

## **Promoting the Transfer of Math Skills to Engineering Statics**

**Dr. Alexander John De Rosa, University of Delaware**

Alexander De Rosa is an Associate Professor in Mechanical Engineering at The University of Delaware. He gained his Ph.D. in Mechanical Engineering from The Pennsylvania State University in 2015, where he worked on experimental combustion research applied to gas turbine engines, and his M.Eng. in Mechanical Engineering from Imperial College London in 2010. Alex's research focuses on the transfer of learning between various courses and contexts and the professional formation of engineers.

**Samuel Van Horne, University of Delaware**

# Promoting the Transfer of Math Skills to Engineering Statics

## Introduction

It has been well documented that students face difficulties in transferring their knowledge and skills learned in prior courses to other areas of the curriculum. These problems with transfer are exacerbated by foundational courses being taught outside the major, as well as the fact that many engineering courses are taught in silos, with little connection being made to the engineering curriculum as a whole. At the same time, engineering graduates are moving into an ever more interdisciplinary workplace where the ability to transfer knowledge across disciplines and contexts is highly valued [1-5].

These problems students face when tasked with transferring knowledge have been cited previously by various authors. In particular, the (in)ability of students to utilize fundamental concepts they learned in mathematics and physics in the context of thermodynamics have been described by both the mathematics and physics communities [6-9]. In these studies, students were shown to have only a cursory understanding of the core math and physics concepts at play to view these ideas as isolated elements lacking connections to other aspects of their curriculum. An important conclusion of this work (that was not studied) was the idea of promoting prior knowledge activation as a mechanism for potentially remedying these issues [6].

In the field of mathematics, Orton [10] provided a summary of student understanding of mathematical concepts and ability to transfer knowledge while noting student difficulties in using fundamental skills such as integration in new contexts outside of mathematics. Orton suggested weaving these important concepts throughout the curriculum as a method for reinforcing these fundamental ideas. Rebello and colleagues [11-13] discussed the idea that students may possess the basic math skills they require, but struggle to see how it applies in new situations. Their work suggested that student knowledge structures needed to be built out to forge the necessary connections to see these applications and allow knowledge to be transferred successfully. Schoenfeld suggested that specifically showing students applied examples and how to recognize similarities in problems might be a remedy to this problem and would aid the development of more expert knowledge structures [13].

Within the field of engineering, there exists a body of literature related to knowledge transfer and applied problem solving. Typically, however, this work is used to evidence problem based teaching approaches [14-16] or to examine the depth of students' conceptual understanding of a topic [17,18]. Further work is, therefore, needed to better enable students to see the connections between their courses and transfer the requisite knowledge and skills from prior classes to other areas of the curriculum, and in their careers.

This investigation aims to address this need by building on a prior study [19] that used a series of think-aloud, problem-solving interviews to assess the barriers and challenges students face in transferring knowledge from prior mathematics courses into an applied engineering setting. In this prior work, participants were tasked with solving a rigid-body equilibrium problem typical of an engineering statics course but which required integration skills, as well as knowledge of the centroid, to solve. In the course of this study it was found that participants could not solve the problem as they could not determine the centroid of the object in question. Participants cited issues such as a lack of applied problems being taught that used centroids, the use of tabulated data for centroids, and forgetting governing equations as major barriers to being able to solve the problem. A majority of participants did, however, believe that being shown a general equation used to calculate centroids would have improved their problem solving success.

Based on this prior study, as well as suggestions in the literature [6,13], we aim to examine the effectiveness of an equation-based prompt used to activate relevant prior knowledge on student problem solving success.

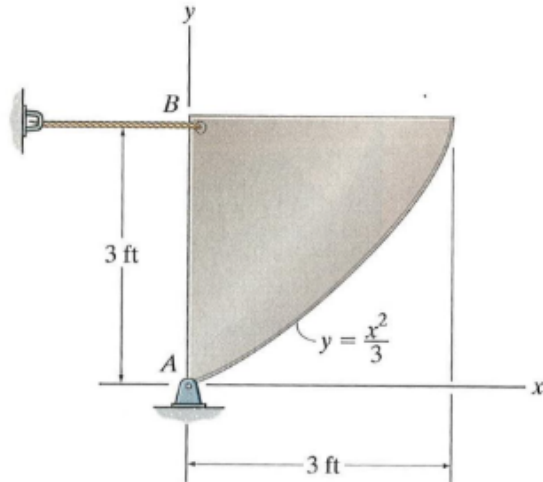
## **Methodology**

Grounded in the results of prior work [19,20,21], a problem solving activity and intervention was devised in which the participants would attempt to solve an engineering statics problem. The statics problem used is detailed in Fig.1a and is classified as a rigid body equilibrium. This particular problem requires integration to find the area of the plate and the location of the centroid and thus requires mathematical skills to be transferred in order to be solved. Based on the suggestions from prior work, if participants struggled to find the location of the centroid, they would be provided with the equation-based prompt shown in Fig.1b. Both the problem used in this study and the equation based prompt come from the Engineering Statics textbook of Hibbeler [22], which is used for the engineering statics class the participants have taken and as such, should be familiar in terms of formatting and language to the participants of the study.

