A Comprehensive Approach to Modeling Dynamic Biological Systems: Enhancing Critical Thinking and Mathematical Problem-Solving in Biomedical Engineering Education

Caleb Wilson Hendrick, University of Maine

Caleb Hendrick, an undergraduate at the University of Maine, majors in biomedical engineering with minors in electrical engineering and mathematics. He serves as a teaching assistant, committed to creating an inclusive learning environment, particularly in the ”Modeling Dynamic Biomedical Systems” (BEN 401) class, and is actively involved in developing course materials. His research focuses on mathematical modeling of complex physiological systems, bridging mathematics and human biology. He is passionate about finding ways to bring mathematical modeling into more fields of research and educational settings.

Prof. Karissa B Tilbury

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Caleb Hendrick, Karissa Tilbury
caleb.hendrick@maine.edu, karissa.tilbury@maine.edu
Chemical/Biomedical Engineering
University of Maine

Abstract:

This work-in-progress project is grounded in a biomedical engineering junior-level course dedicated to modeling biomedical systems. The course and project's primary goal is to integrate fundamental concepts from physics, chemistry, engineering, and mathematics to provide students with a comprehensive foundation for addressing real-world biomedical engineering challenges. Establishing connections and parallels between mathematical methodologies, specifically differential equations, and the constitutive relationships in physics and chemistry are critical in the development of biomedical engineers.

A key objective is developing critical thinking skills in students to tackle real-world biomedical problems. BME problems in this course span multiple domains including: 1) bio-instrumentation, 2) drug kinetics, 3) mechanical systems, and 4) organ models. Undergraduate biomedical engineering students frequently struggle with the intersection of mathematics in these domains as the problems require students to freely recall various techniques to solve systems of differential equations in story-problems. This is in contrast with many differential equations textbooks that emphasize rote memorization methods or provide subtle hints of the particular method and or process to be used to solve pre-written mathematical functions. Within engineering disciplines, it is important for students to actively read story problems or interview stakeholders to identify key constraints, and governing physical and biological conditions to derive their own mathematical functions that describe the potential solution space for engineered solutions. The ability to translate physical constraints and apply prerequisite knowledge from physics and biology are often under-emphasized in mathematics courses. Furthermore, most mathematics courses focus on analytical solutions and do not employ computational tools for numerical approximations which are critical in engineering.

To promote the growth of applied mathematics within BME, we developed a comprehensive text focused on dynamic biomedical systems. The text provides a primer of system characteristics including: 1) linearity, 2) time-invariant, 3) autonomous as well as various different types of input signals (i.e. step, impulse, sinusoidal etc). The text seamlessly interweaves a review of multiple

approaches to solving first and second order differential equations including: 1) Variation of Parameters, 2) Undetermined Coefficients, 3) LaPlace Transforms, and 4) Eigenvectors and Eigenvalues coupled with State Variable format. Alongside these analytical methods, numerical approaches using MATLAB are also outlined. During lectures, students are exposed to complementary instruction leveraging both mini lectures and active learning problem solving using both analytical and numerical approaches that build in complexity from simple word problems to more complex physiological models (i.e. systemic arteries, the body volume compartments, neuron firing). Beyond simply plotting the solution spaces of the mathematical functions, students are asked to write rules to communicate the utility of the models to other stakeholders including healthcare professionals or basic biomedical scientists.

In summary, we have created a unique BME focused text for differential equations and linear algebra that encourages students to harness their knowledge of physics, biology, physiology, engineering, and mathematics to formulate dynamic models of physiological systems. Our overall aim is to enhance students' ability to apply and foster a deep appreciation of the power of mathematics in addressing real-world BME challenges.

Background:

Ordinary differential equations are ubiquitous for understanding various topics and systems studied as part of the undergraduate physics, engineering, chemistry, and biology curriculums [1–4]. Over the past two decades, numerous innovative teaching strategies ranging from teacher-centered didactic and student-centric pedagogical approaches have been implemented with mixed results [5]. Engineering disciplines require the application of ordinary differential equations inherently providing a 'contextual learning' environment; yet the typical engineering undergraduate curriculum employs parallel tracks of 1) Mathematics (i.e. Calculus I-III and Ordinary Differential Equations and Linear Algebra and 2) Engineering-specific courses (i.e. Dynamic Biomedical Systems, Thermodynamics, Kinetics etc). Interestingly, there are a few engineering programs that intimately integrate mathematics with the engineering point of view, several of which have been shared at previous ASEE conferences [6, 7]. In Pennell et al. [6], authors used a mixture of both the "mathematical" and "engineering" perspective in combination with numerical approximations solved in software packages (i.e. MATLAB, LABVIEW) in combination with project-based engineering problems. Overall, they found this strengthened the student's analytical and engineering skill sets [6]. The project-based engineering problems described as part of the mathematics course sequence described in Pennell; however, are generic to all engineering disciplines.

