Multidimensional Aspects of Vector Mechanics Education Using Augmented Reality

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Abstract

The objective of this paper is to provide a holistic summary of ongoing research related to the development, implementation, assessment, and continuous refinement of an augmented reality (AR) app known as Vectors in Space. This Unity-based app was created by the authors and provides a self-guided learning experience for students to learn fundamental vector concepts routinely encountered in undergraduate physics and engineering mechanics courses.

Vectors are a fundamental tool in mechanics courses as they allow for the precise and comprehensive description of physical phenomena such as forces, moments, and motion. In early engineering coursework, students often perceive vectors as an abstract mathematical concept that requires spatial visualization skills in three dimensions (3D). The app aims to allow students to build these tacit skills while simultaneously allowing them to learn fundamental vector concepts that will be necessary in subsequent coursework. Three self-paced, guided learning activities systematically address concepts that include: (a) Cartesian components of vectors, (b) unit vectors and directional angles, (c) addition, (d) subtraction, (e) cross product using the right-hand rule, (f) angle between vectors using the dot product, and (g) vector projections using the dot product.

The authors first discuss the app's scaffolding approach with special attention given to the incorporation of Mayer's principles of multimedia learning as well as the use of animations. The authors' approach to develop the associated statics learning activities, practical aspects of implementation, and lessons learned are shared. The effectiveness of the activities is assessed by applying analysis of covariance (ANCOVA) to pre- and post-activity assessment scores for control and treatment groups. Though the sample sizes are relatively small (less than 50 students), the results demonstrate that AR had a positive impact on student learning of the dot product and its applications. Larger sample sizes and refinements to the test instruments will be necessary in the future to draw robust conclusions regarding the other vector topics and operations. Qualitative feedback from student focus groups conducted with undergraduate engineering students identified the app's strengths as well as potential areas of improvement.

<u>Keywords</u>

Active learning; AR; augmented reality; cross product; dot product; mechanics; physics; statics; technology; vectors; visualization

1. Introduction

Vectors refer to parameters that possess two independent properties, namely, magnitude and direction. Vectors can be represented in both mathematical and geometric forms, and are commonly used to quantify physical phenomena such as position, electromagnetic fields, force, velocity, and weight [1-4]. Students typically first encounter vector mechanics in a physics course at either the high school or college level. Vector mechanics may be regarded as a threshold concept [5] because, once a student masters them, it marks a transformational milestone in the student's ability to understand critical knowledge necessary for subsequent learning at higher levels [6]. Accordingly, undergraduate engineering students subsequently encounter vectors again in engineering mechanics courses where the fundamental concepts are critical to solve advanced problems involving static and dynamic equilibrium.

Educators are cognizant that students must be able to understand and apply vector mechanics to be successful in physics and mechanics courses, and to advance through higher-level coursework such as those found in civil and mechanical engineering curricula. Since students often struggle to learn vector mechanics [4], educators have used a variety of instructional methods including traditional textbook-based learning [7], spreadsheets [8], personalized adaptive learning [9], interaction simulations [10], virtual reality [11], and augmented reality [4]. The latter is the focus of this paper.

Augmented reality (AR) is a steadily growing technology that superimposes virtual enhancements onto a user's view of their actual environment in real time. These enhancements include static images, 2D and 3D objects, dynamic computer-generated simulations, audio narration, and other special effects to create an immersive and interactive experience for the user. AR has been implemented in numerous educational settings to enhance the learning experience of students studying STEM [12]. The findings of Ropawandi et al. [13] demonstrated that AR technology significantly boosted the comprehension of 11^{th} grade students' understanding of electrical principles in an experimental group as compared to a control group. The disparity between the groups was predominantly pronounced in the students' knowledge of abstract physics concepts. A quasi-experimental study also found that the integration of AR movies into online teaching activities for physics enhanced students' comprehension of fundamental principles [14]. Similarly, an intervention by Cai et al. [15] showed that AR in physics classrooms can increase students' self-efficacy by improving their understanding, higher-level cognitive skills, knowledge application, and communication.

