

# Students' Difficulties in Understanding the Fundamental Concepts and Limitation of Application of Appropriate Equations in Solving Heat Transfer Problems

#### Dr. Amir Karimi, The University of Texas at San Antonio

Amir Karimi, University of Texas, San Antonio Amir Karimi is a Professor of Mechanical Engineering at The University of Texas at San Antonio (UTSA). He received his Ph.D. degree in Mechanical Engineering from the University of Kentucky in 1982. His teaching and research interests are in thermal sciences. He has served as the Chair of Mechanical Engineering (1987 to 1992 and September 1998 to January of 2003), College of Engineering Associate Dean of Academic Affairs (Jan. 2003-April 2006), and the Associate Dean of Undergraduate Studies (April 2006-September 2013). Dr. Karimi is a Fellow of ASEE, a Fellow of ASME, senior member of AIAA, and holds membership in ASHRAE, and Sigma Xi. He has served as the Chair of ASEE Campus Representative at UTSA, ASEE-GSW Section Campus Representative, and served as the Chair of ASEE Zone III (2005-07). He chaired the ASEE-GSW section during the 1996-97 academic year.

#### Dr. Randall D. Manteufel, The University of Texas at San Antonio

Dr. Randall Manteufel is an Associate Professor of Mechanical Engineering at The University of Texas at San Antonio (UTSA). He has won several teaching awards, including the 2012 University of Texas System Regents Outstanding Teaching Award. and the 2013 UTSA President's Distinguished Achievement Award for Teaching Excellence, the 2010, 2014, 2018 and 2019 College of Engineering Student Council Professor of the Year Award, 2008 Excellence in Teaching Award for College of Engineering, and 2004-2005 Mechanical Engineering Instructor of the year award, 1999 ASEE-GSW Outstanding New Faculty Award. Dr. Manteufel is a Fellow of ASME with teaching and research interests in the thermal sciences. In 2015-2016, he chaired the American Society for Engineering Education Gulf Southwest section and in 2018-2019 he chaired the Academy of Distinguished Teaching Scholars at UTSA. He is a registered Professional Engineer in Texas.

## Students' Difficulties in Understanding the Fundamental Concepts and Limitation of Application of Appropriate Equations in Solving Heat Transfer Problems

## Abstract

A significant number of undergraduate students have challenges when trying to understand the derivation and application limits of the thermal resistance concept, as well as recognizing the reasons why dimensionless parameters are used in many of formulas presented in the heat transfer textbooks. This paper provides an examination of the underlying causes of student misunderstandings and the instructional strategies used to improve student learning. The concept of thermal resistance is defined for flat walls, cylindrical shells, and spherical shells, under the assumption of one-dimensional, steady-state heat transmission in materials with uniform thermal conductivity and no heat generation. Therefore, the thermal resistance calculations have limitations, particularly in situations when there is heat generation. Dimensional analysis is frequently simplified, causing students to disregard the theoretical foundation for establishing dimensionless parameters. The paper summarizes the areas in which students encounter challenges and provides examples that can be employed to aid students in establishing a more comprehensive concept of the theoretical derivations and grasp of the limitations in solving heat transfer problems.

## Introduction

Almost all undergraduate mechanical engineering degree programs require a semester-long course in heat transfer. Some programs might combine it with fluid mechanics or thermodynamics. In our institution, the undergraduate mechanical engineering program mandates a three-semester credithour course in heat transfer, typically taken by students in their junior year. This introductory course covers the three fundamental modes of heat transfer: conduction, convection, and radiation. Topics include the derivation of the transient multi-dimensional heat conduction equation, analytical solutions to steady-state one-dimensional heat conduction, dimensional analysis, solutions to steady-state two-dimensional heat conduction problems, transient heat conduction problems, numerical solutions, forced and free (natural) convection, radiation exchange between surfaces, and heat exchangers.

Students enrolled in the heat transfer course are required to possess foundational knowledge in several prerequisite topics, including: differential and integral calculus, ordinary differential equations (ODEs), partial differential equations (PDEs), the first and second laws of thermodynamics, evaluation of thermodynamic properties, viscosity and boundary layer concepts, laminar and turbulent flows, dimensional analysis and dimensionless parameters, integral and differential fluid momentum equations, friction and pressure drag forces, major/minor head loss and friction factor, Bernoulli's equation, and basic numerical methods.

A prerequisite quiz is administered during the first week of the semester to assess students' understanding of key prerequisite topics. This quiz covers various concepts, including basic differentiation and integration, distinguishing the differences between ODEs and PDEs, solving second-order ODEs, applying boundary conditions to general solutions of second-order ODEs, using separation of variables method to simplify PDEs into ODEs, simplifying mass balance,

energy balance, and entropy balance equations for closed and open system thermodynamics problems, understanding the relationship between the average velocity of fully developed laminar fluid flow in a pipe and the velocity at the center of the pipe, and conducting dimensional analysis. An example of the prerequisite quiz is provided in the Appendix of this paper for reference.

