# On Teaching and Learning the Fundamentals of L'Hopital's Rule in Visual and Intuitive Ways

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With more than 30 years of combined experience in th

# Work-in-Progress: On Teaching and Learning the Fundamentals of L'Hopital's Rule in Visual and Intuitive Ways

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## Abstract

In today's educational landscape, students increasingly seek to grasp basic understanding of mathematical concepts. It is imperative that teaching approaches evolve to meet this demand, especially in the realm of mathematics where there is a gap between understanding the theory and applying its relevant techniques. This paper focuses on addressing the challenges students face when learning calculus, particularly the L'Hopital's Rule. While students may learn how to apply it, many lack a fundamental understanding of its origins, missing the crucial "why" behind its application.

The primary objective of this paper is to deepen students' understanding of L'Hopital's Rule by incorporating visualization and intuition techniques with mathematical approaches. We explore the background of L'Hopital's Rule and present three different ways to explain the essence of the rule by: a) "zooming-in" on the intersection of two functions, b) employing the Taylor Series, and c) utilizing Infinitesimal Calculus. By using these approaches, it becomes easier to grasp the essence of L'Hopital's Rule, which is about comparing how fast two functions change as they approach a particular point. Moreover, to provide a comprehensive perspective we share some practical textbook examples that highlight the applications of L'Hopital's Rule. This is followed by a real-life engineering problem that utilizes L'Hopital's Rule.

To assess the effectiveness of the new approach for learning L'Hopital's Rule we conducted an in-class anonymous questionnaire. 58 students responded. The results clearly show that understanding the concept of L'Hopital's Rule is either important or very important to students. Most of them praised the visualization and intuition approach for teaching the rule. Even though we did not used activities and exercises, students felt that more hands-on activities and in-class exercises could be

very helpful as well. In general, they liked traditional presentations, but were not as excited as we thought about power point presentations. Surprisingly, most of them prefer not to read textbooks but rather to be taught. Overall, our assessment reveals that students greatly preferred visualizations as an additional learning method. This approach not only enhances their comprehension of L'Hopital's Rule but also underscores their preference for a visual learning experience. The results are consistent with our multi-year experience in evaluating similar methods across various engineering related courses like Control Systems, Digital Signal Processing, Computer Algorithms, and Physics.

It is important to note that this work should be considered work in progress. It is not intended to replace traditional textbook chapters but rather to serve as an invaluable add-on resource. Our goal is to assist educators in teaching and empowering students in learning complex mathematical concepts in a more engaging and meaningful manner.

# Introduction

Addressing the common challenges that students often face in understanding a fundamental grasp of L'Hopital's Rule is vital, and therefore it is important to explore these misunderstandings by employing effective teaching strategies. It is our view that it is essential to incorporate clear, visually engaging, and intuitive methods to understand what is behind the Rule, particularly in mathematics courses.

This paper aims to offer visual, and intuitive explanations of L'Hopital's Rule. The central goal is to help students establish a solid foundation for comprehending L'Hopital's Rule, with a conscious effort to minimize reliance on memorization. We want to ensure that students understand the concept and not only use it a purely problem-solving technique. In this paper we present three different ways to explain L'Hopital's Rule by: a) "zooming-in" on the intersection of two functions, b) employing the Taylor Series, and c) utilizing infinitesimal calculus. We also present a real-life engineering problem that utilizes L'Hopital's Rule.

To assess the effectiveness of the new approach for learning L'Hopital's Rule we conducted an in-class anonymous questionnaire. 58 students responded. The results clearly show that understanding the concept of L'Hopital's Rule is either important or very important to students. Most of them praised the visualization and intuition approach for teaching the rule. Overall, the assessment shows that students greatly preferred visualizations as an additional learning method. Although this evaluation is limited in scope, the author's extensive experience in evaluating similar methods across various engineering-related topics strongly suggests that this approach holds substantial potential.

This initiative is a crucial part of a more comprehensive effort to simplify teaching and learning of STEM concepts. Its aim is to be a valuable educational resource for both teachers and students, encouraging deep understanding through alternative learning experiences. The effort covers various courses, including Physics/Mechanics, Calculus, Statics, Control Systems, Digital Signal Processing, Probability, Estimation, and Computer Algorithms. The larger scale project, as it relates to calculus concepts, intends to develop and integrate engaging games, relevant 3D puzzles and brain teasers, captivating animations, real-world intuitive illustrations and demonstrations, short video clips, hands-on activities (including virtual reality and augmented reality experiences), collaborative teamwork and communication exercises, small-scale inquiry-based research, as well as engaging presentations and peer-based learning.

It should be noted that this work should be considered as work in progress. It is intended to serve as an add-on resource, and not as a replacement to textbook chapters. Our goal is to assist educators in teaching and help students in learning complex mathematical concepts in a more engaging and meaningful manner.

## The Problem

We have all learnt, used and taught the powerful L'Hopital's Rule. We were told its simple and powerful "bottom line" which states that, for functions $f(x)$ and $g(x)$ which are differentiable on an open interval,

$$\text{If} \quad \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{or} \quad \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\infty}{\infty}, \quad \text{then} \quad \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

But like many students, some of us were not shown why it is true, or at least how to get some intuition/visualization for it. For example, when we try to take the limit of the ratio of the function of $f_1(x) = x + 2$ and the function of $f_2(x) = 2x + 3$ as $x$ approaches infinity, we simply apply L'Hôpital's rule to get:

$$\lim_{x\to\infty}\frac{f_1(x)}{f_2(x)} = \lim_{x\to\infty}\frac{x+2}{2x+3} = \lim_{x\to\infty}\frac{\frac{d}{dx}(x+2)}{\frac{d}{dx}(2x+3)} = \lim_{x\to\infty}\left(\frac{1}{2}\right) = \frac{1}{2} \tag{1}$$

Another example, when $x$ approaches 0:

$$\lim_{x\to 0}\frac{1-e^x}{x^3+x} = \lim_{x\to 0}\frac{\frac{d}{dx}(1-e^x)}{\frac{d}{dx}(x^3+x)} = \lim_{x\to 0}\frac{-e^x}{3x^2+1} = -1 \tag{2}$$

We also learnt to apply the Rule twice (or more times), for example:

$$\lim_{x\to\infty}\frac{f_1(x)}{f_2(x)} = \lim_{x\to\infty}\frac{3x^2+4x+1}{5x^2+2x+3} \tag{3}$$

As $x$ approaches infinity, both the numerator and denominator approach infinity. So, we differentiate both the numerator and denominator to get:

$$\lim_{x\to\infty}\frac{\frac{df_1(x)}{dx}}{\frac{df_2(x)}{dx}} = \lim_{x\to\infty}\frac{6x+4}{10x+2} \tag{4}$$

Again, as $x$ approaches infinity, both the numerator and the denominator approach infinity. So, we differentiate both the numerator and denominator again to get:

$$\lim_{x\to\infty} \frac{\frac{d^2 f_1(x)}{dx^2}}{\frac{d^2 f_2(x)}{dx^2}} = \frac{6}{10} = \frac{3}{5} \tag{5}$$

We did it, we taught it, but we have not learned or taught why it is so. This question led us to try to deepen students' understanding of L'Hôpital's Rule by incorporating multiple visualization and intuition techniques.

