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Reflections on a ”Math Disaster”: the Role of Instructor Confusion in the Classroom

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the Role of Instructor Confusion in the Classroom

Abstract

When enacting active learning pedagogies such as problem-based learning or responsive teaching, instructors require students to make mistakes and admit to and grapple through confusion. Students are often reported to be resistant to active learning, and it is important for instructors to develop epistemic empathy for their students’ affective responses to confusion in the classroom. In this work, I report on two class sessions of a higher-level engineering elective in which I elicited and responded to student confusion in one class session, and then in the next, which I initially described as a “math disaster,” made technical mistakes and became confused myself. Through reflective practice on these experiences with confusion, I developed heightened empathy with students who are uncomfortable making mistakes in class, learned to use my own mistakes to model engineering practices, and re-framed my perspective on what it means to be a “good” engineering instructor. This work illustrates the benefits of incorporating reflective practice into the professional development of engineering instructors.

Introduction

Several reform pedagogies require students to grapple with confusion in the classroom, framing confusion as a positive indication of progress towards understanding [1]. For example, in productive failure [2], students grapple with a complex problem, initially explore incorrect solution paths, and eventually, with help from their instructor, collaboratively consolidate their work into the canonically correct solution. Responsive teaching involves presenting students with a problem and then responding to student thinking as they work through it [3]. Problem-based learning, initially developed for medicine [4] but later adapted to other areas, including chemical engineering [5], frames learning around problems, which students learn the material through solving. There are many reports on the effectiveness of these and other active learning pedagogies [6]-[9], but also reports of student resistance to active learning [10]-[14], which can lead to lower student evaluations that may disincentivize instructors from adopting these practices. One important aspect of addressing this issue is attending to the affective responses of the players involved to confusion.

Previous research has focused on students’ affect, often dismissing their discomfort with confusion as a deficit to be overcome by the individual student. Less common is the perspective that it is the responsibility of instructors to develop epistemic empathy with students’ academic experiences, which have created a legitimate fear of the consequences of failure [15]. Part of the process of developing epistemic empathy is for instructors to interrogate our own relationship with confusion; it is difficult to ask students to wade into messy, complex problems when their instructors are not comfortable letting students see them doing the same in the classroom.

Indeed, in response to the submitted abstract for this paper, one reviewer wrote, “there are many repetitions of ‘confusion’ and ‘failure’ that tend to block a positive expectation for the outcomes… Starting with ‘confusion’ to build up the learning experience seems controversial.” It is precisely this encultured perception of confusion that this work aims to challenge. Inevitably, students will be confused and struggle with problems throughout their education and beyond into their professional practice, and they must learn to embrace this confusion instead of seeing it as a sign of personal deficiency [16]. One goal of this work is to examine student and instructor discomfort with confusion and mistakes, and to contribute towards reframing confusion as a positive step towards understanding, as opposed to a negative to be overcome.

This work is based on the author's own teaching experience. As this is a story, in part, about the power of vulnerability in making and admitting mistakes, those experiences will be described in the first person. In teaching an upper-level undergraduate and graduate electrochemical engineering elective course, I made a series of math mistakes in working through an example problem in class. In response to these mistakes, and my confusion in trying to resolve them, I became flustered, apologizing to the class multiple times and later referring to the class as a "math disaster." This paper describes the process of reflecting on this experience, transforming my own perception of my mistakes from being a "disaster" indicating deficiencies in my teaching to being an opportunity to model a productive response to confusion for students.

Background

Student Confusion in the Classroom

Many STEM classes are taught using a traditional, passive, lecture-based structure, in which an expert instructor presents material to students [17]-[19]. On homework and exams, students are then asked to recreate an algorithmic problem-solving technique they have been taught, resulting in a single numerical answer [20]. There are a variety of criticisms of this teaching practice, including concerns related to equity [21] and student engagement [22]. In addition, by presenting knowledge to students instead of allowing them to work through a state of not-knowing to construct it themselves, lecture-based classes do not allow students to productively engage with the material as scientists and engineers [23]. The importance of making a place for confusion in the classroom has been studied in science [24]-[26] as well as in engineering [1], [27]. Making space for confusion is a key aspect of several active learning pedagogies; for example, responsive teaching is centered on responding to student confusion about a topic they are grappling with [3].

Students are often reported to be resistant to active learning pedagogies [10]-[14]. One aspect of this resistance may originate in a discomfort with being wrong or struggle; in attempting to relieve that discomfort, instructors may revert to more passive pedagogies, thus limiting student opportunities to learn [28], or instead dismiss the discomfort as a fault of the student. Epistemic empathy on the part of instructors has been leveraged to frame responding to student feelings, not just ideas, as something to be attended to using responsive teaching [28], [29].

Jaber *et al.* cultivated epistemic empathy in teachers by engaging them in a professional development program as science learners [15]. Their experiences, including confusion and frustration, helped them to understand how their students feel as learners, and thus improved their ability to teach responsively. Just as can students, experts can feel a sense of social-emotional risk in not-knowing [30]. Understanding those feelings can help us to reframe confusion both for instructors and for students. In this work, we are interested in instructor confusion within the context of the class they are instructing, which adds an additional dimension to social-emotional risk by changing the witness to the confusion from researchers or other instructors to the instructor's own students.

Reflective Practice

Kolb's experiential learning theory serves as a model for how practitioners can learn and grow from an experience through reflective practice [31]. It begins with a stage of concrete experience, which serves as the foundation for learning, and is then followed by reflective observation, in which the learner intentionally reflects on the concrete experience, abstract conceptualization, in which they generalize what they have learned, and a planning stage in which they plan for the next concrete experience. These steps are also referred to as "experiencing, reflecting, thinking and acting" [32]. In education, reflective practice cycles have been applied to designing learning activities that will lead students through the cycles of reflective practice [33]-[36], as well as to instructors working to improve their teaching [37]-[41]. In this work, it is used for the latter purpose, in which the concrete experience of the first stage, and which is planned for in the last, is a class session of a university engineering course.

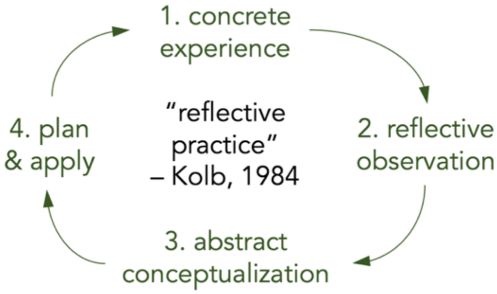


Figure 1. Illustration Kolb's reflective practice framework [31].