A semi-structured interview protocol was developed around this activity in which the participants would “think-aloud” [23-26] and verbalize their problem solving process as they completed the problem solving activity. Both this interview protocol, and the associated data analysis, were grounded in the knowledge transfer framework of Belenky & Nokes [27,28], detailed in Fig.2 and which breaks down the knowledge transfer process into five stages:

1. Generating the Frame
2. Activating Knowledge & Interpreting the Environment
3. Arriving at the Current Representation
4. Solution Generation
5. Solution Evaluation

A steel plate ( $\gamma=490 \text{ lb/ft}^3$ ) has a thickness of 0.5 in and is supported by a pin at A and a rope at B. Determine the magnitude of the reaction forces at the pin and the tension in the rope.



(a) Engineering statics problem used in think-aloud interviews [22].

**Centroid of a Volume.** If the body in Fig. 9-3 is made from a homogeneous material, then its density  $\rho$  (rho) will be constant. Therefore, a differential element of volume  $dV$  has a mass  $dm = \rho dV$ . Substituting this into Eqs. 9-2 and canceling out  $\rho$ , we obtain formulas that locate the *centroid C* or geometric center of the body; namely

$$\bar{x} = \frac{\int_V \tilde{x} dV}{\int_V dV} \quad \bar{y} = \frac{\int_V \tilde{y} dV}{\int_V dV} \quad \bar{z} = \frac{\int_V \tilde{z} dV}{\int_V dV} \quad (9-3)$$

These equations represent a balance of the moments of the volume of the body. Therefore, if the volume possesses two planes of symmetry, then its centroid must lie along the line of intersection of these two planes. For example, the cone in Fig. 9-4 has a centroid that lies on the  $y$  axis so that  $\bar{x} = \bar{z} = 0$ . The location  $\bar{y}$  can be found using a single integration by choosing a differential element represented by a *thin disk* having a thickness  $dy$  and radius  $r = z$ . Its volume is  $dV = \pi r^2 dy = \pi z^2 dy$  and its centroid is at  $\tilde{x} = 0, \tilde{y} = y, \tilde{z} = 0$ .

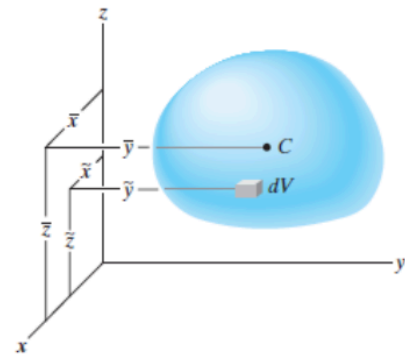
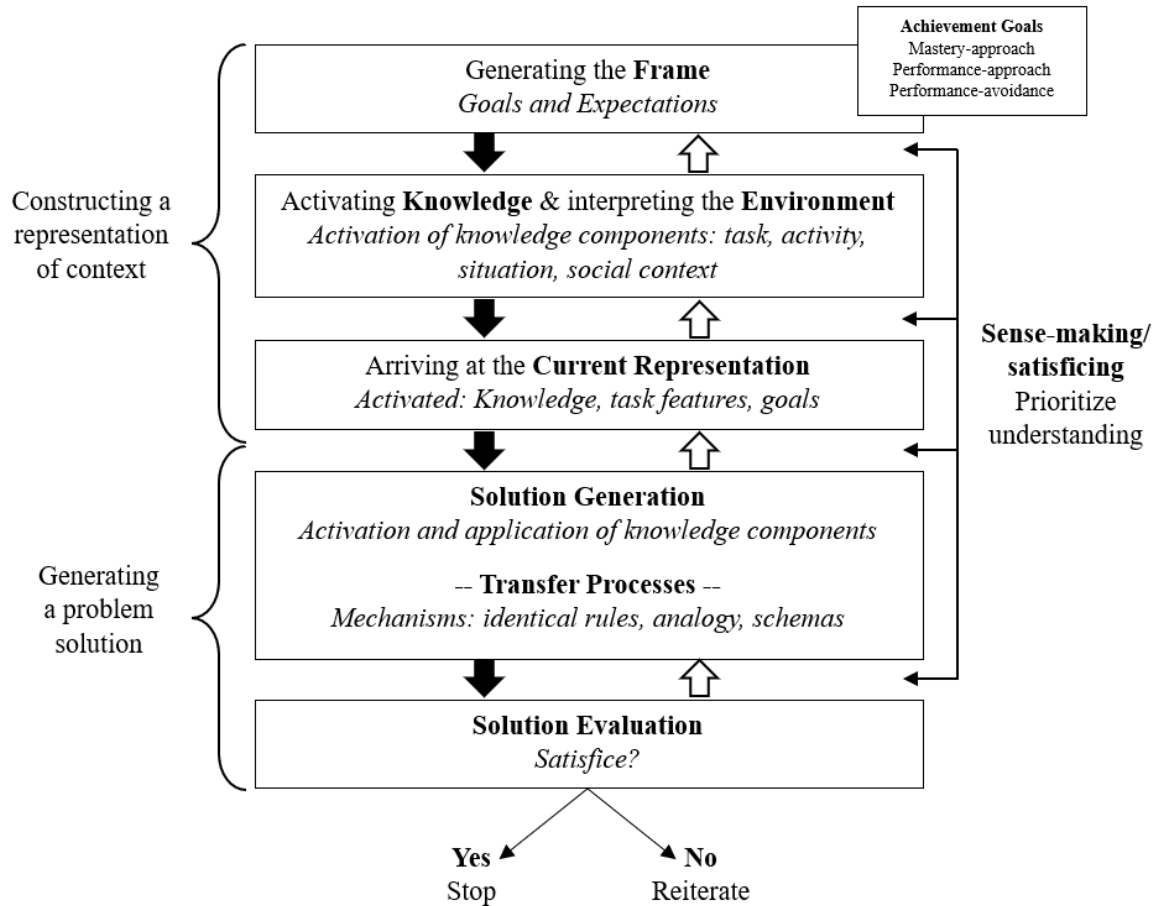