Several AR applications have focused on learning vectors in the context of 3D geometry [16], electric forces [4], gravitational forces [1], as well as other physical forces and their Cartesian components [17]. The study discussed herein details an AR app known as Vectors in Space [18] that was developed by the authors. The app addresses several foundational concepts of vectors (Cartesian components, unit vectors, and directional angles) as well as vector operations (addition, subtraction, cross product, and dot product). The authors selected those specific vector concepts based on their experiences teaching students who have struggled to learn and visualize vectors in 3D. The objective of the research study was to assess the effectiveness of the app and the integrated learning activities in terms of students' knowledge retention and usability. Throughout this paper, the abbreviation "2D" refers to a vector that lies in a Cartesian x-y plane, while "3D" refers to a vector in space with components in x, y, and z directions.

2. Goal of the AR-based Learning Activities

The overall goal of the learning activities was to improve undergraduate engineering students' understanding of the following vector concepts in a 3D space:

1. Cartesian components of vectors
2. Unit vectors
3. Directional angles
4. Vector addition and subtraction using the triangle law
5. Cross product using the right-hand rule
6. Dot product to determine the angle between two vectors
7. Dot product to compute the projection of a force onto a line

The authors created paper-based learning activities in the form of guided inquiry [19] to address each of the concepts. The activities were integrated into the app by designing the app and activities in parallel. Both were designed without context (i.e. not specific to any STEM course). This decision was intended to broaden the range of potential users. Accordingly, the learning activities can be deployed in college and high school settings, and in various courses such as physics, linear algebra, and statics. Details of the app design and the integrated learning activities are described in the subsequent sections.

3. App Design

A critical design requirement was to deploy the app using a headset such that users' hands and arms are free for gesturing. Accordingly, the authors selected Magic Leap One (ML1) to satisfy this requirement and due to the existing availability of these devices at the authors' institution. Content development was accomplished using the Unity® game engine due to (a) its compatibility with ML1 devices and (b) on-campus training and apprenticeship programs for undergraduate student programmers who developed the app. Two self-paced, narrated modules were developed within the Vectors in Space app. Each module and the corresponding subtasks performed by the user are summarized in Table 1.

Table 1. Modules and subtasks in the Vectors in Space AR app

Module	Subtasks
Introduction to 3D Vectors	1. Create a right-handed Cartesian coordinate system in 3D 2. Create one vector in 3D space 3. Explore the vector components 4. Explore the unit vector 5. Explore the directional angles
Vector Operations	1. Create two vectors in 3D space 2. Add the two vectors using the triangle law 3. Subtract the two vectors using the triangle law 4. Perform the cross product using the right-hand rule 5. Compute the angle between two vectors using the dot product 6. Compute parallel and normal components of a force using the dot product

The first module, "Introduction to 3D Vectors", focused on learning vector fundamentals such as magnitude and direction. Users are given the option to first create vectors in a Cartesian plane (2D) before investigating vectors in 3D space. The second module, "Vector Operations", addressed vector addition, subtraction, cross product, and dot product.

Due to the narrated guidance within the app, users can use it as a standalone teaching tool or integrate it into a comprehensive activity. The latter will be discussed in section 4 (Integrated Learning Activities). Mathematical notation is consistent with that utilized in the textbook [20] adopted in the authors' statics courses. The US Customary system of units are used throughout the app. Mayer's principles of multimedia for e-learning [21] are followed to maximize student engagement and learning gains. Specific examples are described in Table 2 and the corresponding point-of-view images are presented in Figure 1.