Results of the prerequisite quiz often show some students have difficulty with differentiation. Some students have errors is solving indefinite integrals, by not including the constant in the integration result. More students have difficulty to solve the problems related to differential equations. Not all students were able to solve numerical methods or thermodynamics problems correctly. Most students cannot explain the reason why the Moody diagram for friction factors is express in terms of dimensionless parameters.

There has been a gradual decline in students' grasp of course material, attributed to several factors, including: 1) easy access to solution manuals, 2) online tutoring services, 3) neglect of reading the textbook, 4) increased absence from lectures, 5) decreased attention to homework, 6) grade inflation in prerequisite courses, and 7) increased class sizes [1] - [5]. Most of these factors are beyond the instructor's control. Efforts have been made to address some of these challenges, which have been reported in engineering educational conferences [1] - [5].

During the COVID-19 pandemic, all classes were conducted online from March 2020 through August 2021. Teaching the heat transfer course in fall 2021 and spring 2022 revealed that many students lacked commitment to attending lectures or diligently solving homework assignments. Additionally, it was observed that many students lacked the prerequisite knowledge acquired during the pandemic period. Consequently, in two sections of the heat transfer course taught in fall 2021, over half of the students were failing after the first two mid-term exams, whereas typically only around 15% would fail before the pandemic [6], [7].

From spring 2020 through summer 2021, some instructors appeared to have been lenient in assessing student knowledge due to the COVID-19 pandemic. During this period, students took many exams online, leading to concerns about widespread cheating compared to face-to-face exams. Maintaining the academic integrity of courses became challenging for instructors [8], [9].

To enhance student performance, quizzes were introduced as an active learning tool starting in fall 2021 [10]. These quizzes carried a weight of 2% bonus points added to the final exam grade to incentivize student participation in class activities. Most quizzes were short, taking less than 10 minutes, with an emphasis on students demonstrating the solution process. Solutions were collected, and instructors immediately reviewed them, addressing any student questions. For longer problems requiring 10 to 20 minutes, quizzes were administered near the end of class, allowing sufficient time for students to submit their solutions and leave the class. The solutions were then posted online after the class and briefly discussed at the beginning of the following class.

To address attendance issues, the University's "Instructor Initiated Drop policy" [11] was adapted for the heat transfer course in fall 2022. This policy empowered instructors to drop any student who exceeded either the absence (4 times) or missed assignment (3 sets) limits, as outlined in the course syllabus. The implementation of this policy led to improved class attendance and students completing their homework assignments. Consequently, student learning improved, and pass rates increased [11].

## Student difficulties with heat transfer topics and concepts

While teaching an undergraduate heat transfer course for many years, it has been noticed that more students have difficulties in the following areas:

- Misapplying thermal resistance relations
- Solving differential equations associated with steady-state, one-dimensional heat conduction problems in rectangular, cylindrical, and spherical coordinates
- Properly applying boundary conditions
- Grasping the limitations of heat conduction solutions
- Understanding the fundamental principles of dimensional analysis widely used in fluid mechanics and heat transfer
- Comprehending the utilization of empirical equations for forced convection problems, where the Nusselt number is expressed as a function of Reynolds and Prandtl numbers, Nu = fn (Re, Pr)
- Verifying the applicability ranges before employing empirical relationships in solving convection heat transfer problems

This paper provides a few examples related to student difficulties used in a heat transfer class.

Example 1

Thermal resistance relations are bases on one-dimensional steady-state heat conduction in a medium involving no heat generation. For a plane wall, the temperature profile is linear and thermal resistance is expresses as  $R_{th} = L/k$ , where L is the wall thickness, and k is the thermal conductivity of the wall. The following problem has been used to help clarify these concepts.

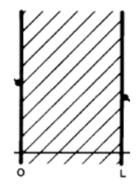
Consider a plane wall, 30 cm thick, having a thermal conductivity, k = 20 W/m.°C. The following expression is given for the temperature profile in the wall.

where, T is in measured in °C, and x in m. Considering a steady state process with uniform volumetric heat generation in the wall, determine;

- (a) The temperatures at x = 0 and x=L, in °C.
- (b) The temperatures at x = 0 and x=L, in °C.
- (c) The rate of heat flux at x = L
- (d) The rate of volumetric heat generation in the wall.

Almost all students evaluated T (0) = 245 °C and T (L) = 20 °C, by inserting the values of x in the temperature profile equation. However, many students used the following equation to evaluate the heat flux incorrectly.

$$q' = \frac{T(0) - T(L)}{R_{th}} = \frac{T(0) - T(L)}{L/k} = 15,000 \, W/m^2$$



One way to evaluate the heat flux at x = L is to use the following relationship.

$$q' = -k \left[ \frac{dT}{dx} \right]_{x=L} = -k (-5000 L) = 30,000 W/m^2$$

Most students did not know how to approach solving the part (c) of the pop quiz problem. Since the pop quiz was a part of class learning activity, the instructor was able to point out the students' misunderstanding on misuse of thermal resistance and provide the solution procedure for part (c) of the problem the following way. If one simplifies the general heat conduction equation assuming on dimensional steady-stated heat conduction, constant k, and uniform volumetric heat generation, it reduces the following relationship:

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 = (-5000) \, K/m^2 + \frac{\dot{q}}{20 \, W/m.K}.$$

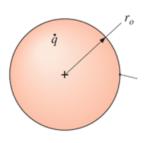
Solving for  $\dot{q}$ , it yields  $\dot{q} = 100,000 W/m^3$ . If this problem was assigned as a homework assignment, some students submit wrong solution and would not check the correct solution. We found the pop quizzes as a part of class active learning are effective in improving student learning.