Research Questions

This study focuses on two class sessions of my course, in which I elicited my students' confusion intentionally as part of reflective teaching in the first, and in the second, became confused about the solution to an example problem. Reflecting on these two episodes and their implications for my instructional practice led to the following research questions:

1. In what ways does confusion manifest in an interactive classroom environment?
2. How can an instructor engage in reflective practice to make sense of and shift their framing of their confusion and mistakes?

Methods

Participants, Positionality, and Context

This work is part of an ongoing ethnographic research project in which I serve as an instructor in the chemical engineering department at a private, research-focused university while studying the factors impacting instruction in the department from within. For this paper, I focus on one aspect of that project, which is the course I designed and led as an instructor in the fall semester of 2023. The course was a one-semester senior-level chemical engineering elective titled "Electrochemical Engineering." The course had 15 students, most undergraduate seniors, but also included several juniors and several graduate students. I am a white woman born in the U.S., have earned bachelor's and PhD degrees in chemical engineering, and am currently working as a post-doctoral fellow in engineering education. My dissertation topic centered electrochemistry for battery applications, which motivated the course design. For my post-doctoral research, I am mentored by a scholar of chemical engineering education, and study instructor decision-making in adopting active learning pedagogies. The course described here was my first experience with designing a semester-length course, and also with being the lead instructor for such a course. In my post-doctoral research, I study and advocate for the use of active learning pedagogies, but insecurity in my role as a new instructor influenced my comfort with the process of enacting those pedagogies, which contributes to the phenomenon reported in this paper.

Data and Analysis

I use an autoethnographic approach to explore instructor experiences with confusion. Specific data sources include my instructor field notes on teaching experiences, personal reflection memos, and notes that I took during and immediately following meetings with mentors. I analyze the data by organizing it into the phases of the reflective practice framework, using an iterative process to identify themes within each phase.

Findings

The findings center two consecutive class sessions, the third and fourth of the semester, and describe one cycle of reflective practice (see Figure 1) for each day. In the first cycle, I elicited student confusion by posing problems and asking students to work through them with their classmates. In the second cycle, I had similar intentions, but made several mistakes in my own solutions to that day's problems and became confused as I tried to explain my (incorrect) work to students. In the following sections, I describe my reflections on these two different manifestations of confusion in my classroom, the first on the students' part and the second on my own, using Kolb's framework.

Cycle 1: A "successful" implementation of responsive teaching

Figure 2 summarizes the first cycle of reflective practice highlighted in this work. Below, I describe the context, including the problems planned for the class period, and then elaborate each of the four steps of Kolb's reflective practice.

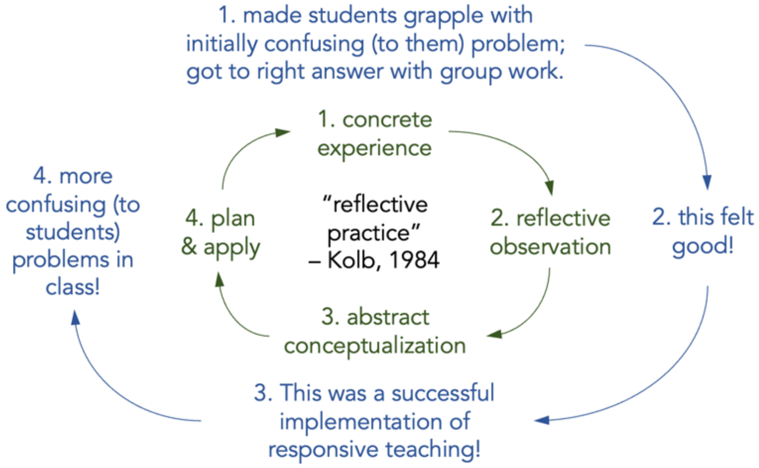
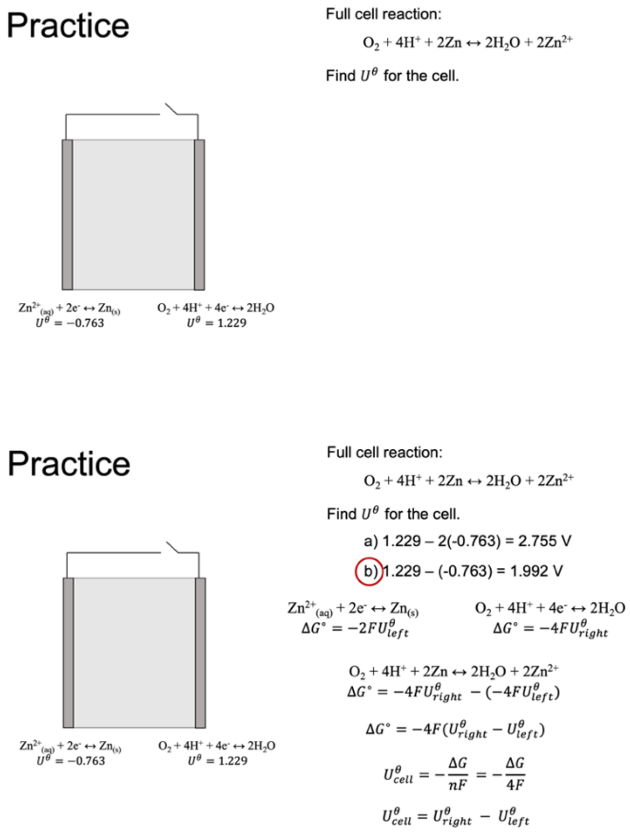


Figure 2. The first cycle of the reflective practice framework applied to the third day of the electrochemical engineering elective course.

Context

This was the third day of class, a Wednesday. On the first day we went through course goals, plans, and shared expectations and began to introduce basic principles. On the second day we began thermodynamics, introducing the equilibrium cell potential; I sensed that there was some confusion about the meaning of the potential, so for this third class period, I planned several problems to help students grapple with the meaning of the potential. Based on the second day of class, they knew that full-cell reactions are the sum of two half-reactions, one oxidation and one reduction, multiplied by stoichiometric factors such that the electrons cancel. They also knew that half-reactions are written as reduction reactions (and thus the standard potential given in tables is the reduction potential) and the full-cell equilibrium potential is the reduction potential of the right reaction minus that of the left (or, equivalently, the sum of the reduction potential of the reduction reaction and the oxidation potential, which is the negative of the reduction potential, of the oxidation reaction).

