The Future of Engineering Education
2024 Annual Conference & Exposition
Oregon Convention Center
Portland, OR . June 23 - 26, 2024
ASEE
Paper ID #42504

Integrating Precalculus into Calculus II and Its Outcomes

Dr. Meiqin Li, University of Virginia

Dr. Li obtained her Ph.D. in Applied Mathematics from Texas A&M University-College Station in 2017. Dr. Li holds a strong interest in STEM education. For example, she is interest in integrating technologies into classrooms to bolster student success, creating an inclusive and diverse learning environment, and fostering student confidence by redeveloping course curricula and assessment methods, etc. Beyond this, her research intertwines numerical computation, optimization, nonlinear analysis, and data science.

Stacie Pisano, University of Virginia

After receiving a Master of Science in Electrical Engineering from Stanford University, Stacie Pisano worked as an Electrical Engineer and Technical Manager at AT&T and Lucent Technologies Bell Laboratories for 16 years, designing and developing telecommunications equipment. She has been teaching at University of Virginia since 2002, and is currently the Director of the Center for Applied Math.

Jennifer Felder Marley, University of Virginia

Jennifer Marley is an Associate Professor of Applied Mathematics at the University of Virginia. She received the B.S. degree in Electrical Engineering from North Carolina State University and M.S. and Ph.D. degrees in Electrical Engineering: Systems from the University of Michigan.

Anne M Fernando, University of Virginia
Prof. Lindsay Wheeler, University of Virginia

Dr. Lindsay Wheeler is Senior Associate director and Assistant Professor in the Center for Teaching Excellence at the University of Virginia. Dr. Wheeler develops, implements, and researches educational development programs and courses to support instructors in redesigning courses.

Integrating Precalculus into Calculus II and Its Outcomes

Introduction

In many calculus courses, a deficiency in precalculus skills frequently poses obstacles to student achievement. At the University of Virginia, lots of students embark on calculus courses immediately after high school graduation, each with varying degrees of precalculus readiness. As experienced instructors of Calculus over many years, we've consistently observed that weak precalculus proficiency often undermines student success. Many students repeatedly commit precalculus-related errors in their calculus coursework, resulting in heightened academic hurdles and diminished self-assurance. Furthermore, in the aftermath of the COVID-19 pandemic, we've noted a decline in student preparedness for Calculus compared to previous years. Despite these challenges, practical limitations such as extended graduation timelines and financial constraints have hindered the implementation of a dedicated precalculus course.

To address these obstacles, the aim of this study is to understand the impact of accessible precalculus practice opportunities to all students, with the goal of enabling them to enhance their precalculus skills without feeling overwhelmed. This objective was achieved by integrating precalculus instruction into the curriculum of Calculus II and assessing its outcomes.

Literature Review

There is acknowledgement that the math course entry point in curricula for engineering students may differ among students based on socio-economic and minority classifications. Those who are first generation college students, Underrepresented Minority (URM)[1] students, or those with fewer academic opportunities in secondary education may have different levels of preparedness than others who may place out of one or many of the calculus sequence courses and into courses beyond Calculus III into their engineering curricula. To address retention among all students, prior studies have considered the impact of Mathematics, and in particular, success in the Calculus sequence as a metric that may predict retention or graduation with particular focus on these underserved groups. Many engineering programs have Calculus I as the first required math course across undergraduate curricula, others start at Calculus II. Studies show that Pre-Calculus may have impact on retention, that is, a course not usually required at the college level may determine success for completion. Some studies point to college algebra as a reasonable prerequisite and that pre-calculus be the first required class for an engineering curriculum.

One study was unable to make strong conclusions due to low sample size. However, trends support addressing the math sequence and addressing the pre-calculus entry point for Calculus mastery and retention success. While some statistical analyses showed no significant improvement in retention and graduation when pre-calculus was the entry point, meaningful positive trends were observed [1]. Studies have been made on what might address socio-economy or educational inequity issues for entry points into engineering [2]. While some studies point to standard best practices for success in the calculus sequence, especially in the first year with Calculus I and II, as potential predictors of further success for engineers, definitive results for retention can be mixed,

[1] Defined as students who identify as Black, Hispanic, Native-Hawaiian, or Pacific Islander (i.e., groups historically excluded, marginalized, and underrepresented in U.S. higher education broadly and in STEM specifically)

including in cases where majority or lesser risk students are not differentiated from at risk students in engineering [3]. One study succinctly stated the growing concerns in this area: "The national need to transform STEM education is paramount, as evidenced by the persistent gap in STEM degree attainment between whites and minorities, which continues to be a wide chasm despite greater numbers of minority students entering into STEM studies as compared to ten years ago. This gap may be attributed in part to the systemic problem of a growing number of minority undergraduate students who are unable to continue their STEM studies because of their inability to pass pre-calculus, the gateway to calculus, which is a requirement" [4].

Where pre-calculus is identified as connected to engineering retention, the impact is greater for those who are from minoritized backgrounds in higher education. "Moving pre-calculus from a gatekeeper to a gateway course has implications for broad program impact in achieving equity in the workforce and has significant impact for increased throughput to graduation of STEM majors, especially Hispanic women and English Learners (ELs)." [4]

Notably, an elaborate study was implemented to address challenges to the Precalculus to Calculus II sequence through case studies [21]. Other mentions of Pre-Calculus towards Calculus success were discussed in the research on math efficacy. Math self-efficacy refers to an individual's beliefs about their ability to understand math concepts and solve related problems. Some studies report that feelings about math are an important component of general engineering self-efficacy, and across the country engineering retention rates are often low and highly correlated with calculus performance [5]. Pre-Calculus and Calculus are acknowledged as indicators of retention for engineering students, in particular for students who have been historically excluded or marginalized in higher education in the U.S.

Given the significant impact on retention and inclusion that the Calculus sequence has in engineering curricula, research has focused on identifying the specific aspects of college-level mathematics courses and of engineering programs in general that contribute to or detract from student success. One study found that both years of study and choice of major were contributing factors to the self-efficacy and motivation of students in calculus. Students earlier in their college career and pursuing STEM degrees had higher self-efficacy and expectations of their math performance [6]. This knowledge should inform pedagogical approaches in the Calculus sequence to foster self-efficacy, which is known to impact learning and overall success. Another study found that the amount of unmet financial need, whether students were admitted directly to an engineering program, length of study at an institution, and age at time of study were factors explaining why students did (or did not) graduate with an engineering degree. Moreover, they found that these factors impacted students identifying as female and/or URM more significantly. Furthermore, the first mathematics course into which students are placed and their overall preparation for college-level math courses impacted the likelihood of completing an engineering degree [7]. This highlights the challenge of identifying a specific barrier to completing an engineering degree and motivates interventions designed to improve student persistence across all student groups. Perhaps the most relevant finding in support of the research discussed in this paper is that increasing the frequency of asking students to retrieve precalculus skills improves their retention across subsequent semesters, better preparing them for courses later in their plans of study and to complete their engineering degrees [8]. It should be noted that while that study investigated the frequency of retrieval in a precalculus course, it follows that incorporating space retrieval within later mathematics courses would benefit students.

Considering the impact of calculus courses on retention and persistence within engineering programs, there is a clear need for intervention strategies to aid students in these courses, ensure their preparation for future courses, and help them build connections with their chosen career. Themes from a comprehensive literature review revealed that many research efforts in improving calculus courses focused on improving the relevance of these courses, strengthening the connections between mathematics and engineering faculty and coursework, increasing the use of active learning strategies, and utilizing non-traditional modes of teaching (e.g. project- and problem-based learning or peer teaching) in calculus. While there are consistent themes among a variety of institutions, a consistent and accessible set of course materials for such interventions remains lacking [3].

Among the most time- and resource-intensive interventions is creating new curricula within engineering programs. Our prior work is one such approach, in which three "tracks" were created to meet the needs of students with different mathematics preparation levels [9]. Additionally, the mode of instruction within individual courses should also be considered when designing larger curricula. In one study, the impact on meeting student psychological needs of large, in-person, active vs hybrid online vs traditional lecture courses were investigated. This study illuminated the importance not only of understanding how each mode of teaching impacted student autonomy, connection to material, motivation, and ability to participate, but also that each had unique challenges to meeting the needs of students [10]. Another approach identified in the literature is implementing first-year math curricula that are designed to highlight to students early in their studies the importance of these courses in their chosen engineering disciplines. Traditional math topics such as systems of equations, trigonometry, and sinusoids, were taught in the context of analyzing engineering systems such as robotics and electrical circuits [11]. With a similar goal of more closely integrating mathematics and engineering content, others co-taught mathematics courses among math and engineering/physics/chemistry faculty [12]. Each of these examples requires not only coordination among various courses but also among departments, involving significant administrative challenges.

Several institutions have instead redesigned specific calculus courses to improve outcomes of subsequent mathematics and engineering courses. An example is designing Calculus I around a semester-long rollercoaster design problem. In completing this project, students iteratively solved an authentic problem and learning calculus in an authentic context [13]. A different approach is to teach Calculus II in three modules, each of which asks students to solve a different engineering design problem. Each module contains a variety of motivating examples, as well as 'mini projects'. The goal was to improve the relevance of Calculus II material to engineering students and increase their motivation [14]. Another team implemented a blended precalculus/calculus corequisite course, as opposed to a separate precalculus course, to improve retention [15]. Course design can take a much broader approach, addressing not only broader choices of learning objectives, but also the active learning strategies involved in the daily lessons. A complete redesign of Calculus II created new content emphasizing relevant applications and reducing the scope of a traditional course, active learning modules for each class period, and a 'community of practice' to offer this new course [16].

