rather than the radius of the shaft to calculate the polar moment of inertia. Remind them that the stress calculation is based on the shaft radius, as this is where the torque induces internal stress.

Understanding the Gear Ratio's Effect on Torque and Angle of Twist: Some students may be confused about why the larger gear carries higher torque while the smaller gear has a greater angle of twist. This is an opportunity to explain the inverse relationship: the gear with the larger radius transmits more torque, while the smaller radius results in a proportionally larger twist due to the same applied torque.

Clarifying the Angle of Twist in Sections BE and CF: Students often find it challenging to understand why sections BE and CF twist the same amount as the gears at points B and C. It is important to explain that in a continuous shaft system, the boundary conditions dictate that free ends cannot rotate independently from connected ends. Consequently, if the gear at point B twists by a certain angle, the connected section BE must also twist by the same angle, as they are part of the same rotational system.

It's also important to emphasize that torque should be calculated from the input end, while the angle of twist must be measured from the fixed end to ensure accurate calculations.

Addressing these questions thoroughly helps students gain a deeper understanding of the relationships between torque, stress, and angle of twist, and ensures they approach their measurements and calculations with a clearer sense of purpose.

Discrepancies Between Calculated and Measured Values of Angle of Twist

In the context of the angle of twist demonstration, students may observe discrepancies between the calculated and measured values of the angle of twist. Understanding the reasons behind these differences is crucial for developing accurate measurement techniques and enhancing students' critical thinking skills in engineering. Inaccuracies in measuring the angle of twist can stem from factors such as:

> **Unwanted Deflections:** The lightweight nature of the components in the current setup and inadequate support for the shaft leads to inaccuracies in measurements. Due to lack of necessary rigidity under applied torque, the shaft experiences undesired bending and lift which distorts the angle of twist, resulting in measurement errors that deviate from the theoretical values. Using heavier-duty bearings and components and adding a central bearing to the shaft would create a more stable foundation, minimizing unwanted movements.
>
> **Quality of Measuring instruments:** Measurement inaccuracies may result from using a low-quality torque wrench. Upgrading to a higher-precision torque wrench designed for low torque applications would improve accuracy, providing more reliable and consistent torque application throughout the experiment. Furthermore, measuring the angle of twist in the current setup can be subjective and challenging, as it relies on visual estimation of the wrench arm's movement relative to a protractor. This method introduces potential for human error and inconsistencies in measurement. Adopting a more objective and precise measurement technique, such as an angle protractor, would eliminate the subjectivity and enhance the accuracy of the readings.
>
> **Measurement Errors:** Accurate measurement of shaft lengths and the radius to the point of contact between gears is crucial, as any discrepancies can affect torque calculations and the angle of twist.
>
> **Assumptions in Calculations:** The calculations assume the shaft material is homogeneous and isotropic, meaning it has uniform properties throughout. However, real materials may exhibit variations in density, yield strength, or other mechanical properties, resulting in discrepancies in the observed angle of twist. Additionally, simplified geometric models used in calculations may overlook complexities in the system, such as welds or joints, which can alter the torsional response and further contribute to differences between calculated and measured values.
>
> **Material Behavior Under Load:** Calculations for angle of twist often assume linear elasticity; however, some materials may exhibit non-linear elastic behavior under torsional loads, leading to deviations from predicted values.
>
> **Accumulation of Errors:** Cumulative measurement errors can arise from each individual measurement (such as lengths and diameters), and when these errors accumulate, they can significantly influence the final calculated angle of twist. Additionally, the uncertainty associated with each measurement affects the overall accuracy of the calculations, making it

crucial to understand how these uncertainties propagate through the calculations to accurately interpret the results.

Recognizing and addressing discrepancies between calculated and measured values of the angle of twist is crucial for student learning. This discussion promotes critical evaluation of measurement techniques, consideration of calculation assumptions, and reflection on the complexities of material behavior.

Discussion and Future Design Considerations

While this low-cost demonstration does not perfectly correlate theory with experiment, it remains valuable for enhancing student understanding. Its simplicity and accessibility make it a useful educational tool that fosters active learning, encourages investigation, and enhances student comprehension of complex engineering concepts in several keyways.

First, it provides a tangible and visual representation of how torque and stress behave in a real-world system, allowing students to observe these concepts in action. Although discrepancies between theoretical and experimental results may arise due to the limitations of the setup, students can still gain hands-on experience in measuring and analyzing torsional effects, which deepens their conceptual grasp of material behavior and engineering principles.

