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Toward Better Understanding of the Fundamental Theorem of Calculus

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With more than 30 years of combined experience in th

Work-in-Progress: Toward Better Understanding of the Fundamental Theorem of Calculus

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Abstract

In teaching calculus, there is often insufficient emphasis on the profound connection between integration and differentiation, as illuminated by the fundamental theorem of calculus (FTOC). All too frequently, students view integration simply as the inverse operation of differentiation without fully understanding the foundational logic behind this relationship. This shallow comprehension encourages a formula-driven mindset, where learners only apply predetermined rules for both operations, diminishing their grasp of the theorem's importance.

In this paper, we attempt to elucidate the FTOC using a visual and intuitive approach. Our primary goal is to promote a foundational understanding of the FTOC. The theorem's explanation is segmented into two distinct parts. The first part of the FTOC asserts that if you take the derivative of an integral with a variable upper limit, you return to the original function. Put more simply, it links the processes of differentiation and integration, illustrating that they are inverse operations. The second part underscores the profound relationship between integration and antiderivatives, especially in the context of definite integrals.

In this paper, we review the concept of inverse functions and provide a brief historical overview of the FTOC. A pivotal aspect of our presentation is visualizing the FTOC, a cornerstone in the realm of mathematical analysis. We delve into the relationship between differentiation and integration, highlighting why these two operations are frequently regarded as inverses of one another. Through this exploration, we aim to offer readers a comprehensive understanding of how these foundational mathematical processes are intertwined. This discussion is supplemented by a step-by-step visual and graphical explanation of the concept, accompanied by real-life examples.

We introduced the new approach to students in a classroom setting and gathered their feedback for a deeper understanding. To ensure honest and unbiased feedback, we used anonymous questionnaires. This method gave us detailed insights into the

effectiveness and reception of the new approach. Our evaluation of this novel technique shows its potential in boosting student understanding. Using this strategy, students not only intuitively understand the FTOC, but also indicate a favorability towards visual learning modalities. Based on feedback from 58 students, 69% deem the comprehension of the FTOC as “important” or “very important”, and 81% prefer visual learning approaches.

It is crucial to highlight that this project is still a work in progress. It is not intended to replace traditional textbook chapters or topics; instead, it serves as a supplementary tool for both educators and learners. Our goal is to assist instructors in conveying knowledge and to help students grasp the FTOC concept in a more comprehensible and relevant manner. This project is part of a broader initiative. Our experience in assessing STEM-related subjects, such as control systems, digital signal processing, computer algorithms, and physics, underscores the potential of this approach.

1 Introduction

When instructing calculus, the profound link between integration and differentiation, as revealed by the fundamental theorem of calculus (FTOC), is sometimes not given enough attention. Many times, students perceive integration merely as the reverse process of differentiation, lacking a complete comprehension of the fundamental logic that underlies this connection. This limited understanding fosters a reliance on formulas, leading learners to apply predetermined rules for both operations, take for granted the relationship between integration and differentiation, and diminishing their appreciation for the theorem's significance.

This paper aims to simplify the explanation of the fundamental theorem of calculus (FTOC) for both instructors and students, employing a clear, intuitive, and example-based approach to enhance understanding. It addresses the main teaching/learning challenge of explaining why integration (area) is the opposite of derivation (slope). Visual illustrations are utilized to establish a solid foundation for a deeper grasp and appreciation of the theorem, focusing on the fundamental understanding and emphasizing the connection between integration and differentiation. The explanation of the theorem is divided into two well-known distinct segments. The first segment of the FTOC states that if you take the derivative of the integral of a function with a variable upper limit (indefinite integral), you revert to the original function. In other words, we try to highlight and explain the inverse nature between differentiation and integration. The second segment emphasizes the profound relationship between integration and anti-derivatives as it relates to definite integrals.

It is crucial to emphasize that this project is currently in development (work in progress). Its purpose is not to substitute conventional textbook explanations but to function as a valuable supplementary resource for instructors and students. Our aim is to support educators in teaching and enable students to grasp intricate mathematical concepts in a more captivating and meaningful way. This initiative is part of a larger project, drawing on our extensive experience in evaluating comparable methods in diverse engineering-related subjects such as control Systems, digital signal processing, computer algorithms, and physics. Our findings strongly indicate that this approach has significant potential.

