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Learning Tool to Enhance Understanding of Stress States and Mohr's Circle

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Work-In-Progress: Learning Tool to Enhance Understanding of Stress States and Mohr's Circle

Abstract

Mohr's circle is commonly introduced in an introductory Mechanics of Materials course as a graphical tool to analyze stress and strain in materials. Yet, many students find this graphical tool challenging to understand and apply. It is an abstract graphical visualization of stress and strain that may not seem directly related to the stress state of a physically loaded structure. To strengthen and deepen students' understanding, a MATLAB app has been developed and a handheld tool has been designed which allows students to explore stress transformations and Mohr's circle through experiential learning. When the handheld tool is manufactured and linked to the custom, publicly available MATLAB app, real-time feedback of the stress state of the physically loaded structure will be provided to students. The handheld tool has been designed to allow students to load the structure in axial, torsional, and flexural loading. This paper outlines the functionality and features of the app, including 2D and 3D Mohr's circle generation and stress state visualization. It also outlines the design of the initial prototype of the handheld tool including details of material selection, electronic configuration, and cost. Future work includes manufacturing of the handheld tool, expansion of capabilities of the app, and a study assessing the tool's effectiveness.

Introduction

First devised as a method to visualize 2x2 and 3x3 matrices by Christian Otto Mohr in the late 1800's, Mohr's circle has since become a foundational, visual tool for mechanics students working to understand the stresses at play at derived points in materials [1]. Undergraduate engineering students are commonly introduced to Mohr's circle in their Mechanics of Materials class as an analytical tool included in the lessons on stress transformations. The basic idea behind Mohr's circle is that normal and shear stresses on a plane within a material depend on the orientation of that plane [2]. Through graphical representation, Mohr's circle simplifies the process of reorienting a given planar section of material to obtain the normal and shear stresses at the new orientation. It also enables the student to identify the principal orientation and identify the principal stresses, all without the need for extensive calculations. Despite the seemingly straightforward approach of the Mohr's circle method, many students find this graphical approach abstract and struggle to apply this relationship to stress states of physically loaded structures [4].

Central to the curriculum in engineering is the intent to link the physics, mathematics, and theory being taught in the classroom to phenomena in the real world. First credited as an educational theory to John Dewey in 1938, experiential learning advocates for direct experiences with the topics being studied, rather than simply discussing and considering them [3]. To strengthen their understanding through direct experience, a MATLAB app has been developed which allows students to explore the connections between Mohr's circle and the orientation of a stress element and location on a loaded structure. To further deepen this educational experience, a handheld tool has been designed to translate direct, physical inputs imposed on the tool by the user into the app. Design, implementation, and effectiveness of similar computational and physical tools have been the subject of prior research [1, 4, 5, 6], yet their implementation in classrooms remains limited. This work aims to improve the accessibility of an interactive tool for stress transformation analysis by offering a cost-effective and easily manufactured handheld tool coupled with a publicly available user-friendly app.

Functionality and features of the app

A MATLAB GUI has been developed through MathWorks MATLAB App Designer [7]. The user interface of the application is designed with a tab-based navigation system. As shown in Figure 1, the user can easily navigate between five selections - 2D, 3D, cuboid, cylinder, and hollow cylinder. The 2D and 3D selections are designed to introduce students to Mohr's circle in two- and three-dimensions. The cuboid, cylinder, and hollow cylinder selections are designed to strengthen a student's connection of the abstract graphical visualization of Mohr's circle to the stress state of a loaded structure.