When changing an entire course is not possible or not necessary for a program, an alternative is modifying specific calculus course content to foster deeper connections between students and their coursework. Examples of this include having engineering faculty visit mathematics courses to present applications of course material and including authentic examples of engineering in mathematics courses [3]. Without changing the content of a traditional Calculus I and Calculus II courses, another approach was to integrate a variety of engineering applications such as robotics, power systems, and bioengineering in the examples within these courses, as well as pair students with engineering mentors. The goal of this approach was to help students appreciate the value of calculus in engineering [17]. A third approach was to have students complete hands-on laboratory experiments during their mathematics course [18].

Though they seek a common goal, approaches such as redesigning courses, redesigning entire engineering programs, modifying the content of a specific course, and offering support resources such as mentoring and learning communities vary in terms of preparation time, level of buy-in from multiple instructors, and amount of required financial and time resources. Some may be better suited to different institutions or programs. *The novelty of the intervention offered in this paper is threefold. First, compared to more extensive approaches, it requires a relatively low amount of preparation on the part of instructors. Second, to the best of our knowledge, the approach of integrating precalculus concepts directly in the content of Calculus II through spaced repetition and recall has not been investigated. Third, few of the interventions described above offer evidence of efficacy or impact. The present study assesses the impact of precalculus skills on performance on Calculus II material.*

Research Questions

This paper aims to incorporate precalculus into the curriculum of Calculus II to strengthen students' precalculus skills and investigate its outcomes. We seek to address the following research questions (RQ):

1. How, if at all, do students' precalculus skills improve throughout the semester with this intervention? Do such improvements differ based on sociodemographic characteristics (i.e., gender, URM, first generation)?
2. To what extent, if any, does the integration of precalculus and prior precalculus/calculus knowledge associate with Calculus II performance?
3. How do students perceive the intervention? Do these perceptions differ based on factors such as sociodemographic, precalculus experience and calculus experience?

This research project is supported by HHMI Inclusive Excellence 3 (IE3) Learning Community at the University of Virginia.

Context

This IRB approved study occurred at the University of Virginia, a large, research-intensive, public, predominately white institution (PWI) in the Mid-Atlantic region of the U.S. At our institution, students apply to and are accepted directly into the School of Engineering & Applied Science. All engineering students are required to have credits for Single Variable Calculus II and Multivariable Calculus as part of the engineering curriculum, regardless of major. The Center for Applied Mathematics (APMA) within the School of Engineering & Applied Science (SEAS) is a group of passionate educators. They teach a variety of math courses to both graduate and undergraduate students from SEAS and conduct research on effective teaching methods and redesign course curricula to better meet students' need. Almost all students in APMA courses are pursuing engineering degrees, with a few students enrolled from outside SEAS. Instructors teaching APMA courses have Teaching Assistants (TAs) to help with grading, run office hours, and engage students during in-class group work. These TAs are either graduate students from other engineering departments or undergraduate engineering majors, who receive pedagogical support through a 1-credit Teaching Methods course (see [20] for a review of the course and its impact).

Because of the math requirements for engineering majors, the Center for APMA administers an online Placement Test (PT) to all incoming engineering students prior to the beginning of the Fall semester each year. Along with students' AP calculus scores obtained from high school, PT helps to place students at the best calculus level for them.

In APMA 1090-Calculus I, there are a couple of weeks of pre-calculus review at the beginning of the semester, but there is no such review in APMA 1110-Calculus II. Thus, we endeavored to weave pre-calculus skills into Calculus II and to provide an incentive for Calculus II students to invest in improving these skills.

Study Design and Methods

Intervention

To address challenges in covering pre-calculus content in Calculus II, we created a mastery-based component of precalculus where students learned and were assessed on precalculus skills throughout the semester. Our goal was to help the students re-activate their previously learned pre-calculus skills that they had forgotten and to learn the skills that they had not learned before. As such, our research team developed a comprehensive list of precalculus learning objectives such as simplifying expressions, applying Laws of Logs, determining limits, etc. The pre-calculus part in the PT was regulated by such list of learning targets. We also created five pre-calculus assessments, each comprising thirty multiple-choice questions, with one point awarded for each correct answer. These assessments were meticulously designed to align with the specified learning objectives outlined in the PT pre-calculus content and format (Table 1).

Table 1: Alignments for the learning objective, "can solve quadratic inequalities (N)"

Pre-calculus in PT	**Learning Version**	**Assessment 1**
Q10: Solve for x: $x^2 - 2x > 3$ A. $(-\infty,-1) \cup (3,\infty)$ B. $(-\infty,-2) \cup (-1,\infty)$ C. $(-1,3)$ D. $(-2,-1)$	Q10: Solve for x: $x^2 + 3x - 10 < 0$ A. $(-\infty,-5) \cup (2,\infty)$ B. $(-\infty,-5) \cup (5,\infty)$ C. $(-\infty,\infty)$ D. $(-5,2)$	Q10: Solve for x: $x^2 - 9x < -14$ A. $(-\infty,9) \cup (14,\infty)$ B. $(-\infty,2) \cup (7,\infty)$ C. $(2,\infty)$ D. $(2,7)$
Assessment 2	**Assessment 3**	**Assessment 4**
Q10: Solve for x: $x^2 + 9x > -14$ A. $(-7,-2)$ B. $(-\infty,2) \cup (7,\infty)$ C. $(-\infty,-7) \cup (-2,\infty)$ D. $(-\infty,-2)$	Q10: Solve for x: $x^2 + 2x < 8$ A. $(-\infty,-4) \cup (2,\infty)$ B. $(-\infty,-2) \cup (4,\infty)$ C. $(-2,4)$ D. $(-4,2)$	Q10: Solve for x: $x^2 + 7x + 12 > 0$ A. $(-\infty,\ \infty)$ B. $(-\infty,-4) \cup (-3,\infty)$ C. $(-4,-3)$ D. $(0,\infty)$

One of the five assessments was the “Learning Version”. With it, we displayed one question per class day and gave a short in-class review/demonstration of the skills involved. The four remaining assessments were used during the four designated assessment weeks during the semester within each of which students had an opportunity to pass one of four assessments (Table 1). The Pre-Calculus assessment was worth 5% of the final grade for Calculus II. If a student passed one of the assessments with a score of 24 (80%) or more, they would receive the full 5% toward their final grade.

Although we did not publish the four assessment versions to students, we did provide information about the learning targets for each question and each student was informed of their results for each question. They were permitted to view their missed questions during office hours. We enforced this security so that students couldn’t just memorize each question type and so that we can use the same assessments in future semesters.

Participants

A total of 263 students were enrolled in six sections of APMA 1110-Single Variable Calculus II and participated in the study in whole or in part. These sections were taught by four female instructors, all co-authors of this paper, with teaching experience ranging from seven to twenty-one years. Students in APMA 1110 in Fall 2023 were majority male, non-URM, and continuing generation (Table 2). These sociodemographic breakdowns are mostly like those of all engineering students as well as of all undergraduates at the University of Virginia, except for gender.

Table 2:
Sociodemographic overview of APMA 1110 participants as compared to the engineering school and institutional demographics

Demographic		APMA 1110 n (%)	Engineering n (%)	Institution n (%)
Gender	Male	161 (61.5)	1,998 (66.2)	7,473 (43.4)
	Female	101 (38.5)	1,018 (33.8)	9,732 (56.6)
Race	URM	49 (18.6)	434 (14.4)	2,583 (15.0)
	Non-URM	214 (81.4)	2,583 (85.6)	14,235 (82.7)
Generational status	First generation	53 (20.2)	447 (14.8)	2,635 (15.3)
	Continuing generation	210 (79.8)	2,570 (85.2)	14,565 (84.6)

Data Sources & Collection

Students’ placement test score, four pre-calculus assessment scores, APMA 1110 – Single Variable Calculus II final exam score, end-of-semester survey, and sociodemographic information served as the data used to answer the research questions. The handling of collected data strictly adheres to the university's policy regarding data management and regulated by IRB.

The placement test (PT) comprises thirty pre-calculus questions, along with fifteen questions each for Calculus I and Calculus II. Each question carries one point, resulting in a total of 60 points. The pre-calculus questions within PT were meticulously crafted in accordance with specific learning objectives, covering essential pre-calculus skills, particularly those areas that have historically proven to be weak for students. The outcomes of PT were shared with Calculus instructors, enabling them to pinpoint and address pre-calculus weaknesses of their students highlighted by the pre-calculus segment of the placement test.

The four precalculus assessments consisted of thirty multiple-choice questions each, with questions designed to mirror each other across assessments and align with the pre-calculus content in PT, based on the specified learning targets (Table 1). The Calculus II final exam, spanning three hours, was comprehensive in nature and encompassed various question formats, including multiple-choice, matching, and free-response problems. It covered essential topics in APMA 1110-Calculus II, such as integration skills, calculus applications like hydrostatic forces and moments, and series. All pre-calculus and calculus questions underwent multiple rounds of development and revision by the APMA 1110 instructors.