In this work in progress paper, we present a biomedical engineering themed approach firmly rooted in modeling of physiological systems that compliments existing parallel instruction of ordinary differential equations and linear algebra while providing students the opportunity to apply biomedical engineering skills involving the following: 1) Assembly of appropriate information from stakeholder interviews or written problem statements to create appropriate box diagrams and system input system output diagrams, 2) Analyze information to identify and define the system boundaries, assumptions, appropriate variables, and the appropriate conservation or accounting equations of the extrinsic property or properties of interest, 3) Calculate using analytical and numerical techniques, and 4) Finalize using graphical approaches that highlight the

quantitative and qualitative impacts of system variables for the identification of the design constraints or potential allowable tolerances.

Dynamic biomedical systems within biomedical engineering can include multiple domains ranging from modeling of physiological systems used to create 'virtual experiments' to test new theories of physiological responses. Alternatively, biomedical models may be used to identify design constraints for medical devices required to restore normal physiological function (i.e. insulin pumps, dialysis machines) or protective equipment (i.e. fighter jet pilot helmets) to minimize injuries.

Redesigned Dynamic Biomedical Systems Course Structure:

To address the breadth of dynamical biomedical systems for adequate preparation career preparation of junior biomedical, we re-developed a core junior course at New England land, sea, and space grant public institution. Key elements of the course revision included: 1) New course notes that provided step-by-step annotated analytical and numerical solutions to several physiological-based dynamic biomedical systems, 2) Pre-class reading quizzes, 3) Hands-on small group (3-4 students/group) in class problems, 4) Additional structured office hours run by a senior undergraduate biomedical engineering student teaching assistant for near-peer mentoring, 5) Multiple quizzes during the semester, and 6) Holistic in-person one-on-one final exam presentation modeled as an interaction between a project manager and employee or academic advisor and student researcher to assess the student's engineering habits of mind [8].

New Course Notes & Pre-class Reading Quizzes:

New PDF searchable course notes were co-developed by a senior undergraduate student assistant and the course instructor the summer before the course offering. Appendix A summarizes each of the topical areas of the course notes. The notes are built in complexity both in terms of physiological systems and mathematical concepts. The course notes assumed no prior knowledge of System Characterization beyond the ability for students to identify if the system is open or closed, and the extensive property of interest based on pre-requisite course knowledge. Chapter One is focused on System Characteristics (i.e. Dynamic vs. Static, Casual, Time-Invariant, Linear, Stability) and various System Driving Forces. These topics were new to most of the students enrolled in the course and were taught in both the context of rigorous mathematical proofs as well as more applied qualitative arguments forming the basis of the Dynamic, Linear, Time-invariant systems [Dynamic LTE] leveraged throughout the rest of the course. Chapter Two reviewed various analytical (Variation of Parameters, Underdetermined Coefficients, LaPlace Transforms, and Eigenvectors & Eigenvalues coupled with State Variable Format) with step-by-step solutions for students to review pre-requisite knowledge from their ordinary differential equations and linear algebra course. Appendix B & C provides an example of step-by-step analytical solution and numerical approaches to similar "mathematical" type ordinary differential equation.

Once the analytical approaches were reviewed, the course notes provided examples of using MATLAB function, ode45, to numerically solve the same differential equations. The numerical MATLAB instruction was also supplemented with online self-paced training courses: 1)

MATLAB On-ramp and 2)Solving Ordinary Differential Equations with MATLAB as most of the students had minimal or no experience in the MATLAB coding environment. Chapters Three-Five transitioned the students from "mathematical" differential equations into dynamic biomedical engineering systems with increasing complexity ranging from single compartment/component to multi-compartment/component systems. Appendix D provides an example of a single compartment physiologically relevant example problem with step-by-step instructions.