Example 2

Many students have difficulty deriving a general solution for the temperature distribution in radial direction for cylinders and spheres, considering one-dimensional heat conduction with or without heat generation. The following is an example of a quiz problem assigned in the heat transfer course.

Consider steady-state, one-dimensional heat conduction in the radial direction in a solid sphere having constant thermal conductivity and uniform volumetric heat generation,  $\dot{q}$ .

- a) Simplify the general heat conduction for this problem,
- b) Solve the resulting differential equation to obtain a general solution for temperature distribution in the sphere



Most student solved the part (a) of the problem correctly and presented the following expression as the answer

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot{q}}{k}$$

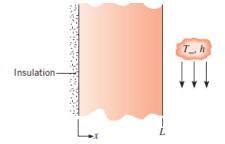
The solution to part (b) should have been very easy, since the second order ordinary differential equation could have been solved simply by integration. Unfortunately, many students cancelled the two  $r^2$  appearing on the left-hand side of the equation and presented it as:  $\frac{d^2T}{dr^2} = -\frac{\dot{q}}{k}$ , before starting the process for deriving a general solution. In order to improve students' mathematical skills, the following question was posed as a pop quiz, allowing students to brain storm with immediate neighbors, if they wished: Take the derivative of the terms appears in the parenthesis

of  $\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)$  and explain if the two  $r^2$  terms appearing in the following expression can cancel each other.  $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)$ ?

## Example 3

The following is a problem concerning boundary conditions.

Consider steady-state, one dimensional heat conduction in a plain wall having constant thermal conductivity and uniform volumetric heat generation,  $\dot{q}$ . The surface at x = 0 is insulated and the surface at x = L is exposes to convective fluid flow where the heat transfer coefficient h and the fluid temperature  $T_{\infty}$  are known.



- a) Simplify the general heat conduction for this problem,
- b) Solve the resulting differential equation to obtain a general solution for temperature distribution in the wall,
- c) State the appropriate boundary conditions at x = 0 and x = L in order to evaluate the constants in the general solution for this problem.

Most students were able to modify the general heat conduction equation as applied to this problem and presented the following expression:

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

Most derived a correct expression for general solution of the differential equation and presented it as:

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2$$

However, many students expressed the two boundary conditions as  $T(0) = T_1$  and  $T(L) = T_2$  instead of expressing them correctly as:  $\frac{dT(0)}{dx} = 0$  and  $-k \frac{dT(L)}{dx} = h[T(L) - T_{\infty}]$ , respectively. In more recent semesters we have used these kinds of examples as a part of active learning activities during the lecture period and observed student learning has improved.

#### Why heat transfers relations are presented as dimensionless parameter

A course in fluid mechanics is typically a prerequisite for a course in heat transfer. In the fluid mechanics course, students are introduced to Buckingham's Pi theorem and the concept of dimensional analysis. Since it is expected that students are exposed to dimensional analysis when they have taken a course in fluid mechanics, the topic is not included in some popular heat transfer textbooks [12], [13], but it is still covered in other textbooks [14] - [17].

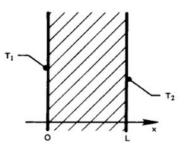
#### Example 4

Dimensional parameters are used throughout heat transfer textbooks; hence it is helpful to explain and motivate the use of dimensional analysis with the following example.

For one dimensional steady-state heat conduction in a plain wall with no volumetric heat generation T(x) can be expressed as:

$$T(x) = T_1 + (T_2 - T_1)(x/L)$$

Can you plot T(x) vs. x on a single graph for all possible values of  $T_1$ ,  $T_2$ , and L?



If that is not possible, the above temperature profile can be presented in dimensionless form as:

$$\frac{T(x) - T_1}{T_2 - T_1} = \theta = \frac{x}{L} = \eta$$

It is easy to plot  $\theta$  vs  $\eta$ , since at x = 0,  $\eta = 0$  and  $\theta = 0$ . At x = L,  $\eta = 1$  and  $\theta = 1$ .

#### **Dimensional Analysis**

Buckingham's Pi theorem states that if there are n variables in a physical problem and these variables contain j primary dimensions. The equation relating all the variables can be rearranged into k = (n - j) dimensionless parameters. The primary dimensions in heat transfer problems are typically mass represented by m, length represented by L, time represented by t, and temperature represented by T. There are several methods available to convert the dimensional parameters in a physical problem into dimensionless group. The most common method introduced is the method of repeating variables. The process of developing dimensionless  $\pi$  groups is describe in the following example.