Moreover, the demonstration encourages critical thinking as students investigate the causes behind the differences between theoretical predictions and experimental observations. This encourages them to explore factors such as material properties, measurement errors, and system design—critical skills for future engineers. By engaging with this demonstration, students not only reinforce their understanding of fundamental concepts but also develop problem-solving skills by identifying potential sources of error and considering improvements, such as component upgrades or more precise measurement techniques. These skills are just as important as the specific outcomes of the demonstration itself.

To enhance the effectiveness of the demonstration and achieve a significantly better correlation between theory and experiment, several improvements should be made. First, using heavier-duty components will enhance stability and reduce unwanted movement. Upgrading to a high-quality torque wrench and using an angle protractor will also improve the accuracy of measurements, leading to more reliable results. Currently, the gears and shaft nut are secured using set screws with flat-ground surfaces to minimize slippage. However, welding these components would provide better resistance to higher torque. The shaft was previously secured using a set screw through the bearing, but this method led to excessive stress and a crack in the bearing. To improve stability, the fixed end of the shaft (Point D) should be secured to the mounting block using a weld joint. These changes will enhance the experimental setup, improving its ability to correlate theoretical calculations with real-world observations.

Demonstration 3

This demonstration closely mirrors Demonstration 2, with the key distinction being the use of three shafts instead of two. Using three shafts instead of two enhances students' understanding of rotational mechanics in a more complex system. The apparatus is depicted in Figure 3. Detailed specifications of the equipment are provided in **Error! Reference source not found.** (See Appendix A).

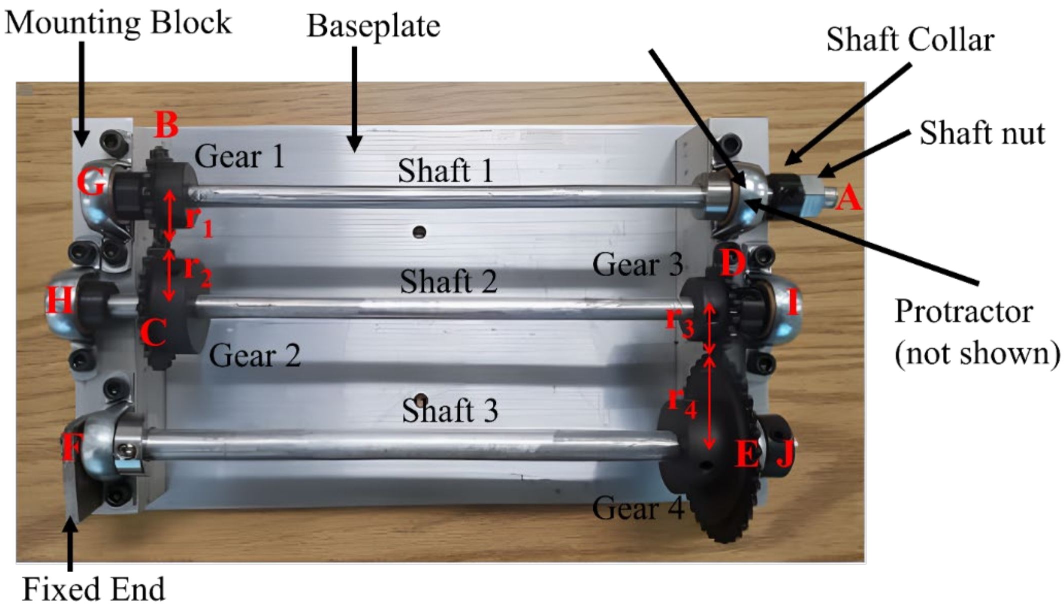


Figure 3: Demonstration #3 setup.

Experimental Procedure

Students start by measuring essential parameters—shaft diameters, lengths, and gear radii—and, after instructor verification, recording these values in

Table *3* (See Appendix A). Using these measurements and a 45 lb-in torque applied at Point A, they calculate the torque and torsional stresses in each shaft segment (AB, BG, CD, CH, DI, EF, and EJ), noting variations based on each segment's length, diameter, and material properties. They then calculate the theoretical angle of twist and compare it to experimental results, observing how torque induces a progressive twist along the shaft system. All values are documented in

Table *3* (See Appendix A). The experimental values varied within ±10% across multiple runs conducted by different students, indicating acceptable repeatability despite the lack of formal statistical analysis. Finally, students assess the accuracy of their calculations, considering factors that may cause real-world deviations.

Discussion

In addition to the student challenges related to measurements and calculations identified in Demonstration 2, this setup introduces a unique source of confusion. Specifically, students may have difficulty understanding the torque distribution on the shaft CD, which connects the two gears. One challenge is determining which gear to use when calculating the torque in this shaft and identifying the appropriate gear ratio for transferring torques between the shafts. Since torque flows from the input end to the output end through the gears, it is crucial to select the correct gear for accurate calculations and to emphasize that the torque in the shaft remains consistent. The reasons for discrepancies between the calculated and experimental values of the angle of twist in this demonstration are similar to those in Demonstration 2. As for design considerations, the same recommendations from Demonstration 2 apply to this setup as well.