## Related Work

In the context of grasping concepts, students often struggle to experience the "Aha! moment." They express a desire for more hands-on, experiential, visual, intuitive, and enjoyable methods (such as game-based), along with technology-based information available on the web.

This visual and intuitive approach for teaching STEM concepts is not novel. Tyler DeWitt [1] addressed this issue by teaching isotopes through an analogy involving cars with minor modifications. This analogy illustrates that isotopes are essentially the same atom, possessing an identical number of protons and electrons but differing in the number of neutrons, akin to cars with minor changes (e.g., color). Several calculus textbooks incorporate visual explanations, as exemplified by references [2–11]. Notably, the work of Apostol and Mamikon from Caltech [11] is particularly intriguing, as they successfully explained the integration of certain functions without relying on mathematical formulas. Other books, such as [12, 13], have contributed to the understanding of concepts in "Control Systems" and the fundamentals of "Newton's Laws of Motion."

Other successful attempts to teach basic understanding using interactive animations, clear explanations, and sometimes visual and intuitive methods can be found in 3Blue1Brown [14] where almost all animations on this platform are made using a custom open-source Python library named Manim. On his YouTube channel, Sanderson brings learners into the world of mathematics with clear visualizations and comprehensible explanations. Aviv Censor [15] focuses on clear explanations with mathematical proofs using visual examples. GeoGebra [16] is an interactive visual animation tool for teaching and learning mathematics. Khan Academy [17] offers multiple sets of online videos that assist students in better understanding concepts and techniques in STEM. "Teaching Math Without Words" [18] presents a visual approach to learning math, where readers can also find clear explanations on the importance of visual learning. A refreshing notice by the American Mathematical Society regarding friendly and inviting learning of math can be found in [19].

# Three Different Ways to Explain the Essence of L'Hopital's Rule

## a) "Zooming-in" on the Intersection of Two Functions

When "zooming-in" around a specific point on a graph of a function, the curve near the point looks more like a straight line from which one can obtain the derivative (slope) of the function at that point, as can be observed in Figure 1:

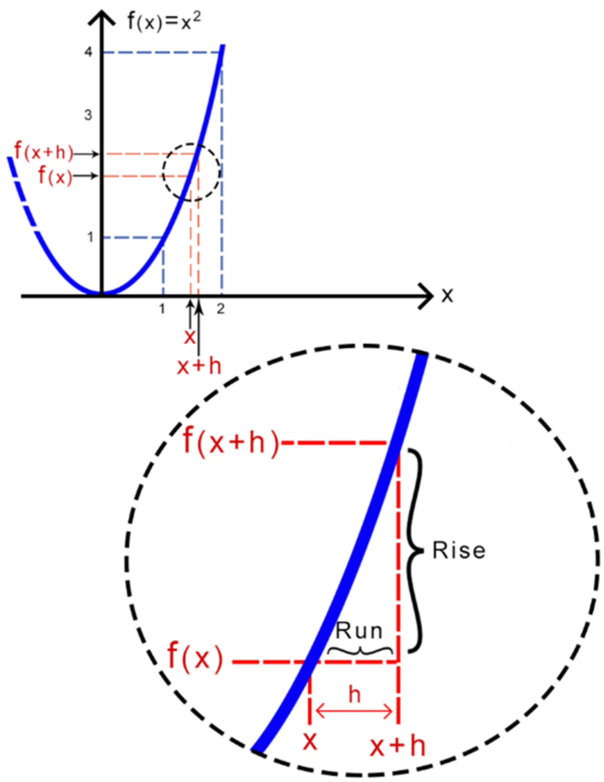


Figure 1: Visualizing the idea of "zooming-in"

**Example 1:**

Observe the functions $\sin(x)$ and $x$ as $x \to 0$ the ratio of the two functions at $x = 0$ becomes indeterminate (due to $\frac{0}{0}$). However, when "zooming-in" near the (0,0) point of the graphs of the functions, they not only have the same value at $x = 0$ but also the ratio of the functions near (0,0) is getting closer to 1 as we keep "zooming-in" (Figure 2). For example, expressing angles in radians we obtain:

$\sin(x)$ @ $x = 1$ is approximately 0.841

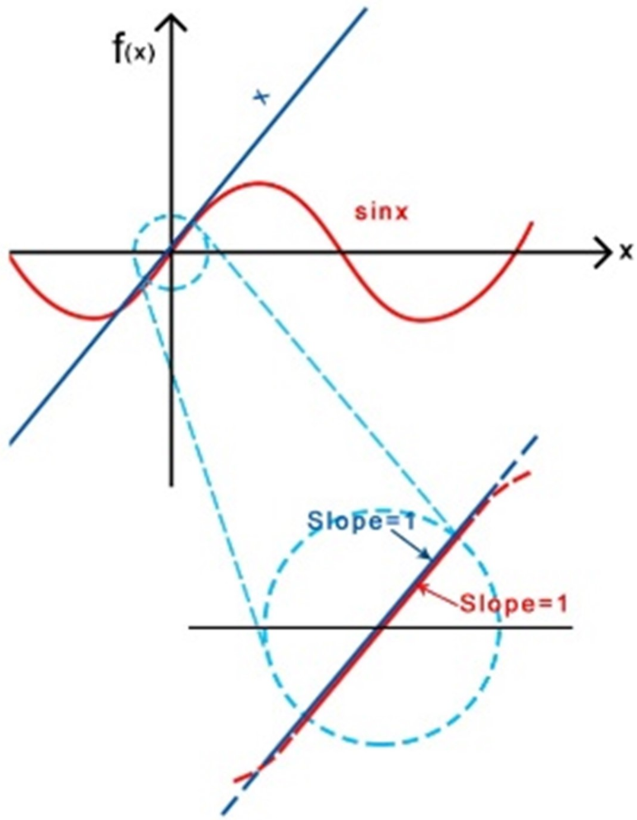


Figure 2: "Zooming-in" on the functions $\sin(x)$ and $x$ near $x = 0$

$\sin(x)$ @ $x = 0.1$ is approximately 0.0998

$\sin(x)$ @ $x = 0.01$ is approximately 0.0099998

$\sin(x)$ @ $x = 0.001$ is approximately 0.000999999666

Clearly the ratio $\dfrac{\sin(x)}{x}$ approaches 1 as we get closer and closer to $x = 0$. In addition, we notice that the ratio of the *slopes* of the two functions at $x = 0$ equals to 1 as well. So we can visually see that:

$$\lim_{x\to 0} \frac{\sin(x)}{x} = \lim_{x\to 0} \frac{\frac{d}{dx}\sin(x)}{\frac{d}{dx}x} = 1 \tag{6}$$

**Example 2:**

The following example (Figure 3) shows the same idea from a different point of view. We can tell that as $x$ approaches the value of 1 (for both functions), the function $2x - 2$ approaches the value of zero twice as fast as the function $x - 1$. The ratio of

