The Future of Engineering Education
2024 Annual Conference & Exposition
Oregon Convention Center
Portland, OR . June 23 - 26, 2024
Paper ID #44228

Correlating Common Errors in Statics Problem Solving with Spatial Ability

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Work in Progress: Correlating Common Errors in Statics Problem Solving with Spatial Ability

Abstract

Solving engineering problems is more than simply solving equations. It requires a clear visual representation of the problem, e.g. a schematic or diagram, before analysis is possible. In statics, for example, generating an appropriate free-body diagram is a critical step in the process. Spatial visualization skills (SVS) may play a critical role in developing these free-body diagrams properly. Many concepts in statics rely on the ability to visualize the effects of various force vectors on the equilibrium of an object. Understanding the direction of force and moment vectors is key to mastering these concepts.

Without strong SVS, students may rely on other tactics such as identifying superficial patterns to help solve the problem without a good understanding of the underlying concept. When confronted with a new situation, they are unable to properly "extrapolate" the memorized examples, so their misconceptions are revealed.

Students with strong SVS may be able to more easily interpret graphical representations (such as vector addition) leading to a stronger understanding of the underlying concepts. However, high visualizers are not guaranteed to grasp the concepts if they do not spend the effort to connect with the material.

In this study, we studied the potential correlation between a student's spatial ability and the types of errors made in solving fundamental statics problems. The two exam problems selected for the study focus on calculating resultant force and resultant moment in 2D. If certain types of errors are indicative of low spatial ability, identifying these types of errors will be the first step in developing activities to help correct these misconceptions. We performed statistical analyses on the datasets and found that the error prevalence rates for higher in students with low and medium SVS skills compared to the high-level mastery students. We also performed one-way ANOVA and looked at the correlation between two problems sets. Our results indicate that there is a correlation between the SVS skill levels and the problem outcomes, i.e., low visualizers master concepts at a lower rate than high visualizers.

Based on our findings, we infer that activities involving physical manipulatives and/or virtual 3D models may improve conceptual understanding for low visualizers, including the development of hands-on lab experiments.

Introduction

Engineers need to be able to visualize a problem by formulating a schematic, a model, or an equation to analyze and solve. In statics, for example, sketching an appropriate free body diagram (FBD) is a critical step in the solution process. Spatial visualization skills (SVS) may play a critical role in developing the FBD properly [1]. Many concepts in statics rely on the ability to visualize the effects of various force vectors on the equilibrium of an object. An accurate understanding of the direction of force and moment vectors is vital to mastering these concepts. Without strong SVS skills students may need to rely on memory, examples solved in class, or pure mathematical ability to solve the problem (such as attempting a cross product for solving a 2D moment problem).

SVS are attributed to student success in engineering courses along with significant improvement in learning and retention [1], [2]. Along with excellence in mathematics and physics, SVS can aid in high rate of success in STEM [3]. The NSF report [3] recommends that measures of spatial ability be included in talent assessment tests. Furthermore, Ha and Fang [4] report that there is an inherent correlation to understanding fundamental concepts such as FBDs, moments, vector representations and the spatial ability of the students.

Higley et al. [5] used three measures to predict student success in a statics course: mathematics, spatial reasoning, and conceptual knowledge related to statics. The authors were able to cluster certain errors and associate them with spatial reasoning, which is a starting point and can be expanded to further characterize the errors and attribute them to a deficiency in one or more specific areas, such as mathematical or spatial ability.

Davishahl et al. [6] presented the construct of representational competence, described as the ability to think and communicate via visual representations of the problem, and its role in student success in Statics. Students that used a more representational approach developed a deeper understanding of the concepts compared to students who relied solely on equations and algebraic manipulations, gaining only a superficial understanding of the concepts. Wood et al. [7] reported that the spatial ability of the students improved significantly in statics course, which indicates that students are developing and applying their SVS as they learn to solve problems in statics.

Sorby et al. [1] found a strong correlation between spatial skills and drawing FBDs. Students with low SVS struggled to interpret word problems into the proper FBDs necessary to solve. There is compelling evidence that instructors should work towards integrating spatial ability training and testing in foundational courses [1], [6].