I planned two practice problems to help with understanding the equilibrium potential of half-reactions and full-cell reactions. In the first practice problem, stoichiometrically one reaction must be multiplied by a factor of two for the electrons to be balanced in generating the full-cell reaction; however, the standard reduction potentials are reported on a per mole of electrons basis, so the half-cell potentials are not scaled by this stoichiometric factor to calculate the full-cell potential. The question is shown in Figure 3a, where students are asked to find the cell potential. Figure 3b shows the same slide with the steps to solve the problem, each of which appear one at a time with animation. I planned to show the slide in Figure 3a, give the students time to work with neighbors, and then ask them for their answers and explanations before stepping through Figure 3b.



Practice

Full cell reaction:

$O_2 + 4H^+ + 2Zn \leftrightarrow 2H_2O + 2Zn^{2+}$

Find U^θ for the cell.

$Zn^{2+}_{(aq)} + 2e^- \leftrightarrow Zn_{(s)}$
$U^\theta = -0.763$

$O_2 + 4H^+ + 4e^- \leftrightarrow 2H_2O$
$U^\theta = 1.229$

a.

Practice

Full cell reaction:

$O_2 + 4H^+ + 2Zn \leftrightarrow 2H_2O + 2Zn^{2+}$

Find U^θ for the cell.

a) 1.229 – 2(-0.763) = 2.755 V

[✓ b)] 1.229 – (-0.763) = 1.992 V

$Zn^{2+}_{(aq)} + 2e^- \leftrightarrow Zn_{(s)}$
$\Delta G^\circ = -2FU^\theta_{left}$

$O_2 + 4H^+ + 4e^- \leftrightarrow 2H_2O$
$\Delta G^\circ = -4FU^\theta_{right}$

$O_2 + 4H^+ + 2Zn \leftrightarrow 2H_2O + 2Zn^{2+}$
$\Delta G^\circ = -4FU^\theta_{right} - (-4FU^\theta_{left})$

$$\Delta G^\circ = -4F(U^\theta_{right} - U^\theta_{left})$$

$$U^\theta_{cell} = -\frac{\Delta G}{nF} = -\frac{\Delta G}{4F}$$

$$U^\theta_{cell} = U^\theta_{right} - U^\theta_{left}$$

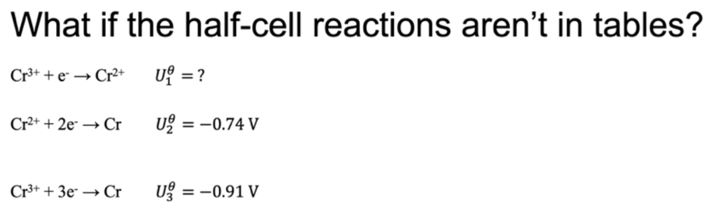
$Zn^{2+}_{(aq)} + 2e^- \leftrightarrow Zn_{(s)}$
$U^\theta = -0.763$

$O_2 + 4H^+ + 4e^- \leftrightarrow 2H_2O$
$U^\theta = 1.229$

b.

Figure 3. The first practice problem on the third day of class as given (a) and with the solution shown (b).

For the second problem, I planned to move on to calculating an unknown half-cell reduction potential from other known half-cell reduction potentials (Figure 4a). In this example, stoichiometrically, reaction 1 (which we want to find, and which involves 1 electron) can be generated by subtracting reaction 2 (which involves 2 electrons) from reaction 3 (which involves 3 electrons), with both reactions having a stoichiometric factor of unity. However, because the reduction potential of each reaction is reported on a per mole of electrons basis, the actual potentials of reactions 2 and 3 as written are $2U_2^\theta$ and $3U_3^\theta$, and the potentials of the reactions as written are the quantities that must be subtracted to find the reduction potential of reaction 1. Thus, unlike in the previous problem, in this case the lack of balance of the electrons needs to be accounted for, so the correct answer is that $U_1^\theta = 3U_3^\theta - 2U_2^\theta$. I planned to ask them what the reduction potential of reaction 1 should be, with the expectation that many students would account for stoichiometry of molecular species, not of electrons, resulting in an initial answer of $U_1^\theta = U_3^\theta - U_2^\theta$. After soliciting their initial reactions, assuming that I had correctly predicted their mistake, I planned to give them time to work through it in small groups. Figure 4b shows the same slide as Figure 4a with the steps to solve the problem, each of which appear one at a time with animation. It shows two ways to solve the problem, the first mathematical, by calculating the Gibbs energy of each reaction and translating it to potential, and the second by understanding the stoichiometric basis for the standard reduction potentials.



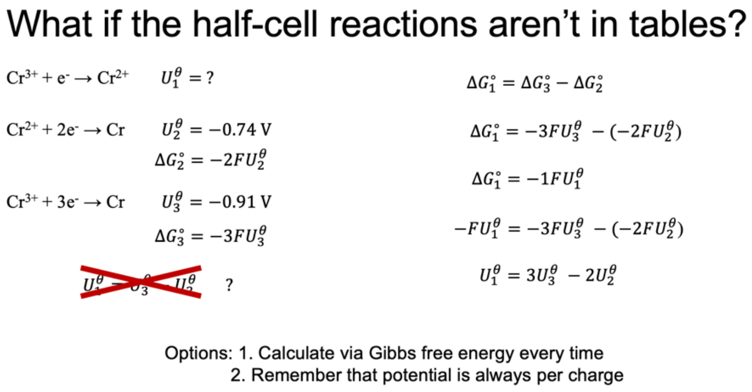
What if the half-cell reactions aren't in tables?

$Cr^{3+} + e^- \rightarrow Cr^{2+}$ $U_1^\theta = ?$

$Cr^{2+} + 2e^- \rightarrow Cr$ $U_2^\theta = -0.74$ V

$Cr^{3+} + 3e^- \rightarrow Cr$ $U_3^\theta = -0.91$ V

a.