The course was fully coordinated by the course coordinator, Prof. Stacie Pisano, a seasoned Calculus instructor with more than twenty years of experience, who spearheaded the research project discussed in this paper. All students had access to identical teaching materials, including class notes, worksheets, and homework assignments. Additionally, they took uniform precalculus assessments and Calculus II exams in person across all sections.

The end-of-semester survey was administered to students during the last class of the semester via Qualtrics. It comprised 11 Likert scale questions aimed at gauging students' perceptions of the precalculus intervention. In the survey, students were asked to self-report the number of successful attempts they made on precalculus assessments, defined as achieving a score of at least 24 out of 30 (80%). These survey questions were crafted by the APMA 1110 instructors and reviewed by an expert in science education and survey design.

Study Methods

In Fall 2023, out of 263 students enrolled in APMA 1110, 262 attempted at least one precalculus assessment, with one student not participating. These 262 students were categorized into eight groups based on their performance on the assessments and the number of attempts made. Passing the assessment, defined as achieving a score of at least 24 out of 30 (80%), resulted in placement into groups labeled "P1" through "P4" depending on the number of attempts required to pass, for example, “P3” refers to the group of students who passed the assessment on their third attempt. Conversely, students who did not pass were classified "NP1" through "NP4" based on the number of attempts made without success, for example, "NP2” refers to the group of students who attempted the assessment twice but did not pass. The symbol "NP" collectively denotes students who did not pass the assessment, comprising "NP1" through "NP4" and the one who did not take any attempt, while the symbol “P” denotes the students who passed the assessment, comprising "P1" through "P4".

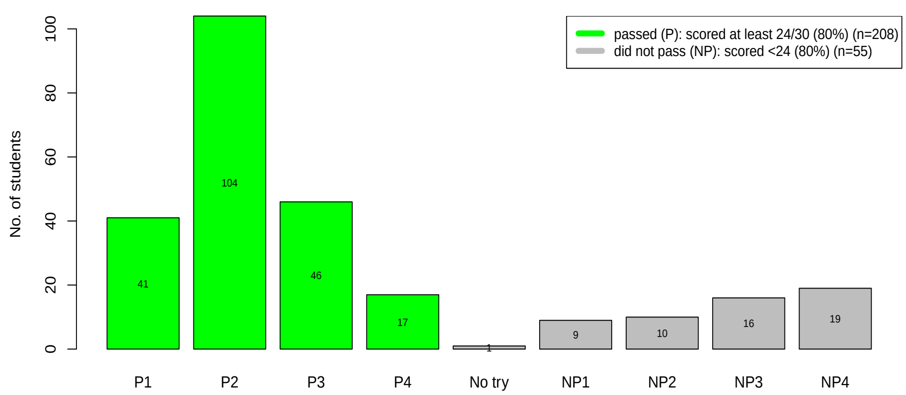


Figure 1: Counts of students who did (green) and did not (grey) pass the precalculus component by the number of attempts

As illustrated in Figure 1, out of the 262 APMA 1110 students who attempted the assessment, 208 (79.4%) achieved a score of 80% or higher on at least one of their four precalculus assessments (designated as "P"). Among these students, 41 accomplished this in just one attempt, while 104 required two attempts, 46 needed three attempts, and 17 utilized all four attempts. However, 55 students fell short of reaching the 80% threshold (designated as "NP"). It is noteworthy that some of these students did not utilize all four attempts, with one student abstaining from making any attempt.

We utilized an embedded mixed method approach with a quantitative emphasis [19] to collect and analyze survey and performance data, which is described in detail below.

To address research question 1, we undertook two main approaches. Firstly, we scrutinized pre-calculus assessment scores to discern any progression across multiple attempts, according to different groups (P1, P2, P3, P4, NP1, NP2, NP3, NP4) and demographic factors such as gender, generational status, and URM status. Secondly, we examined the evolution of students' precalculus proficiency from the "prior" to "post" stages. The precalculus component score from PT served as the "prior" mastery benchmark, while the student's last attempt score served as the "post" mastery indicator. In our analysis, we applied statistical techniques such as bar plots, paired t-tests, and one-way ANOVA. We examined whether all assumptions were met before applying the tests; if not, we used the corresponding non-parametric tests as the alternatives. For instance, we used paired t-tests when the sample size exceeded 30 or when the sample data displayed a relatively normal distribution. In cases where these conditions were not met, we utilized the nonparametric Wilcoxon test for paired sample data.

To explore research question 2, we pursued three approaches. Firstly, we examined whether there were significant differences in Calculus II final exam scores among different groups (P1, P2, P3, P4, NP) and based on demographic factors such as gender, generational status, and underrepresented minority status. Secondly, we investigated the linear relationship between the number of attempts students made on the assessment and their APMA Calculus II final exam scores. Lastly, we explored the relationship between students' participation in the PT and their Calculus II final exam scores, along with the relationship between different component scores of PT and their Calculus II final exam scores. Statistical techniques such as boxplots, one-way ANOVA (or Kruskal-Wallis test as the alternative), linear regression, and correlation tests were applied to analyze the data, with assumption being checked before applying the tests if there is any.

To answer research question 3, we used a subset of the 262 students who completed the end-of-semester perceptions survey (n=183, 69.8%). This data set included students' Likert survey responses, self-reported number of attempts to pass the precalculus assessment (PA), final Cal II exam scores, and sociodemographics. Because of the smaller sample size, we did not have large enough sub-groups of first generation and URM students for comparison; therefore, we created an 'underserved'[2] category that included students who identified as first generation, students who identified as Black or Hispanic, or any combination of these sociodemographics. All other students were identified as 'not underserved'. An exploratory factor analysis (EFA) was conducted to

[2] We use the term 'underserved' to identify students who fall in this group as those who have historically been excluded, marginalized, or not served by is of higher education.

identify factors within the 11 Likert questions and used the resulting factor, presented in the results, in subsequent analyses. These perceptions were analyzed by similar statical tools as mentioned above as well as by different factors such as “not underserved” vs “underserved”, precalculus experience and calculus experience.

Data Analysis and Results

Precalculus improvements throughout the semester (RQ1)

A total of 262 students took at least one attempt of the precalculus assessment. These students were categorized into two groups: "passed (P)" and "not pass (NP)". Within the "passed" group, there were subgroups P1, P2, P3, and P4, while the "not pass" group consisted of subgroups NP1, NP2, NP3, and NP4.

Firstly, we examined pre-calculus assessment scores to recognize any improvement across multiple attempts, according to different groups (P1, P2, P3, P4, NP1, NP2, NP3, NP4) and demographic factors such as gender, generational status, and URM status.

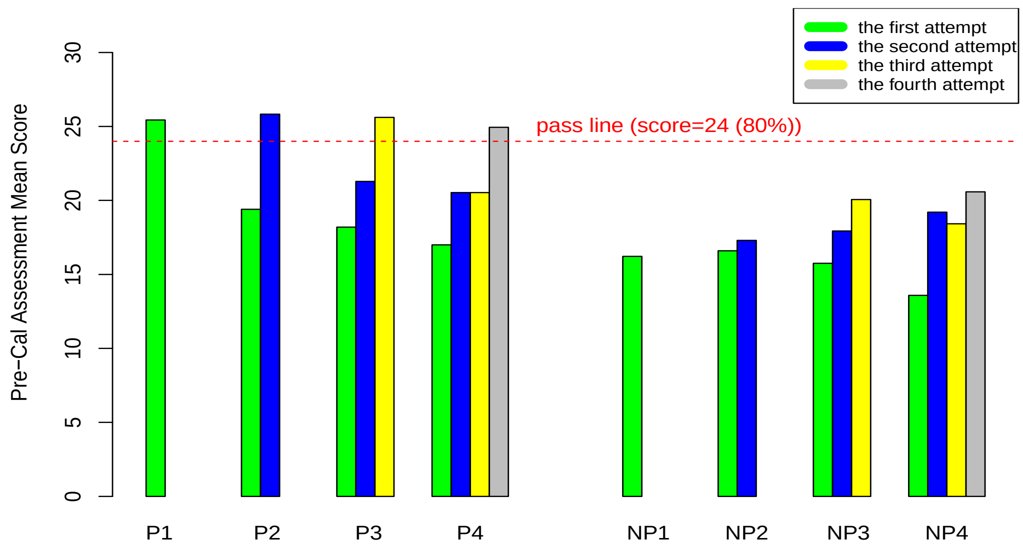


Figure 2: Pre-Calculus assessment mean score through different attempts for different groups

Based on the depicted graph (Figure 2), it is evident that improvements were observed through different attempt levels for both categories, namely "passed (P)" and "not pass (NP)". From the statistical analysis conducted (Table 3), there were significant improvements in assessment performance through the different attempts for subgroups P1, P2, and P3 within the "passed (P)" category. However, for subgroup P4, although the average score remained the same between the second and third attempts, it was still significantly evident that students performed better throughout the semester compared to their performance during the first attempt. In contrast, the improvement was not as pronounced for students within the "not pass (NP)" category. For instance, there was no statistically significant improvement for students in subgroup "NP2". Nonetheless, it is worth noting that most students who did not pass the assessment still exhibited a significant improvement when comparing their first attempt to their last attempt. Notably, the subgroup that did not pass the assessment but utilized all four attempts had the lowest average score for the first attempt. This could suggest that students in the "NP4" subgroup faced greater challenges when acquiring precalculus knowledge.