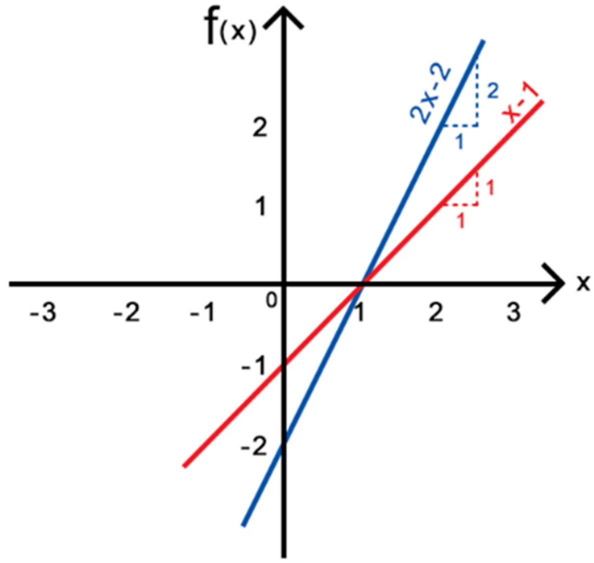


Figure 3: Visualizing L'Hopital's Rule with different slopes.

the functions is: $\dfrac{2(x-1)}{x-1} = 2$, *which is the same as the ratio of the slopes* at $x = 0$ (i.e., $2/1 = 2$).

**Examples 3 and 4:**

To get a better insight for the "zooming-in" idea, we show two additional sets of two different functions, one near the (0,0) and the other near (1,0), where *the ratio of the slopes is the same as the ratio of the functions.* They are shown in Figures 4 and 5.

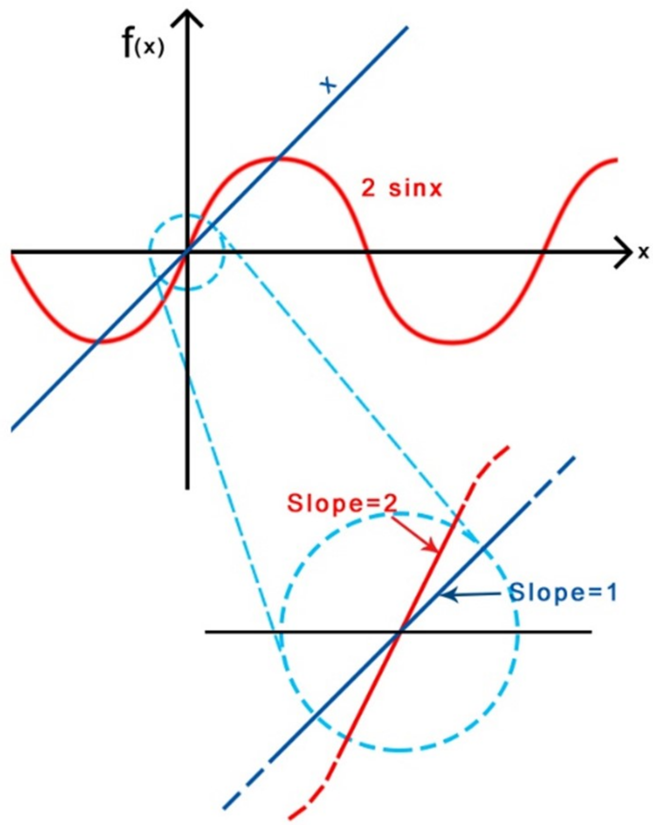


Figure 4: “Zooming-in” on two functions with 0 values at their intersection.

## b) Employing the Taylor Series

For two different functions that can be expanded using Taylor Series we can write:

$$f_1(x+\Delta x)\Big|_{x=x_1} = f_1(x)\Big|_{x=x_1} + \frac{df_1(x)}{dx}\Big|_{x=x_1}\Delta x + \left(\frac{1}{2}\right)\frac{d^2 f_1(x)}{dx^2}\Big|_{x=x_1}(\Delta x)^2 + \dots \quad (7)$$

and

$$f_2(x+\Delta x)\Big|_{x=x_1} = f_2(x)\Big|_{x=x_1} + \frac{df_2(x)}{dx}\Big|_{x=x_1}\Delta x + \left(\frac{1}{2}\right)\frac{d^2 f_2(x)}{dx^2}\Big|_{x=x_1}(\Delta x)^2 + \dots \quad (8)$$

If $f_1(x)\big|_{x=x_1} = f_2(x)\big|_{x=x_1} = 0$ and if we take only the first derivative term of the Taylor expansion of the functions we get:

$$\frac{f_1(x+\Delta x)}{f_2(x+\Delta x)}\Big|_{x=x_1} \approx \frac{f_1(x)\Big|_{x=x_1} + \frac{df_1(x)}{dx}\Big|_{x=x_1}\cdot\Delta x}{f_2(x)\Big|_{x=x_1} + \frac{df_2(x)}{dx}\Big|_{x=x_1}\cdot\Delta x} \quad (9)$$

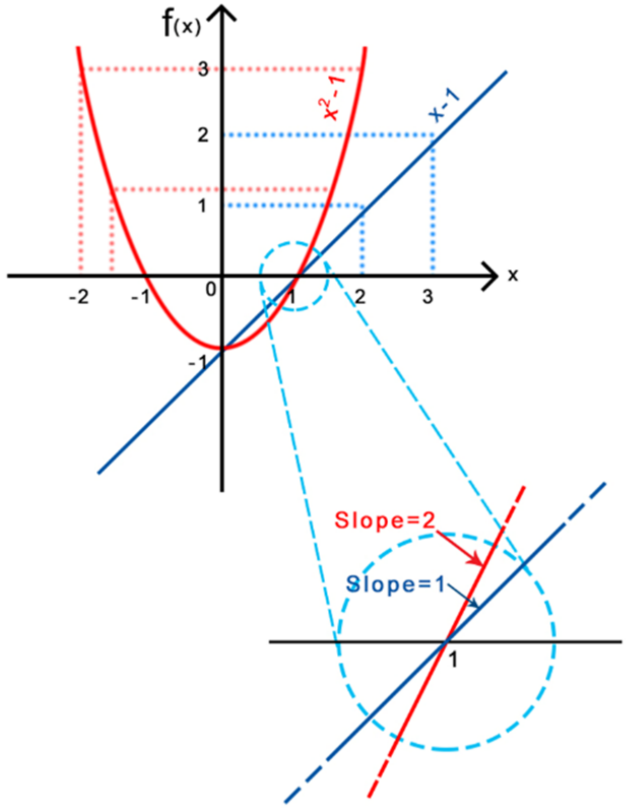


Figure 5: "Zooming-in" on two functions with 0 values at their intersection.

$$\left.\frac{f_1(x+\Delta x)}{f_2(x+\Delta x)}\right|_{x=x_1} \approx \frac{0+\left.\frac{df_1(x)}{dx}\right|_{x=x_1}\cdot\Delta x}{0+\left.\frac{df_2(x)}{dx}\right|_{x=x_1}\cdot\Delta x} = \frac{\left.\frac{df_1(x)}{dx}\right|_{x=x_1}}{\left.\frac{df_2(x)}{dx}\right|_{x=x_1}} \tag{10}$$

And so:

$$\lim_{x\to x_1}\frac{f_1(x)}{f_2(x)} = \lim_{x\to x_1}\frac{\frac{df_1(x)}{dx}}{\frac{df_2(x)}{dx}} \tag{11}$$

In short:

$$\lim_{x\to x_1}\frac{f_1(x)}{f_2(x)} = \lim_{x\to x_1}\frac{f_1'(x)}{f_2'(x)} \tag{12}$$

**Example:**

Refer to [20]. Evaluate:

$$\lim_{x\to 2}\left(\frac{2}{\log x}+\frac{x}{x-2}\right) \tag{13}$$

We start by rewriting the expression as:

$$\frac{2}{\log x} + \frac{x}{x-2} = \frac{2(x-2) - x\log x}{(x-2)\log x} = \frac{(2x-4) - x\log x}{(x-2)\log x}. \tag{14}$$

As $x \to 2$, we can use Taylor series expansion around 2.