The interface for the 2D and 3D selections exhibit the same features with the 3D selection incorporating added complexity for the three-dimensional analysis. As such, the 3D selection is discussed below and is shown in Figure 1 for illustrative purposes. The 3D selection prompts students to input the state of stress of a three-dimensional differential cube. Specifically, it prompts students to input values for normal stresses along the x-, y-, and z-directions (σ_x, σ_y, and σ_z) and shear stress acting on the x-y, y-z, and z-x planes (τ_{xy}, τ_{yz}, and τ_{zx}). The 'Simulate' button activates the calculation process and generates the 3D Mohr's circle, differential cube, output table, and stress matrix. The output table displays the three principal stresses and absolute

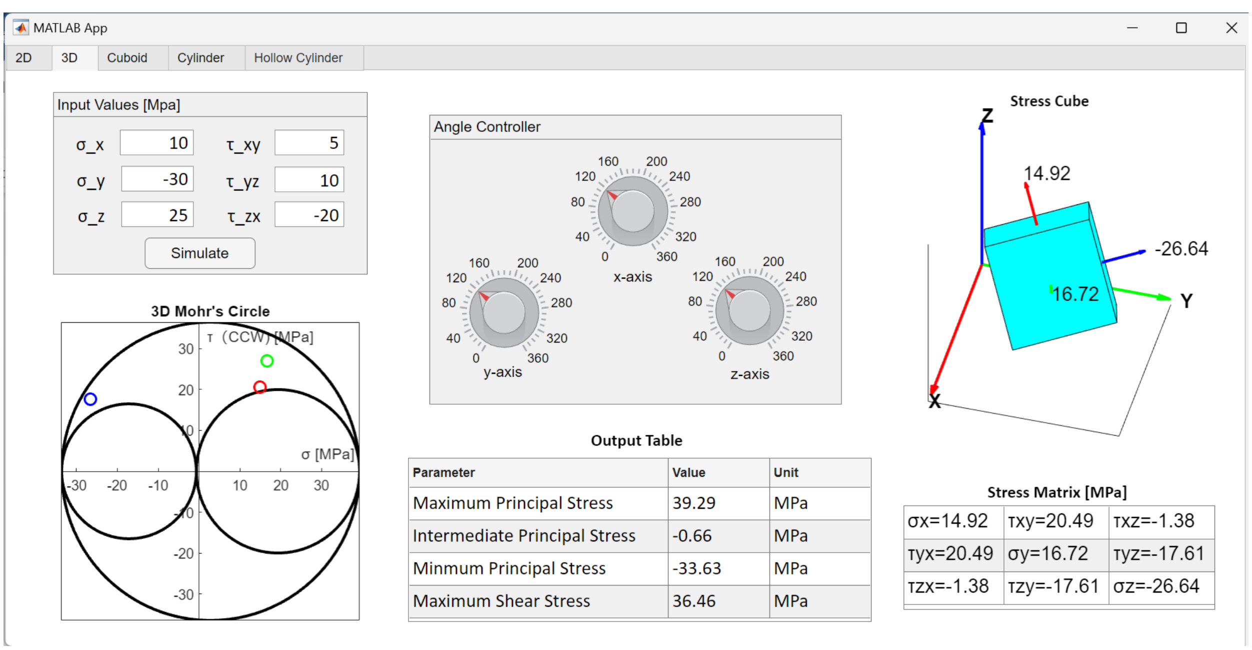


Figure 1. Image of interface for 3D selection showing input parameters, stress cube, angle controllers, 3D Mohr's circle, output table, and stress matrix.

maximum shear stress. Initially, the stress matrix and differential cube will display the user inputted values. The differential cube displays the normal stresses on the positive cube faces. Students use the dials available in the interface to vary the transformation angle of the differential element about the x-, y-, and z- axes. As the transformation angle varies, the app updates the stress matrix values and the corresponding value of normal stress on the rotating differential cube.

Recall the rotation of the differential cube about a principal axis will correspond to a circular arc shown on the 3D Mohr's circle. However, when the axis of rotation is not a principal axis, the stress state of the differential cube will lie between the three Mohr's circles [1]. To clearly show this for students, the normal and shear stresses for the positive cube faces (+x, +y, and +z) are displayed as small red, green, and blue circles, respectively, on the 3D Mohr's circle. These circles are updated as the transformation angle is varied. Note that the magnitude of the resultant shear stress vector on each positive cube face is utilized to plot the small circles [1, 6].