Fig. 9-3

(b) Equation based prompt from Statics textbook [22]

Figure 1: Engineering problem and prompt used in this study



**Figure 2: Sense-making framework of knowledge transfer. Adapted from [28].**

This framework was chosen based on its agreement with the authors understanding and conceptualization of the knowledge transfer process, and for the structure it provided in developing the interview protocol - questions and stages of the protocol were based in stages of the chosen framework. The think-aloud interviews were conducted individually and in-person by one member of the research team. Any artifacts produced by the participants - typically paper-based solutions to the problem - were recorded as well as an audio only transcript of the interview.

### Participants

Statics problems such as that considered here (Fig.1a) are taught as part of the first semester, sophomore year Statics course at UD and as such, any student from the second semester, sophomore year and above was eligible to participate in this study. As such, the research population initially consisted of 5 undergraduate mechanical engineering students. One male, sophomore level student and four juniors (three male, one female) chose to participate in this phase of the study.

## **Data Analysis**

Data was analyzed using thematic analysis [29,30] with both the participant solutions to the problem and interview transcripts being analyzed using a rubric based on both the stages of the knowledge transfer framework (Fig.2), the problem solving process, and the themes observed in prior work such as the language and symbols used by the participants [19]. Emergent themes were also developed from the data. Two members of the research team independently read and analyzed each transcript and solution before integrating the resulting themes and patterns into a single dataset.

## **Positionality Statement**

While the research team is split by gender (male/female), we recognize that we are all white and come from positions of privilege. Our decision to investigate the topic of knowledge transfer is driven by our belief that students *should* be able to complete tasks such as that we set for them here. This belief provides a clear direction for the research which is further refined by our choice to use a particular framework of knowledge transfer (that we personally agree with). We also recognize our position of power in asking our students to solve problems in front of us - there have been several instances of students seemingly wanting to perform well in front of the research team or feeling embarrassed that they could not solve the problem in front of us. While steps have been taken to mitigate all of these factors, such as explaining how participation was voluntary and not tied to any class or grading as this was purely a research exercise, we recognize that these factors impact the results of this study and that the findings we present are highly contextualized.

## **Findings - Equation Based Prompt**

Five mechanical engineering undergraduate students completed think-aloud interviews using the equation based prompt. One male, sophomore level student and four juniors (three male, one female) chose to participate in this phase of the study. All had GPAs above 3.2 (4.0 scale) and had recorded either an A or B grade in both Engineering Statics and Calculus. Of these five participants, only one was able to solve the problem correctly without guidance. None of the other students (four) were able to solve the problem, even after the provision of the prompt.

In terms of the stages of the knowledge transfer framework; (1) Generating the frame; students correctly identified the need to solve for the sum of the forces and moments in order to determine the correct answer. Comments such as “*It looks like statics [problem]... I'm gonna have to do sum of the moments. Start with a freebody diagram...*” were typical of student thinking at this stage.