The expansion of $\log x$ around 2 becomes:

$$\log 2 + (x-2)/2 - (x-2)^2/8 + O((x-2)^3) \tag{15}$$

Substituting this into our expression yields:

$$\frac{(2x-4) - (x-2)(\log 2 + \frac{x-2}{2} - \frac{(x-2)^2}{8}) + O((x-2)^3)}{(x-2)(\log 2 + \frac{x-2}{2} - \frac{(x-2)^2}{8}) + O((x-2)^3)}. \tag{16}$$

The term $O((x-2)^3)$ represents the remainder of the Taylor series and includes all the terms of order $(x-2)^3$ and higher. As $x$ approaches 2, these terms become negligible, and the function's behavior is dominated by the lower order terms.

Thus, the limit can be reduced to:

$$\lim_{x\to 2}\left(\frac{-2 + \frac{x-2}{2} + O(x-2)}{1 + \frac{x-2}{2} + O(x-2)}\right) = -2. \tag{17}$$

## c) Utilizing Infinitesimal Calculus

In infinitesimal calculus, L'Hopital's Rule provides a powerful technique for evaluating limits that result in indeterminate forms. Consider two functions, $f(x)$ and $g(x)$, that are differentiable in the neighborhood of a point $a$, where

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0 \tag{18}$$

Then, as we know, using L'Hopital's Rule

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}. \tag{19}$$

To deepen our understanding, let's introduce an infinitesimal quantity $\varepsilon$.
From the graph (Figure 6) we can see that:

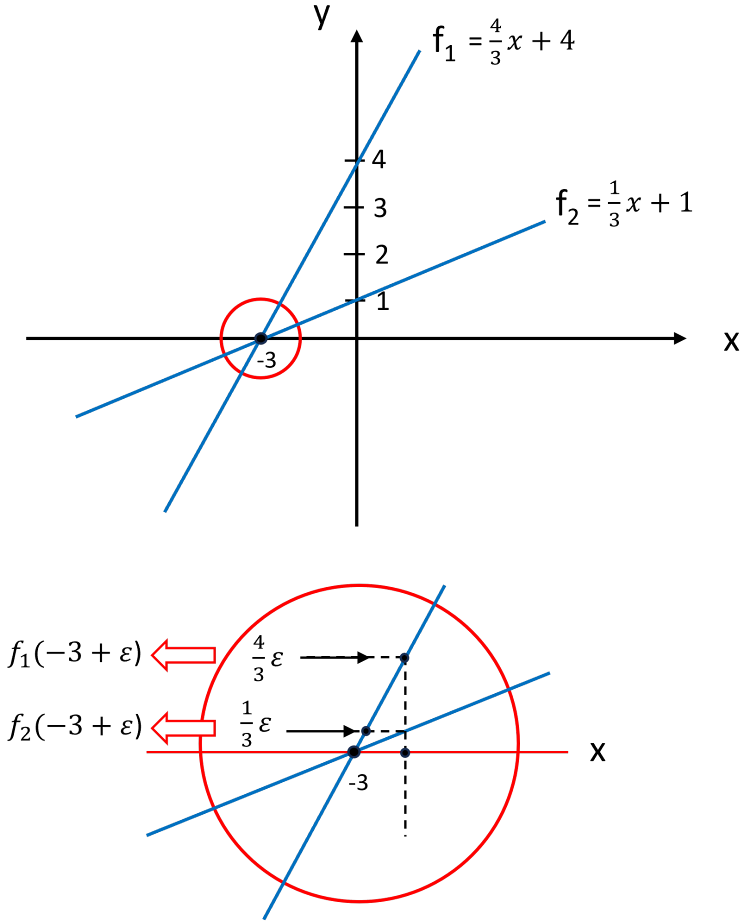


Figure 6: Using infinitesimal calculus to visualize L'Hopital's Rule.

**Example:**

$$\frac{f_1(-3)}{f_2(-3)} = \text{“}0/0\text{”} \tag{20}$$

i.e., the ratio is indeterminate.

We observe from Figure 6:

$$f_1(-3+\varepsilon) = \left.\frac{df_1(x)}{dx}\right|_{x=-3} \cdot \varepsilon \tag{21}$$

$$f_2(-3+\varepsilon) = \left.\frac{df_2(x)}{dx}\right|_{x=-3} \cdot \varepsilon \tag{22}$$

And so:

$$\frac{f_1(-3+\varepsilon)}{f_2(-3+\varepsilon)} = \frac{\frac{df_1(x)}{dx}\Big|_{x=-3} \cdot \varepsilon}{\frac{df_2(x)}{dx}\Big|_{x=-3} \cdot \varepsilon}$$

$$= \frac{\frac{df_1(x)}{dx}\Big|_{x=-3}}{\frac{df_2(x)}{dx}\Big|_{x=-3}} = \frac{\frac{4}{3}}{\frac{1}{3}} = 4 \tag{23}$$

# Engineering Example

Consider the following scenario: Engineers are attempting to characterize an unknown electrical device by studying the ratio between its voltage and its current at different temperatures. Utilizing a multitude of experimental data points, they have derived polynomial expressions for both the voltage $V$ (in volts) and the current $I$ (in amps) as functions of the temperature $T$ (in degrees Celsius).

The fit polynomials to the data were determined to be:

$$V(T) = T^3 - 5T^2 + 6T \tag{24}$$

$$I(T) = T^4 - 2T^3 + 3T^2 - 4T \tag{25}$$

When attempting to evaluate the ratio $\frac{V(T)}{I(T)}$ at $T = 0$, they encounter an indeterminate ratio of $\frac{0}{0}$. To resolve this, they apply L'Hopital's Rule, which involves differentiating the function of the numerator $V(T)$ and the function of the denominator $I(T)$ with respect to $T$ followed by evaluating the limit of this new ratio as $T$ approaches 0.