Students were assigned to read and work through the example problems ahead of class time and complete a reading quiz using an online learning management system. Before each class, the instructor reviewed the overall class performance on each of the reading quiz questions and reviewed as needed during class. Pre-class reading quizzes were graded where scores of 70% or higher were replaced with 100%.

Hands-on Small Group Modeling Activities:

The course was taught in an active learning classroom equipped with both whiteboards and large TV monitors with oval shaped tables with chairs for 4-5 students. The total class size for the Fall 2024 semester was 20 students, so generally students sat in groups of 4 based on their own selection. The in-class activities were designed to build on the course notes and provide additional opportunities for students to analytically and numerical solve LTE ordinary differential equations. Once students mastered the basic mechanics of analytical and numerical solution approaches, the in-class problems focused more on physiological systems. Over the course of the semester, student teams completed in-class exercises focused on the following physiological systems: 1) Arterial Vascular Dynamics, 2) Body Volumes, 3) Neural Activity of a single neuron using Hodgkin Huxley Model and 4) Blood Alcohol Dynamics. The physiological modeling assignments contained both an in-person group based portion as well as an individual homework component. The individual homework component focused more on the synthesis and interpretation of the modeled phenomenon. Examples of an in-class and individual post-class assignments can be found in Appendix E & F. The combination of the in-class and individual assignments refined students' engineering habits of mind (Eng. Habits of Mind ref) Lucas & Hanson, particularly in the description and justification of 'what if' scenarios focused on altering various parameters of the system. Full justification of the results of the 'what if' scenarios required the students to describe using mathematical principles rather than solely relying on graphical solutions derived from the numerical solutions to improve mathematical rigor.

Undergraduate Student Learning Assistant for Near-Peer Mentoring:

The senior undergraduate student author of this WIP was critical for the implementation of the course revisions. The undergraduate student attended every lecture period and actively engaged students during problem solving sessions. In addition to class periods, the undergraduate learning assistant provided two separate two-hour problem solving sessions each week for the students to attend. During these class sessions, he provided additional problem-solving strategies and practice opportunities.

Individual Student Performance Assessment:

Students were assessed multiple times throughout the semester in the form of four 75-minute quizzes given during class. The quizzes were modeled based on in-class activities and homework assignments. The quizzes consisted of two-parts; part one was closed-book and focused on analytical approaches whereas part two required students to use MATLAB to write appropriate functions to numerically solve dynamic biomedical systems. Each quiz was worth 7.5% of the overall grade and provided students frequent and low-stakes opportunities to confirm their own understanding. The final individual assessment was based on a more extensive take-home final exam problem that students were given 24-hours to complete and post both a PowerPoint presentation outlining their engineering problem solving strategy, system diagram, solution approach, and graphical solutions as well as all MATLAB scripts written to numerically solve the system. Students booked 15-minute appointments with the course instructor and presented their PowerPoints one-on-one during the final exam week. Students were provided an opportunity to correct any errors during the presentation as well as describe how they may have altered their approach if they had erred. Ultimately, students were assessed using the engineering habits of mind rubric as described in [8]. The final exam accounted for 30% of the overall grade.

Results & Discussion:

Quantitative Student Assessment:

Quantitatively, student success was based on the average student scores in homework, quizzes, and final exam grades. The average student performance on homework assignments was 89%, higher than the individual student performance on individual quizzes or final exam. This was not surprising as during homework assignments students worked collaboratively without specific time-constraints and had open-access to both the instructor and the undergraduate learning assistant questions. Interestingly, the final exam average of 86.5% was higher than any quiz average despite being more complex.

The increasing quiz average as seen in Figure 1 over the course of the semester and the fact that the final exam scores were the highest on average further supports that students grew in their ability to solve and communicate solutions to dynamic biomedical systems. Interestingly, the standard deviation of the final exam was the smallest indicating that the overall student performance was coalescing around scores that demonstrate proficiency.