(2) Activating Knowledge & Interpreting the Environment; four of five students recognized the need to use mathematical skills to solve the problem but only three students observed the need to find the centroid. Three of the participants clearly had memorized the centroid locations for

simple shapes such as squares and triangles; “*So like a triangle is 1/3*” but could not recall or formulate an equation to find the centroid of more complex geometries.

(3) Arriving at the Current Representation; all five participants drew a correct free body diagram of the problem and set up their equations for the sum of the forces correctly. Only three students wrote correct equations for the sum of the moments however.

(4) Solution Generation; three participants solved for the area correctly while only two managed to determine the weight of the plate correctly. Several students referred to using tabulated data or simpler shapes in other courses to find the centroid and this lack of practice with equations being a barrier to success in solving the problem used in this study which does not use a simple shape; “*So you have areas which you can find by, by just like simple shapes. And then those have like known centroids. And then you can just do sum of centroid times area divided by sum of area for this because your thing is modeled by an equation, you can't do that. So my dilemma now is remembering the formula.*”

(5) Solution Evaluation; the only student to obviously display reflective and evaluative practice was the individual who correctly solved the problem without any additional guidance (male, sophomore); “*This does make sense. An important part of engineering problems is asking yourself if it does make sense. This does make sense because we know that as we go more left by the equation, we're taking off more and more mass. So then it would be on the left side of the exact midpoint of three feet. So it'd be less than 1.5 feet, and I'm, I'm thinking this is like 1.125...*”. In this quote the participant is comparing their answer for the location of the centroid to a square. They correctly reason that their numerical answer should be less than 1.5 feet based on this reflection. The participant displays higher level, critical thinking skills in doing this that were not displayed by the other students. Also of note is the fact that this individual was able to link their understanding of centroids to similar topics such as distributed loads, which use the same mathematical formulations, again an ability that was unique to this individual.

Two of the other participants displayed interesting behaviors when presented with this problem. One of them (female, junior) stated the following when presented with the prompt: “*So, since I don't know how to use that, and I don't remember how I was taught irregular shapes. I'm just going to, like, fudge it and be like this is a regular shape because it's close enough to a triangle that I'm like, this seems relatively okay.*” As a first approximation, this choice of treating the shape as a triangle is perhaps an ok assumption. The final answer will however be incorrect. Again this quote speaks to the students' use of simple shapes in other classes and lack of mathematical understanding of centroids in general. The fact that this student is still able to make some headway in solving the problem without a correct numerical answer is good, however, in that it is indicative of a degree of conceptual thinking at a greater level than the other participants. One other participant (male, junior, older than typical college students) did not

display higher level thinking skills compared to the other students but did give the impression that they would have continued trying to solve this problem for an extended period - their interview had to be terminated by the researcher simply based on the time it was taking. This observation only relates to a single student but is clearly

Finally, The equation based prompt used with these participants (Fig. 1b) was examined and participants were asked whether this prompt was helpful or not as part of the interview process. Generally we observed that this prompt was unhelpful and perhaps even created additional confusion amongst the participants than prompting their prior knowledge. Comments from the participants often stated that the prompt added confounding ideas that they found difficult to resolve. Examples of these confounding issues included the inclusion of equations in multiple dimensions when only the location of the centroid in the horizontal direction is required to solve this problem, as well as the labels and symbols used in the mathematical prompt being different from those presented in the problem.