Applying L'Hopital's Rule, the derivatives of the numerator and denominator are:

$$V'(T) = \frac{d}{dT}(T^3 - 5T^2 + 6T) = 3T^2 - 10T + 6 \tag{26}$$

$$I'(T) = \frac{d}{dT}(T^4 - 2T^3 + 3T^2 - 4T) = 4T^3 - 6T^2 + 6T - 4 \tag{27}$$

Using L'Hopital's Rule we obtain:

$$\lim_{T\to 0} \frac{V'(T)}{I'(T)} = \lim_{T\to 0} \frac{(3T^2 - 10T + 6)}{(4T^3 - 6T^2 + 6T - 4)} = \frac{6}{-4} = -\frac{3}{2} \tag{28}$$

Figure 7 illustrates the discrete experimental data of the voltage and the current at various temperatures, and their polynomial approximations as functions of temperature. It is evident that at $T = 0$ the ratio of the two functions is indeterminate.

Refer to Figure 8 for a detailed view around $T = 0$. This figure clearly demonstrates that, at $T = 0$, the ratio of the voltage slope to the current slope is $-\frac{3}{2}$.

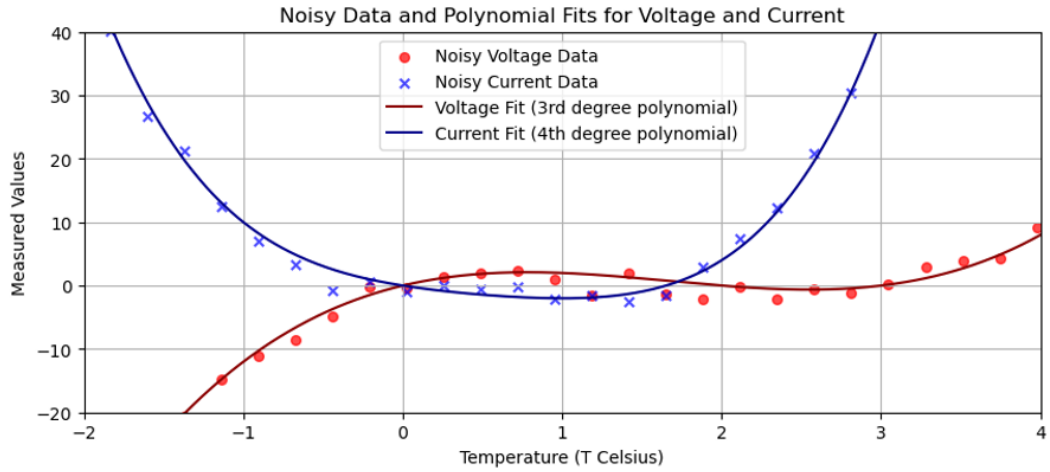


Figure 7: Polynomial approximations using experimental data of the voltage and the current. The polynomials intersect at $T = 0$, where the ratio of the functions is indeterminate.

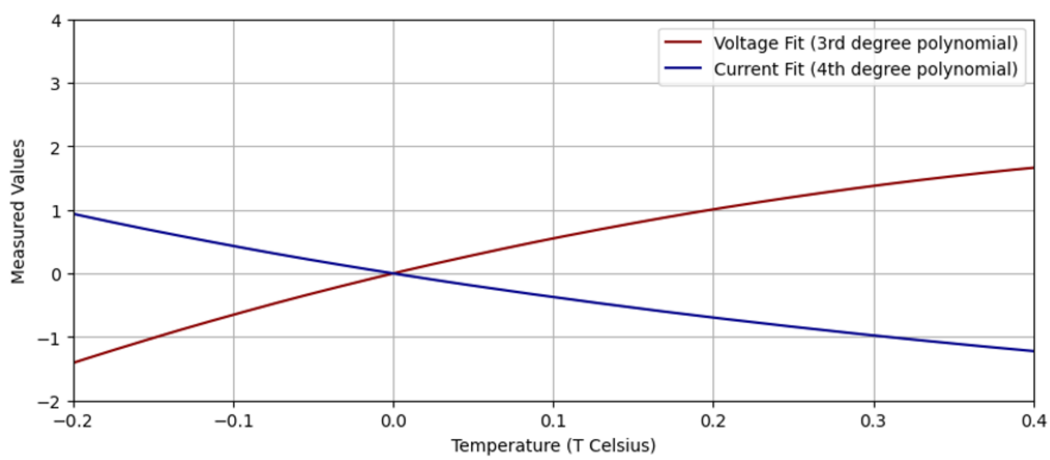


Figure 8: Zooming around $T = 0$.

# Assessment

To assess the effectiveness of the new approach for learning L'Hopital's Rule we conducted an in-class anonymous questionnaire. 58 students responded. The results clearly show that understanding the concept of L'Hopital's Rule is either important or very important to students. Most students praised the visualization approach for teaching L'Hopital's Rule. Even though we have not used activities and exercises, students felt that more hands-on activities and in-class exercises could be helpful. In general, they like traditional presentations, but not as excited as we thought about power point presentations. Surprisingly most of them prefer *not* to read texbooks. Overall, we feel that visualizations were very well accepted and preferred by students as supplementary way for learning.

The following is a summary of results of what students consider important or very important (Table 1). More details are provided in the Appendix.

| **Aspect** | % |
|---|---|
| Understanding the concept of L'Hopital's Rule | 67.9 |
| Visualizing the concept of L'Hopital's Rule | 71.4 |
| Introduction through visual examples | 66.1 |
| Introduction through hands-on activities | 64.3 |
| Engagement in class exercises | 80.4 |
| Learning using traditional presentations | 50.0 |
| Learning through Power Point presentations | 46.4 |
| Reading relevant chapter in the book | 39.3 |

Table 1: Students' feedback importance of different learning aspects for L'Hopital's Rule

# Conclusion

Drawing insights from our exploration of L'Hopital's Rule, this paper underscores the significance of integrating intuitive and visual learning methods into the conventional mathematical curriculum. Our approach, which bridges the gap between conceptual understanding and techniques being used, focuses on enhancing the understanding of L'Hopital's Rule through innovative teaching techniques.

We emphasize that the methods and examples presented in this paper are designed to *complement, not replace*, traditional calculus textbooks. They aim at enriching existing educational material. By introducing these teaching tools, we hope to make complex mathematical concepts more accessible and understandable for students.

This work, still in its developmental stages, offers a fresh perspective on teaching calculus. We are preparing multiple engineering-related problems that utilize L'Hopital's Rule, specifically in the fields of electrical, mechanical, and civil engineering. This effort addresses a notable gap, as most textbook examples of L'Hopital's Rule do not include real-life engineering examples. We encourage educators to consider adopting these methods in their relevant educational contexts, and contribute new ideas to this evolving field. Our hope is that this paper will inspire further innovation in the way mathematics is taught and learned, making it a more intuitive and relatable subject for students across various disciplines.

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## References

[1] Tyler DeWitt. *What are Isotopes?* YouTube video. 2012. URL: https://youtu.be/EboWeWmh5Pg.

[2] Colin Conrad Adams. *Zombies and Calculus*. Princeton, NJ: Princeton University Press, 2014. ISBN: 9780691161907. URL: https://press.princeton.edu/books/hardcover/9780691161907/zombies-and-calculus.

[3] Kenn Amdahl and Jim Loats. *Calculus for Cats*. Broomfield, CO: Clearwater Pub., 2001.

[4] Robert Ghrist. *Funny Little Calculus Text*. U of Pennsylvania, 2012.