Table 3: Results of comparing the mean scores through different attempts for different groups

category	**subgroup**	**n**	**test used**	**attempt i → attempt j**	**mean difference $\mu_j - \mu_i$**	**p value**
P	**P2**	104	paired t-test	attempt 1 → attempt 2 ***	6.423077	p<2.0e-16 ***
	P3	46	paired t-test	attempt 1 → attempt 2 ***	3.086957	p<2.0e-16 ***
				attempt 2 → attempt 3 ***	4.326087	p<2.0e-16 ***
				attempt 1 → attempt 3 ***	7.413043	p<2.0e-16 ***
	P4	17	Wilcoxon test	attempt 1 → attempt 2 **	3.529412	.0022 **
				attempt 1 → attempt 4 ***	7.941176	.00027 ***
				attempt 2 → attempt 4 ***	4.411765	.0003 ***
NP	**NP2**	10	Wilcoxon test	attempt 1 → attempt 2	0.7	.635
	NP3	16	Wilcoxon test	attempt 1 → attempt 2	2.1875	.1
				attempt 2 → attempt 3	2.125	.073
				attempt 1 → attempt 3 *	4.3125	.025 *
	NP4	19	Wilcoxon test	attempt 1 → attempt 2 ***	5.631579	.00019 ***
				attempt 2→ attempt 3	-0.7894737	.2693
				attempt 3 → attempt 4 *	2.157895	.0204 *
				attempt 1 → attempt 4 ***	7	.00028 ***

****significant, p<.001; **significant, p<.005; *significant, p<.05.*

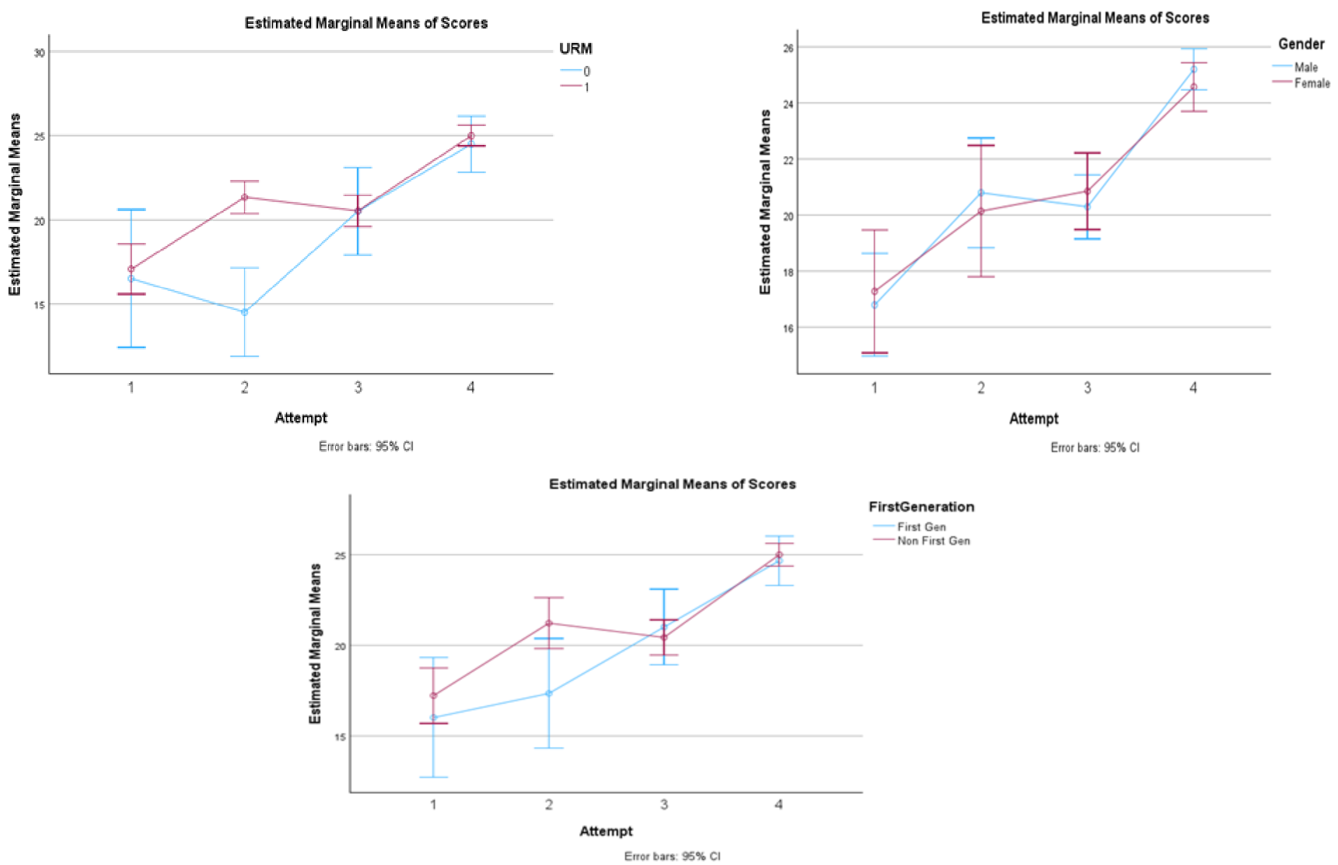


Figure 3: Trends in passing scores for P4 students (n=17) disaggregated by sociodemographics.

To further explore the improvement for students, we examined at trends in attempt scores by the subgroup (e.g., P1, P2) by gender, generational status, and URM.

For students that passed in two attempts (n=104), we found they significantly improved their scores from attempt 1 to attempt 2 ($p < .05$). However, we found no significant differences in improvement between attempts 1 and 2 when comparing groups by gender, first generation status, or URM. In other words, both male and female students who took two attempts to pass made significant improvements between the two attempts, but there were no differences between male and female in scores. These trends were similar for students that passed in three attempts (n=46). While the number of students that took four attempts is small (n=17), we observed descriptive differences in how students' scores changed over time for URM students compared to non-URM students. These differences were not observed for either male and female students or for first generation and continuing generation students (Figure 3).

Secondly, we examined whether there was a significant improvement of students' precalculus proficiency from the "prior" to "post" stages. The precalculus component score from PT served as the "prior" mastery indicator, while the student's last attempt score served as the "post" mastery indicator. Both scores were obtained using the same precalculus learning target list with different versions of problems aligned with each other, so they were comparable.

A total of 241 students completed both the placement test (PT) and at least one attempt of precalculus assessment in Fall 2023. Out of these students, 199 successfully passed the assessment while 42 did not. Upon examining the mean scores of PT for all subgroups within the "not pass (NP)" category, we noticed that they were all quite similar. Therefore, we treated all students who did not pass the assessement as one group ("NP") without further subgroup division. Note that the number of students from each group here is different since some students enrolled in the course did not take PT.

From Figure 4 below, it became evident that students who passed the precalculus assessment (P1, P2, P3, P4) exhibited improved "post" mastery compared to their "prior" mastery. Conversely, students who did not pass the precalculus assessment (NP) did not demonstrate much gain in knowledge from the intervention when compared to their initial "prior" knowledge.

Table 4: Results of comparing "prior" and "post" Precal mastery for different groups

group	n	mean difference $\mu_{post} - \mu_{prior}$	p value
P1	38	4.18	.000633 ***
P2	99	5.42	p<2.0e-16 ***
P3	45	4.42	p<3.3e-08 ***
P4	17	5.24	.0054 **
NP	42	0.29	.7472

****significant, p<.001; **significant, p<.01*

The findings (Table 4) from applying paired t-tests for each group further confirmed the observations made from Figure 4. It was evident, that students who achieved a score of 80% or above in PA experienced a significant improvement in their skills following the intervention. On the other hand, the analysis revealed that students who did not reach the 80% threshold did not demonstrate significant progress because of the intervention (Table 4).