### Findings - Class Notes Based Prompt

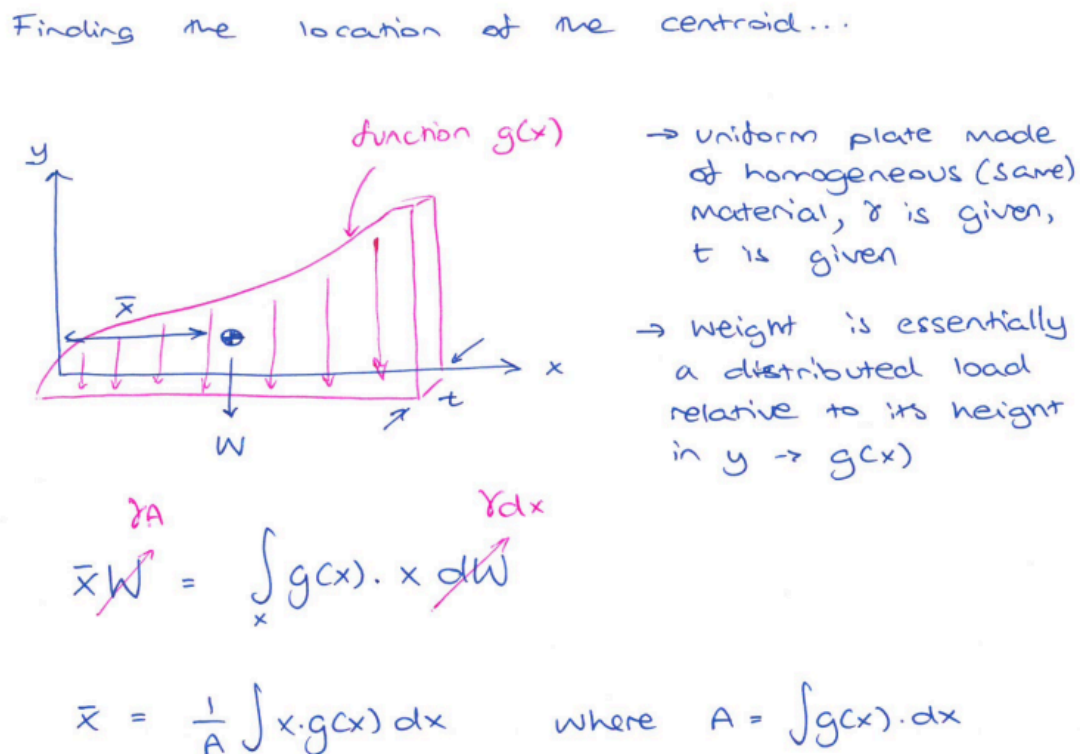


Figure 3: Prompt based on Statics course notes [31].

Based on peer debriefing and discussion within the research team, the decision was made after conducting the five initial interviews using the equation based prompt to change the intervention to utilize a more applied prompt rather than a purely mathematical formulation. To this end, an excerpt of the notes from the Statics class taught in the mechanical engineering department at



UD was chosen for use as the prompt when students struggled to find the centroid. The reasoning for using this prompt was the belief within the research team that the mathematical prompt was too abstract and that a more applied prompt would be more relevant to the students in that it used terminology that they would have used in class and could be more effective in activating prior knowledge. The excerpt of the class material provided to the students as this applied prompt is detailed in Fig.3. Aside from the change in the material used to prompt the participants, the interview protocol and procedure remained the same. Six mechanical engineering undergraduates participated in this second phase of the study. All six of these students were male; five were sophomores and one was a junior. All participants except one had GPAs above 3.3/4.0 and had received A and B grades in prior statics and mathematics courses. Four of these students were able to solve the problem correctly, although all of them required provision of the prompt in order to do so. The other two participants were shown the prompt but still failed to solve the problem.

(1) Generating the frame: students correctly identified the need to solve for the sum of the forces, moments and centroid in order to determine the correct answer. Comments such as “*[It is] A static reaction forces problem but it's also got complex geometry. So I believe that's going to have to require me finding the centroid of it*” were indicative of student thinking at this stage.

(2) Activating Knowledge & Interpreting the Environment: all six students recognized the need to use mathematical skills to solve the problem as well as the need to find the centroid. Again, several students made comments about using simple shapes in the past and knowing centroid locations for objects such as squares and triangles. “*I'm trying to remember how to do centroids of like with complex geometries because we've done like squares is easy, that's the center and we've done triangles.*”

(3) Arriving at the Current Representation: five of the six participants drew a correct free body diagram of the problem and set up their equations for the sum of the forces and moments correctly. The two students (different) who had initial problems with setting up their equations of static equilibrium were eventually able to solve the problem after being given the prompt.

(4) Solution Generation: all six participants solved for the area of the plate correctly but only four correctly determined the weight of the plate. All six participants required the prompt to be provided in order to progress to a calculation of the centroid but only four participants could solve the entire problem correctly after being shown the prompt. The two students who still could not solve the problem were not able to correctly interpret the prompt and apply the given equation for the centroid in this case, although both were close to being able to do so (minor math errors in their method in each case).

(5) Solution Evaluation: none of the students in this phase of the study using the applied prompt displayed significant reflective or evaluative practice concerning their answers. Note that this was despite questioning at the end of the problem solving process as to the correctness of their solution. Answers such as “*why wouldn't it be*” or “*I could check with, like the professor, ask if, like it was supposed to be like that or [ask] a TA if they did like a similar process to me*” reflect the kind of responses that students gave to this prompting.

Again, participants were asked how they felt about the prompt at the end of the process and what about it aided their thinking in solving the problem. It appeared that the division by area to find the centroid was something of a triggering condition in this regard; “*I remembered I need to do  $X$  times something but like, I felt like I was missing dividing by something because like if I, because I didn't remember correctly, which was the area*”.