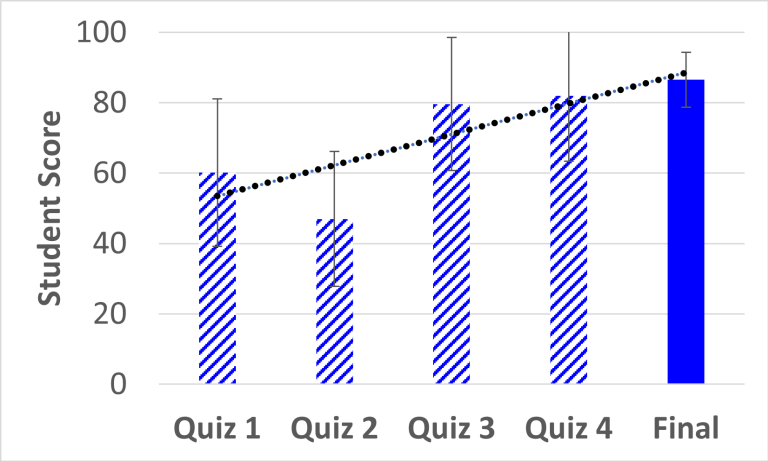

Figure 1: Average (n = 20) student performance on quizzes and final exam. Error bars are standard deviation.

Semi-quantitative Assessment of Student Attitude Towards Course:

The end of semester course evaluations were used as a semi-quantitative assessment of student attitude towards the course. Of the 20 students enrolled in the course, 7 students completed the end of semester course evaluation and specific attention was given towards the following standard course evaluation questions using a 5-point Likert scale where 1 is not at all and 5 is very much.

Questions focused on intellectual growth:

- Question 1: How much were you encouraged to think for yourself? Average: 4.57, Median 5.0.
- Question 2: How much did this course challenge you intellectually? Average: 5, Median 5.
- Question 3: How much did you learn from this course? Average: 4.29, Median 4.0.

Overall these student responses indicate that the course revisions fostered a rich learning environment that effectively stimulated students' intellectual growth.

Questions focused on course materials & class structure:

- Question 4: Were the class meetings profitable and worth attending? Average 4.45, Median 4.0.
- Question 5: What is your overall rating of the primary readings? Average: 3.33, Median; 3.5.

These scores suggest that students valued the in-class hands-on problem solving approach to lectures. Interestingly, there was only moderate satisfaction with the primary readings. Therefore, we have initiated an exploration into how to improve the course notes to improve student satisfaction.

Qualitative Student Responses:

In addition to the official university-sponsored evaluation of the course, students were voluntarily asked to respond to the following three questions:

1. Please describe how your perception of math as it is applied in biomedical engineering was altered as a result of Dynamic Biomedical Systems?
2. Please describe why modeling is important to the field of biomedical engineering?
3. Please describe how Dynamic Biomedical Systems challenged your way of thinking?

Overall student responses to question 1 highlighted a positive shift in their perception of the relevance of mathematics in the context of the biomedical engineering program. One particular student wrote:

> *'I realized that there were actual, legitimate, beneficial applications of differential equations. Before I thought differential equations were a niche, general form of math, now I can see how they can be applied to model physiological systems.'*

In response to question two, student responses consistently underscore the significance of modeling within the biomedical engineering domain. Students appreciated how previous knowledge acquired in engineering courses (conservation and accounting equations) were interwoven with biology, physics, and chemistry needed to model the dynamics of biological systems. One student wrote the following quote highlighting both the impacts of modeling in terms of hypothesis testing and medical device design:

> *'Modeling is where you have the most range to tweak and perfect your ideas and theories. Otherwise there is the possibility of being entirely off based, and even potentially harming someone or something in human/animal trials should that be the next step. Then there is also the side of modeling where the next step isn't trials but rather just gaining a better understanding of biological systems and this is the best method that exists at this point in time.'*

Finally in response to question 3, students generally expressed a shift in how they approach problem solving highlighting the importance of numerical approaches to create graphical representations of how the various components of the human body interact. Specifically, one student wrote the following:

> *'Dynamic Biomedical Systems helped open my mind to the idea of different ways to learn and test ideas. While hands-on testing is important, knowing that models can be used to rapidly test and explore concepts that would be unfeasible to perform in the real world (and/or at a large scale) allows me to have an additional tool in my pocket for future endeavors.'*

Conclusion:

In summary, we have created a unique BME focused text for differential equations and linear algebra that encourages students to harness their knowledge of physics, biology, physiology, engineering, and mathematics to formulate dynamic models of physiological systems. Our overall aim is to enhance students' ability to apply and foster a deep appreciation of the power of mathematics in addressing real-world biomedical engineering challenges.