[5] Hergé. *The Calculus Affair: The Adventures of Tintin*. London: Methuen Children's, 1992.

[6] W. Kelley. *The Complete Idiot's Guide to Calculus, 2nd Edition*. S.l.: DK, 2006.

[7] Clifford A. Pickover. *Calculus and Pizza: A Cookbook for the Hungry Mind*. Hoboken, NJ: John Wiley, 2003.

[8] Bonnie Averbach and Orin Chein. *Problem Solving through Recreational Mathematics*. Mineola, N.Y.: Dover Publications, 2000.

[9] Kalid Azad. *Math, Better Explained*. 2014.

[10] Oscar E. Fernandez. *Everyday Calculus: Discovering the Hidden Math All around Us*. Princeton: Princeton UP, 2014.

[11] Tom Apostol. *A Visual Approach to Calculus Problems*. ENGINEERING SCIENCE NO. 3. 2000. URL: http://www.mamikon.com/VisualCalc.pdf.

[12] Daniel Raviv. *Math-Less Physics: A Visual Guide to Understanding Newton's Laws of Motion*. n.p.: CreateSpace Independent Publishing Platform, 2016. ISBN: 9781535567046.

[13] Daniel Raviv and George Roskovich. *Understood: A Visual and Intuitive Approach to Teaching and Learning Control Systems*. n.p.: CreateSpace Independent Publishing Platform, 2014. ISBN: 9781500288884.

[14] Grant Sanderson. *3Blue1Brown*. YouTube channel. YouTube. 2024. URL: https://www.youtube.com/c/3blue1brown (visited on 01/20/2024).

[15] Aviv Censor. *Differential and Integral Calculus 2*. YouTube video. Lecture series by Dr. Aviv Censor, Technion - International School of Engineering. 2015. URL: https://youtu.be/oV0INtZT_n0?list=PLW3u28VuDAHLWNxKyfoBQSVBp-fhWrDr0.

[16] *GeoGebra: Interactive Visual Animation Tool for Mathematics*. Software tool. An interactive tool for teaching and learning mathematics. 2023.

[17] *Khan Academy: Online Educational Platform*. Online resource. Platform offering multiple sets of videos and resources in STEM. 2023.

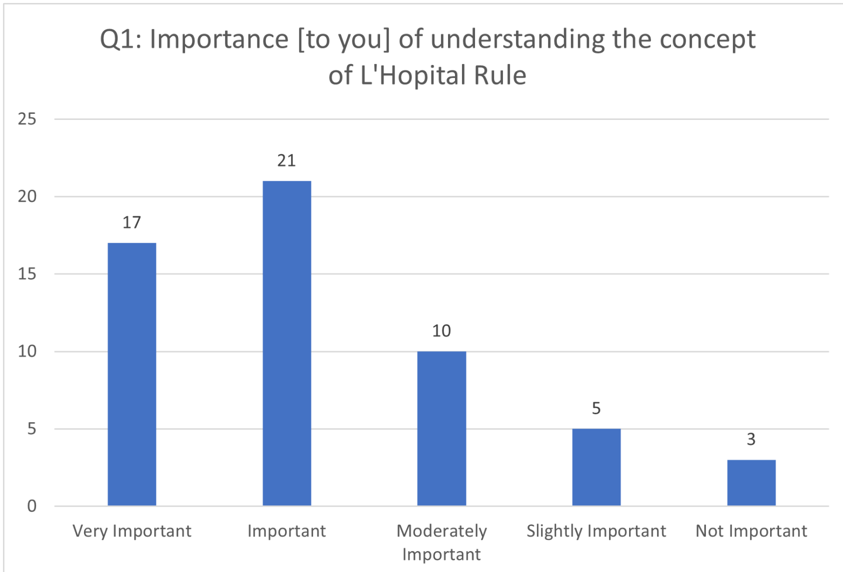
[18] Matthew Peterson. *Teaching Math Without Words, A Visual Approach to Learning Math*. YouTube video. Matthew Peterson, CEO of MIND Research Institute, speaks at TEDx Orange Coast. June 2011. URL: https://youtu.be/7odhYT8yzUM.

[19] Scott Hershberger. "The World of YouTube Math Communication". In: *Notices of the American Mathematical Society* 69.10 (1790).

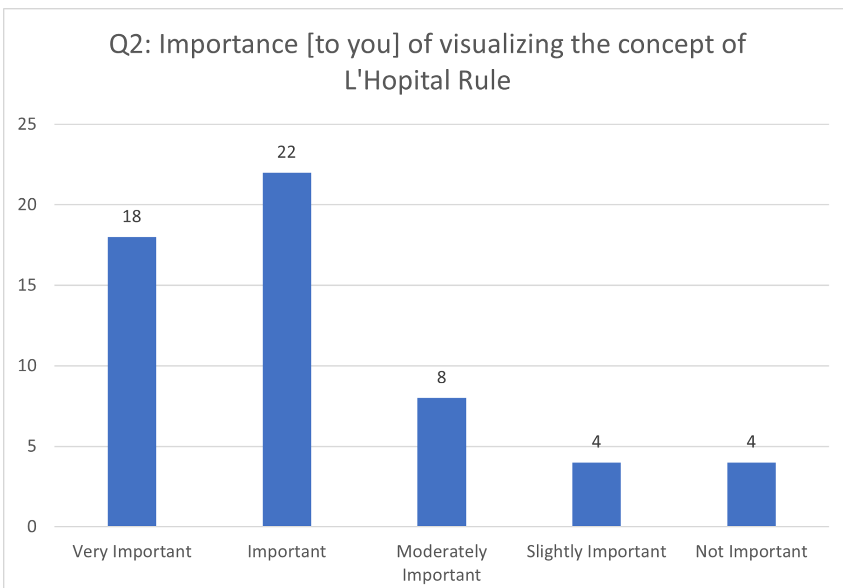
[20] John Palmieri. *Taylor Series*. https://sites.math.washington.edu/~palmieri/Courses/2012/Math135/taylor.pdf. Accessed: 2024-01-22. 2012.