The cuboid, cylinder, and hollow cylinder selections exhibit the same features for different geometries. As such, the hollow cylinder interface is discussed below and shown in Figure 2 for illustrative purposes. For the hollow cylinder selection, students input the outer and inner radii, height of the cylinder, and applied torque. The torque is assumed to be applied uniformly at the ends of the hollow cylinder. The 'Simulate' button activates the calculation process, rendering both a 3D model of the structure and a corresponding cross-section diagram. To identify a location of interest within the hollow cylinder, a student selects a specific height using the slider feature on the right-side of the 3D model and clicks on a radial location within

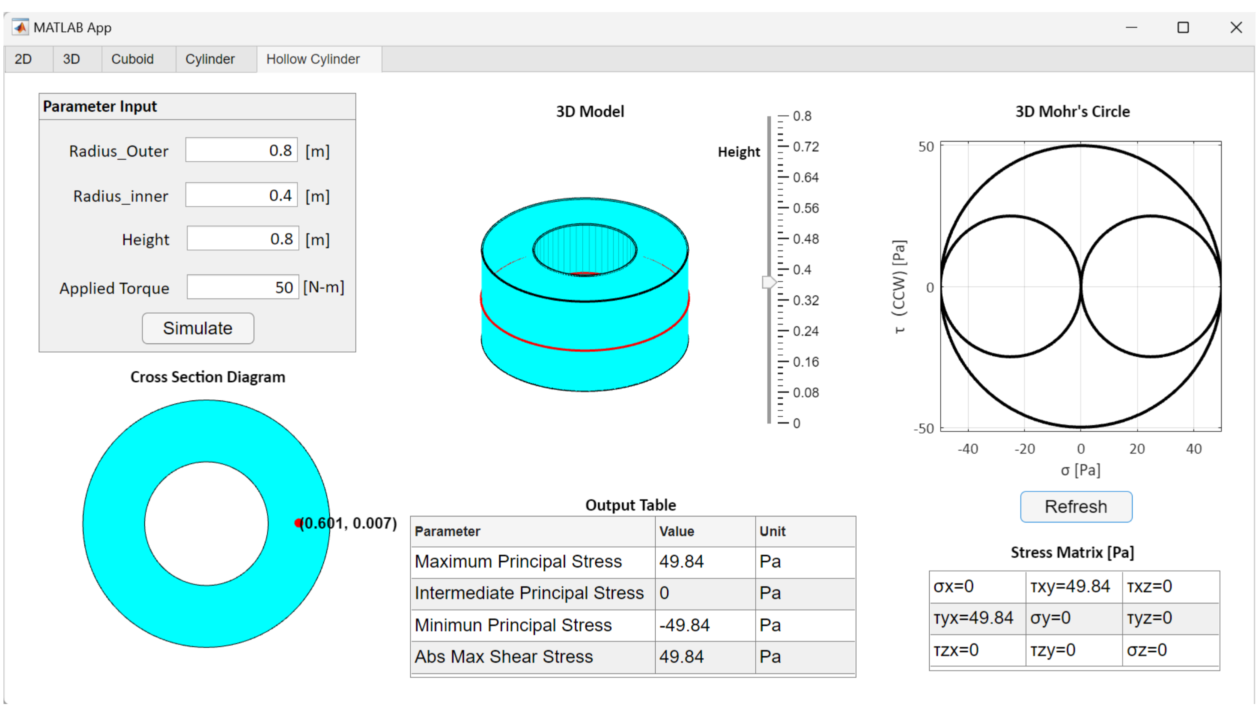


Figure 2. Image of interface for hollow cylinder selection showing parameter inputs, 3D model, 3D Mohr's circle, cross-sectional diagram, output table, and stress matrix.

the cross-sectional diagram. This identifies a specific coordinate within the space of a 3D model to explore the stress state.