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$Cr^{3+} + e^- \rightarrow Cr^{2+}$ $U_1^\theta = ?$

$Cr^{2+} + 2e^- \rightarrow Cr$ $U_2^\theta = -0.74$ V

$\Delta G_2^\circ = -2FU_2^\theta$

$Cr^{3+} + 3e^- \rightarrow Cr$ $U_3^\theta = -0.91$ V

$\Delta G_3^\circ = -3FU_3^\theta$

$\cancel{U_1^\theta = U_3^\theta - U_2^\theta}$?

$\Delta G_1^\circ = \Delta G_3^\circ - \Delta G_2^\circ$

$\Delta G_1^\circ = -3FU_3^\theta - (-2FU_2^\theta)$

$\Delta G_1^\circ = -1FU_1^\theta$

$-FU_1^\theta = -3FU_3^\theta - (-2FU_2^\theta)$

$U_1^\theta = 3U_3^\theta - 2U_2^\theta$

Options: 1. Calculate via Gibbs free energy every time
2. Remember that potential is always per charge

b.

Figure 4. The second practice problem on the third day of class as given (a) and with the solution shown (b).

After these two problems, I prepared slides covering how to correct for concentration and temperature using activity and entropy, which sets up the topic for the fourth day of class, Pourbaix Diagrams.

1. Concrete Experience

Presented with the problem in Figure 3a, some students tried to account for stoichiometry by scaling the potential of the zinc reaction by a factor of two, and others said the potentials should be subtracted without a stoichiometric correction. I gave them time to think about it and calculate the potential using the Gibbs energy, and after working with their neighbors they all agreed that it must be the latter. After this agreement was reached, we walked through the solution together using the slide shown in Figure 3b and I emphasized that I think of the potentials as "per mol of electrons" so that each potential already accounts for an equal number of electrons and does not have to be re-balanced.

For the second problem (Figure 4a), multiple students immediately said that $U_1^\theta = U_3^\theta - U_2^\theta$, which is what I expected. After this initial reaction, I did not agree or disagree with that answer, but gave them time to work through it with their classmates, and walked around as they did so. I was asked a few questions about Gibbs energy and potential, but for the most part students worked without intervention. As I walked around, I heard two different explanations. When we came together, I first asked one student to explain his work; he went through a Gibbs energy calculation process that exactly mirrored what I had on the slide, so as he explained his work I clicked through the animations; other students were nodding and agreed. I then asked a second student to explain her work, and she said she didn't do any math, but realized that U_2^θ is per mole of electrons and must be multiplied by 2 to account for the actual stoichiometry represented, so the actual potential of reaction 2 as written is $2U_2^\theta$, and analogously for U_3^θ the potential is $3U_2^\theta$, so the potentials that must be subtracted to find U_1^θ are $3U_3^\theta - 2U_2^\theta$, the same answer as reached by the first student. Other students also agreed with this reasoning; I validated both and explained that these are two different ways of thinking about it that are equally physically grounded, and we moved on. My perception was that the students were all satisfied and felt comfortable with the concepts covered by the problems.

We then covered the remaining slides as planned.

2. Reflective Observation

In my field notes written immediately after this class period, I described what happened with both problems, and after the description of the second (Figure 4), I wrote "this in particular felt good, like we grappled (with a problem) and got somewhere." Because this was only the third day of class, I had previously felt that I was still getting used to being a lead instructor and the classroom dynamics of this group of students. I noted that after I returned to my office after this class, I told a collaborator that "I feel like I'm finally getting into the swing of it." I was also proud that I had correctly predicted how my students would respond to the problem, in that they would not initially account for the electrons in the second problem and then would realize that they needed to upon being given time to work through it; this made me feel like I was in control of how the class would proceed, orchestrating student confusion.

3. Abstract Conceptualization

Upon reflecting on this experience, I felt that it was a successful implementation of responsive teaching. In both cases, I posed a problem to students, listened to their responses, gave them time to work together to reach an answer, and then tailored the follow-up stage to their reasoning. It also confirmed to me the effectiveness of active learning, because students made progress towards conceptual understanding of the potential through grappling with these problems.

4. Plan & Apply

After this successful teaching experience, I didn't feel the need to make any changes to my teaching practice. I felt validated as a good engineering instructor, and that my teaching was aligned with research-based pedagogical techniques like responsive teaching. I planned to continue to incorporate challenging problems for students to grapple with in class.

Cycle 2: A "math disaster"

The next cycle of reflective practice occurred during and after the following class period. Figure 5 includes the first cycle (from Figure 2) and adds the steps of the second cycle. I first describe the context, in particular the example problems I planned to solve in class, and then elaborate each step of the second cycle of reflective practice.