# Appendix

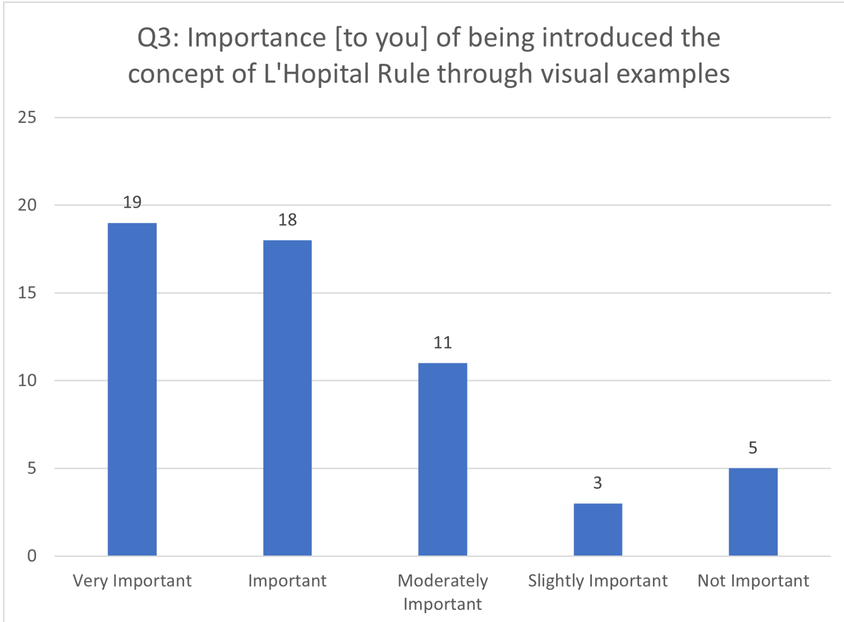
## Q1



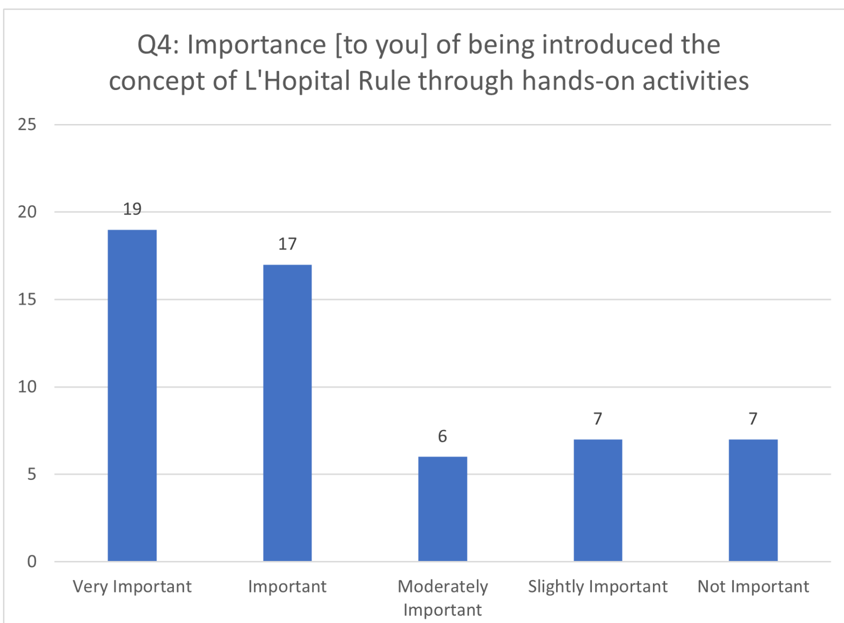

## Q2



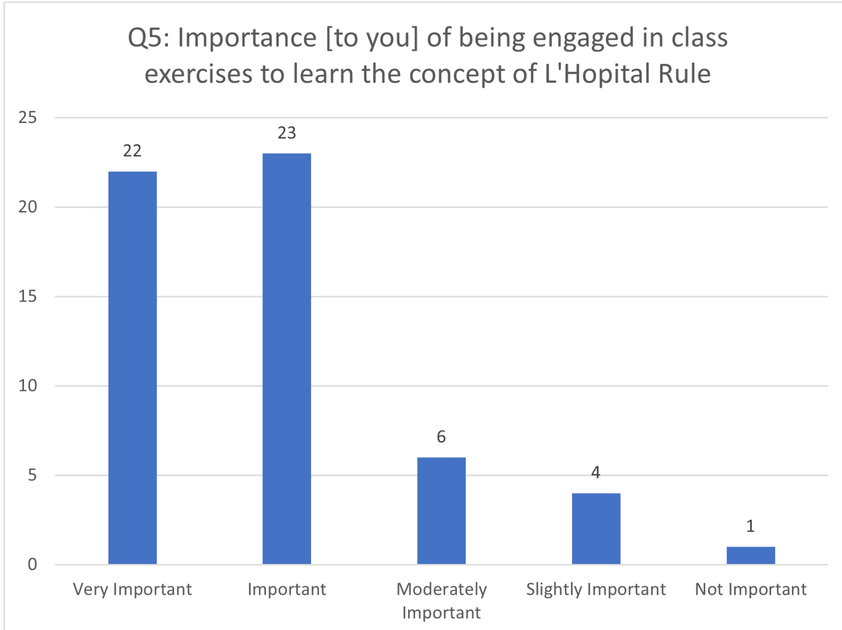
## Q3


Q3: Importance [to you] of being introduced the concept of L'Hopital Rule through visual examples
25
20
15
10
5
0
19
18
11
3
5
Very Important
Important
Moderately Important
Slightly Important
Not Important

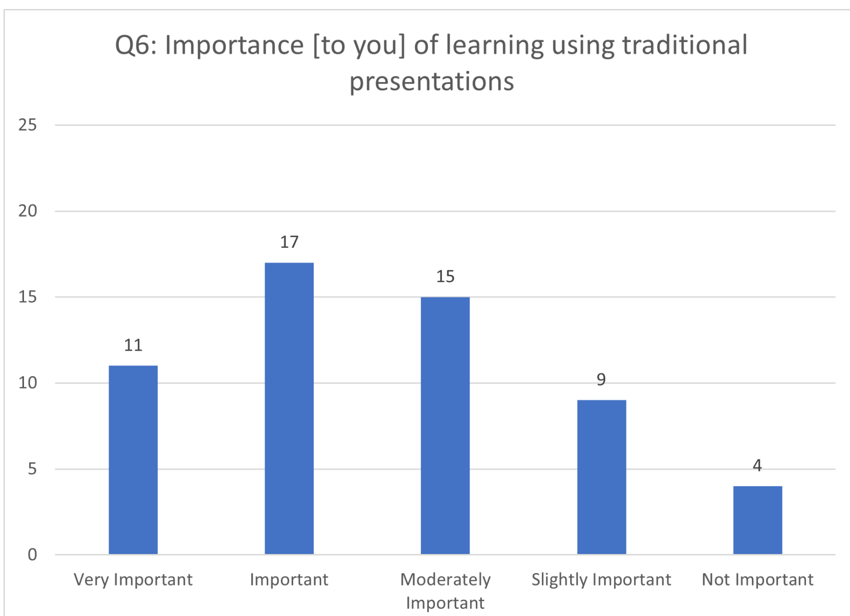
## Q4


Q4: Importance [to you] of being introduced the concept of L'Hopital Rule through hands-on activities
25
20
15
10
5
0
19
17
6
7
7
Very Important
Important
Moderately Important
Slightly Important
Not Important