By clicking on the "Refresh" button, the 3D Mohr's circle, output table, and stress matrix are updated for the selected coordinate. The output table displays the three principal stresses and absolute maximum shear stress. These resultant values are essential for exploring stress states for different user-selected locations and their connection to potential failure of the material. For example, a student can sequentially select the inner and outer radius of the hollow cylinder. Upon selecting “Refresh”, the 3D Mohr’s circle will grow, displaying to students not only the increase in the shear stress at the new location of interest, but also the increase in the maximum principal stress. Through user-driven exploration, the location of the maximum principal stress in the material can be identified and further connected to failure criteria. Note that while the height selection does not currently impact the output results, this feature has been established for use in future work.

The app alone serves as a useful tool for exploration of stress transformations and Mohr’s circle. It directs students to connect these fundamental ideas to loaded structures. To deepen this connection and strengthen the learning experience, a handheld tool has been designed to translate direct, physical loads imposed on a material by the user into the app. Therefore, rather than having students input loads and geometric parameters, users will explore the stress state physically imposed on a material in axial loading, torsion loading, and bending.

Design of the handheld tool

A SolidWorks rendering of the handheld tool is shown in Figure 3. The handheld tool will translate physical deformations of a sample material into inputs in the app. Inspired by the device created by Moller et al [4], the handheld tool will capture axial, torsional, and bending deformations each with a rosette of strain gauges. The strain gauge signals will be amplified by a microcontroller housed in one of the device handles and passed to the app running on a PC via USB.

While the device created by Moller et al utilized an arm-mounted apparatus to lockout undesired deformation modes during operation [4], the handheld tool will accomplish this by way of removable pieces (the central steel shaft and pins) and interlocking elements (slots and holes in the central steel shaft and keyed features in the handles). The removable central steel shaft with a machined slot and removable pin are shown in Figure 4(a). Figure 4(b) shows the other end of the central steel shaft with a machined hole and another removable pin. The pins couple the handle and sample material to the central steel shaft. The keyed rectangular feature shown in Figure 4(c) couples the handles to the sample material.

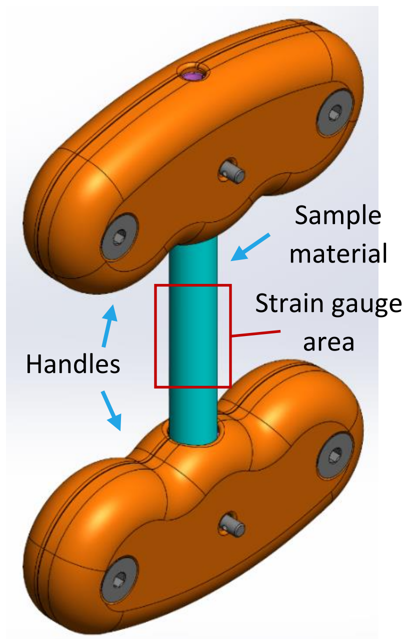
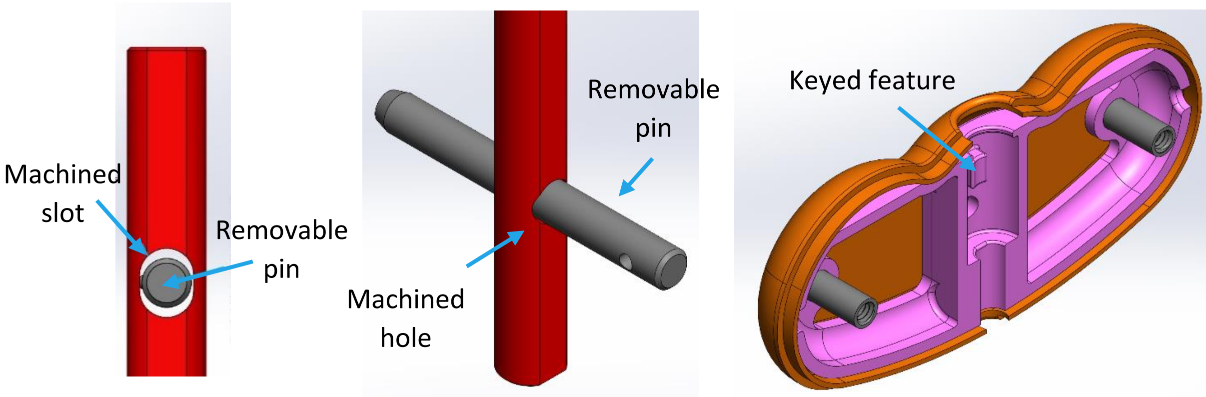