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Appendix A A Condensed Version of The Table of Contents Within Class Notes

Appendix B Solving For an Analytical Solution Example From Text

Example 2 of variation of parameters method on a D.E.:

$$\frac{dy(t)}{dt} = ky(t) + 6$$

First find homogeneous solution:

$$\frac{dy(t)}{dt} - ky(t) = 0$$
$$y_h(t) = Ce^{kt}$$

Parameterize constant C for the equation:

$$y_h(t) = C(t)e^{kt}$$

Place into original D.E.:

$$\left[C(t)e^{kt}\right]\frac{d}{dt} = kC(t)e^{kt} + 6$$

Differentiate the equation (hint: use product rule):

$$\frac{dC(t)}{dt}e^{kt} + kC(t)e^{kt} = kC(t)e^{kt} + 6$$
$$\frac{dC(t)}{dt}e^{kt} = 6$$

Solve for C(t):

$$\frac{dC(t)}{dt} = 6e^{-kt}$$
$$C(t) = \int 6e^{-kt}\,dt$$
$$C(t) = -\frac{6}{k}e^{-kt} + C_1$$

Place back into original function where C was parameterized, to get solution:

$$y(t) = (-\frac{6}{K}e^{-kt} + C_1)e^{kt}$$

Appendix C Solving For an Numerical Solution Example from text

Example 2 given the following D.E. with k = 2, and an initial condition I.C. = 3. The function can be written in MATLAB as:

$$\frac{dy(t)}{dt} = -ky(t) + 2 \tag{C.0.1}$$

```
function dydt = myODE1(t,y)
k = 2; %[s]^-1
dydt = -y*k + 2;
end
```

Now to solve the differential ODE45 is used as it is the base solver implemented in MATLAB for simple differential equations. The function is called with the time frame with the I.C. and plotted as:

```
IC = 3;
tspan = linspace(0,10); %time span from 0 to 10 [s]
[t,y] = ode45(@myODE1,tspan,IC); %solving the function with ODE45
    solver
```

```
plot(t,y)
xlabel('t [s]')
ylabel('y(t)')
ylim([0 4])
xlim([0 10])
```

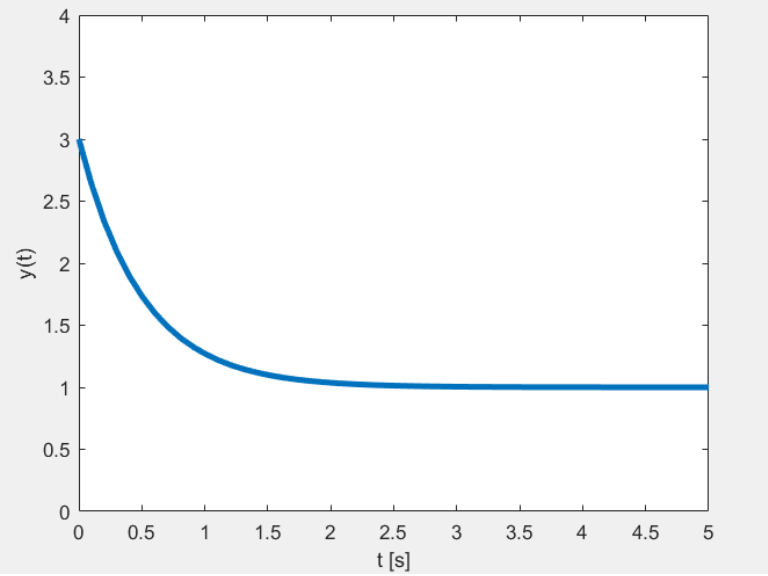

Figure 2: Example 2 differential equation plotted from 0 to 5 seconds. (This figure was made in MATLAB.)

Appendix D Example Of Liver With Reaction Kinetics

Adding complexity to modeling systems is looking more closely at what is occurring in said system. Take the liver for example, we would like to look at a type of metabolism. We want to determine the diffusion of the substance into the liver then the metabolism of the substance. In fig. 3 we can see that the arteries and veins will carry the substance in the blood to the liver, where then it is metabolized.