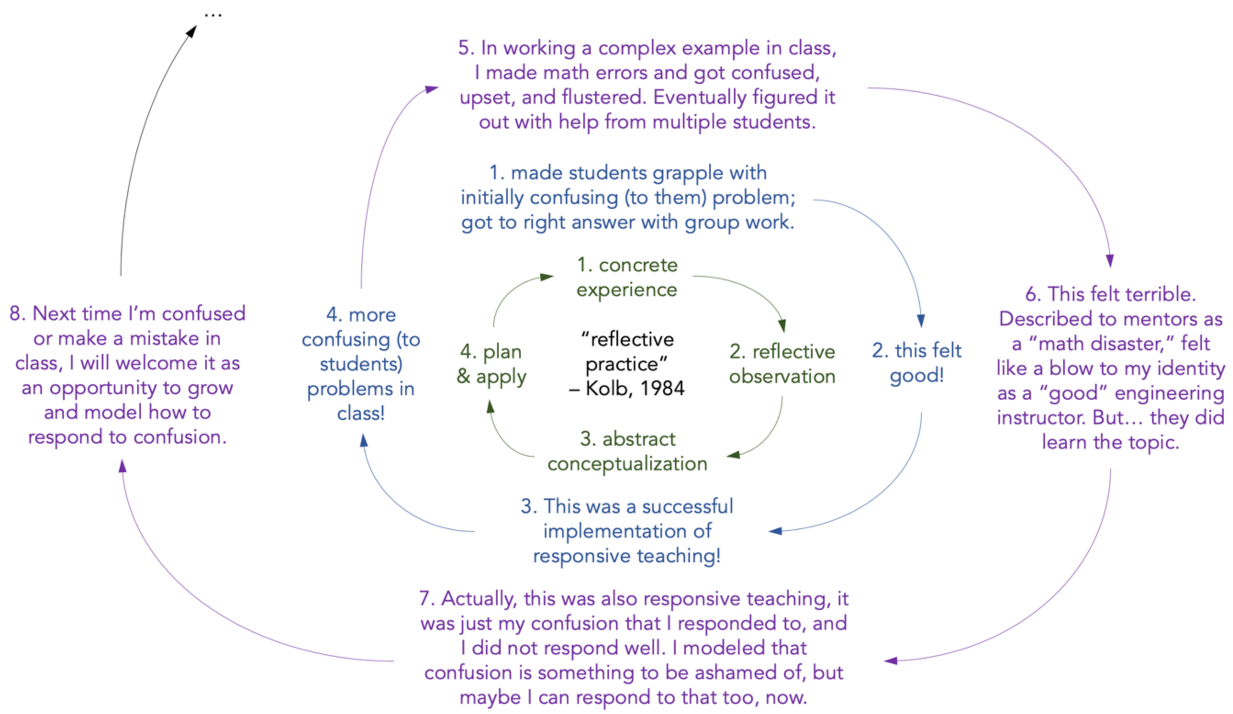


Figure 5. The first and second cycles of reflective practice applied to the third and fourth days of the electrochemical engineering elective course.

Context

This was the fourth day of class, a Monday. I planned to review concepts related to electrochemical potential and methods for calculating the relationship between potential and concentration, and then spend most of the class applying these concepts to the derivation of Pourbaix diagrams, which describe the regions of stability of different species as a function of potential and pH. To do this, I planned to begin by deriving the reference lines describing the stability of H^+ *vs* H_2 and O_2 *vs* H_2O conventionally present on all Pourbaix diagrams. The correct derivation is shown in Figure 6a. In the version of the derivation that I prepared and showed in class (Figure 6c), I assigned a stoichiometric coefficient of +2 to H^+, despite H^+ being a reactant and therefore having a negative stoichiometric coefficient. However, I also omitted a negative sign from the definition of the pH, and these errors negated each other, leading to the correct relationship between potential and pH and therefore the correct diagram. A similar error is present in the derivation of the O_2 *vs* H_2O line. Mistakes are highlighted in yellow in Figure 6c. Following the derivation of the diagram, I planned a discussion of how to determine which region on the diagram corresponds to which species.

Next, I planned to have the whole class work together with me to derive the potential-pH relationship for the first two zinc reactions:

1. $Zn^{2+} + 2e^- \leftrightarrow Zn$, and

2. $Zn^{2+} + H_2O \leftrightarrow ZnO + 2H^+$.

The first reaction does not involve H^+ and therefore the corresponding stability line does not depend on the pH, while the second does not involve electrons and therefore the corresponding stability line does not depend on the potential. For these reactions, before showing any math, I planned to ask the students to discuss which factors (potential and/or pH) influence the stability and therefore how the line should appear (horizontal, vertical, or sloped). After working through each calculation with them, I planned to ask them to decide which species was stable on which side of each line.

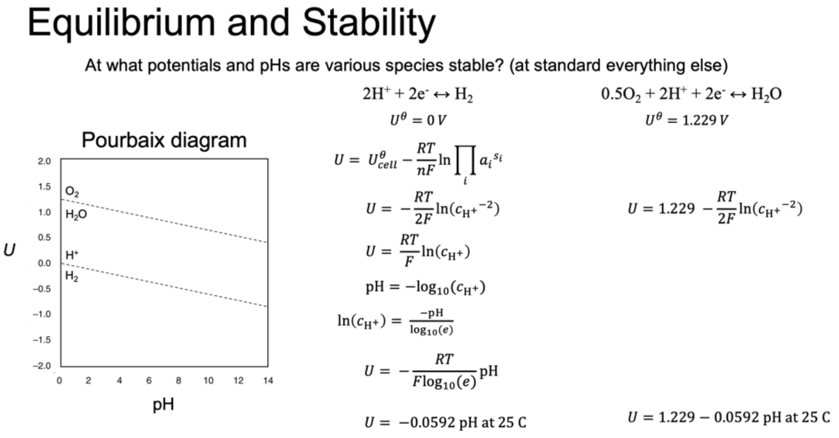
Finally, for the third reaction:

3a. $ZnO + 2H^+ + 2e^- \leftrightarrow Zn + H_2O$

(which was initially written on the slide as an oxidation reaction,

3b. $Zn + H_2O \leftrightarrow ZnO + 2H^+ + 2e^-$,

counter to convention), I planned to provide students with the Gibbs energy of reaction ($\Delta G^{\circ}_{\mathrm{Rx}} = -91.91$ kJ for the oxidation reaction) and ask them to work together to derive the stability line while I walked around the room. The correct derivation of the stability lines for each of these three zinc reactions is shown in Figure 6b, but the slide I prepared (Figure 6d) had additional errors, highlighted in yellow. Given the relationship between equilibrium cell potential and $\Delta G^{\circ}_{\mathrm{Rx}}$ ($U^{\theta}_{cell} = -\frac{\Delta G}{nF}$), a $\Delta G^{\circ}_{\mathrm{Rx}}$ of -91.91 kJ should result in a positive cell potential, but I had dropped a negative sign. However, if I had followed convention and written the reaction as a reduction reaction, $\Delta G^{\circ}_{\mathrm{Rx}}$ would have been positive and therefore resulted in the correct negative equilibrium cell potential. Finally, I planned to have students discuss how to assign different species to regions on the Pourbaix diagram generated by the derived stability lines.



Equilibrium and Stability

At what potentials and pHs are various species stable? (at standard everything else)