Table 2. Typical multimedia design principles utilized in the Vectors in Space AR app

Principle	**Action**	**App-specific Example**
Signaling	Provide visual cues or visual highlights to identify important information	The narration directs the user to focus on the unit vector (labeled "**e**") of the larger position vector, **A**. A partially transparent blue prism is displayed around the unit vector to identify its three Cartesian components, as shown in Figure 1(a).
Spatial contiguity	Include printed text adjacent to their matching graphics	The vector equation and magnitude of vector **A** is displayed at the tip of its corresponding arrow. Its unit vector, "**e**", is displayed with the three directional angles, all adjacent to their virtual graphic in 3D space; Figure 1(b).
Embodiment	Create or animate objects to reflect humanesque motions	Users can opt to have guidance from an animated virtual hand that overlays the user's right hand and slowly curls its fingers while the user simultaneously performs the right-hand rule on two vectors; Figure 1(c).
Segmenting	Separate an activity into smaller, self-paced units	As shown in Figure 1(d), each module is divided into several tasks as well as distinct vector operations. Once a user selects a task and vector operation, the corresponding buttons remain highlighted throughout the activity. Self-pacing is enabled using navigation buttons that include "Continue", "Go Back", "Watch Again", and "Try Again."
Temporal contiguity	Simultaneously display graphics and corresponding narration	All instructions and steps within the app are narrated to provide guidance to the user. This was accomplished by recording more than 500 individual audio files.

Table 2 (continued)

Principle	Action	App-specific Example
Personalization	Use conversational style for words and narration	Except for technical terms related to vectors, all words and narration utilize an informal tone. In addition, 14 sound effects are used to lighten the user experience. For example, a clip of audience clapping is played when a user correctly answers a quiz question within the app.
Voice	Use a human voice instead of computer-generated	An author of this paper provided all narration since he is currently teaching statics. It was believed that the familiarity of the voice heard during the lectures would make the app more relatable.