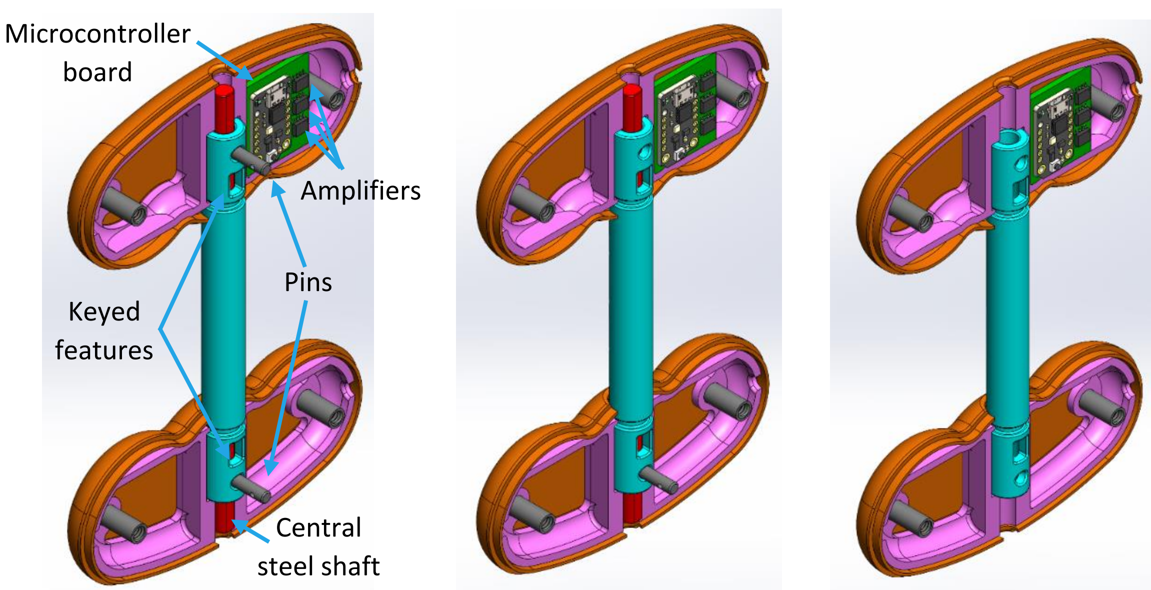
Figure 3. SolidWorks rendering of the handheld tool.

When the central steel shaft and both retaining pins are in position within the handheld tool as shown in Figure 5(a),



(a) (b) (c)

Figure 4. SolidWorks rendering of the central steel shaft slot with pin (a), central shaft hole with pin (b), and keyed feature of the steel inner handle structure (c).



(a) (b) (c)

Fig. 5. SolidWorks renderings of configurations for axial deformations (a), torsional and axial deformations (b), and all three deformation modes (c).

torsion and bending deformations will be prevented by the pins but the machined slot in the central steel shaft will allow axial deformations. When one pin is removed and the other remains installed as shown in Figure 5(b), torsional and axial deformation will be possible. When the central shaft and both pins are removed as shown in Figure 5c, axial, torsional, and bending deformation will be possible.