$2H^+ + 2e^- \leftrightarrow H_2$

$U^{\theta} = 0\,V$

$$U = U^{\theta}_{cell} - \frac{RT}{nF}\ln\prod_i a_i^{s_i}$$

$$U = -\frac{RT}{2F}\ln(c_{H^+}{}^{-2})$$

$$U = \frac{RT}{F}\ln(c_{H^+})$$

$$\mathrm{pH} = -\log_{10}(c_{H^+})$$

$$\ln(c_{H^+}) = \frac{-\mathrm{pH}}{\log_{10}(e)}$$

$$U = -\frac{RT}{F\log_{10}(e)}\mathrm{pH}$$

$U = -0.0592$ pH at 25 C

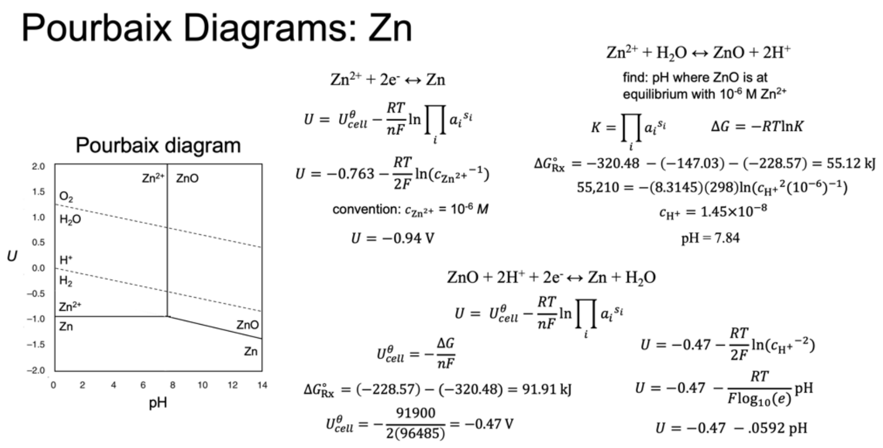
$0.5O_2 + 2H^+ + 2e^- \leftrightarrow H_2O$

$U^{\theta} = 1.229\,V$

$$U = 1.229 - \frac{RT}{2F}\ln(c_{H^+}{}^{-2})$$

$U = 1.229 - 0.0592$ pH at 25 C

a.



Pourbaix Diagrams: Zn

$Zn^{2+} + 2e^- \leftrightarrow Zn$

$$U = U^{\theta}_{cell} - \frac{RT}{nF}\ln\prod_i {a_i}^{s_i}$$

$$U = -0.763 - \frac{RT}{2F}\ln({c_{Zn^{2+}}}^{-1})$$

convention: $c_{Zn^{2+}} = 10^{-6}\ M$

$$U = -0.94\ \mathrm{V}$$

$Zn^{2+} + H_2O \leftrightarrow ZnO + 2H^+$

find: pH where ZnO is at equilibrium with 10^{-6} M Zn^{2+}

$$K = \prod_i {a_i}^{s_i} \qquad \Delta G = -RT\ln K$$

$$\Delta G^{\circ}_{\mathrm{Rx}} = -320.48 - (-147.03) - (-228.57) = 55.12\ \mathrm{kJ}$$

$$55{,}210 = -(8.3145)(298)\ln({c_{H^+}}^2(10^{-6})^{-1})$$

$$c_{H^+} = 1.45\times10^{-8}$$

$$\mathrm{pH} = 7.84$$

$ZnO + 2H^+ + 2e^- \leftrightarrow Zn + H_2O$

$$U = U^{\theta}_{cell} - \frac{RT}{nF}\ln\prod_i {a_i}^{s_i}$$

$$U^{\theta}_{cell} = -\frac{\Delta G}{nF}$$

$$\Delta G^{\circ}_{\mathrm{Rx}} = (-228.57) - (-320.48) = 91.91\ \mathrm{kJ}$$

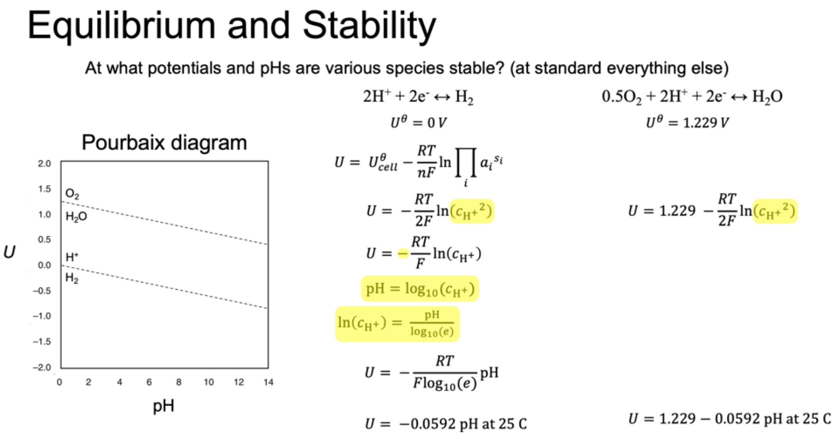
$$U^{\theta}_{cell} = -\frac{91900}{2(96485)} = -0.47\ \mathrm{V}$$

$$U = -0.47 - \frac{RT}{2F}\ln({c_{H^+}}^{-2})$$

$$U = -0.47 - \frac{RT}{F\log_{10}(e)}\mathrm{pH}$$

$$U = -0.47 - .0592\ \mathrm{pH}$$

b.



Equilibrium and Stability

At what potentials and pHs are various species stable? (at standard everything else)

$2H^+ + 2e^- \leftrightarrow H_2$

$$U^{\theta} = 0\ V$$

$$U = U^{\theta}_{cell} - \frac{RT}{nF}\ln\prod_i {a_i}^{s_i}$$

$$U = -\frac{RT}{2F}\ln({c_{H^+}}^2)$$

$$U = -\frac{RT}{F}\ln(c_{H^+})$$

$$\mathrm{pH} = \log_{10}(c_{H^+})$$

$$\ln(c_{H^+}) = \frac{\mathrm{pH}}{\log_{10}(e)}$$

$$U = -\frac{RT}{F\log_{10}(e)}\mathrm{pH}$$

$$U = -0.0592\ \mathrm{pH\ at\ 25\ C}$$

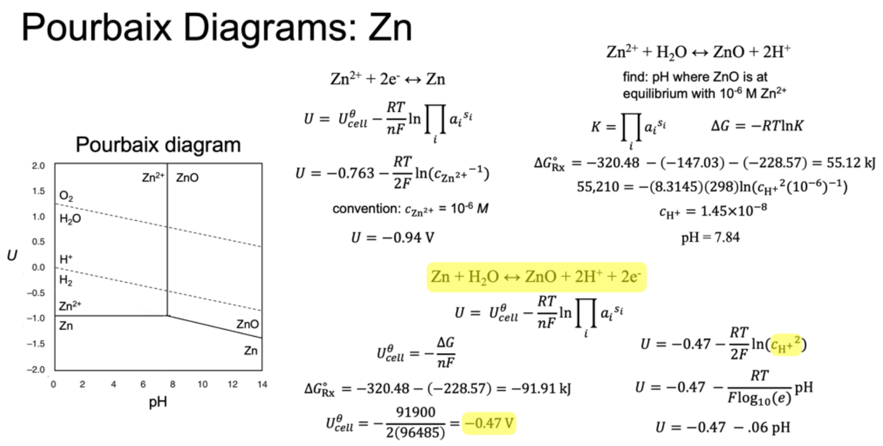
$0.5O_2 + 2H^+ + 2e^- \leftrightarrow H_2O$

$$U^{\theta} = 1.229\ V$$

$$U = 1.229 - \frac{RT}{2F}\ln({c_{H^+}}^2)$$

$$U = 1.229 - 0.0592\ \mathrm{pH\ at\ 25\ C}$$

c.