In this study we investigate the effect of spatial ability on comprehension of fundamental statics concepts, specifically force vectors and moment of a force. Students are divided into three groups based on SVS: low, medium, and high. Exam scores for these three groups are compared and analyzed through one-way ANOVA. We hypothesized that differences between SVS groups would be observed when finding moments of forces, but not when finding the resultants of force vectors.

In addition to overall performance on the exam, various types of errors on each problem will be tracked and categorized. The prevalence of each type of error among the three SVS groups (low, medium, and high) are determined and compared. We hypothesized that errors related to direction of resultant vectors and moments will be more prevalent among low visualizers, and errors related to calculating the magnitude of these vectors and moments will not be correlated with SVS.

Methods

All second-year engineering students at Stevens Institute of Technology are required to take Statics & Introduction to Engineering Mechanics. Math prerequisites for the course include two semesters of calculus. Force vectors and moment of a force are covered in the first two weeks of the semester, and students are tested on these concepts in the first exam (around week 4 or 5). Two exam problems from the first exam in Fall 2022 are used in this study.

In problem 1, students were asked to resolve three force vectors into components, calculate the magnitude and orientation of the resultant, and finally sketch the resultant force vector (see Figure 1). This problem was used to assess mastery of the following fundamental skills:

1. resolving a vector into components,
2. adding vectors, and
3. visualizing the graphical form of a vector from a vector expression.

Problem 1.

Three co-planar forces are acting on a bracket as shown in the figure below. Let $\varphi = 65°$.

a. Express each of the three forces in Cartesian vector form.
b. Determine the magnitude and orientation of the resultant force. Be sure to specify which angle you are calculating.
c. Sketch the resultant force vector (as accurately as possible). Clearly label the magnitude and angle of the resultant force.

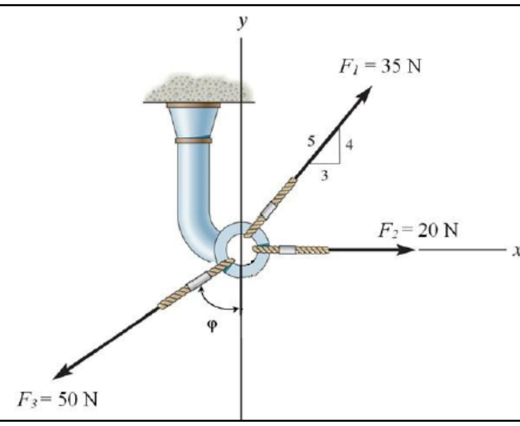

Figure 1. Exam problem on force vectors.

The errors identified for each of the above skills are summarized in Table 1 below. Minor calculation errors were not included in the analysis. Additionally, no errors in vector addition were observed.

Table 1. Errors identified for problem 1 on force vectors.

Fundamental Skill	Errors Identified
Resolve a vector	S - Incorrect sign on one or more components F - Incorrect value of one or more components
Add vectors	N/A
Sketch a vector	Q - Vector sketched in quadrant inconsistent with vector expression A - Angle indicated on sketch inconsistent with calculated angle

In problem 2, students were asked to determine the moment of each force about a given point, then find the magnitude and direction of the resultant moment (see Figure 2). Using the scalar approach to calculating 2D moments, this requires that students are able to:

1. Identify which force components do and do not cause a moment about the given point.
    a. Ability to recognize vertical forces that cause moment.
    b. Ability to recognize horizontal forces that cause moment.
2. Find the moment arm distance for each force component.
3. Determine the direction of each moment of force.
4. Add to find the resultant moment.

Problem 2.

Three coplanar forces are acting on the structure as shown, where $F_1 = 200$ N, $F_2 = 150$ N, and $F_3 = 275$ N. For this system solve for the following:

a. Determine the moment of each of the three forces about point A.

b. Determine the resultant moment about point A.

c. Determine the moment of each of the three forces about point B.

d. Determine the resultant moment about point B.

Be sure to include the correct sign with your answers. Note that counterclockwise moments are positive.