In our assessment of the new approach, we introduced it to students in a classroom setting and collected their feedback through anonymous questionnaires to ensure honesty and impartiality. This evaluation method provided us with detailed insights into the effectiveness and reception of the approach. Our findings indicate that this method holds substantial potential in enhancing student understanding. Through this strategy, students not only intuitively grasp the fundamental theorem of calculus (FTOC) but also express a preference for visual learning modalities. Analyzing feedback from 58 students, we observe that 69% deem the comprehension of FTOC as “important” or “very important,” while 81% favor visual learning approaches. This comprehensive assessment underscores the positive impact of the new approach on student learning experiences. For details please refer to the Appendix.

A bit of historical perspective

The fundamental theorem of calculus (FTOC) traces its origins to the 17th century when two eminent mathematicians, Sir Isaac Newton and Gottfried Wilhelm Leibniz, independently contributed to its formulation. Newton, an English mathematician and physicist, developed calculus to comprehend motion and change, introducing the fundamental concept of integration and anti-derivative. Concurrently, Leibniz, a German mathematician, independently crafted his calculus notation, including the integral symbol ($\int$). Although the FTOC was not explicitly stated in its modern form by Newton or Leibniz, their foundational contributions paved the way for its eventual refinement. Later mathematicians such as Augustin-Louis Cauchy and Bernhard Riemann in the 19th century further formalized the theorem, establishing a rigorous framework for the relationship between differentiation and integration. The FTOC has since evolved into a vital mathematical tool, playing a pivotal role in diverse applications across science and engineering.

Related work

In the pursuit of grasping complex concepts, students often encounter challenges in conceptual understanding. There is an obvious desire among students for pedagogical approaches that are hands-on, experiential, visually engaging, intuitive, and even enjoyable. The advent of technology has further amplified this craving, with students seeking accessible web-based information to supplement their learning journeys.

In the domain of calculus, numerous textbooks [1–10] have embraced visual explanations, with notable contributions from Apostol and Mamikon [10]. Their work stands out for explaining the integration of certain functions without heavy reliance on mathematical formulas, marking a noteworthy departure from conventional instructional methods.

Expanding on the incorporation of visual and intuitive methodologies, the fields of "Control Systems" Physics have seen insightful contributions from works such as [11, 12]. In the digital domain, content creators like 3Blue1Brown [13] leveraging the open-source Python library Manim for interactive animations, have made significant strides in teaching foundational STEM concepts. It stands out for its clear visualizations and comprehensible explanations, effectively bringing learners into the world of mathematics.

[14] further exemplifies the successful fusion of clear explanations, mathematical proofs, and visual examples to enhance understanding. GeoGebra [15] has emerged as a valuable interactive animation resource for teaching and learning mathematics. Khan Academy [16], through its diverse collection of online videos, has become a cornerstone in assisting students in comprehending STEM concepts and techniques. [17] takes a visual approach to learning mathematics, offering clear explanations and emphasizing the significance of visual learning.

In the spirit of fostering a friendly and inviting learning environment for mathematics, the American Mathematical Society's noteworthy contribution can be found in [18]. This comprehensive exploration of diverse pedagogical approaches underscores the evolving landscape of STEM education and highlights the crucial role of visual, intuitive, and interactive methods in enhancing the learning experience.

2 The Concept of Inverse Function

In simple words: Applying a function f and then its inverse f^{-1} results in the original value back again. Using mathematical notation:

$$f(a) = b \Leftrightarrow f^{-1}(b) = a$$

In [19] the basic idea behind inverse function is visually explained. Here we summarize the explanations.

Refer to Figure 1. Envision a function represented as a "gray box" at the top, featuring a green square as input and an orange circle as output. The inverse function, depicted in the bottom "gray box," accepts the orange circle as input and transforms it back into the green square as output.