## Example 5

For fully developed flow in a pipe, the friction factor, f (a dimensionless parameter), is a function of  $\rho$ , V, D,  $\mu$ ,  $\varepsilon$ , where  $\rho$  is the density of fluid, V is the average velocity, D is the diameter of pipe,  $\mu$  is viscosity, and  $\varepsilon$  is the wall roughness:  $f = fn (\rho, V, D, \mu, \varepsilon)$ . Using dimensional analysis, show that the f is a function of Reynolds number (*Re*) and  $\varepsilon/D$ .

- 1. List the dimensional parameters involved in the problem. Since f is a dimensionless parameter, there are n =5 dimensional parameters ( $\rho$ , V, D,  $\mu$ ,  $\epsilon$ ) involved in this problem.
- 2. The primary dimensions in each parameter are:  $\rho$  ( m. L<sup>-3</sup>), V (L t<sup>-1</sup>), D (L),  $\mu$  (m L<sup>-1</sup>t<sup>-1</sup>),  $\epsilon$  (L).
- 3. The primary dimensions in this problem are m, L, and t (j =3 dimensions). Therefore, based on Buckingham's Pi theorem, the equation relating all the variables can be rearranged into k = (n-j) = (5-2) = 2 dimensionless parameters ( $\pi$  groups).
- 4.  $\rho$ , V, D contain all primary dimensions m, L, and t. Therefore, they can be used as repeating parameters to determine the remaining two  $\pi$  groups (f is already a dimensionless parameter in this problem).

- 5. The primary dimensions contained in viscosity,  $\mu$  along with the those for the repeating parameters  $\rho$ , D and V are used first to obtain the first dimensionless parameter  $m^0 L^0 t^0 = \mu \rho^a D^b V^c = (m L^{-1} t^{-1}) (m L^{-3})^a (L)^b (L t^{-1})^c$
- 6. Equating the exponents of m, L, and t on both side of equation results in three simultaneous equations in term of three unknown a, b, c
  - m:0 = 1 + a + 0 + 0yieldsa = -1L:0 = -1 3a + b + cyieldsb + c = -2t:0 = -1 + 0 + 0 cyieldsc = -1 and b = -1

Therefore

$$\pi_1 = \mu \rho^{-1} D^{-1} V^{-1} = \frac{\mu}{\rho V D} = \frac{1}{Re}$$
 or  $\frac{\rho V D}{\mu} = \text{Re}$ 

7. The primary dimensions contained in surface roughness,  $\varepsilon$  along with the those for the repeating parameters  $\rho$ , V and D are used first to obtain the second dimensionless parameter

 $m^0 L^0 t^0 = \epsilon \rho^{a'} D^{b'} V^{c'} = (L) (m L^{-3})^{a'} (L)^{b'} (L t^{-1})^{c'}$ 

- 8. Equating the exponents of m, L, and t on both side of equation results in three simultaneous equations in term of three unknown a', b', c'
  - m:0 = 0 + a' + 0 + 0yieldsa' = 0L:0 = 1 3 a' + b' + c'yieldsb' + c' = -1t:0 = 0 + 0 + 0 c'yieldsc' = 0 and

Therefore

$$\pi_2 = \mu \rho^0 D^{-1} V^0 = \frac{\varepsilon}{D}$$
$$f = fn \left(Re, \frac{\varepsilon}{D}\right)$$

For some problems, the repeating variables method is cumbersome and Lienhard et.al have developed an alternative method, presented in their Heat Transfer textbook [14]. The alternative approach is called the method of functional replacement. The following describes the procedure used functional replacement method in developing dimensionless parameters.

- 1. List the parameters in the problem and count their total number, n
- 2. List the units for of each of the n parameters.
- 3. Identify the units appears in the list of parameters and count the number of such units, j. Each unit must appear at least twice in the list of parameters. If one of units appears only once, then reduce it to a more basic unit. For example, W = J/s.
- 4. Calculate the number of expected dimensionless parameters ( $\pi$  groups), in the problem: k = (n j)
- 5. Identify the repeating parameters that has one of the units. Avoid choosing dependent or independent variables as repeating parameters (such as x, r, t, etc, in heat transfer problems).
- 6. At each step choose one of the repeating parameters to eliminate one of the units.

The following shows the steps taken in obtaining the dimensionless parameters for Example 6, using the functional replacement method.

1. Listing the parameters in the problem and counting their total number, n

$$f = fn \left[ \rho \left( \frac{kg}{m^3} \right), \mathbb{V} \left( \frac{m}{s} \right), \mathbb{D} \left( m \right), \mu \left( \frac{kg}{m \cdot s} \right), \varepsilon \left( m \right) \right], \quad n = 6$$

- 2. The units associated in this problem are m, kg, an s, j=3
- 3. K = n j = 6 3 = 3. Therefor applying the functional replacement method should produce three (3) dimensionless.
- 4. In the following procedure, at each step one of the parameters are used to eliminate one of the units.

Step 1: D is used to eliminate m

$$f = fn \left[ \rho D^{3}(kg), V/_{D}(1/_{S}), \mu D(kg/_{S}), \varepsilon/_{D}(1) \right]$$

Unit (1) is used to identify a dimensionless parameter.