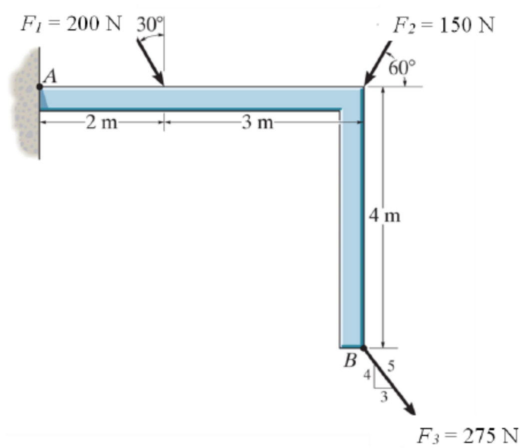

Figure 2. Exam problem on moments of forces.

The errors identified for each of the above skills are summarized in Table 2 below. Again, minor calculation errors were not included in the analysis.

Table 2. Errors identified for problem 2 on moments of forces.

Fundamental Skill	Errors Identified
Resolve force vector	F - Did not resolve forces at all
Recognize which force components cause moments	O - Omitted one or more force components contributing to resultant moment (e.g. calculated moments due to vertical force components only) Z - Included one or more force components that should be zero moment
Recognize that moment arm is perpendicular to force	D - Incorrect moment arm distance, or did not identify moment arms at all
Determine the direction of the moment of force	S – Incorrect sign on one or more moments of forces
Add moments	C - Assigned i- and j- directions to one or more moments

Spatial visualization skills were assessed using the Purdue Spatial Visualization Test: Rotations (PSVT:R), a timed standardized test of mental rotations [8]. The PSVT:R developed by Bodner and Guay is a widely used test for assessing the spatial ability in students within the engineering education research domain. As part of the Introduction to Engineering Design course, this test is administered to all first-year engineering students during the first week of class to assess their spatial ability. The passing threshold is typically set at 60% or 70% to identify students with low spatial ability. At Stevens, thresholds of 70% and 80% are used to separate students into groups of low, medium, and high spatial ability.

Results

Students were divided into three groups, based on their PSVT:R score. Low visualizers were defined by scores below 70%, the medium visualizers scored between 70% to 79%, and the high visualizers achieved scores of 80% or higher. In this cohort (n=136), the percentage of students with low, medium, and high SVS was 24%, 30%, and 46%, respectively, as seen in Figure 3.

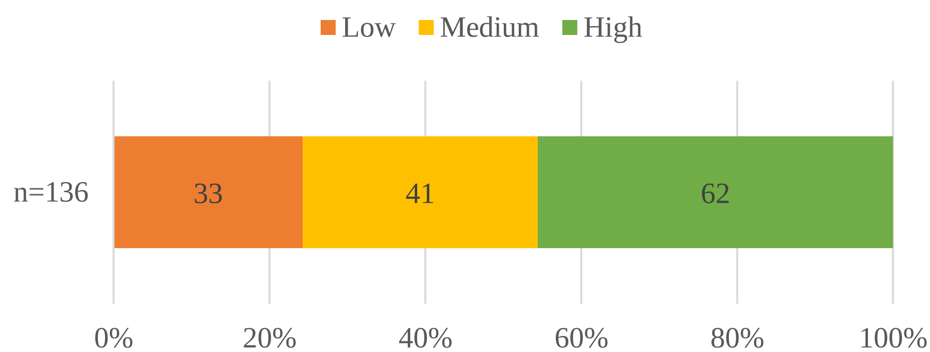

Figure 3. Distribution of students with low SVS, medium SVS, and high SVS.

The distribution of scores on the exam problems for low, medium, and high visualizers are shown in Figure 4. Unsurprisingly, students performed better on problem 1 (force vectors) than problem 2 (moments of forces). For both problems, the average score increases with visualization skill. Descriptive statistics are provided in Table 3. Scores for each problem are out of 10 points.