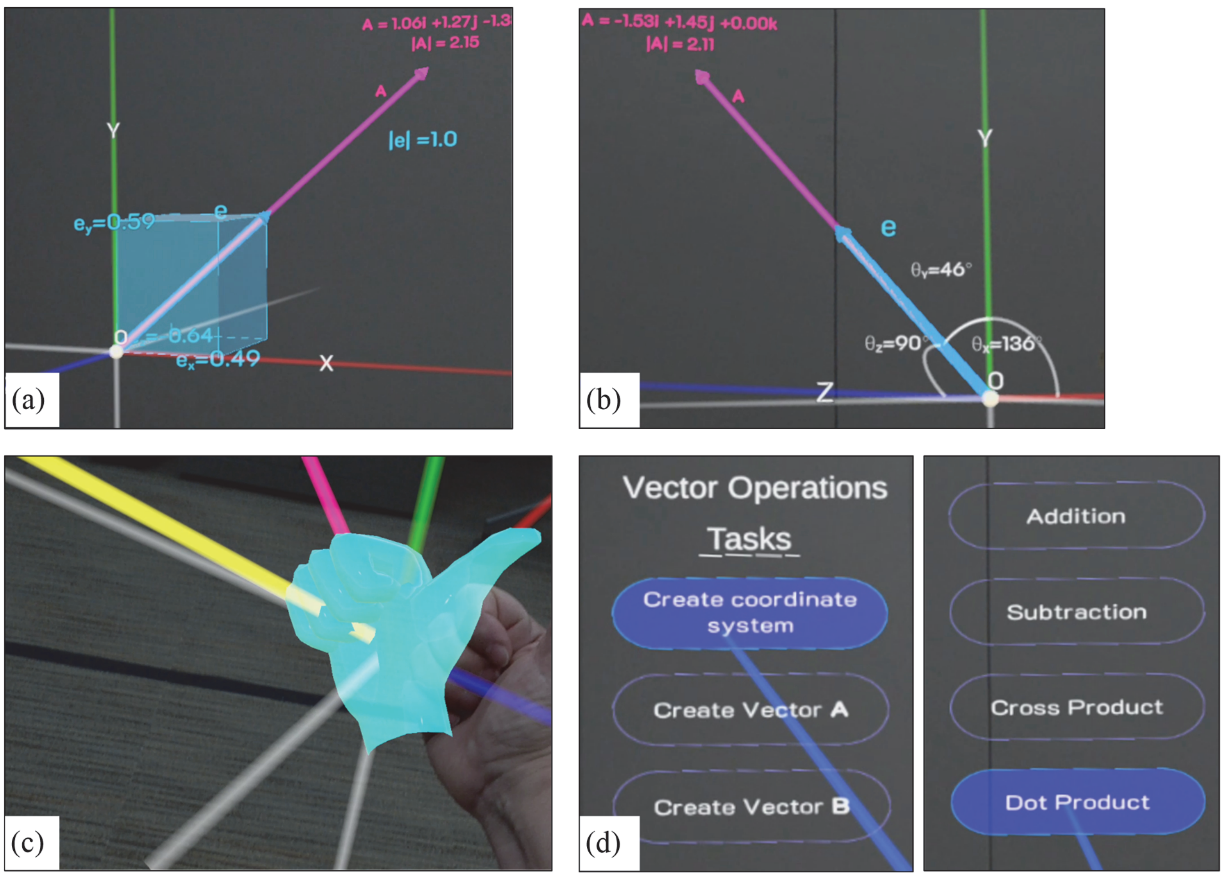

Figure 1. Point-of-view images demonstrating the principles of multimedia utilized in the app design; (a) signaling, (b) spatial contiguity, (c) embodiment, and (d) segmenting

Throughout the app's activities, true/false quiz questions are presented to the user to prompt them to recall recent information. Since rapid feedback can improve learning [22], the user is immediately informed whether their answer is correct using an audible message as well as sound effects. Regardless of whether the user answered the question correctly, the correct answer is explained to the user. The authors opted to include this explanation to the user such that it reinforces the topic being learned, and since a user could guess randomly.

4. Integrated Learning Activities

Three pairs of integrated learning activities were developed; each pair included a similar activity for control and treatment groups. Details of the activities are presented in Table 3.

Table 3. Summary of learning activities

Activity	**Topics Covered**	**Duration**	**App Module**
2D and 3D Vector Fundamentals	Unit vectors (including **i**, **j**, **k**), components, magnitude, position vectors, directional angles	60 minutes	Introduction to 3D Vectors
Vector Addition, Subtraction, and Cross Product	Application of the triangle law, resultants, right-hand rule	60 minutes	Vector Operations
Dot Product	Angle between vectors, projections, parallel and normal components	60 minutes	Vector Operations

The duration of each activity (limited to 60 minutes) was governed by several factors:

- Feedback from some students who used the AR headsets in previous semesters commented that they exhibited eyestrain after 60 minutes of continuous use.
- The temperature of the AR headset battery pack substantially increases after 60 minutes. The device may become uncomfortable for the user to wear or hold.
- Pre- and post-activity assessments required an average of 15 minutes (30 minutes total). Based on the instructors' observations in previous semesters, students exhibited signs of fatigue and lethargy after 90 minutes. Therefore, the activity was limited to 60 minutes such that the total duration of the class meeting would not exceed 90 minutes.

A typical class session lasted 90 minutes and consisted of a 15-minute assessment ("pre-test") followed by the 60-minute learning activity, and then a subsequent administration of the same 15-minute assessment ("post-test").

Each learning activity was paper-based and was provided to students at the start of the class session. Students were permitted to work with their peers in groups of 2 or 3. Control group activities involved solving computational problems, some of which had been used as test or lecture example problems by the instructors in previous semesters. In the case of the augmented reality activities, the worksheet included several exercises (problems) for each student to complete. The exercises were arranged in order of increasing difficulty. The instructions on the AR activity worksheets provided scaffolding and paralleled the flow of the app such that students could record their work as they progressed through each exercise's tasks. Students were typically asked to first perform calculations on their worksheet, which they would later validate using the app. The activities were initially deployed in the fall semester of 2023, then subsequently refined based on instructor and student feedback. Students using the AR app were each assigned an AR headset at the start of the activity; sharing devices was not necessary.