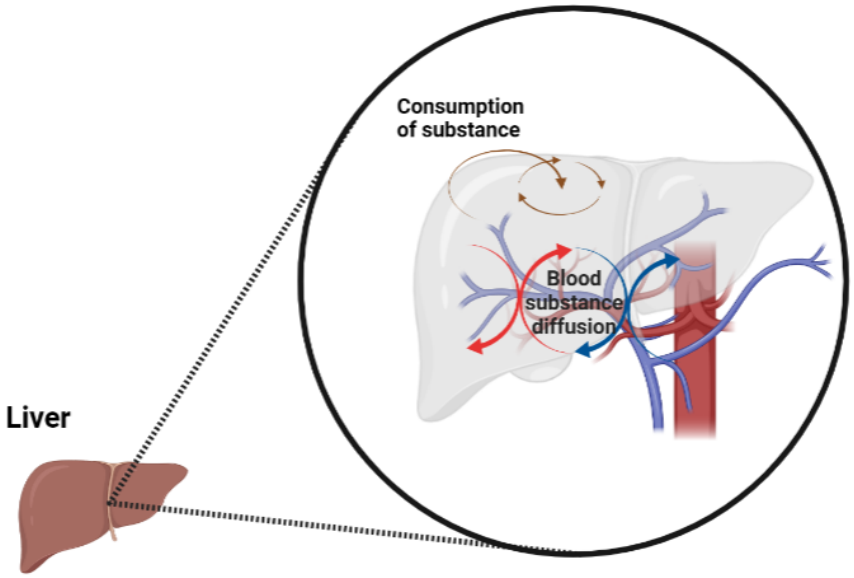

Figure 3: General look at the physiological mechanisms of the liver. (This figure was made in biorender.)

With now seeing the system we can define its boundaries and make assumptions. This will then allow us to start a model for the system we will mathematically define. We then construct the systems diagram with the boundaries specified and the list of assumptions shown in fig. 4.

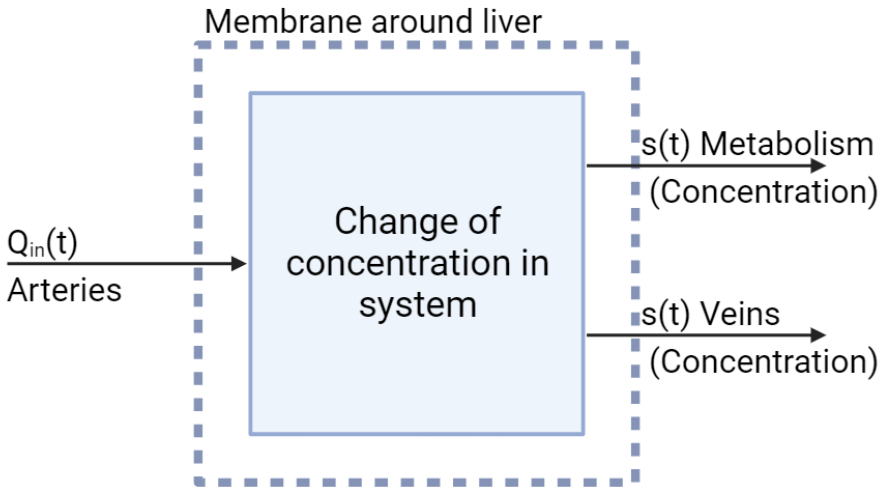

Figure 4: System input output diagram of the liver. (This figure was made in biorender.)

List of assumptions:

- Blood incomprehensible fluid
- Reaction is steady and does not change based off of concentration
- Concentration of substance cannot react outside the liver
- Flow of substance into the bladder is constant.
- Reaction is not dependent on spacial location in the liver
- Constant volume of liver (homogeneous structure)

With the assumptions listed and diagram made the equation can be written in terms of the concentration of substance s(t), where the reaction constant is K_m:

$$\frac{dV_L s(t)}{dt} = Q_{in}s_{in}(t) - Q_{out}s(t) - K_m s(t)$$

Volume is not changing so the it can be pulled out of the differential operator:

$$\begin{aligned} V_L\frac{ds(t)}{dt} &= Q_{in}s_{in}(t) - Q_{out}s(t) - K_m s(t) \\ \frac{ds(t)}{dt} &= \frac{Q_{in}s_{in}(t) - Q_{out}s(t) - K_m s(t)}{V_L} \\ \frac{ds(t)}{dt} &= \frac{Q_{in}s_{in}(t)}{V_L} - \frac{s(t)(Q_{out} + K_m)}{V_L} \end{aligned}$$

List of vairables:

- s(t) = substance concentration
- $s_{in}(t)$ = substance coming in
- Q_in = flow rate in
- Q_in = flow rate out
- K_M = Reaction constant
- V_L = Volume of liver

Now we can solve the D.E. with respect to s(t), keeping in mind that this is a in-homogeneous D.E. so we will use variation of parameters: First find homogeneous solution:

$$\begin{aligned} \frac{ds(t)}{dt} + \frac{s(t)(Q_{out} - K_m)}{V_L} &= 0 \\ s_h(t) &= Ce^{-kt} \\ k &= \frac{(Q_{out} + K_m)}{V_L} \end{aligned}$$

Parameterize constant C for the equation:

$$s_h(t) = C(t)e^{-kt}$$

Place into original D.E.:

$$\left[C(t)e^{-kt}\right]\frac{d}{dt} = -kC(t)e^{-kt} + \frac{Q_{in}s_{in}(t)}{V_L}$$

Differentiate the equation (hint: use product):

$$\frac{dC(t)}{dt}e^{-kt} - kC(t)e^{-kt} = -kC(t)e^{-kt} + \frac{Q_{in}s_{in}(t)}{V_L}$$
$$\frac{dC(t)}{dt}e^{-kt} = \frac{Q_{in}s_{in}(t)}{V_L}$$

Solve for C(t) use integration:

$$\frac{dC(t)}{dt} = \frac{Q_{in}s_{in}(t)}{V_L}e^{kt}$$
$$C(t) = \int \frac{Q_{in}s_{in}(t)}{V_L}e^{kt}\,dt$$
$$C(t) = \frac{Q_{in}s_{in}(t)}{kV_L}e^{kt} + C_1$$

Place back into original function where C was parameterized, to get solution:

$$s(t) = (\frac{Q_{in}s_{in}(t)}{kV_L}e^{kt} + C_1)e^{-kt}$$
$$Where,\ k = \frac{(Q_{out} + K_m)}{V_L}$$

Appendix E In Class

In this in-class activity we will use MATLAB to solve the first order differential equation that was discussed in the cardiovascular modeling background reading. There is real-world patient data that is supplied for you that you will load into MATLAB.

To complete this in-class activity, you will need to download the following files from Brightspace:

- ODESolverStart.m
- ODEfunc2Start.m

Expectations: By the end of the class period, your group will be turning in a single MATLAB script to the appropriate folder on Brightspace as well as a single physical worksheet (this sheet) showing all your work.
Set-up:

1. Complete the MATLAB files, ODESolverStart.m and the ODEfunc2Start.m to solve Equation 8 in the background reading materials. For initial model parameters, we will use the following values:
   - Capacitance: 1.2 mL/mmHg
   - Resistance: 1.0 mmHg·ms/mL

   Hint this means that you will need to create multiple figures:
   - Modeled pressure vs time
   - 2-resistor modeled pressure vs. time.
   - Modeled pressure vs. time and a 2-resistor modeled vs. time on a single plot
2. Now we will explore the effects of changes to the Resistance and Capacitance using the Single resistance based model using a fixed input flow, Qi(t), using Example1 data set. To isolate the effect of altered resistance or capacitance, we will hold either the resistance or capacitance constant. Note that the ODESolverStart.m script contains functions to find the minimum, maximum, and mean of both the modeled and measured data sets. **Complete the following tables:**

(a) Let's hold C constant at 1.2 mL/mmHg and fixed $Q_i(t)$

R (mmHg*s/ml)	Pressure Maximum (mmHg)	Pressure Minimum (mmHg)	Pressure mean (mmHg)	Pulse Pressure $(max - min)$ (mmHg)
0.50				
0.75				
1.0 (BL)				
1.25				
1.5				
1.75				
2.0				

(b) Let's hold C constant at 1.2 mL/mmHg and fixed $Q_i(t)$

C (ml/mmHg)	Pressure Maximum (mmHg)	Pressure Minimum (mmHg)	Pressure mean (mmHg)	Pulse Pressure $(max - min)$ (mmHg)
0.6				
0.9				
1.2 (BL)				
1.5				
1.8				
2.1				
2.4				

(c) Using the data in the previous 2 tables, plot the percentage change in maximum, minimum, mean, and pulse pressures (referenced to baseline values identified in the table) as a function of percentage change in R (referenced to baseline value) in a single plot.