Table 3. Descriptive statistics for Problem 1 (force vectors) and Problem 2 (moment of force) among the low, medium, and high SVS groups.

	n	**P1 - Force Vectors**		**P2 - Moment of Force**	
		M	*SD*	*M*	*SD*
Low SVS	33	8.98	1.19	8.21	2.41
Medium SVS	41	9.06	1.18	8.34	2.28
High SVS	62	9.69	0.60	9.56	1.10

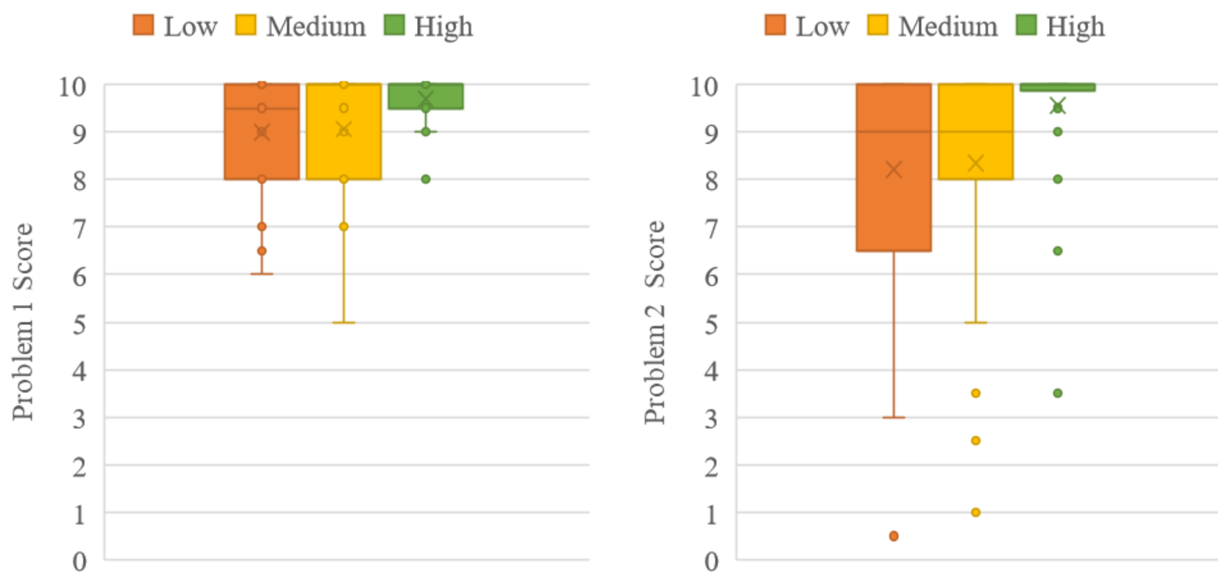

Figure 4. Box plots of scores on Problem 1 (force vectors) and Problem 2 (moment of a force) for low, medium, and high SVS groups.

One-way ANOVA results indicate highly significant differences between groups for Problem 1 (F (2, 133) = 7.366, $p < .001$) and for Problem 2 (F (2, 133) = 7.819, $p < .001$). In addition, the Pearson correlation between these two problem scores indicated a moderate positive correlation (r = 0.5419, $p < .001$) indicating that the SVS skills are translated to both the problems.

To further investigate the differences in conceptual understanding among the SVS groups, errors made on each problem were categorized by type, as previously described in Tables 1 and 2. The frequency of each error type among the three SVS groups for problems 1 and 2 is displayed in Figures 5 and 6, respectively. Error prevalence is lowest for the high SVS group across all error types.