5. Research Methods

Objectives

The objectives of the IRB-approved study were to:

1. Assess the impact of the AR-based activities on students' fundamental understanding of vectors and vector operations
2. Assess the usability and user friendliness of the app
3. Solicit feedback from students and course instructors to identify areas of improvement for the app and the integrated learning activity

Setting

The study was conducted in a statics course during weeks 2-5 in the 14-week fall semester of 2023 at a private university. Students enrolled in the course satisfied Physics I and Calculus I pre-requisites. The statics course includes three 50-minute lectures per week in a synchronous face-to-face format as well as recitation (discussion) sections ranging from 75 to 150 minutes per week. Lecture and recitation sections were scheduled consecutively to ensure a continuous and uninterrupted period of at least 90 minutes to conduct each class activity. The average class size was about 24 students.

Quantitative methods

The study was structured using a nonequivalent groups design utilizing pre- and post-activity assessments [23]. For each vector learning activity described earlier in Table 3, the authors created control and treatment groups using either random assignment or stratification based on a graded test administered in the week leading up to the activity. Groups of unequal size were necessary due to the limited number of AR headsets available to the authors. Control and treatment groups convened in separate locations on campus. Pre-activity assessments were administered at the start of the class session in which the activity was conducted. Students then participated in their respective activity, which was followed by the post-activity assessment.

Qualitative methods

Qualitative data were gathered via observation and focus group interviews. A graduate research assistant conducted the observation during the activity sessions, documenting observations and noting any unique participant behaviors in field notes. These field notes served as the basis for identifying patterns and themes. The course instructors also shared their observations made during the activities and throughout the planning stages leading up to the class activities.

Focus groups were convened to solicit feedback from students who utilized the app during the classroom activities. The objectives were to (a) gauge the usability and user friendliness of the app, (b) identify the app's strengths and weaknesses, and (c) identify potential areas of improvement. Two weeks after the last class activity using the app, student participants were invited to participate in the focus group interviews. Participants were compensated monetarily for their time investment of about 30 minutes.

6. Test Instruments

Pre- and post-activity assessments

Paper-based test instruments were originally adapted from existing vector tests and concept inventories available in the literature [9, 24-30]. A majority of those assessments' test items were limited to vectors in a Cartesian *x*-*y* plane (2D) and were typically developed for students encountering vectors for the first time in a physics or linear algebra course. Figure 2(a) provides an example of a multiple-choice test item similar to that appearing in the Test of Understanding Vectors (TUV) [26]. The test item shows two vectors (**A** and **B**) with unequal magnitudes and different directions in a Cartesian plane. The student must determine the vector sum by selecting an appropriate graphical image from several answer choices that include logical distracters (not shown in Figure 2). The authors of this study administered several questions from those tests in their statics course over the past few years. Many students performed relatively well on test items that assessed students' understanding of *x* and *y* vector components, as well as magnitude and direction of vectors that were limited to a Cartesian plane (2D). The authors concluded that students were likely using their prior knowledge of vectors acquired in their prerequisite physics course. Statics differs from physics coursework in that statics substantially expands the study of vectors to include vectors in a 3D space and more complex vector operations. Therefore, it was necessary for the authors to create new test instruments that included more challenging 2D test items, as well as test items that focused on vectors in a 3D space.

New test items were adapted from misconceptions identified (a) in existing studies for 2D vectors in the literature, (b) in open-ended problems administered on exams in prior offerings of the authors' statics course, and (c) by the authors' experiences and interactions with students while teaching the course over the past 17 years. Figure 2(b) shows a typical, representative example of a test item created by the authors to assess vector addition in a 3D space. Similar to its 2D counterpart, the multiple choice question requires students to determine the vector sum by selecting an appropriate graphical image among several logical distractors (not shown in Figure 2). Aside from the obvious spatial difference (2D vs. 3D), other distinct differences are apparent between the 2D and 3D versions of this test item. The differences and the authors' rationale include:

1. Vectors' orientation: The 2D version provides two concurrent vectors arranged in a tail-to-tail fashion. The authors noted that some students do not recognize that vectors must be oriented tail-to-tail. Thus, the 3D version removes the concurrent aspect in an effort to test students' understanding of how the vectors should be positioned to perform vector addition. This approach is also intended to assess students' spatial visualization skills, which may be improved using augmented reality.