Step 2: V/D (1/s) is used to eliminate s  

$$f = fn \left[ \rho D^3 (kg), \mu D^2 / V (kg), \frac{\varepsilon}{D} \right]$$
(1)

Step 3: Use 
$$\mu D^2/V$$
 to eliminate kg  
 $f = fn \left[ \rho VD \right]_{\mu} = Re_{p}(1) \left[ \epsilon/p \right]_{p}(1)$ 

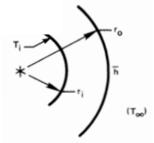
$$f = fn \left[ \begin{smallmatrix} r & r \\ \mu \end{smallmatrix} \right] - \frac{r}{\mu} = Re_D(1), \ \epsilon'_D(1) \right]$$
$$\pi_1 = f \qquad \pi_2 = Re_D \qquad \pi_3 = \epsilon'_D$$

Dimensional analysis serves as an invaluable tool for identifying dimensionless parameters that aid in the development of empirical relations for problems where obtaining analytical solutions proves challenging. Throughout the course, we incorporated short quizzes as learning activities, requiring students to employ the functional replacement method to derive dimensionless parameters for various problems. The functional replacement method was favored due to its efficiency in quickly identifying dimensionless parameters. Few example problems are presented below.

#### Example 6

Consider steady-state, one dimensional heat conduction in the radial direction in a long pipe. The inner surface at  $r = r_i$  is maintained at a constant temperature  $T_i$  and the outer surface at  $r = r_o$  is exposes to convective fluid flow where the heat transfer coefficient h and the fluid temperature  $T_{\infty}$  are known.

a) Identify appropriate parameters describing temperature distribution in the pipe for this problem



b) Assuming that an analytical solution cannot be obtained to express temperature distribution in the pipe, obtain the dimensionless functional equation for temperature distribution in the pipe

The governing differential equation for this problem is expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

The two boundary conditions are:

$$T(r_i) = T_i$$
, and  
 $-k \frac{dT(r_0)}{dr} = h[T(r_o) - T_{\infty}]$ 

Considering the parameters in the governing equation and the boundary conditions, the functional relationship for the temperature distribution, including the units can be presented as

$$(T - T_{\infty})(K) = fn [r(m), r_i(m), r_o(m), (T_i - T_{\infty})(K), k (W/m.K), h (W/m^2.k)]$$

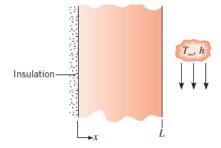
where  $T_{\infty}$  is used as a reference temperature. There are seven parameters, including the dependent variable  $(T - T_{\infty})$  (n = 7). There are three units associated with the seven parameters, m, K, and W (j = 3). Therefore, the dimensional analysis steps should yield (7-3) = 4 dimensionless parameters. Using the steps described earlier for functional replacement method the following dimensionless functional relationship is developed for the temperature distribution in the pipe.

$$\frac{T - T_{\infty}}{T_i - T_{\infty}} = fn \left[ \frac{r}{r_o}, \frac{r_i}{r_o}, \frac{hr_o}{k} = Bi \right]$$

where Bi is the Biot number based on outer diameter.

Example 7

Another dimensional analysis example considers a plate of thickness L (L<< the height and width of the plate) initially at a temperature Ti. One side of plate (x=0) is insulated. The other side of plate is suddenly exposed to a convection environment at  $T_{\infty}$  and h, Write down the governing heat conduction equation, initial condition and boundary conditions for this problem Develop a dimensionless functional equation for the temperature distribution in the wall.



Do not solve the differential equation, use dimensional analysis to identify the dimensionless Pi groups.

The governing differential equation for this problem is expressed as

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

The initial and boundary conditions are expressed as:

$$T(x,0) = T_i$$
$$\frac{\partial T(x,0)}{\partial x} = 0$$
$$-k \frac{\partial T(L,t)}{\partial x} = h[T(L,t) - T_{\infty}]$$

Considering the parameters in the governing equation, initial condition and the boundary conditions, the functional relationship for the temperature distribution, including the units can be presented as:

$$(T - T_{\infty})(K) = fn [x(m), t(s), \alpha (m^2/s, L(m), (T_i - T_{\infty})(K), k (W/m.K), h (W/m^2.k)]$$

where  $T_{\infty}$  is used as a reference temperature. There are eight parameters, including the dependent variable  $(T - T_{\infty})$  (n =8). There are for units associated with the eight parameters, m, K, s, and W (j = 4). Therefore, the dimensional analysis steps should yield (8-4) = 4 dimensionless parameters. Again, using the steps described earlier for functional replacement method the following dimensionless functional relationship is developed for the temperature distribution in the plate.

$$\frac{T - T_{\infty}}{T_i - T_{\infty}} = fn \left[ \frac{x}{L}, \frac{\alpha t}{L^2} = Fo, \frac{hL}{k} = Bi \right]$$

where, Fo and Bi represent Fourier and Biot numbers, respectively.