Conclusion

The demonstrations presented in this paper offer a valuable approach to engaging students in mechanics of materials education, specifically in understanding Ultimate Tensile Strength (UTS), angle of twist, and torque transfer in geared systems. Addressing measurement uncertainties and discrepancies between calculated and measured values enhances student learning by fostering critical thinking, problem-solving, and a deeper understanding of material behavior and real-world engineering complexities. Observations during the experiments indicated that students were actively engaged, frequently discussing challenges and insights with their peers and instructors. Students expressed satisfaction with the hands-on, collaborative approach, appreciating the opportunity to apply theoretical concepts to practical setups while fostering curiosity and teamwork.

Future improvements to the demonstration aim to enhance the correlation between theory and experiment. Using stiffer components, welding the gears to the shaft, and upgrading measurement tools will reduce errors and improve the reliability of results and robustness of the setup. Additionally, implementing a structured assessment methodology in future iterations—such as pre- and post-experiment quizzes to measure conceptual gains, student feedback surveys to gauge perceptions of the experiment's effectiveness, and observational rubrics to assess teamwork and critical thinking skills—will provide objective data on learning outcomes. These refinements will further enhance the educational impact of the demonstration, better preparing students to bridge theoretical calculations and real-world applications in engineering.

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Appendix A

Table 2: Demonstration #2 calculations.

Measurements (inches)					**Polar Moment of Inertia (J) (in^4)**	**Shear Modulus (G) for Carbon Steel (ksi)**
L_{AB}	L_{CD}	r_1	r_2	Shafts 1 and 2 radii (ρ)	$\pi \times \frac{\rho^4}{2}$	Web Lookup
7. 6	3.1	1.43	0.73	0.1875	19.4×10^{-4}	11,460
Torque Calculations (lb-in)						
T_{AB}	$T_{CD} = \frac{T_{AB} \times r_1}{r_2}$				T_{BE}	T_{CF}
45	88.15				0	0
Stress Calculations (psi)						
$\tau_{AB} = \frac{T_{AB} \times \rho}{J}$		$\tau_{CD} = \frac{T_{CD} \times \rho}{J}$			τ_{BE}	τ_{CF}
4349		8519			0	0
Angle of Twist Calculations						
Angle of Twist		**Formula**				**Value (rad)**
ϕ_D		-				0
$\phi_{C/D}$		$\frac{T_{CD} \times L_{CD}}{J \times G}$				0.0123
$\phi_{B/D}$		$\phi_{C/D} \times \frac{r_1}{r_2}$				0.0241
$\phi_{A/D}$		$\phi_{C/D} + \frac{T_{AB} \times L_{AB}}{J \times G}$				0.039
$\phi_{A/D}$ (Calculated)		$\phi_{A/D} \times \frac{180}{\pi}$				**2.23 deg**
$\phi_{A/D}$ (Experimental)						**4 deg**
$\phi_{F/D}$		Same as $\phi_{C/D}$				0.0108

$\phi_{E/D}$	Same as $\phi_{B/D}$	0.021

Table 3: Demonstration #3 calculations.

Measurement (inches)									**J_{AB} & J_{CD} (in^4)**	**J_{EF} (in^4)**	**G (ksi)**
L_{AB}	L_{CD}	L_{EF}	r_1	r_2	r_3	r_4	ρ_1 & ρ_2	ρ_3	$\pi \times \frac{\rho^4}{2}$	$\pi \times \frac{\rho^4}{2}$	Web Lookup
10.5	8.75	9.75	0.73	1.08	0.73	1.4	0.1875	0.25	19.4×10^{-4}	61.3×10^{-4}	3770

Torque Calculations (lb-in)						
T_{AB}	$T_{CD} = \frac{T_{AB} \times r_2}{r_1}$	$T_{EF} = \frac{T_{CD} \times r_4}{r_3}$	T_{BG}	T_{CH}	T_{DI}	T_{EJ}
45	66.6	127.7	0	0	0	0

Stress Calculations (psi)						
$\tau_{AB} = \frac{T_{AB} \times \rho_{AB}}{J_{AB}}$	$\tau_{CD} = \frac{T_{CD} \times \rho_{CD}}{J_{CD}}$	$\tau_{EF} = \frac{T_{EF} \times \rho_{EF}}{J_{AB}}$	τ_{BG}	τ_{CH}	τ_{DI}	τ_{EJ}
4349	6436	5208	0	0	0	0