Pourbaix Diagrams: Zn

$Zn^{2+} + 2e^- \leftrightarrow Zn$

$$U = U^{\theta}_{cell} - \frac{RT}{nF}\ln\prod_i {a_i}^{s_i}$$

$$U = -0.763 - \frac{RT}{2F}\ln({c_{Zn^{2+}}}^{-1})$$

convention: $c_{Zn^{2+}} = 10^{-6}\ M$

$$U = -0.94\ \mathrm{V}$$

$Zn^{2+} + H_2O \leftrightarrow ZnO + 2H^+$

find: pH where ZnO is at equilibrium with 10^{-6} M Zn^{2+}

$$K = \prod_i {a_i}^{s_i} \qquad \Delta G = -RT\ln K$$

$$\Delta G^{\circ}_{\mathrm{Rx}} = -320.48 - (-147.03) - (-228.57) = 55.12\ \mathrm{kJ}$$

$$55{,}210 = -(8.3145)(298)\ln({c_{H^+}}^2(10^{-6})^{-1})$$

$$c_{H^+} = 1.45\times10^{-8}$$

$$\mathrm{pH} = 7.84$$

$Zn + H_2O \leftrightarrow ZnO + 2H^+ + 2e^-$

$$U = U^{\theta}_{cell} - \frac{RT}{nF}\ln\prod_i {a_i}^{s_i}$$

$$U^{\theta}_{cell} = -\frac{\Delta G}{nF}$$

$$\Delta G^{\circ}_{\mathrm{Rx}} = -320.48 - (-228.57) = -91.91\ \mathrm{kJ}$$

$$U^{\theta}_{cell} = -\frac{91900}{2(96485)} = -0.47\ \mathrm{V}$$

$$U = -0.47 - \frac{RT}{2F}\ln({c_{H^+}}^2)$$

$$U = -0.47 - \frac{RT}{F\log_{10}(e)}\mathrm{pH}$$

$$U = -0.47 - .06\ \mathrm{pH}$$

d.

Figure 6. Slides walking through the correct derivation of the Pourbaix diagram reference lines (a) and example Zinc Pourbaix diagram (b), as well as reconstructions of the error-filled versions of those same slides which were erroneously used in class (c, d). Errors in (c) and (d) are highlighted in yellow.

5. Concrete Experience

In the derivation of the H^+ *vs* H_2 and O_2 *vs* H_2O lines, I did not notice my mistakes, and none of the students commented on them. The derivation proceeded uneventfully and was followed by a discussion of how to determine which region on the diagram corresponds to which species. In that discussion, students did not immediately have an answer, but worked through confusion with their peers and eventually correctly decided on labels for the stability regions on the diagram.

For the first two zinc reactions, students discussed which factors influence the stability and therefore how the line should appear and which species was stable on which side of each line. This portion of the lesson went unremarkably, with students volunteering ideas and eventually collaboratively reaching the correct answers.

Finally, for the third reaction, as I listened to students discussing in their small groups how to calculate the stability, I realized that I was missing a negative sign in my own work, but knew from the literature that my final answer was correct, and did not feel that I had time to figure out what had happened. When we came together to go through the answer, I tried to hide my mistake by very quickly clicking through the animations showing the steps of the calculation, not asking students for their own answers, nor asking them if their answers agreed with mine (because I knew they would not, since mine were wrong), nor giving them the chance to digest the process. I then added the line to the diagram and asked them to decide which species was stable on which side of the line. After a period of silence, a student politely asked me why the equilibrium cell potential was negative if $\Delta G^{\circ}_{\mathrm{Rx}}$ was also negative. I acknowledged that she was correct, apologized for the mistake, and then considered the problem for a moment before realizing that I should have written the reaction as a reduction reaction, cancelling out the error. I again apologized and promised to fix the slides immediately after class and send an email to the students.

Perhaps encouraged by his peer having asked a question about my math, another student then asked why, on the previous slide, H^+ had a positive stoichiometric coefficient. After a long pause, because I had not previously noticed this mistake, I said “huh,” and again acknowledge that he was correct. We then spent several minutes troubleshooting, with that student and several others making suggestions, such as that the initial equation was wrong, the reaction was written backwards, or that the slope should indeed be positive. We together realized that each of these was not the correct resolution but could not find an answer. Eventually, we reached the end of the class period, and I said, “I’m really sorry you guys, thank you for your patience,” and again promised to resolve the question after class and send out an email. Several students remained after class, and one eventually noticed the second error, in the definition of pH, which cancelled out the first. I thanked that student and corrected the slides, removing all trace of the mistake-filled slides from both the course Canvas page and my own computer despite this ongoing research project; Figures 6c and 6d are reconstructions based on my notes. I then recorded a video explaining the correct calculations and emailed students with a link to the video within 30 minutes of the end of class. The email I sent to students included the line “I’m so sorry for the confusion – your professors are humans who make dumb mistakes too ☺”

6. Reflective Observation

My first phase of reflection was individual: I began by recording field notes describing the concrete experience. At the end of the retelling, I wrote "SO BAD!! SO PAINFUL!!" and, "I feel like a fraud, like I've been 'found out' as someone who is not a real expert in this. Truly a disaster, worst case scenario. At least everyone was nice." I then returned to my shared office and told a peer that I had had a "math disaster" in class. That afternoon, I traveled to a different city for an interview for a teaching position; in my notes I wrote that the "disastrous" teaching experience "feels extra bad because… I was already feeling unqualified / anxious" for the job interview.