The approximation method or integral method for boundary layer solution of momentum and energy equation for laminar flow over flat plat are no longer presented in the many heat transfer textbooks [12, 13]. These methods have proven to be very beneficial for students in gaining a deeper understanding of why Nusselt number (Nu) is a function of Reynolds number (Re) and Prandtl number (Pr).

Since the textbook we currently use [12] does not cover the approximate method, we have incorporated dimensional analysis as part of active learning class activities. This approach helps students develop an understanding of why the empirical expressions presented in the textbook for forced convection are based on Nu = fn (Re, Pr).

#### Assessment of the effectiveness of interventions

After 20 months of online instruction during COVID-19 pandemic, the heat transfer course offered for the first time in fall 2021 in face-to-face modality again. After a few weeks into the semester, it was noticed that many of students were not attending lectures or solving homework assignments. Additionally, it was observed that many students lacked the sufficient knowledge of the topics covered in the prerequisite courses taken during the pandemic period. Consequently, in two sections of the heat transfer course offered in fall 2021, fifty-two % (52%) of students had average scores of less than 70 after the first two mid-term exams [6], [7]. Student attendance and class participation was very low prior to second exam, even though students could receive up to 2%

bonus points added to their semester grade, based on the total points earned for the pop quizzes given in class.

In order to improve attendance and engage student in class activities, the instructor met with those students who received average scores of less than 70 in the first two exams. Students were advised that they need to put more effort in the course in order to pass the course. The frequency of pop quizzes given during the lectures was increased after the second midterm exam. Doing poorly in one exam did not doom the semester for many students. Based on the grade policy, the lowest midterm exam was being replaced by the average of the other three exams, including the final exam. The instructor also provided an additional incentive to encourage students to put more effort in studying for the course. Five points were added to second lowest midterm exam of any student who scored more than 70 points in the remaining two exams. The same incentive was granted to student taking the course in fall 2022. [11]

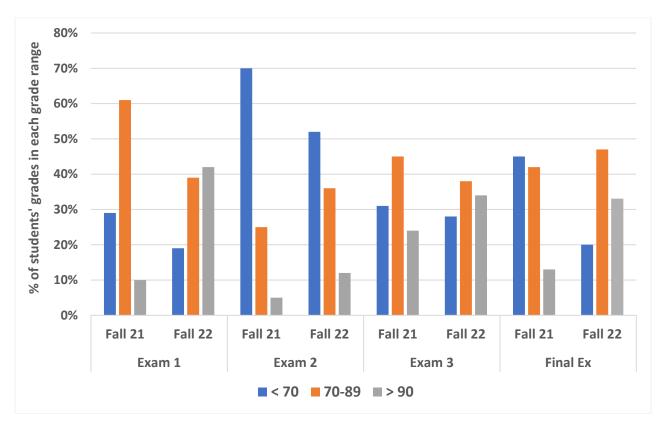
Table 1 presents the grade distributions for all the exams given in the heat transfer courses offered in fall 2021 and fall 2022. The table displays improved student exam performances in fall 2022, when the Instructor Initiated drop policy was in effect, as compared to those in fall 2021, when such policy was not enforced. In all exams, much higher percentage of students received grades of over 90 in fall 2022 as compared to those in fall 2021. Similarly, lower percentages of students received grades of below 70 in fall 2022 as compared to those in fall 2021. For the first exam, 42% of students received grades of over 90 in fall 2022 as compared to only 10% in fall 2021, and 19% of students received grades of below 70 in fall 2022 as compared to 29% in fall 2021. For the second exam, 12% of students received grades of over 90 in fall 2022 as compared to 5% in fall 2021, and 52% of students received grades of below 70 in fall 2022 as compared to 70% in fall 2021. For the third exam, 34% of students received grades of over 90 in fall 2022 as compared to 24% in fall 2021. For the final exam, 13% of students received grades of over 90 in fall 2022 as compared to 31% in fall 2021, and 20% of students received grades of below 70 in fall 2022 as compared to 45% in fall 2021.

Table 1 also shows that both in fall 2021 and fall 2022 much lower percentages of students received grades of over 70 or 90 and much higher percentage of students received grades below 70 in the second exam as compared with other exams. The reason for that is that the second exams were related to heat conduction problems, which required students' mathematical skills developed in such courses covering integral calculus and ordinary differential equations. In both fall 2021 and fall 2022, it was clear that most students had weak background in solving problems that required mathematical skills. The data in Table 1 also shows improved students exam performance in fall 2021 after the second exam, resulting from the instructor's individual meetings with the students receiving low grades in the first two exams. For visual comparison, Fig. 1 presents the percentage of students receiving scores in grade ranges of < 70, 70-89, and > 90 for each exam given in fall 2021 and fall 2022, respectively.

Exams	Semester	# of exams	< 70	70-89	> 90	Ave	Std-Dev.
Exam 1	Fall 21	83	29%	61%	10%	73.99	13.45
	Fall 22	69	19%	39%	42%	83.35	16.17
Exam 2	Fall 21	84	70%	25%	5%	60.16	17.70
	Fall 22	68	52%	36%	12%	62.89	17.03
Exam 3	Fall 21	82	31%	45%	24%	73.51	18.93
	Fall 22	65	28%	38%	34%	74.01	18.96
Final Ex	Fall 21	78	45%	42%	13%	70.95	17.59
	Fall 22	66	20%	47%	33%	78.14	17.29

 Table 1. Comparison of exam performance by students enrolled in Heat Transfer courses in fall semesters of 2021 and 2022

Fig. 1 Percentage of students receiving scores within each grade range of < 60, 70-89, and >90 for each exam given in fall 2021 and fall 2022, respectively.