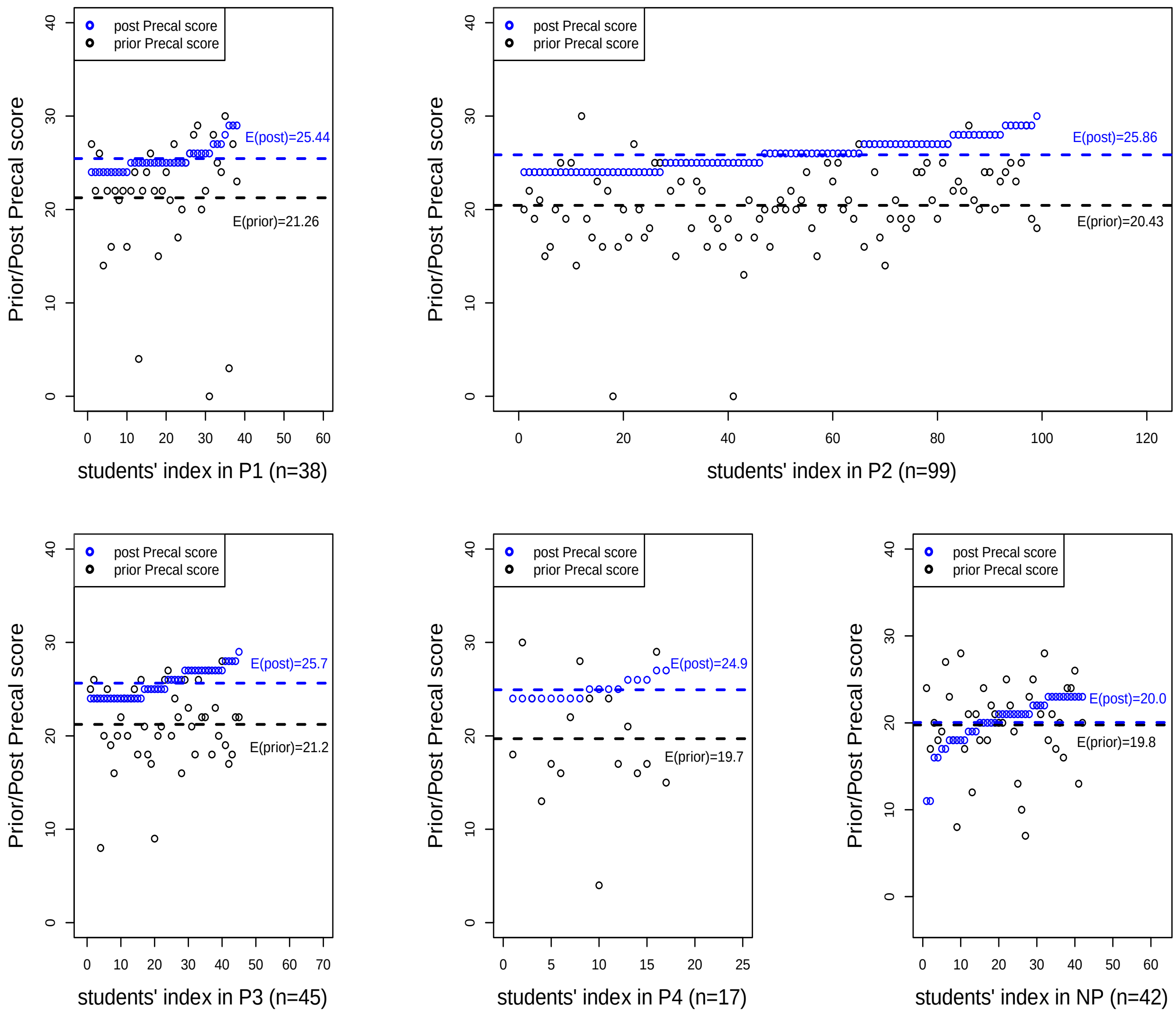


Figure 4: Comparison between students' mastery of "prior" and "post" precalculus knowledge for different groups

Relations between Integration of Precalculus and Prior Precalculus/Calculus Knowledge with Calculus II Performance (RQ2)

To explore relations between the integration of precalculus and prior Precalculus/Calculus knowledge with Calculus II performance, we pursued three approaches.

Firstly, we conducted an analysis to explore whether significant different in Calculus II final exam scores among groups of P1(n=41), P2(n=104), P3(n=46), P4(n=17) and NP(n=55). The final exam, which covered all core topics of the course with a total of 250 points, was used as an indicator of students' mastery of Calculus II skills. We observed that the mean scores decreased as the attempts increased (Figure 5). Additionally, the group of students who did not pass the precalculus assessment had the lowest final exam scores.

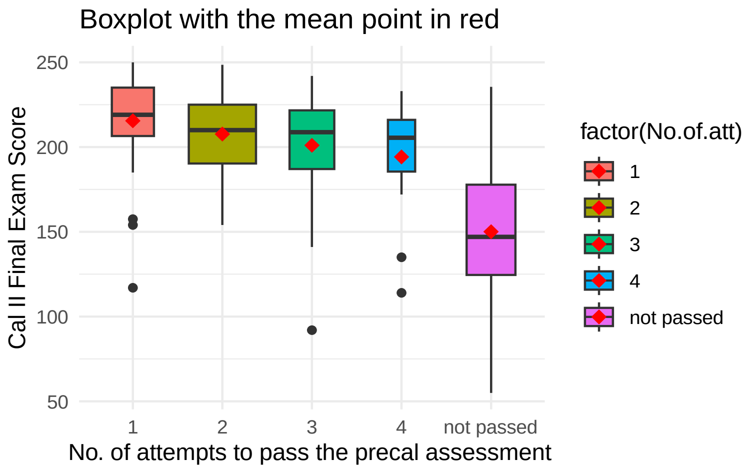


Figure 5: Boxplot of Cal II final exam scores for different groups

To obtain statistically reliable conclusions, we conducted a Kruskal-Wallis test, which revealed a significant difference in the Calculus II final exam scores among the different groups. The obtained p-value was 0, indicating a significant difference. Furthermore, since only the P4 group had a sample size of n=17, while the other four groups had sufficiently large sample sizes, we also conducted a one-way ANOVA test, which yielded the same conclusion as the Kruskal-Wallis test.

To further determine the specific groups that exhibited significant differences, we performed a post hoc test called Tukey's HSD (honestly significant difference) following the ANOVA test. Additionally, we conducted Dunn's test with the Bonferroni method following the Kruskal-Wallis test. Both post hoc tests produced the same conclusions. The results from Tukey's HSD test can be found in Table 5.

Table 5: Results of testing difference of Cal II final exam scores for each pair of groups

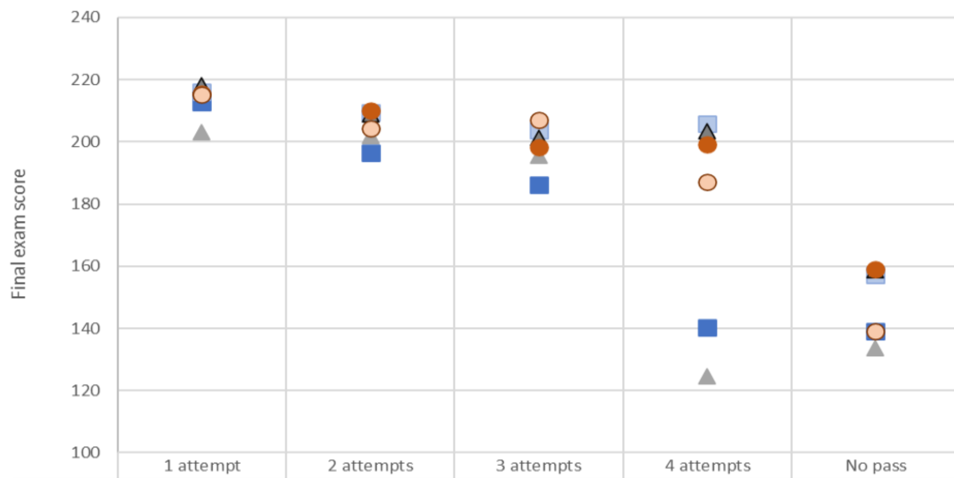
	P4-NP	**P3-NP**	**P2-NP**	**P1-NP**	**P3-P4**	**P2-P4**	**P1-P4**	**P2-P3**	**P1-P3**	**P1-P2**
mean diff	44.12	50.95	57.596	65.46	6.82	13.477	21.34	6.64	14.51	7.87
p adj	p< 3.05e-6 ***	p< 2.05e-13***	p< 1.40e-13***	p< 1.70e-13***	.932	.435	.107	.728	.171	.623

****significant, p<.001. The first row displays the difference of Cal II mean final exam scores for each pair of groups*

According to the analysis, it was determined that students who passed the precalculus assessment during one of the four attempts demonstrated significantly higher mastery of Calculus II skills compared to those who did not pass. However, there was no statistically significant difference in Calculus II mastery among the groups of students who passed the assessment, regardless of the number of attempts they made.

We also looked at differences in exam scores by number of attempts and sociodemographics (Figure 6). No significant differences existed in exam scores for any subgroups who took 1, 2, or 3 attempts to pass the precalculus assessment.

There were significant differences in final exam scores between URM and non-URM students who took four attempts to pass the precalculus assessment, $F(15) = 38.385$, $p<.001$, with URM students performing significantly lower on the final exam than non-URM students. These results were similar for first generation and continuing generation students $F(15) = 33.396$, $p<.001$ (Figure 6). Significant differences also existed in final exam scores between URM and non-URM students who did not pass the precalculus assessments at any point, $F(53) = 4.878$, $p<.032$ (Figure 6). No other significant differences were observed for the NP group.