2. Grid: The 2D version includes a grid with equally spaced horizontal and vertical lines. The authors observed that students would often measure the vectors' magnitudes in the horizontal and vertical directions in terms of the individual squares of the grid. Students would then add the respective components and draw the resultant vector. This analytical method is

correct, but it would not assess students' understanding of the graphical approach (i.e. triangle law). The 3D version is intended to address this concept by requiring students to apply the steps of the triangle law without the benefit of numerical coordinates.

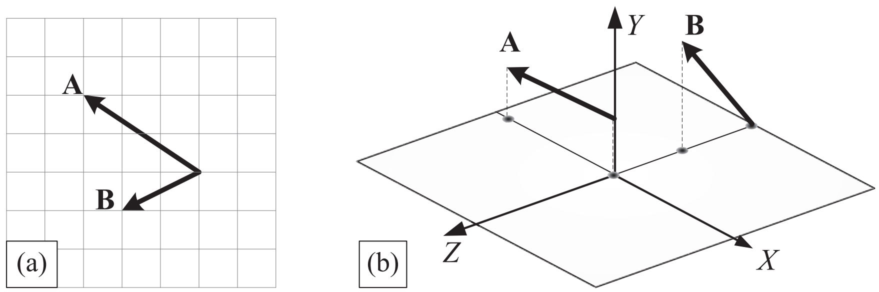

Figure 2. (a) typical 2D vector addition test item appearing on the TUV, adapted from [26]; (b) typical example of a 3D vector addition test item developed in this study

Table 4 summarizes the test instruments and items used during the fall 2023 administration of the learning activities in the statics courses taught by the authors at their institution.

Table 4. Summary of test instruments to assess vector knowledge

Activity	**Topics Assessed**	**Number of Test Items**					
		Total	Spatial domain*		Type†		
			2D	3D	MC	SA	OT
2D and 3D Vector Fundamentals	Magnitude, direction, components, unit vectors, directional angles	11	7	4	10	1	0
Vector Addition, Subtraction, and Cross Product	Triangle law for vector addition and subtraction, right-hand rule	11	4	7	11	0	0
Dot Product	Angle between vectors, parallel and normal components, vector projection onto a line	15	7	8	12	0	3

* Some qualitative test items assessing students' understanding of theoretical vector concepts could be considered either 2D or 3D.
† Test Item Types: MC = Multiple-Choice (dichotomous, close-ended); SA = Short Answer (dichotomous, open-ended); OT = Other (non-dichotomous, open-ended)

Each instrument contained a mixture of qualitative and quantitative problems. Test durations ranged from 12 to 17 minutes depending on the number of test items. For each activity, the pre- and post-activity assessments were nearly identical except for minor differences in the (a) numerical values used in computation problems, (b) the order of the test items, and (c) the order of the answer choices. Scoring of the open-ended test items was conducted by the authors using blind grading. Since the graded assessments were not returned to the students, they did not formally count towards the students' final course grade. However, students' involvement in the activities was considered a contribution to their class participation grade.

Focus groups

Student focus groups were planned with the expectation of asking seven primary questions and potentially up to 22 follow-up questions. One primary question was "*Could you list 3 strengths and 3 weaknesses of using the Vectors in Space app*?" All questions were pre-scripted and read to the undergraduate engineering students by a member of the research team who was unaffiliated with the College of Engineering at the authors' institution. The statics course instructors were not present during these meetings nor were they informed of which students participated. A total of five participants agreed to participate in the focus group interviews, which were conducted in three separate sessions – two sessions each with two participants, and one session with one student alone. Of the 5 participants: 2 were considered high performers, 2 medium performers, and 1 low performer. Their performance level was based upon their actual course grade when the focus group convened.

7. Results and Discussion

Quantitative

Pre- and post-activity test scores were analyzed statistically to identify differences between the post-activity test scores of the treatment and control groups. Analysis of covariance (ANCOVA) was utilized to account for initial differences between the groups that may be reflected in the pre-activity test scores. For brevity, only the descriptive statistics and outcomes of the ANCOVA are summarized in Table 5; the full analysis process is described in an earlier work [31]. An interaction effect was not present for any of the activities' results.