## Summary and conclusion

Some of the areas that undergraduate mechanical engineering students frequently struggle in a heat transfer course are identified in this paper. A significant number of undergraduate students have challenges when trying to understand the derivation and application limits of the thermal resistance concept, as well as recognizing the reasons why dimensionless parameters are used in many of

formulas presented in the heat transfer textbooks. The underlying causes of student misunderstandings were examined in this paper and instructional strategies utilized to improve student learning were described. To address student difficulties in understand heat transfer concepts, example problems used to facilitate a deeper understanding of fundamental principles, aiming to prevent mistakes that stem from flawed conceptual understanding. By incorporating example problems shared in this paper, student comprehension of fundamental concepts and limitations of empirical correlations has improved. Since the resumption of face-to-face classes in the fall of 2021 following the COVID-19 pandemic, student exam grades have seen significant improvements.

## References

- Karimi, A. and Manteufel, R., "Does Student Access to Solution Manual Pose a Challenge?" ASEE-2011-2753, Proceedings of the 2011 ASEE Annual Conference, Vancouver BC, Canada.
- [2] Karimi, A. and Manteufel, R., "Assessment of Student Learning Outcome in an Introductory Thermodynamics course." ASEE-2012-5503, Proceedings of the 2012 ASEE Annual Conference, San Antonio, Texas.
- [3] Karimi A., and Manteufel R., "An Experiment with Flipped Classroom Concept in a Thermodynamics Course," Proceedings of the 2018-ASEE-GSW Section Conference, Paper ID number 3B.3, April 4-6, 2018, Austin, Texas.
- [4] Karimi A. and Manteufel. R., "Alternatives to Textbook Homework Assignment," Proceedings of the 2018- ASEE Annual Conference and Exposition, ID #: 23651, June 24-27, 2018, Salt Lake City, UT.
- [5] Karimi, A., "Instructor Initiated Drop Policy Effect on Student Success," Proceedings of 2020 ASEE-GSW S Section Virtual Annual Conference, April 2020.
- [6] A. Karimi. and R. Manteufel., "Comparisons of Student Performance in Similar Courses prior to, during, and after Online Instruction Due to COVID-19 Pandemic," Proceedings of the 2022 ASEE-GSW Section Annual Conference, Prairie View, Texas, March 16-18, 2022.
- [7] Karimi, A., and Manteufel, R.D., "Students Poor Exam Performance in an Engineering Course after Twenty Months of Online Instruction and Efforts to Improve", Proceedings of 2022 ASEE Annual Conference, Paper ID #37618, Minneapolis, MN, June 26-29.
- [8] Manteufel, R., Karimi, A, and Bhounsule, P., "Use of Phones and Online Tutors to Cheat on Engineering Exams," Proceedings of 2020 ASEE-GSW S Section Virtual Annual Conference, April 2020.
- [9] Karimi, A. Manteufel, R, and. Herbert, J.F., "Challenges in Virtual Instruction and Student Assessment during the COVID-19 Pandemic," Proceedings of 2021 ASEE Virtual Annual Conference and Exposition, Paper ID #34503, July 2021.
- [10] Karimi, A., and Manteufel, R.D., "Engage to Retain: Active Learning at the Conclusion of Traditional Lectures," Proceedings of the 2024 ASEE Gulf-Southwest Annual Conference, West Texas A&M University, Canyon, TX, March 10-12, 2024.
- [11] Karimi, A., and R.D. Manteufel, "Implementation of Instructor Initiated Drop Policy after COVID Pandemic Period to Improve Student Learning and Success," Proceedings of 2023 ASEE Annual Conference, Paper ID #40130, Baltimore, MD, June 25-28, 2023.

- [12] Bergman, T. L. and Lavine, A.S., *Fundamentals of Heat and Mass Transfer*, 8th Edition, Wiley Publishing Co. 2017.
- [13] Cengel, Y.A. and Ghajar, A. J., *Heat and Mass Transfer: Fundamentals and Applications*, 6th Edition, McGraw Hill Publishing CO, 2020.
- [14] Lienhard, J.H. IV, and John, J.H. V, A Heat Transfer Textbook, 5th edition, Dover Publications, 2020.
- [15] Holman, J. P., Heat Transfer, 10th Edition, McGraw-Hill Publishing Co. 2010.
- [16] Kreith, K., Manglik, R. M., and Bohn, M. S., *Principles of Heat Transfer*, 7th Edition, Cengage Learning, 2011.
- [17] Bayazzitoglu, Y. and Ozisik, M.N., *Elements of Heat Transfer*, McGraw-Hill Publishing Co. 1988.