	1 attempt	2 attempts	3 attempts	4 attempts	No pass
FG	212.83	196.5	186.21	140.33	139.18
non-FG	215.97	209.5	203.65	205.71	157.3
URM	203	201.92	195.5	124.5	133.74
non-URM	218.08	208.84	201.38	203.47	158.67
Male	215.66	209.98	198.3	199.1	159.05
Female	215.28	204.33	207.18	187.14	139.12

Figure 6: Results of comparing Students' Cal II final exam scores from different groups based on sociodemographics

Secondly, we conducted correlations test to analyze the relationship between Cal II final exam scores and the number of attempts taken to pass the precalculus assessment. The analysis was performed separately for students who passed the assessment and those who did not. The results indicated a significant correlation (p=.001) between the number of attempts taken and the performance in Cal II for students who passed the PA, with a correlation coefficient of -0.221 (p=.001) (Figure 6). However, no significant relationship (p=.3905) was found between the number of attempts student took and Cal II final exam performance for those who did not pass the assessment. Based on this analysis, we assigned a code of "5" to all students who did not pass the assessment and another correlation test was conducted, revealing a significant correlation (p=0) between the number of attempts students took and Cal II performance, with a correlation coefficient of -0.581 with p=0.

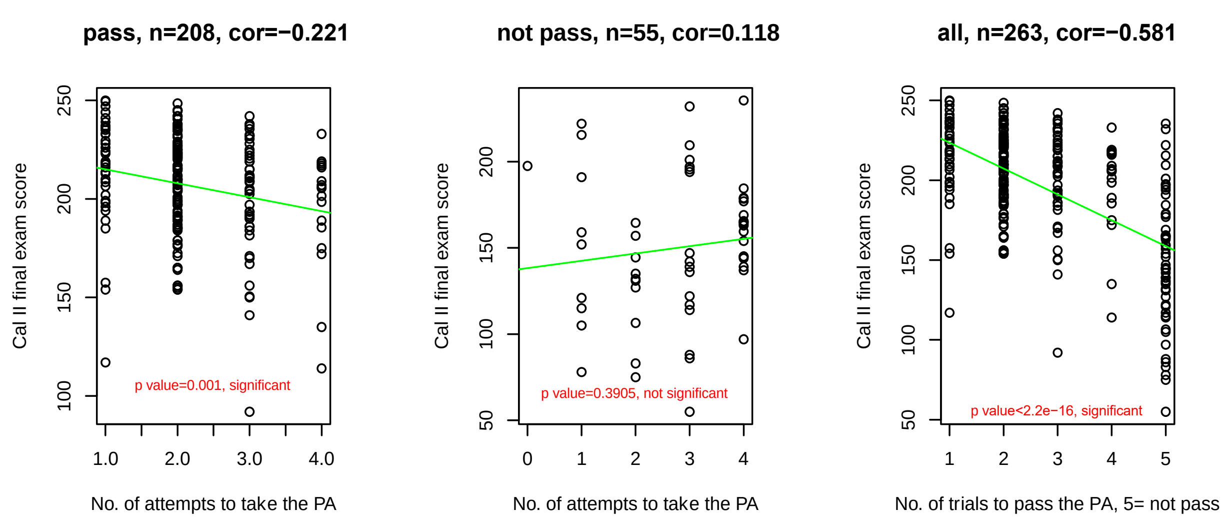


Figure 7: Correlations between the number of attempts and Cal II final exam scores

It is important to note that assigning the code "5" to the group of students who did not pass the assessment does not imply that they were able to pass it within five attempts. Instead, it suggests that if a student couldn't pass the assessment within the given four opportunities, they were more likely to have a lower Cal II final exam score compared to students who did pass the assessment, and the difference in scores was significantly apparent.

Thirdly, we investigated to determine whether there was any relationship between Cal II final exam scores and the PT scores for three different components: PT-Pre (Precalculus), PT-Cal I (Calculus I), and PT-Cal II (Calculus II). Our objective was to determine if there was any correlation between the different component's scores from the placement test and the performance in Cal II. To analyze this, we plotted the scores for each component of the placement test (PT-Pre, PT-Cal I, PT-Cal II) as well as the total PT score against the Cal II final exam scores. We analyzed the data separately for each pair: Cal II final vs PT-Pre, Cal II final vs PT-Cal I, Cal II final vs PT-Cal II, and Cal II final vs PT-total (Figure 8).

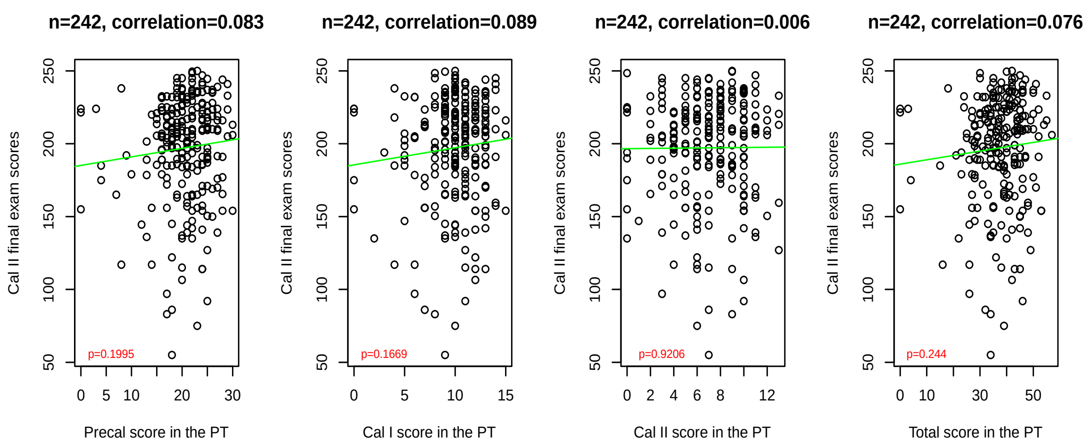


Figure 8: Correlations between PT components and Cal II final exam scores

We then applied correlation tests to determine if there was a significant relationship between each pair of scores. Based on our analysis, we found that all p-values obtained were sufficiently large, indicating that there was no significant evidence to suggest a relationship between any component of PT and Cal II performance. This conclusion is also supported by the correlation table presented below, which further reinforces the lack of correlation between the placement test components and Cal II final exam performance.

Table 6: Correlation table

Cor (n=242)	Final Exam	PT-PreCal	PT-Cal I	PT-Cal II	PT-total
Final Exam	1.000				
PT-PreCal	0.083	1.000			
PT-Cal I	0.089	0.657 ***	1.000		
PT-Cal II	0.006	0.483 ***	0.498 ***	1.000	
PT-total	0.075	0.913 ***	0.829 ***	0.743 ***	1.000

****significant, p<.0001.*

Out of the 263 students enrolled in Cal II, a total of 242 students took PT. We excluded those who did not take the PT from our analysis and conducted a correlation test to generate the correlation table. This table consists of correlation coefficients, with each cell representing the correlation coefficient between different variables. Specified p values were not displayed in Table 6 since they are either too big or 0.

According to the correlation table, it was found that only the different components of the placement test were significantly related to each other, but there was no significant evidence to suggest a relationship between the PT components and Cal II performance. This conclusion contradicted our initial expectation, as we anticipated that a higher performance in the PT would correspond to better performance in Cal II. Several factors may have contributed to this unexpected outcome. It is worth considering that the PT was administered online at the beginning of the summer. Consequently, some students might not have adequately prepared or taken the test seriously, potentially relying on online tools or resources. This lack of preparation and seriousness in taking the PT could have influenced the results and undermined any relationship between PT scores and subsequent Cal II performance.

We also explored the relationship between the participation of PT and Calculus II performance. We divided all students into two groups based on their participation in the placement test (PT). One group consisted of students who took the PT, coded as "1," while the other group included students who did not take the PT, coded as "0."

To analyze this, we employed the Kruskal-Wallis test due to the notable difference in sample sizes. The test yielded a p-value of .006, suggesting that the group of students who took the PT performed significantly better than those who did not take the PT. Additionally, the correlation test resulted a correlation coefficient as 0.191, which, although not particularly large, indicated a significant relationship due to the small p-value of 0.0019. This implied that there was conclusive evidence of a relationship between Cal II performance and the decision to take or not take the PT, namely, a student did not take PT would likely to have significant lower final exam II performance.

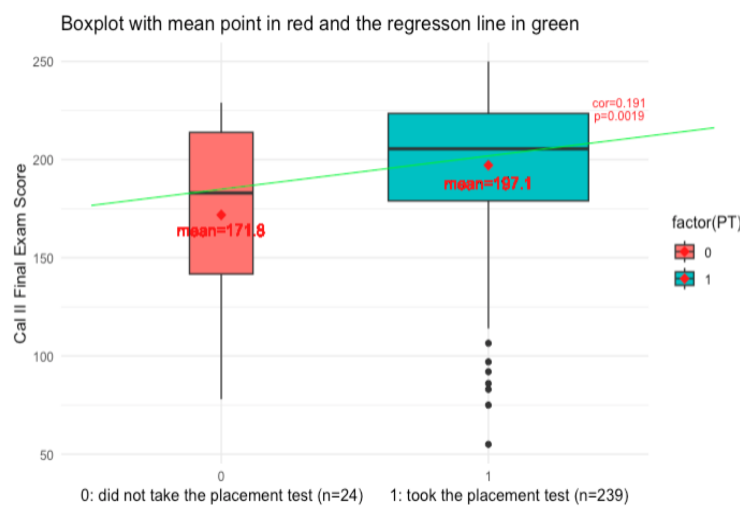


Figure 9: Results of students' Cal II performance based on the participation of PT

Students' Perceptions on the Intervention (RQ3)

Overall, students had mostly positive perceptions of the pre-calculus intervention in supporting their learning (Table 7). Over three-quarters of students agreed or strongly agree that the intervention helped them learn pre-calculus concepts and was effective.