Table 5. Descriptive statistics and summary of ANCOVA results

Activity	Group	Time	*N*	*M*	*SD*	*Var*	*Min - Max*	*p-value*	*Partial* η^2
2D and 3D Vector Fundamentals	Control	Pre	21	56.5	25.7	661.3	8.0 - 89.0	0.292	0.028
		Post	21	67.0	21.2	450.2	16.0 - 89.0		
	Treatment	Pre	22	53.0	16.4	268.1	25.0 - 91.0		
		Post	22	68.2	15.9	253.7	33.0 - 100.0		
Vector Addition, Subtraction, and Cross Product	Control	Pre	11	73.6	18.8	354.7	45.5 - 100.0	0.345	0.050
		Post	11	83.5	12.7	162.3	63.6 - 100.0		
	Treatment	Pre	10	61.8	14.7	216.8	36.4 - 81.8		
		Post	10	81.8	9.6	91.8	63.6 - 90.9		
Dot Product	Control	Pre	22	54.0	27.6	764.2	2.8 - 100.0	0.014	0.145
		Post	22	66.9	21.2	448.1	21.4 - 100.0		
	Treatment	Pre	21	44.0	23.2	537.2	7.1 - 91.6		
		Post	21	68.3	18.0	322.3	21.6 - 92.9		

Table 5 shows that the *p*-values for the first two activities were 0.292 and 0.345, respectively. These are greater than the significance level of 5%, suggesting that the mean post-activity test scores were not significantly different between the control and treatment groups within each activity. It should be noted that the second activity (vector addition, subtraction, and cross product) had relatively small sample sizes of control and treatment groups of 11 and 10, respectively.

The p-value of 0.014 for the dot product activity indicates that the treatment group exhibited a significantly higher mean post-activity test score than the control group. The mean test score of the control group increased from 54.0 to 66.9 (change of 12.9), while the treatment group increased from 44.0 to 60.3 (change of 24.3). Although the treatment (AR) group exhibited an increase nearly twice that of the control group, the size effect was regarded as small as reflected in the relatively low value of 0.145 for *Partial* η^2 [32].

To complement the ANOVA results, a test item analysis was performed on each activity's test instrument. The Kuder-Richarson Formula (KR-20) [33] was used to measure the reliability of the test instruments with dichotomous items, while Cronbach's alpha [34] was computed to gauge the reliability of those with non-dichotomous items. A discriminatory item analysis [35] was performed to provide insight on how well each assessment is able to differentiate between low and high performing students. This was accomplished by calculating the average discrimination index (DI) for each test instrument. Table 6 presents the results of the test instrument analyses. The numerical value of each parameter is provided along with a qualitative interpretation based on generally accepted norms stated in the literature.

Table 6. Summary of test instrument analyses

Test Instrument	**KR-20**		**Cronbach**		**Discrimination Index (DI)**	
2D and 3D Vector Fundamentals	0.744	Reliable	n/a*	n/a*	0.494	Discriminating
Vector Addition, Subtraction, and Cross Product	0.504	Not Reliable	n/a*	n/a*	0.397	Average
Dot Product	0.719	Reliable	0.799	Reliable	0.562	Discriminating

*n/a: Not applicable to instruments without non-dichotomous items.

Of the three test instruments, the instrument assessing vector addition, subtraction, and cross product was not considered reliable based on its KR-20 score of 0.504. Its ability to discriminate between high and low performers was considered average with a DI of 0.397. Both findings may be partially attributable to the small sample size of students who completed the test (only 21). In addition to the analyses, students informed the authors of potential ambiguity in some of the test items. Accordingly, the test items will be revised for clarity in future administrations of the test instruments.

Qualitative

The student feedback received during the focus groups is summarized in Table 7 and is organized according to the students' performance level in the course. "N" represents the number of students in each performance level.