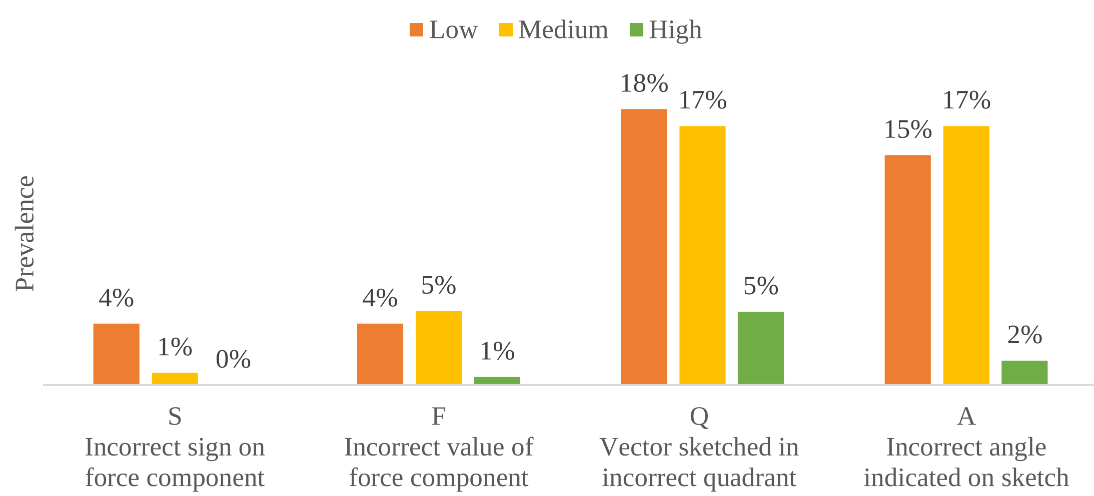

Figure 5. Error prevalence metrics for Problem 1 among low, medium, and high SVS groups.

As seen in Figure 5, a few errors were made in the resolution of the force vectors (S, F). We observed that some students linked $\boldsymbol{F_y}$ to sine of the angle, and $\boldsymbol{F_x}$ to cosine of the angle, without understanding that it is critical to pay attention to how the angle is defined (from horizontal vs. from vertical).

The majority of errors were made in the visualization of the resultant vector (Q, A). Low and medium visualizers often calculated the resultant vector properly for the correct numerical result but were unable to recognize the appropriate direction of the resultant vector when asked to sketch it. Similarly, they were able to calculate an angle to describe the direction of the resultant vector (e.g. using arctan) but were unable to properly indicate which angle they had calculated on a sketch. This points to an over reliance on using equations and formulas to blindly achieve some numerical result without a good understanding of what the values represent.

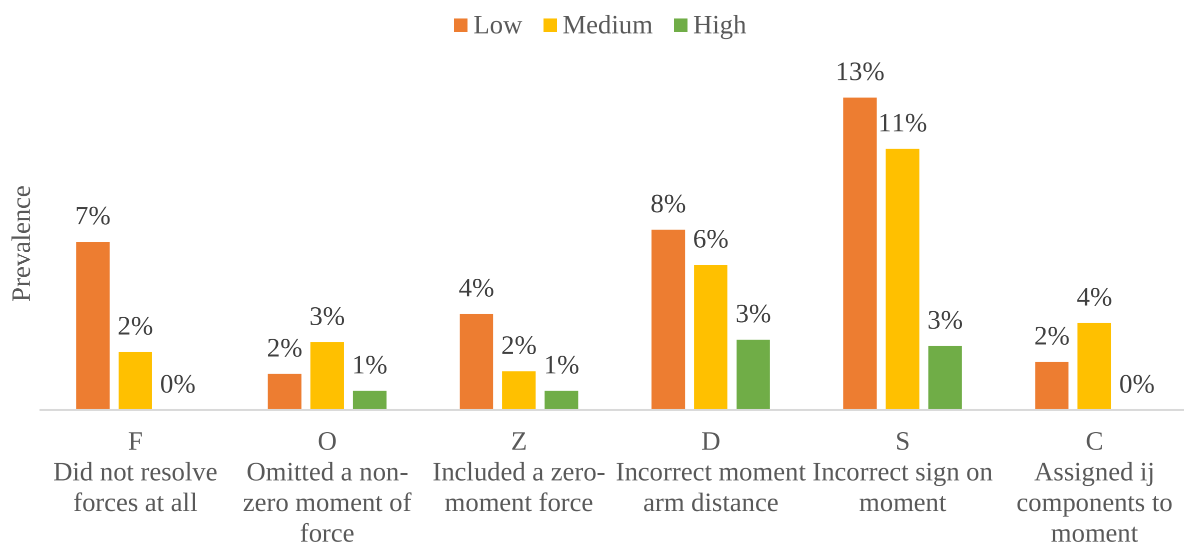

Figure 6. Error prevalence metrics for Problem 2 among low, medium, and high SVS groups.