Most students reflected that: a) incorporation of precalculus into APAM 1110 brought back their precalculus skills, b) multiple attempts to take the precalculus assessments enhanced their precalculus learning, c) the precalculus slides at the beginning of each APMA 1110 class were helpful to review the associated topics, d) the precalculus assessment problems were comprehensive representatives of precalculus skills, e) integrating precalculus into Calculus II helped them to master precalculus skills better, f) integrating precalculus into Calculus II helped them to master Calculus II concepts better, and g) integrating precalculus into Calculus II will help them to better prepare for future APMA courses.

Table 7: *Students' perceptions of the precalculus interventions in supporting their learning.*

Survey questions	Mean (SD)	% agree/ strongly agree
The incorporation of precalculus into APAM 1110 brought back my precalculus skills.	3.92 (.67)	78.3%
I had enough resources to prepare for the precalculus assessments given in APMA 1110 this semester.	3.44 (1.10)	54.9%
Multiple attempts to take the precalculus assessments enhanced my precalculus learning.	3.83 (1.00)	67.9%
The precalculus slides at the beginning of each APMA 1110 class were helpful to review the associated topics.	3.73 (.89)	68.5%
The precalculus assessment problems were comprehensive representatives of precalculus skills.	3.66 (.86)	66.8%
Integrating precalculus into Calculus II helped me to master precalculus skills better.	3.91 (.86)	75.0%
Integrating precalculus into Calculus II helped me to master Calculus II concepts better.	3.70 (.94)	64.1%
Integrating precalculus into Calculus II will help me to better prepare for future APMA courses.	3.75 (.86)	64.7%
Overall, integrating precalculus into APMA 1110 was effective.	3.82 (.86)	75.5%

1-strongly disagree, 2-disagree, 3-neutral, 4-agree, 5-strongly agree

To distill students' perceptions of the intervention from the 11 Likert survey questions, we first conducted an exploratory factor analysis (EFA) using Principal Axis Factoring and an Oblimin rotation with Kaiser Normalization with a 2-factor forced structure following exploration of the data and review of the scree plot. These two factors explained 46% of variance in the data (Table 7). Additional reliability analysis determined that Factor 1, called "Precalc perceptions", was highly reliable, while Factor 2 was not. We therefore created an average "Precalc perceptions" value for each student that was used in subsequent analyses.

Table 8: Exploratory Factor Analysis (EFA) for end-of-semester student precalculus intervention perceptions

Question text	Factor	
	1	2
• Integrating precalculus into Calculus II will help me to better prepare for future APMA courses.	.865	
• Integrating precalculus into Calculus II helped me to master Calculus II concepts better.	.848	
• Overall, integrating precalculus into APMA 1110 was effective.	.839	
• Integrating precalculus into Calculus II helped me to master precalculus skills better.	.671	
• The incorporation of precalculus into APAM 1110 brought back my precalculus skills.	.503	
• The precalculus assessment problems were comprehensive representatives of precalculus skills.	.413	
• Multiple attempts to take the precalculus assessments enhanced my precalculus learning.	.412	
• I had enough resources to prepare for the precalculus assessments given in APMA 1110 this semester.		.688
• REVERSE: The process to prepare for the precalculus assessments was overwhelming.		.599
Factor pre-survey Mean	3.78	3.30
Eigenvalue	4.354	1.264
Percentage of variance	39.165	7.096
Construct reliability	.838	.597

As a measure of overall perceptions, we looked for differences between students' Precalc perceptions mean value by sociodemographics using an independent t-test. Our findings revealed no significant differences in perceptions between male and female students (t (169) = -.794, p =

.428) or between students identified as underserved and non-underserved (t(168) = 1.439, p = .152) (Table 9).

Table 7: *Student perceptions of precalculus intervention as measured by Precalc supports average value*

Sociodemographic		n	Mean (SD)
Overall		184	3.80 (.62)
Gender	Male	111	3.90 (.64)
	Female	60	3.88 (.55)
Underserved	Underserved	46	3.72 (.69)
	Not-underserved	124	3.87 (.59)

We used an ANOVA to identify differences in perceptions by different groups when taking precalculus assessment (P1, P2, P3, P4, NP). Our results revealed significant differences in perceptions among the five groups (F (174) = 6.543, p < .001) (Figure 10). Subsequent Bonferroni post-hoc testing revealed differences in perceptions between students who attempted twice to pass the assessment and those who did not pass. Students who successfully passed the assessment with two attempts exhibited significantly more positive perceptions of the intervention compared to those who did not pass.

We also found significant differences in students' perceptions of the pre-calculus intervention based on their final exam scores (F (179) = 2.466, p = .047) (Figure 11). Further Bonferroni post-hoc analysis revealed significant differences in perceptions of the precalculus intervention between students who received failing grades on the final exam (0-60%) and those who scored in the range of 90-100%.

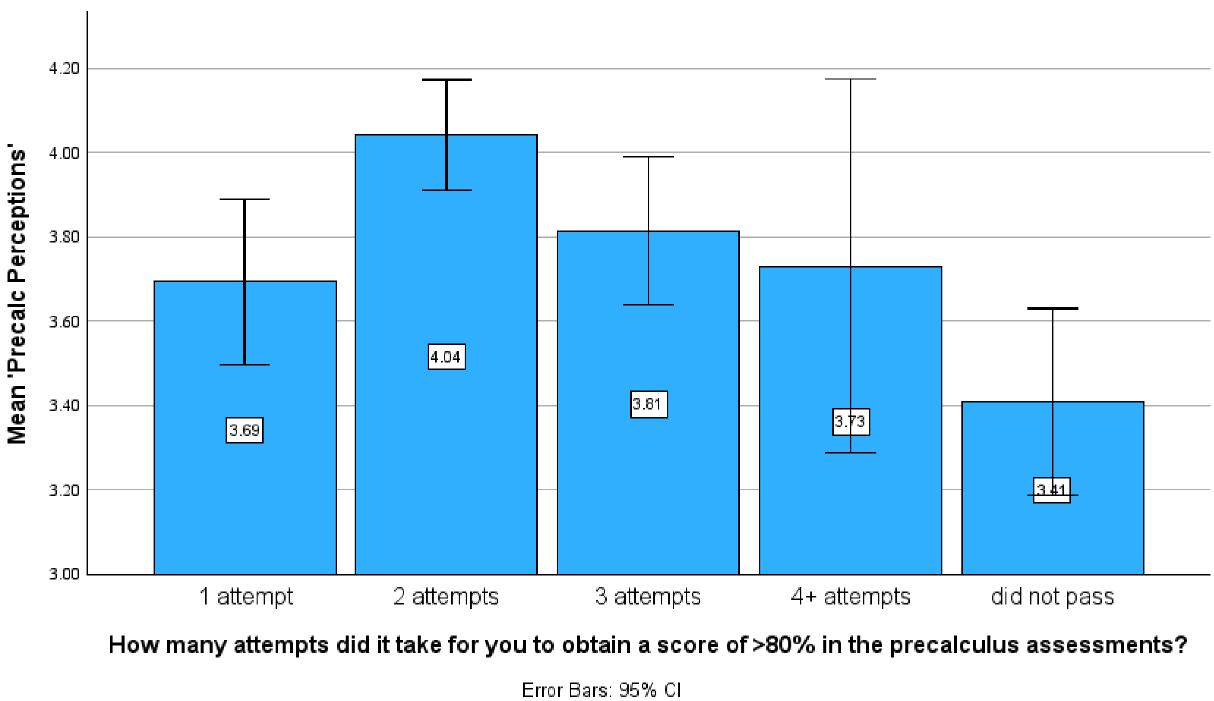


Figure 10: Mean Precalc perceptions of different groups by no of attempts to pass the Precalculus component