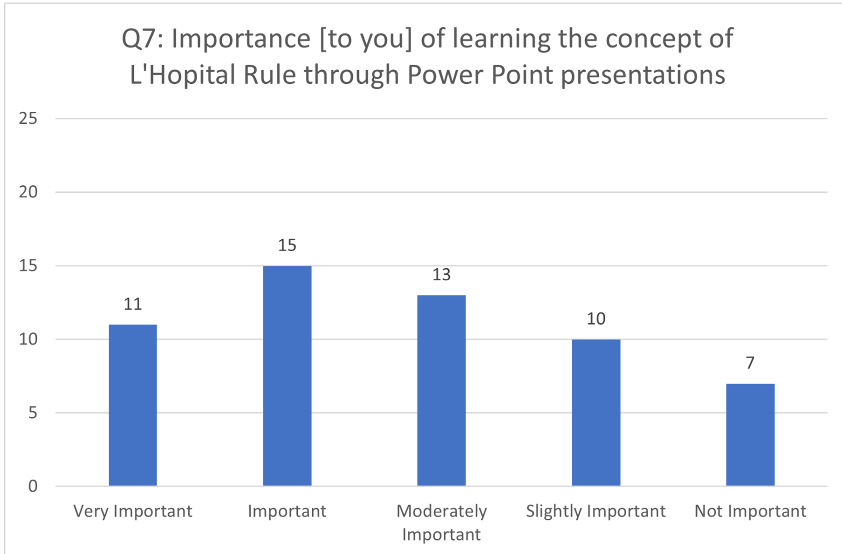
## Q5


Q5: Importance [to you] of being engaged in class exercises to learn the concept of L'Hopital Rule
25
20
15
10
5
0
22
23
6
4
1
Very Important
Important
Moderately Important
Slightly Important
Not Important

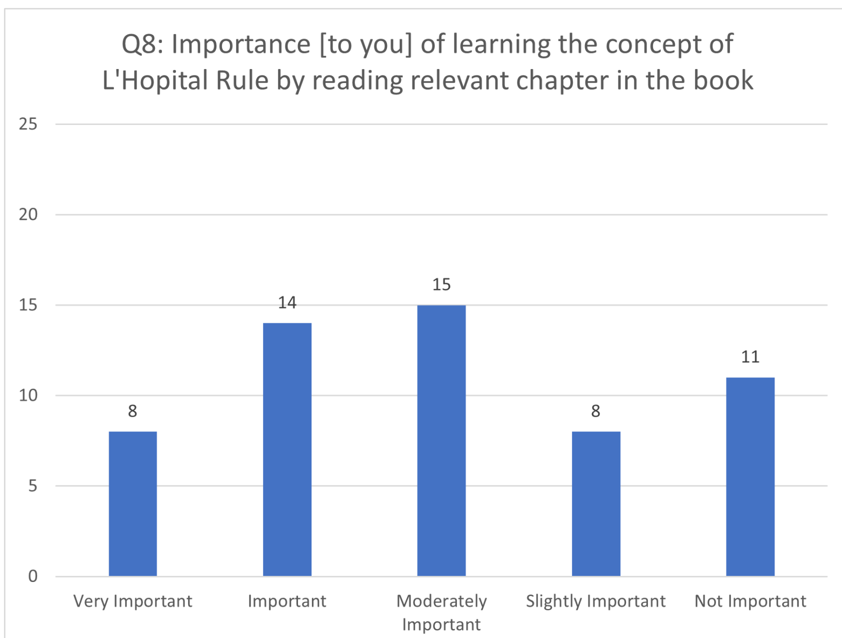
## Q6


Q6: Importance [to you] of learning using traditional presentations
25
20
15
10
5
0
11
17
15
9
4
Very Important
Important
Moderately Important
Slightly Important
Not Important

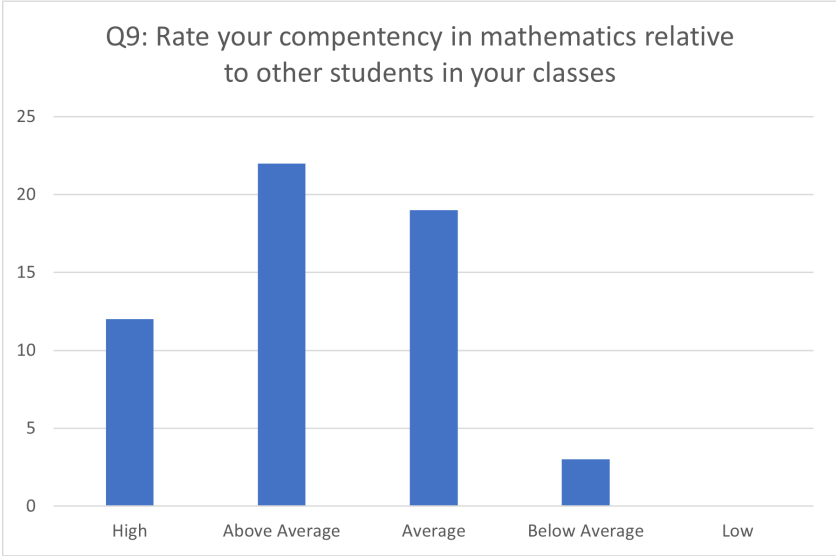
## Q7


Q7: Importance [to you] of learning the concept of L'Hopital Rule through Power Point presentations
25
20
15
10
5
0
11
15
13
10
7
Very Important
Important
Moderately Important
Slightly Important
Not Important

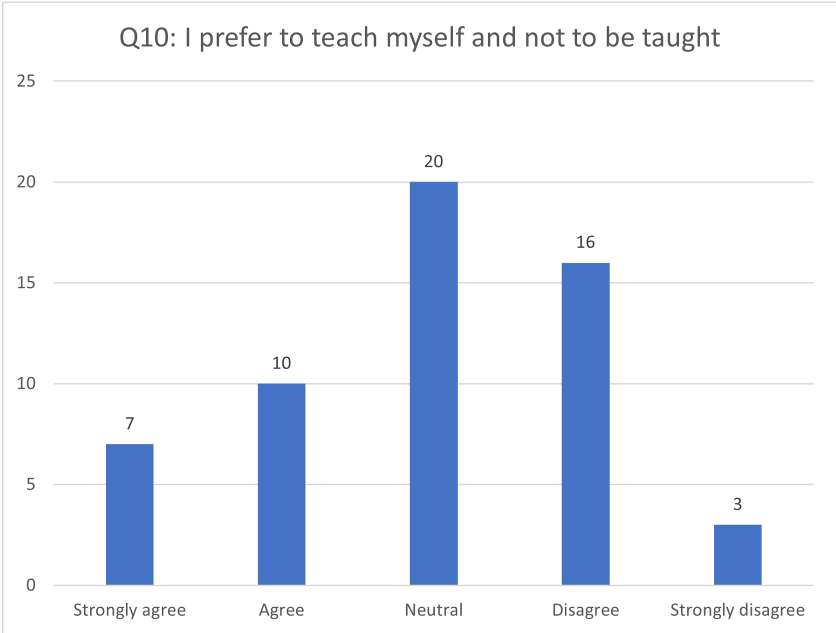
## Q8


Q8: Importance [to you] of learning the concept of L'Hopital Rule by reading relevant chapter in the book
25
20
15
10
5
0
8
14
15
8
11
Very Important
Important
Moderately Important
Slightly Important
Not Important

## Q9


Q9: Rate your compentency in mathematics relative to other students in your classes
25
20
15
10
5
0
High
Above Average
Average
Below Average
Low

## Q10


Q10: I prefer to teach myself and not to be taught
25
20
15
10
5
0
7
10
20
16
3
Strongly agree
Agree
Neutral
Disagree
Strongly disagree