As seen in Figure 6, sign errors (S) were the most prevalent across all SVS groups in the calculation of moments of force. Visualizing the direction of the rotational effect of a force about a given point posed a challenge for many low- and medium-visualizers, and even some high-visualizers.

One of the most serious conceptual errors was not resolving the forces at all (F). It is interesting to note that high visualizers did not make this type of error. Another error that we encountered that was specific to the low and medium SVS groups was the assignment of i- and j- directions to certain moments of forces (C). This demonstrates a lack of comprehension about the concept of a moment of a force as a rotational effect with a directionality that is either clockwise or counterclockwise (in the 2D case).

Discussion

Significant differences in the exam scores for the low, medium, and high SVS groups were observed. In problem 1, error rates for the computational portion (resolving forces and adding force vectors) were very low among all SVS groups, ranging from 1% to 5%. For the graphical portion (sketching the resultant vector), however, error rates increased dramatically, up to 18% for the low SVS group. This trend highlights the significance and need for spatial visualization skills in students.

It is important to recognize that the majority of students, regardless of spatial ability, were able to solve both problems completely accurately, indicating that low SVS does not prevent a student from mastering a concept, but perhaps makes it more difficult to do so. Our preliminary results indicate that low visualizers master concepts at a lower rate than high visualizers. Conversely, strong SVS skills do not guarantee mastery of new concepts. Without some effort to connect with the course material, even students with strong math and spatial abilities will not perform well in the course. This is evidenced by the handful of high visualizers that did not score well on problem 2 (see Figure 4).

It is evident from Figures 5 and 6 that the primary error prevalences in low SVS ability students are incorrect angle marking for the resultant force vectors, incorrect signs for the moments, incorrect force components (including sine/cosine swapping), and incorrect moment arms. These errors intuitively indicate that improving the spatial ability would help students understand the problems better and solve them successfully. This will also translate to real world problem solving and is not just limited to a course or a grade. To help overcome these issues, and help improve the spatial ability of the students, instructors could introduce a physical [9] or simulation component [10] to certain fundamental concepts. Further an introduction of lab activities which focus on a more qualitative understanding of topics such as moments, moment arms, direction of the moment, etc. rather than focusing on quantitative analysis. These types of labs are currently being developed and implemented at Stevens.

Hands-on activities involving physical manipulatives could help students gain a deeper conceptual understanding and break away from simply memorizing examples. For instance, as mentioned in the previous section, we observed that students seemed to think that F_y would always be equal to $F \sin\theta$, regardless of where the angle was defined, most likely because the angle is commonly defined relative to the horizontal axis in most examples. This points to the importance of using varied examples when introducing new concepts.

Another common misconception was that only vertical force components generated moments, and horizontal force components always generated zero moment. Again, this likely stems from the setup of a typical example problem. Sorby et al. [1] observed a similar situation with drawing the reactions at a roller support, where students assumed a vertical reaction force even when the roller was placed on a vertical wall.

Future Work

One major limitation of the study is the potential growth in the student's spatial ability over the span of time between taking the PSVT:R and taking the statics course. To address this, the PSVT:R may be administered at the start of the statics course for a more representative measure of the student's SVS. Further the PSVT:R test could be administered at the end of the course to measure whether the Statics course helped to improve SVS.

Tracking mastery of various concepts in later exams could also provide insight into whether students with lower SVS simply need more time to grasp the concept. For example, finding moment of a force is a fundamental skill necessary for performing truss and frame analyses, so exams given later in the semester can be used to track mastery of the earlier concepts. This could be further expanded to follow-on courses, such as Mechanics of Materials, Dynamics, and Design of Machine Components, to better understand whether mastery of various concepts was achieved and retained. It would be especially interesting to see how students handle 3D problems in these follow-on courses, since the Statics course primarily focuses on 2D problems only.

Preliminary results from this study have helped to identify common misconceptions, which can be used to inform the development of hands-on activities and physical demonstrations for the course. As mentioned previously, lab activities with a more qualitative focus have been successfully piloted at Stevens. Based on positive student feedback, plans to develop additional qualitative labs and exercises are underway. Comparisons of different course implementations (with and without the lab activities) can be used to evaluate the efficacy of the labs in helping students achieve mastery of statics concepts across all spatial skill levels.

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