## Appendix Heat Transfer Prerequisite Quiz

#### Solve the following math problems:

1. (3 points)  $y = 5 - 5x^{-2} - 3x^{-1} + 12x + 3x^{2} + 6e^{2x}$   $\frac{dy}{dx} =$ 

2. (3 points)  

$$\int (4 + 1/x + 2x - 3x^2 - 6e^{-2x}) dx =$$

3. (3 points)  
$$\int_{x_1}^{x_2} \left( 4 + \frac{1}{x} + 2x - 3x^2 - 6e^{-2x} \right) dx =$$

4. (3 points)  
$$\int_{1}^{2} (4 + 1/x + 2x - 3x^{2} - 6e^{-2x}) dx =$$

#### 5. (3 points)

What is the difference between an ordinary differential equation and a partial differential equation?

6. (4 points) Find a general solution for  $\frac{d^2y}{dx^2} - 4y = 0$ 

7. (4 points)

Find a general solution for  $\frac{d^2 y}{dx^2} + 4y = 0$ 

8. (5 points)

Find a general solution for  $\frac{d^2y}{dx^2} - 4y = 5x + 2$  and show that your solution satisfies the differential equation

9. (5 points) The general solution for a second order differential equation is given as  $y = C_1 x^2 + C_2 x + 5$ Given the boundary conditions:

$$\frac{dy}{dx} = 2 \quad @ \qquad x=0$$
  
y = 4 @ x=1  
evaluate C<sub>1</sub> and C<sub>2</sub>

10. (5 points)

Use the separation of variables method to reduce the following second order partial differential equation to two ordinary differential equations.

$$\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial z}{\partial t} = 0$$

11. (3 points)

Can separation of variables method be used to reduce the following second order partial differential equation to two ordinary differential equations? Give the reasoning.

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} - 5Z\frac{\partial z}{\partial y} = 0$$

12. (5 points)

Use the Newton-Raphson numerical method to estimate a root of  $f(x) = 5 - 5x^{-2} + 2x + 3x^2$ . Use x= 0.5 as a first estimate.

13. (5 points)

Use centered difference approximation to estimate first and second derivatives of  $y=e^{2x}$  at x=0.5, numerically. Use a step size of 0.1 (h=0.1).

14. (5 points)

Integrate the following function both analytically and using numerical method using Simpson's 1/3 rule, with n=4.

 $\int_{0}^{4} (4-x)^2 dx =$ 

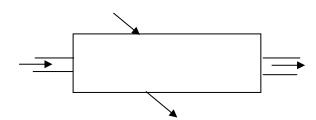
The following relations are equations for mass rate balance, 1<sup>st</sup> law of thermodynamics and second law of thermodynamics for control volumes. **Please use these equations to answer questions 15 through 20.** 

$$\frac{dm_{cv}}{dt} = \sum_{i} \dot{m}_{i} - \sum_{e} \dot{m}_{e}$$
(A)
$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_{i} \dot{m}_{i} \left( h_{i} + \frac{V_{i}^{2}}{2} + gz_{i} \right) - \sum_{e} \dot{m}_{e} \left( h_{e} + \frac{V_{e}^{2}}{2} + gz_{e} \right)$$
(B)

$$\frac{dS_{cv}}{dt} = \sum_{j} \frac{\dot{Q}_{j}}{T_{j}} + \sum_{i} \dot{m}_{i} s_{i} - \sum_{e} \dot{m}_{e} s_{e} + \dot{\sigma}_{cv}$$
(C)

15. (5 points)

Consider the schematic drawing of a general control volume shown bellow. Place  $m_{cv}$ ,  $E_{cv}$ ,  $S_{cv}$ ,  $\dot{m}_i, \dot{m}_e, \dot{Q}_{cv}, \dot{W}_{cv}, h_i, h_e, s_i, s_e$  in appropriate locations on the diagram



## Fig A

## 16. (6 points)

Consider the control volume in Fig. A (question 15) and equations A through C. Assuming a steady state process, simplify equations for mass rate balance, 1<sup>st</sup> law of thermodynamics and second law of thermodynamics for control volumes.

## 17. (6 points)

Consider equations A through C. Write down the general equations for mass balance, 1<sup>st</sup> law of thermodynamics and second law of thermodynamics for a closed system.

## 18. (4 points)

What is the purpose of second law of thermodynamics?

#### 19. (3 points)

For fully developed laminar flow in a pipe how does the average velocity compare to the velocity at the center of the pipe.

#### 20 (3points)

Explain why the Moody diagram for friction factor of fully developed flow in circular pipes are presented in dimensionless parameters.

## 21. (10 points)

For fully developed flow in a pipe, the friction factor, f (a dimensionless parameter), is a function of  $\rho$ , V, D,  $\mu$ ,  $\epsilon$ , where  $\rho$  is the density of fluid, V is the average velocity, D is the diameter of pipe,  $\mu$  is viscosity, and  $\epsilon$  is the wall roughness:  $f = fn (\rho, V, D, \mu, \epsilon)$ . Using dimensional analysis, show that the f is a function of Reynolds number (Re) and  $\epsilon/D$ . What do f, and Re number represent?