How does changes in R affect various aspects of pressure (minimum, maximum, mean, and pulse)?

(d) Now repeat part c, but this time plot the percentage change in maximum, minimum, mean, and pulse pressures (referenced to baseline values identified in the table) as a function of percentage change in C (referenced to baseline value) in a single plot.

How does changes in C affect various aspects of pressure (minimum, maximum, mean, and pulse)?

3. Analytically demonstrate that the modeled system is linear (Hint: Use law of additivity and homogeneity).

Appendix F Out of Class

In this homework assignment, we are going to continue working with the cardiovascular modeling in-class activities and dig a little deeper to continue our exploration. Be sure to have your MATLAB files accessible and notes from the in-class exercise to assist you in completing this assignment.

1. So far in our modeling journey, we have used modes to perform 'what if' kind of experiments. Models can also be used for system identification. For example, if you have measured pressure and flow data you may interpret the physiological status of the cardiovascular system in terms of resistance and capacitance. The model that you completed in class calculate the arterial pressure, P a (t), given measured volumetric flow rate, Q i (t) and parameter values R and C.

   As a first step, adapt the code to create a plot of the measured P a (t) and modeled P a (t) using the measured data from Example1.txt file (Patient 1). After you adapt the code, alter both the Resistance and Capacitance values such that the modeled data fits the measured data better. (Hint: your model is very simple so it will not be an exact match. A good fit is determined by visual inspection with matching peaks and valleys of the pressure waveform.

(a) Complete the following table using the data from Examples 1-3 which corresponds to Patients 1-3. Notice that R and C are provided for Example data sets 2 and 3. Use these fits as models to consider how good the visual fit needs to be.

	R Estimated (mmHg*s/mL)	C Estimated (mL/mmHg)	SVR (mmHg*s/mL)
Example 1			
Example 2			
Example 3			

(b) Generate and label plots of Patient 1-3 measured and modeled arterial pressure vs. time plots. Clearly label the modeled vs. measured patient data and all axes.

(c) How does the modeled parameter R compare with the Systemic Vascular Resistance (SVR)? Hint: Remember SVR = mean measured pressure/mean measured flow.

(d) Let's assume that Example 1 is representative of a normal patient's systemic arterial circulation. How do the physiological conditions corresponding to Example 2 and Example 3 differ from this normal condition? Hint: A complete answer will include potential diseased states associated with the altered R and C values provided in part a.

2. Now that you have model the system and considered both the 'what if' and the systematic characterization, you are tasked with creating a set of rules to help guide a physician in distinguishing abnormalities in arterial pressure caused by changes in arterial compliance (C) vs. changes in arterial resistance (R) if the input flow rate, Q i (t) remains constant for all conditions. Hint: Be sure to cite figures that you have generated from the model to support your rules.

3. Now we are going to model the 3-element Windkessel model (see Cardiovascular background reading for more information). This model adds a third element, a resistor, to improve the fit of the model vs. measured arterial pressure dynamics. Modify your existing MATLAB code to include this additional resistance. Assign the following values for each of the components:

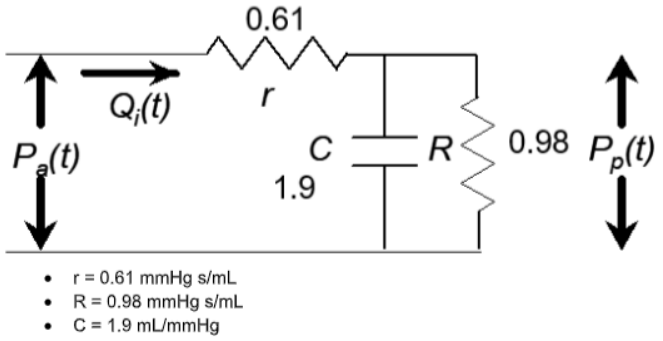

Figure 5: RC circuit for Windkessel model

Use the following values of:

- r = 0.61 mmHg*s/mL
- R = 0.98 mmHg*s/mL
- C = 1.90 mL/mmHg

(a) Plot the modeled vs. measured arterial pressure curves and clearly label the axes and the curves.

(b) How does the parameter values compare to the parameter values in the 2-element Windkessel model? Hint what does (r + R) in the 3-element Windkessel model mean? How does the capacitance values compare?