Table 7. Summary of student feedback obtained during focus groups

Level	*N*	Feedback
High	2	• AR was helpful for visualizing the vectors in 3D space. • The utility of the app is recognized, but with some weaknesses. • Creating the vectors in the precise location specified on the activity worksheet was challenging and took too much time. • The narrated instructions were long and lacked functionality to speed up. • The exercises were too simple; more advanced examples were desired.
Medium	2	• (Similar comments as the high performers) • More time was spent trying to learn AR than desired. • Future AR exercises could be useful in courses.
Low	1	• The visualization of the calculations helped on exams. • Learning about vectors with more application-based examples were desired. • It was possible to visualize objects while sitting, instead of moving around. • The AR headset hurt their head after a while.

Table 8 presents typical difficulties faced by the instructors and the research team throughout the research study, including the implementation of the activities in their statics classes.

Table 8. Challenges encountered and lessons learned during planning and implementation

Challenge	Lessons Learned
Only 11% of the students enrolled in Statics opted to participate in the focus groups. The low participation rate limited the feedback available to improve the app and learning activities.	In general, monetary compensation is an insufficient incentive for students to participate in an optional endeavor. Provisions for earning extra credit to improve students' course grade may be a stronger motivator.
Minimal peer collaboration occurred when students were using the AR headsets since they are intended to be worn by a single user.	Learning activities should be peer-based such that students can visualize and simultaneously interact with the *same* digital objects. Future apps will facilitate peer collaboration in a digital environment using mixed reality and multiplayer mode.
The novelty effect of using the AR technology during students' first AR experience detracted from their ability to follow and focus on a structured learning activity.	A short, 20 - 30 minute introductory session is needed such that students can experience the AR technology before a formal learning activity in class. This session can also identify students who experience adverse effects of AR.
Since students were required to complete handwritten worksheets as part of their AR activity, some students were reluctant to ambulate since they required a table to write on.	Providing clipboards allows students to write their work while simultaneously ambulating during the AR experience. Ideally, the activities should be self-contained lessons within the app and without paper-based instructions or other supplementary material.

8. Conclusions and Future Work

This paper presented ongoing research related to the development, implementation, and assessment of the Vectors in Space augmented reality app. This app provides undergraduate engineering students with the opportunity to learn the fundamentals of vectors and their operations, including addition, subtraction, cross product, and dot product. Three pairs of learning activities were created to enable students in control and treatment groups to strengthen their knowledge of vectors. Control groups were engaged in problem solving using peer collaboration, while students in treatment groups utilized the augmented reality app. The research design included quantitative data in the form of pre-and post-activity tests analyzed using ANCOVA, while qualitative data was obtained from student focus groups and the course instructors during the planning and implementation of the activities. Using the results obtained from the first execution of the study in fall 2023, several conclusions can be drawn:

1. Two of the AR learning activities (Vector Fundamentals; Vector Addition, Subtraction, and Cross Product) did not demonstrate significantly higher mean post-activity scores than their control group counterpart.
2. While controlling for the effect of pre-activity test score, the AR learning activity for Dot Product demonstrated a significantly higher mean post-activity test score than that of the control group. The effect size of 0.145 was considered low.
3. The test instrument for Vector Addition, Subtraction, and Cross Product was not considered reliable. Small sample sizes and ambiguity in several of the test items may have contributed to this result.
4. Five students participated in the focus groups to provide feedback on the app and the integrated AR learning activities. High performers sought more challenging examples and the ability to accelerate through the app. Medium and low performers provided mixed feedback.

The vector test instruments presented herein will undergo additional refinement to improve their reliability with the anticipation of creating a concept inventory [36] for 3D vectors and their associated applications. The study will be repeated with larger sample sizes to improve the statistical power of the findings. Future work on the app is also expected to include a comprehensive evaluation [37] as well as a focus on quality of experience (QoE) [38]. Moreover, the effect of embodiment on learning vectors in an AR setting is another aspect to be considered. Future work will explore whether students comprehend the fundamentals of vectors more effectively in a highly embodied learning environment. Effects on students' motivation and cognitive load are expected to be quantified using established survey instruments [39, 40].

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