## **Discussion**

Findings suggest that undergraduate mechanical engineering students struggle to determine the centroids of complex geometries i.e. shapes that require application of a mathematical equation to find the location of the centroid rather than geometries for which the centroid location might be tabulated or otherwise determined.

The use of a mathematical, equation-based-prompt to promote problem solving success and determination of the centroid was unsuccessful. It is possible however that the form of this prompt was confusing and that a simpler, fully mathematical prompt may be successful in helping students solve the problem - the prompt used in this case came from a textbook example and contained extraneous information concerning the centroid locations in additional dimensions for example. This additional information may have posed additional cognitive load and should perhaps be removed as it is not required to solve the problem (though the equation does not differ in these dimensions, only the symbols change, i.e.,  $x,y,z$ ).

An applied prompt based on prior course notes was more successful in enabling participants to find the centroid and solve the problem - four out of six undergraduate students were able to correctly solve the statics problem when the applied prompt was provided. It was unclear, however, if the prompt was successful in its intended purpose of activating prior knowledge and whether participants fully understood the conceptual and mathematical underpinning of the prompt. It is possible that the participants were able to copy the “pattern” and elements of the prompt rather than use it based on a full understanding of the mathematics. Some students appeared to only need to see the form of the equation to proceed to a solution, suggesting this equation is something they do not use frequently enough to have internalized. Clearly this uncertainty is problematic in terms of the goals of this study which are to activate existing prior knowledge in order to allow its transfer, rather than reteach concepts or allow students to copy a problem solving method. Evidence that the prompt was not fully understood includes the fact

that two of six participants still could not solve the problem even with the prompt, and that several other students asked clarifying questions regarding the prompt and seemed to indicate in their verbalizations that they were filling in terms in the equations rather than grasping them conceptually. Further work will be undertaken in the future to examine student understanding of centroids and the mathematics behind their determination.

A common theme in why student participants were not able to solve the problem was the lack of mathematical formulations for concepts such as the centroid being used in their courses. A majority of participants cited using either simple shapes (squares, triangles, etc.) or tabulated data to determine centroids in the past. This lack of practical application of mathematical concepts calls into question the existence of the correct mathematical prior knowledge in the minds of the students if it is not learned deeply or reinforced in subsequent classes. One of the participants also referred to this lack of use of various concepts in the program; *“Because it's statics. We've done a lot more complicated stuff since then. I think maybe this emphasizes the point of your of like that statement that I read that I agreed with. Like, it's difficult to transfer knowledge between courses. I think this kind of shows that like, you learn the specific knowledge for the course. And then like kind of when the course is over like or like when the exams are over, you kind of maybe don't retain it.”* While this statement indicates a welcome degree of metacognition on behalf of the student, it does not change the fact that this participant was unable to solve the problem and indicated that these important concepts are not being utilized in other areas of the curriculum or reinforced (a claim made by the participants and that we agree with based on our observations of the curriculum) - mechanisms suggested as best practices in the literature [9-13].

The chosen theoretical framework of knowledge transfer used in this study (Fig.2) includes a solution evaluation stage at the end of the problem solving process in which the participant reflects on their solution. This evaluative practice was rarely observed here, however, and when participants were prompted to examine the accuracy of their solution, the resulting answers were unconvincing. Notably, the only student who displayed any meaningful reflections during their problem solving process was the only participant who fully solved the problem without aid. This student was also the most clear in describing their solution method and demonstrated the most thorough understanding of all the concepts involved in solving the problem. This observation and the general lack of reflective practice among the unsuccessful problem solvers, coupled with results from prior work [19] which also suggested the importance of reflection in solving the problem, seem to indicate that an increased emphasis on teaching evaluative and reflective skills could improve problem solving ability. It is also possible that this lack of reflective practice is a symptom of the lack of conceptual understanding held by the participants in that it may not be possible to evaluate an answer which one does not fully understand the origin of.

Persistent problems with (English) units and a lack of utilizing a formal problem solving method were also observed in both implementations of the prompt. When participants did try to calculate the area of the plate, only three students used the specific gravity and the symbol gamma ( $\gamma$ ), which would have been physically correct, while four students used the density and the symbol rho ( $\rho$ ). While this use of terminology did not always lead to a numerical error in the solution it is notable that only students who used density (and  $\rho$ ) made mistakes in calculating the weight of the plate. Typically these students followed an “SI units approach” in which the density is multiplied by both volume and acceleration due to gravity to find a weight. These fundamental issues with units and misunderstandings concerning the English system of units were observed in prior work [20] and indicate the lack of true conceptual understanding of basic physical principles that are taught early in the engineering program. Again a possible cause of these errors is the lack of follow-through in using the concepts taught in lower level courses; *“we also don't tend to use as many British units so that would be I don't think that's a slug, or it's either a slug or a, is it a slug?”*.