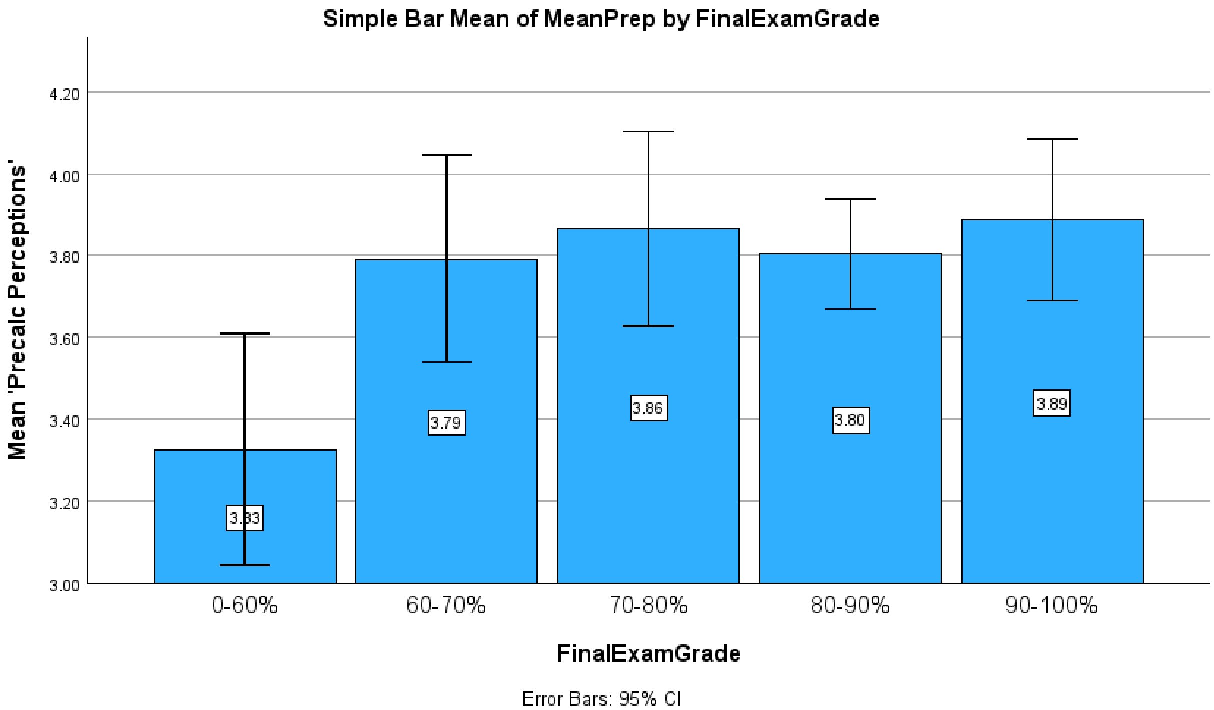


Figure 11: Mean Precalc perceptions by Calculus II final exam scores

Through this analysis, it is evident that students who performed poorly in Calculus II and those who did not pass the precalculus assessment exhibited the least favorable perceptions of the intervention. There may be an overlap between these two groups of students. Although not explored in this paper, delving into the underlying reasons for this disparity would offer an intriguing avenue for future research.

Conclusion

This paper primarily focuses on investigating the incorporation of mastery-based component of precalculus into the APMA 1110-Single Variable Calculus II course. In Fall 2023, precalculus assessments were integrated into all sections of APMA 1110. Students were provided with a comprehensive list of precalculus learning targets to review and enhance their precalculus skills. They were then given the opportunity to take the precalculus assessment up to four times during the semester, with the goal of achieving a minimum score of 80%.

Regarding the improvements made in precalculus skills throughout the semester because of this incorporation, majority students demonstrated significant improvements over multiple attempts, with only a small portion of those who did not pass the assessment not showing significant improvement. It is important to note that all students who passed the assessment showed a significant gain in precalculus skills when comparing their assessment score to their pre-calculus score from the placement test.

When examining the relationship between the incorporation of precalculus and performance in Calculus II, we found that students who required fewer attempts to pass the precalculus assessment tended to perform better in Calculus II, while those who did not pass the assessment displayed the weakest performance. Additionally, URM students who passed the assessment with four attempts or did not pass it performed significantly worse in Calculus II compared to their non-URM counterparts. Moreover, students who did not take the precalculus placement test exhibited significantly weaker performance in Calculus II compared to those who took the test. However, no evidence was found to suggest a direct relationship between precalculus placement test scores and Calculus II final exam scores.

Most students reflected positively on the effectiveness of the incorporation of precalculus material in general. However, it is worth noting that the students who performed poorly in Calculus II or did not pass the precalculus assessment had the least favorable perceptions.

Through this study, it has become apparent that we need to understand more about the group of students who did not pass the assessment and the URM group. It is important to identify the specific reasons and challenges these groups may have faced. Did they lack confidence, or did they not dedicate enough time to prepare? Did they feel a sense of belonging in the course? Were there sufficient resources available to support their success? These questions provide directions for future research to assist these students.

Moreover, there are other interesting topics that warrant further investigation. For example, how much time did students spend to pass the precalculus assessment? What were the common resources they used to prepare for the assessment? What factors contributed to some students passing with only one attempt, while others were unable to pass even after four attempts? Why did the not-passing students not use the full set of four opportunities? Even these questions may not have definitive answers, exploring them will assist calculus instructors in continually researching effective ways to support their students in (re)learning foundational precalculus skills that enable future success in undergraduate engineering.

Acknowledgement

We are grateful for the funding support from the HHMI Inclusive Excellence 3 Learning Community at the University of Virginia, the contributions made by Prof. Monika Abramenko to this project, and the comments provided by the reviewers. These factors all played a crucial role in the successful completion of this research.

References

[1] L. B. Massey, *et al*., “Success in Engineering Study of Under-Prepared Students,” in *Proceedings of the 2020 First-Year Engineering Experience Conference*, East Lansing, Michigan, 2020.

[2] E. Gallagher, *et al*., “Poverty and Guidance: Challenges and Opportunities in Mathematics Preparation for Engineering,” in *Proceedings of the 2020 ASEE Virtual Annual Conference*, Virtual Online, 2020.

[3] M. K. Watson, *et al.*, “A Systematic Review of Models for Calculus Course Innovations,” in *Proceedings of the 2019 ASEE Annual Conference & Expositio*n, Tampa, Florida, 2019.

[4] J. Viera, *et al*., “Shifting Pre-Calculus from a Gatekeeper to a Gateway Course,” in *Proceedings of the 2019 CoNECD - The Collaborative Network for Engineering and Computing Diversity Conference*, Crystal City, Virginia, 2019.

[5] M. K. Watson and S. T. Ghanat, “Exploring Math Self-Efficacy Among First-Year Civil Engineering Majors,” in *Proceedings of the 2020 First-Year Engineering Experience*, East Lansing, Michigan, 2020.

[6] R. A. George, *et al*., “Self-beliefs of Success for College Calculus Students,” in *Proceedings of the 2020 ASEE Virtual Annual Conference*, 2020.

[7] N. Mariano, *et al.*, “Quantitative Analysis of Barriers to Completion of Engineering Degrees for Female-identifying and Underrepresented Minority Students,” in *Proceedings of the 2018 CoNECD - The Collaborative Network for Engineering and Computing Diversity Conference*, Crystal City, Virginia, 2018.

[8] R. F. Hopkins, *et al.*, “Board 121: Retrieval Practice and Spacing: Effects on Long-Term Learning among Engineering Precalculus Students,” in *Proceedings of the 2018 ASEE Annual Conference & Exposition*, Salt Lake City, Utah, 2018.

[9] S. Pisano, *et al.*, “Redesigning the Calculus Curriculum for Engineering Students,” in *Proceedings of the 2018 ASEE Annual Conference & Exposition*, Salt Lake City, Utah, 2018.

[10] P. R. Norton, K. A. High, and W. Bridges, “Towards Creating Motivationally Supportive Course Structures for Introductory Calculus,” in *Proceedings of the 2020 ASEE Virtual Annual Conference,* Virtual Online, 2020.

[11] J. C. Minor, E. A. Stephan, and A. T. Stephan, “Implementing an Engineering Math Curriculum Sequence: Preliminary Results and Lessons Learned,” in *Proceedings of the 2021 ASEE Virtual Annual Conference*, 2021.

[12] C. Lam, M. Danforth, and R. Hughes, “Lessons from a Lower-division Mathematics Co-teaching Sequence,” in *Proceedings of the 2020 ASEE Virtual Annual Conference*, 2020.

[13] S. Mayoral, *et al.*, “Work in Progress: Implementing Elements of Engineering Design into Calculus,” in *Proceedings of the 2021 ASEE Virtual Annual Conference*, 2021.

[14] L. Gawarecki, Y. Dong, and G. Rablau, “Redesigned Application-oriented Integral Calculus Curriculum,” in *Proceedings of the 2018 ASEE Annual Conference & Exposition*, Salt Lake City, Utah, 2018.

[15] D. M. Olsen, *et al.*, “Promoting Success of Undergraduate Engineering Students Through Curricular Improvements in First-year Mathematics Courses,” in *Proceedings of the 2021 ASEE Virtual Annual Conference Content Access*, Virtual Conference, 2021.

[16] D. Bullock, J. Callahan, and J. B. S. Cullers, “The Crux: Promoting Success in Calculus II,” in *Proceedings of the 2018 ASEE Annual Conference & Exposition*, Salt Lake City, Utah, 2018.

[17] N. Kaabouch, *et al.*, “Retention in Engineering Programs: Integrating Real World Problems in Calculus Courses To Increase Student Learning and Interest in Engineering,” in *Proceedings of the 2013 North Midwest Section Meeting*, Fargo, ND, 2013.

[18] A. Galbraith, *et al.*, “Hands-on Laboratory Exercises for Engineering Applications of Mathematics Course,” in *Proceedings of the 2019 FYEE Conference*, Penn State University, Pennsylvania, 2019.

[19] J. W. Creswell, and V. L. P. Clark, “Designing and Conducting Mixed Methods Research,”. Sage Publications, 2017.

[20] G. Guadagni, H. Ma, and L. Wheeler, “The Benefit of Training Undergraduate Teaching Assistants,” in *Proceedings of the* 2018 *ASEE Annual Conference & Expositio*n, Salt Lake City, Utah, 2018.

[21] E. Johnson, *et al.*, “Addressing Challenges to the Precalculus to Calculus II Sequence through Case Studies: Report based on the National Science Foundation Funded Project Precalculus through Calculus II,” *MAA Press, Volume 92, 2022.*