Deformation in each of the three modes will be captured via a full Wheatstone bridge composed of four VPG 240UZA strain gauges. The signal from each set of gauges will be run through a AD627BRZ-R7 amplifier before being digitized by an Atmel ATSAMD21 microcontroller and passed to a connected PC. The 240UZA gauges are designed to measure deformation on the

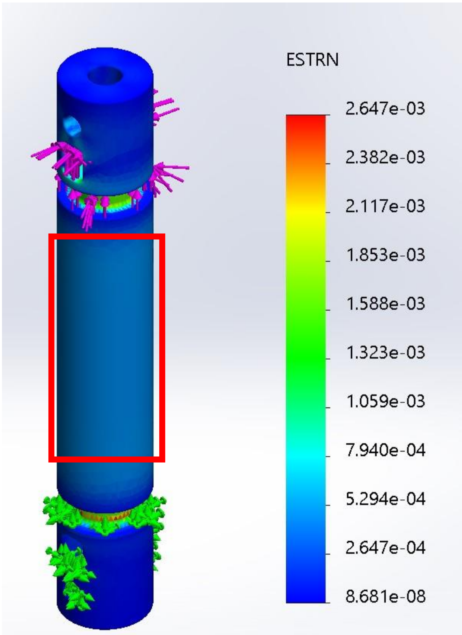


Fig. 6. Strain results for nylon substrate under 40 lbf tension.

Table 1. Estimated cost of handheld tool.

Component	Qty	Cost [$]	Total [$]
Central steel shaft	1	7.52	7.52
Sample material	1	0.77	0.77
Handle - inner	4	5.86	23.45
Handle - shell	4	5.00	20.00
Retaining pin	2	4.30	8.60
Binding barrel	4	3.34	13.37
Microcontroller board	1	8.95	8.95
Amplifier	3	11.97	35.91
Strain gauges	12	2.50	30.00
			148.57

order of hundreds of microstrain. A finite element analysis (FEA) was performed in SolidWorks on various sample materials with varying sizes in tension and torsion to determine a suitable material and geometry for the desired strain levels. Analysis of a nylon substrate 5 in. in length with inner and outer diameter of 0.26 and 0.75 inches, respectively, is shown in Figure 6. Under a tensional load of 40 lbf. the strain in the strain gauge area (indicated by the red box) was within the range of the gauges. Higher areas of strain were observed around the features interacting with the inner steel handle structure. Comparable results were observed under a torsional load of 30 in-lbf. As such, a nylon substrate will be utilized as the sample material in the handheld tool.

The component costs for the initial prototype of the handheld tool are shown in Table 1 and estimated to total approximately $150. Currently, the components of greatest cost include the amplifier, strain gauges, and material for the handles. Cost savings are anticipated as prototypes are tested, device production is scaled to manufacturing levels, and hardware is finalized. Machining and assembly costs are still to be determined.

Future Work

The app is under continual development for improvements in ease of use and expanded functionality. Future updates for the app include modeling of point loads and stress concentrations. The first prototype of the handheld tool is currently in development. When linked to the app, a new tab will be populated with the geometry and applied load from the physical device. Students will explore stress states at selected points similar to the cuboid, cylinder, and hollow cylinder simulations. This feature will be paramount for creating connections between actual physical loads, 3D Mohr's circle, and failure criteria.

Once manufacturing and testing of the handheld tool are complete, a study of the effectiveness of the app and the app coupled with the handheld tool will ensue. The study will measure improvement in students' intuitive understanding of the connection between the load's magnitude and stress, their ability to anticipate the way in which stress varies with transformation angle, their capability to identify principal stresses, and their proficiency in calculating maximum shear stress. Results from this study will be utilized in future iterations of the app and handheld tool.

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