## **Conclusion**

The use of a prompt to prime prior knowledge transfer and promote problem solving success was examined using a think-aloud interview protocol in which participants solved an engineering statics problem. Initially, five undergraduate mechanical engineering students attempted to solve a rigid body equilibrium problem that required mathematical skills such as integration and the determination of the location of the centroid in order to be solved correctly. Based on prior work, a mathematical, equation based prompt was prepared to aid the students in solving the problem, should they need it.

Only one of the five students was able to successfully solve the statics problem without the need for the equation-based prompt. None of the other four students could solve the problem, even with the aid of the supplied equation. Based on these results, the decision was made to alter the prompt to use a more applied equation based on the course notes from an engineering statics class. A further six undergraduate mechanical engineering students were then recruited into the study to test this new prompt. Four of these students were able to solve the problem, although all required the new prompt to be provided in order to do so.

Key findings from this study relate to the lack of conceptual understanding of both mathematics and physics displayed by the students and which hampered their problem solving success. Many participants made reference to the fact that the skills required to solve this problem, in particular the determination of the location of the centroid, were not repeated throughout the curriculum or used in later courses after they had been initially introduced. Participants also displayed a lack of reflective and evaluative thinking that may have enabled them to assess the accuracy of their solution and notice errors in their work. This lack of evaluation and reflection is perhaps linked to the lack of true conceptual understanding of the principles at play.

Future work will examine student conceptual understanding of centroids as well as their understanding of the mathematical formulations used to find the centroid. The use of reflective practices in problem solving and their impact on problem solving success will also be explored.

### **Acknowledgements**

Support for this work was provided by the National Science Foundation under Award No. 2301341. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. Research work was conducted under institutional IRB protocols, IRB#1965654. The authors would also like to thank Dr. Jenni Buckley for providing copies of her Engineering Statics class notes for use in this work.

### **References**

1. ABET, "Criteria for Accrediting Engineering Programs, 2020 – 2021 | ABET," ABET, 2021. <https://www.abet.org/accreditation/accreditation-criteria/criteria-for-accrediting-engineering-programs-2020-2021/> (accessed Jan. 29, 2024).
2. Moghaddam, Y., Demirkan, H. and Spohrer, J. T-Shaped Professionals: Adaptive Innovators. Hampton, NJ: Business Expert Press, 2018.
3. N. A. of Engineering and N. A. of Engineering, The Engineer of 2020: Visions of Engineering in the New Century. Washington, DC: The National Academies Press, 2004. doi: 10.17226/10999.
4. ASEE, Transforming Undergraduate Education in Engineering: Phase 1 Synthesizing and Integrating Industry Perspectives, ASEE, 1, May 2013. Accessed: Jan. 29, 2024. [Online]. Available: <https://tuee.asee.org/phase-i/report/>
5. Van der Heijden, B. (2002). Prerequisites to guarantee life-long employability. Personnel review, 31(1), 44-61. doi: 10.1108/00483480210412418.
6. Pollock, E. B., Thompson, J. R., and Mountcastle, D. B., "Student Understanding Of The Physics And Mathematics Of Process Variables In P- V Diagrams," AIP Conf. Proc., vol. 951, no. 1, pp. 168–171, Nov. 2007, doi: 10.1063/1.2820924.
7. Loverude, M. E., Kautz, C. H., & Heron, P. R. (2002). Student understanding of the first law of thermodynamics: Relating work to the adiabatic compression of an ideal gas. American journal of physics, 70(2), 137-148. doi: 10.1119/1.1417532.
8. Meltzer, D. E. (2004). Investigation of students' reasoning regarding heat, work, and the first law of thermodynamics in an introductory calculus-based general physics course. American Journal of Physics, 72(11), 1432-1446.
9. Orton, A. (1983). Students' understanding of integration. Educational Studies in Mathematics, 14(1), 1-18. doi: 10.1007/BF00704699.