Several days later, I had a regular meeting with a mentor, who asked how teaching was going. I said, "I had a math disaster on Monday, but it's okay," and tried to redirect the conversation, but he asked what I meant by a math disaster. I explained the experience, trying to minimize its importance due to shame at having made such simple, elementary mistakes. He then asked "so why is that a disaster? Would you like to unpack that?" and I, for the first time, considered that it might not have been such a disaster. We immediately, together, moved into the abstract conceptualization phase of reflective practice.

7. Abstract Conceptualization

I considered my practice of asking students to work through confusing problems in class, and my happiness the previous day (see *Cycle 1*) when I asked a question that initially stumped my students because it gave us an opportunity to grapple with the concepts at play. I knew that students often struggle with showing confusion, because they have been enculturated to believe that being a good student requires them always to know the correct answers. I have never judged students who make mistakes as poor students, or understood why they judge themselves as such, but I realized that in this instance, I had judged myself as a poor instructor for making mistakes, just as my students do to themselves. This experience and realization helped me to develop epistemic empathy with students who are uncomfortable with being wrong in class, because I had felt the same way.

Why was it that when my students made a mistake, it was a sign that they were on a trajectory of learning, but when I made a mistake, it was a sign that I was "a fraud" or "unqualified"? One difference is that of positionality: I had positioned myself as an authority, an instructor who is expected to know the material perfectly and not make mistakes, while students were expected to make mistakes. Using responsive teaching practices when students were confused in the first cycle, I thanked them for sharing their (incorrect, though I did not tell them so at the time) ideas, listened to their thinking, and allowed them to work together, viewing the learning experience as a success when we, together, reached the correct answer. However, I did not respond to my own confusion the same way. I apologized for sharing incorrect ideas, modeling to my students that making mistakes is something negative, which should be apologized for, instead of a part of learning, and something positive that should be welcomed as an opportunity for growth. In telling another mentor, who has extensive experience and expertise in responsive teaching, about this reflection, he joked, "you shouldn't apologize for making mistakes, but you should apologize for apologizing for making mistakes" because of the attitude towards confusion that I had modeled for my students. Upon further reflection, I believe the pedagogical mistake of apologizing for a mathematical mistake is simply another kind of mistake that I can treat as an opportunity for another kind of learning: my own development as an instructor.

8. Plan & Apply

After this reflection, I resolved to work to respond differently to future mistakes I will certainly make as an instructor. Instead of attempting to protect my instructional authority by hiding the mistake, or apologizing and treating the mistake as an indication of lack of skill, I will recognize that making mistakes and responding to them is an essential part of the learning process, and that by making a mistake in front of the class, I have an opportunity to model for my students how they might work through their own confusion.

Discussion

For active learning pedagogies like productive failure, problem-based learning, and responsive teaching to be effective, students must attempt problems they do not initially know how to solve, make mistakes, and learn from them. However, the process of being publicly confused in the learning environment can be more challenging than many instructors realize. Even as a researcher studying engineering education, and as a proponent of active learning, I was still resistant to publicly admitting my own uncertainty in my classroom. Once my mistakes were acknowledged, I apologized for them multiple times, and framed the class session as a "disaster." The experience of being the one in the classroom who is confused helped me to gain epistemic empathy with student discomfort with admitting confusion.

Through reflective practice, I realized that one source of my discomfort was a feeling that I had failed at meeting my own expectations of a good engineering instructor. The implications of a good/bad binary in instruction has been explored in the K-12 context: the result was called the "baggage of the binary" because when instructors are busy judging themselves for practices they deem "bad" and holding themselves to standards defining what they deem "good" practices, they are not reflectively interpreting the coexisting positive and negative consequences of those practices, which may shift with context [42]. Through my own reflection, I came to realize that my ingrained ideas of "good" instructing included always knowing everything about the topics I taught, and I uncritically held myself to that standard, judging myself when I did not meet it.

However, this conception was out of step with both the disciplinary practices [23] of engineering, which require confronting problems with unknown solutions, and with what I want to model for students. By apologizing repeatedly for making mistakes, I modeled to my students that initial failure in addressing a problem is a disaster to be ashamed of, not something to welcome as an opportunity to work towards eventual success. I implicitly positioned myself above my students because I expected them to ask questions and make mistakes but did not allow the same from myself. I missed an opportunity to teach my students that, as engineers, they will often come across problems they don't know how to solve, and that it's possible to use that confusion as the first step towards learning something new.

None of these realizations occurred immediately, or even with several days of individual reflection. It was only in a meeting with a mentor three days (including one additional class period) after my "math disaster" that I began to reframe the experience. I was in the relatively unusual position of teaching my first university class while being supported by a structured mentorship relationship with an experienced engineering educator and engineering education researcher. Few, if any, of my colleagues have similar support, and are instead evaluated using high-stakes, summative student evaluations. I wonder how many times my colleagues have had teaching experiences that could have served as enriching opportunities for development in their pedagogical practice and instead dismissed those experiences, as I almost dismissed my “math disaster.” This story of pedagogical growth illustrates the benefits of reflective practice for engineering educators, which instructional communities can work to scaffold for themselves and for new instructors.

Conclusions

Many active learning pedagogies are built upon students grappling with confusing problems, admitting when they don’t know something, and learning from their mistakes. This experience can be challenging for students who are accustomed to lecture-based courses where correctness is prioritized, which contributes to the commonly reported student resistance to active learning. In this work, I reflect on two class sessions of an electrochemical engineering course I designed and led as my first experience as a lead instructor for a semester-length university course. In the first session, I elicited student confusing using two practice problems, and worked with students to reach conceptual understanding of the topic. I viewed this class session as a successful implementation of responsive teaching. In the second session, I made multiple mistakes in solving a problem, which led me to feel flustered and try to hide my mistakes. Despite eventually resolving the confusion, the experience left me doubting my expertise in the topic and my ability as an instructor. I apologized to students repeatedly, and I described the class to a mentor as a “math disaster.” Upon reflection, I realized that I had missed an opportunity to model for my students how to productively work through mistakes and had instead modeled that mistakes are something that they should be ashamed of and try to hide. Through my experience with being confused in front of my class, I gained epistemic empathy for my students’ experience with active learning pedagogies like responsive teaching, which will help me to design for and respond to student confusion in the future. The insight I gained from the experience would not have been possible without mentored reflection, to which I had access by serving as an instructor while conducting post-doctoral research with a scholar of engineering education. Teaching-specific mentorship structures for new instructors could help others to reflect on, and to learn and grow from, their own teaching experiences.

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