Angle of Twist Calculations		
Angle of Twist	**Formula**	**Value (rad)**
ϕ_F	-	0
$\phi_{E/F}$	$\frac{T_{EF} \times L_{EF}}{J_{EF} \times G}$	0.0539
$\phi_{D/F}$	$\phi_{E/F} \times \frac{r_4}{r_3}$	0.1034
$\phi_{C/F}$	$\phi_{D/F} + \frac{T_{CD} \times L_{CD}}{J_{CD} \times G}$	0.1832

$\phi_{B/F}$	$\phi_{C/F} \times \frac{r_2}{r_1}$	0.2710
$\phi_{A/F}$	$\phi_{B/F} + \frac{T_{AB} \times L_{AB}}{J_{AB} \times G}$	0.3356
$\phi_{A/F}$ (Calculated)	$\phi_{A/F} \times \frac{180}{\pi}$	**19.2 degrees**
$\phi_{A/D}$ (Experimental)		**30 degrees**
$\phi_{G/F}$	Same as $\phi_{B/F}$	0.266
$\phi_{H/F}$	Same as $\phi_{C/F}$	0.18
$\phi_{I/F}$	Same as $\phi_{D/F}$	0.1
$\phi_{J/F}$	Same as $\phi_{E/F}$	0.052

Table 4: Demonstration #1 specifications and part list.

Part and Specification list for Demonstration #1			
Property	**Details**	**Property**	**Details**
Wire Type	ASTM B3 110 soft-temper grounding copper wire	Force Gauge Type	Nidec FG-3008
Brand	ARCOR	Price	$337
Source	McMaster-Carr	Accuracy	±0.3% F.S.
Spool Weight	¼ lb	Resolution	0.1 lbf
Price per Spool	$11	Max Capacity	110.0 lbf
Diameter Tolerance	±0.0004"		
Wire Part #s	8873K28, 8873K26, 8873K24, and 8873K22		

Table 5: Demonstration #2 specifications and part list.

Part and Specification list for Demonstration #2				
Item	**Qty.**	**Description**	**Part #**	**Total Price ($)**
Gear 1	1	Steel roller chain sprocket for ANSI 25 chain, 36 teeth	2737T261 McMaster	30
Gear 2	1	Steel roller chain sprocket for ANSI 25 chain, 18 teeth	2737T121 McMaster	17
Shaft 1 and 2	2	3/8" Carbon steel with 60 HRC hardness, 12" long	3FUA8 Grainger	80
Mounted Sleeve Bearings	4	Oil-embedded with steel housing and copper bearing	3813T1 McMaster	36
Baseplate	1	Aluminum 10.25" x 6" x ¼"	8975K437 McMaster	22
Shaft Collars	4	303 SS with set screws	6462K14 McMaster	24
Protractor	1	Plastic protractor	Amazon	2
Mounting Block	2	Aluminum 4.5" x 1" x 1"	9008K14 McMaster	8
Shaft Nut	1	11/16" Seel	95036A037 McMaster	7
Torque Wrench	1	ACDelco ARM601-3 Digital 8" wrench (3.7 to 37 ft-lbs.)	Amazon	88

Table 6: Demonstration #3 specifications and part list.

Part and Specification list for Demonstration #3				
Item	**Qty.**	**Description**	**Part #**	**Total Price ($)**
Gear 1 & 3	2	Steel roller chain sprocket for ANSI 25 chain, 18 teeth	2737T121 McMaster	34
Gear 2	1	Steel roller chain sprocket for ANSI 25 chain, 24 teeth	2737T171 McMaster	21
Gear 4	1	Steel roller chain sprocket for ANSI 25 chain, 36 teeth	2737T261 McMaster	30
Shaft 1 and 2	2	3/8", 12" long 6061 Aluminum	8974K24 McMaster	5
Shaft 3	1	1/2", 12" long 6061 Aluminum	8974K28 McMaster	3

Mounted Sleeve Bearings	4	3/8” Oil-embedded with steel housing and copper bearing	3813T1 McMaster	36
Mounted Sleeve Bearings	2	1/2” Oil-embedded with steel housing and copper bearing	3813T2 McMaster	9
Shaft Collars	4	303 SS with set screws	6462K14 McMaster	24
Shaft Collars	2	303 SS with set screws	6462K16 McMaster	15
Protractor	1	Plastic protractor	Amazon	2
Mounting Block	2	Aluminum 6.5” x 1.5” x 1”	8975K52 McMaster	20
Shaft Nut	1	11/16” Steel	95036A037 McMaster	7
Torque Wrench	1	ACDelco ARM601-3 Digital 8” wrench (3.7 to 37 ft-lbs.)	Amazon	88