10. Rebello, N. S., Cui, L., Bennett, A. G., Zollman, D. A., & Ozimek, D. J. (2017). Transfer of learning in problem solving in the context of mathematics and physics. In *Learning to solve complex scientific problems* (pp. 223-246). Routledge.
11. Cui, L., Rebello, N. S., & Bennett, A. G. (2006, February). College students' transfer from calculus to physics. In *AIP Conference Proceedings* (Vol. 818, No. 1, pp. 37-40). American Institute of Physics. doi: 10.1063/1.2177017.
12. Rebello, N. S. (2013). Facilitating Transfer of Learning and Problem Solving in Physics. *Bulletin of the American Physical Society*, 58.  
Available: <https://meetings.aps.org/Meeting/PSF13/Session/E1.4>
13. Schoenfeld, A. H. (2014). *Mathematical problem solving*. Elsevier. Accessed: Jan. 29, 2024. [Online]. Available: <https://www.elsevier.com/books/mathematical-problem-solving/schoenfeld/978-0-12-628870-4>
14. Martin, T., Rayne, K., Kemp, N.J., Hart, J., & Diller, K.R. (2005). Teaching for Adaptive Expertise in Biomedical Engineering Ethics. *Science and Engineering Ethics*, Vol. 11(2), pp. 257-276.
15. Martin, T., Rivale, S.D., & Diller, K.R. (2007). Comparison of Student Learning in Challenge-based and Traditional Instruction in Biomedical Engineering. *Annals of Biomedical Engineering*, Vol. 35, pp. 1312–1323.
16. Rayne, K., Martin, T., Brophy, S., Kemp, N. J., Hart, J. D., & Diller, K.R. (2006). The Development of Adaptive Expertise in Biomedical Engineering Ethics. *Journal of Engineering Education*, Vol. 95(2), pp. 165-173.
17. Passmore, L., & Litzinger, T., & Masters, C. B., & Turns, S., & Van Meter, P., & Firetto, C., & Zappe, S. (2010, June), Sources Of Students' Difficulties With Couples And Moments In Statics Paper presented at 2010 Annual Conference & Exposition, Louisville, Kentucky.  
<https://doi.org/10.18260/1-2--16198>
18. Litzinger, T. A., Meter, P. V., Firetto, C. M., Passmore, L. J., Masters, C. B., Turns, S. R., & Zappe, S. E. (2010). A cognitive study of problem solving in statics. *Journal of Engineering Education*, 99(4), 337-353. <https://doi.org/10.1002/j.2168-9830.2010.tb01067.x>
19. De Rosa, A. J., Reed, T. K., & Arndt, A. E. (2023, June). Work in Progress: Promoting the Transfer of Math Skills to Engineering Statics. In 2023 ASEE Annual Conference & Exposition.
20. De Rosa, A. J. (2020, June). Examining Knowledge Transfer Between Thermodynamics and Mathematics. In 2020 ASEE Virtual Annual Conference Content Access. Accessed: Jan. 29, 2024. [Online]. Available: <https://peer.asee.org/examining-knowledge-transfer-between-thermodynamics-and-mathematics>
21. De Rosa, A. J., Serbin, D., & Lee, S. (2019, November). Facilitating Cross-Course Connections & Knowledge Transfer between Engineering Thermodynamics and Mathematics (WIP). In 2019 Fall Mid Atlantic States Conference. Accessed: Jan. 29, 2024.

[Online]. Available:

<https://peer.asee.org/facilitating-cross-course-connections-knowledge-transfer-between-engineering-thermodynamics-and-mathematics-wip>

22. Hibbeler, R. C. (1997). *Engineering Mechanics: Statics*. 12th. Upper Saddle River, NJ: Pearson/Prentice-Hall.
23. Vygotsky, L. S. (1962). *Thought and language*. (E. Hanfmann & G. Vaker Eds., Trans.). Cambridge, MA: MIT Press.
24. Ericsson, K. A., & Simon, H. A. (1980). Verbal reports as data. *Psychological Review*, 87(3), 215–251.
25. Charters, E. (2003). The use of think-aloud methods in qualitative research: An introduction to think-aloud methods. *Brock Education Journal*, 12(2).
26. Cowan, J. (2019). The potential of cognitive think-aloud protocols for educational action-research. *Active Learning in Higher Education*, 20(3), 219-232.  
<https://doi.org/10.1177/1469787417735614>
27. Nokes-Malach, T. J., & Mestre, J. P. (2013). Toward a model of transfer as sense-making. *Educational Psychologist*, 48(3), 184-207. DOI: 10.1080/00461520.2013.807556
28. Nokes, T. J., & Belenky, D. M. (2011). Incorporating Motivation into a Theoretical Framework for Knowledge Transfer. *Psychology of Learning and Motivation*, 109–135. doi:10.1016/b978-0-12-387691-1.00004-1
29. Creswell, J. W., & Poth, C. N. (2016). *Qualitative inquiry and research design: Choosing among five approaches*. Sage publications.
30. Braun, C., Clarke, V., Hayfield, N., Davey, L., & Jenkinson, E. (2023). Doing reflexive thematic analysis. In S. Badger-Charleson & A. McBeath (Eds.) *Supporting research in counseling and psychotherapy* (pp. 19–38).
31. Buckley, J. (2023). *Centroids [Class handout]*. University of Delaware, MEEG210.