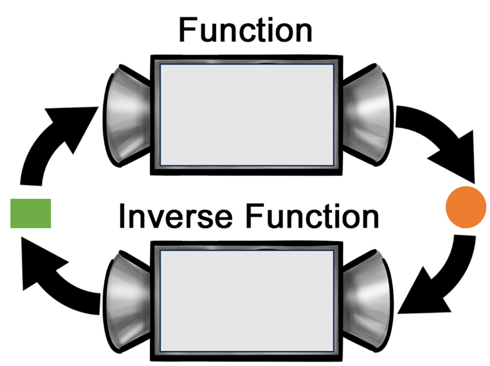


Figure 1: Basic visualization of a function and its inverse

The following cartoon (Figure 2) [20] presented below explains the fundamental concept of an inverse function. This illustrative example draws from real-life experiences, infused with a humorous and imaginative essence. The initial "function" represents an inadvertent action, cleverly countered by its deliberate "inverse function."

The following is an illustration where the inverse is non-existent:

Picture yourself at a barber shop. Regrettably, should the barber trim more than anticipated, there is no way to reverse the haircut damage. Figure 3 serves as a clear illustration of a haircut function lacking an inverse.

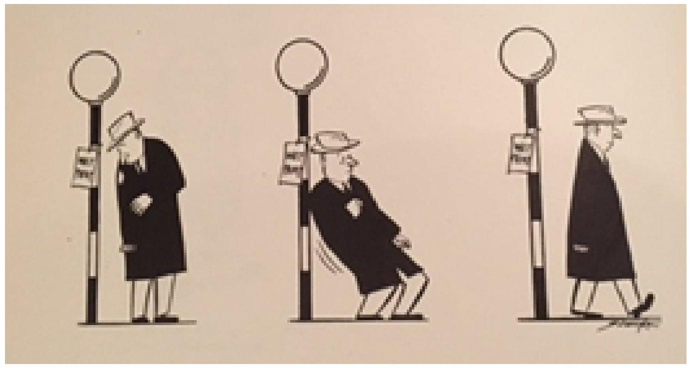


Figure 2: Inverse function: An inspiring cartoon [20]

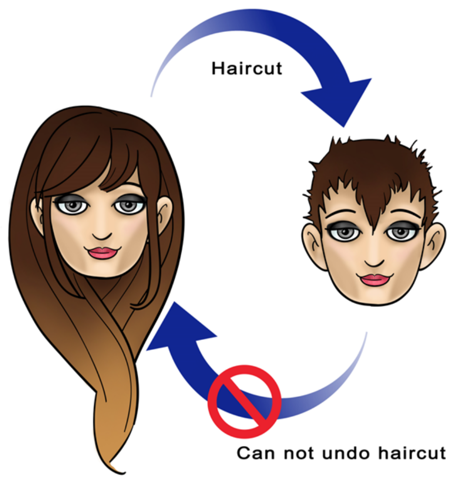


Figure 3: Haircut - Demonstrating the absence of an inverse function

3 Explanation of the FTOC

The fundamental theorem of calculus is a powerful theorem that establishes the connection between differentiation and integration. We explain the two parts of the FTOC:

FTOC Part I: It asserts that if you have a function defined as the integral of another function with a variable upper limit (i.e., $F(x) = \int [a, x]\, f(t)dt$), then its derivative is equal to the original function: $F'(x) = f(x)$. Figure 4. Visually shows this idea.

FTOC Part II: This segment affirms that if a function is continuous on a closed interval $[a, b]$, then there exists an antiderivative of that function, represented as $F(x)$, such that the definite integral of the function over the interval $[a, b]$ is equal to the difference between the antiderivative at the endpoints: $\int [a, b]\, f(x)dx = F(b) -$

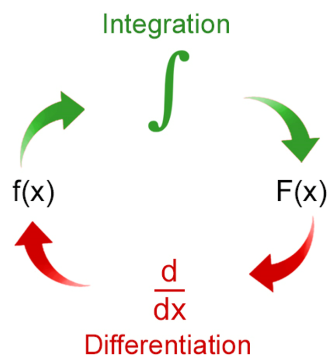


Figure 4: Visualization of FTOC

$F(a)$.

3.1 A Basic Derivation of the FTOC

The area $F(x)$ under the graph of $f(x)$ is the sum of infinitely small areas, i.e., $F(x) = \sum f(x)dx$. The change in $F(x)$, i.e., the change in the area at a specific x is $f(x)$.

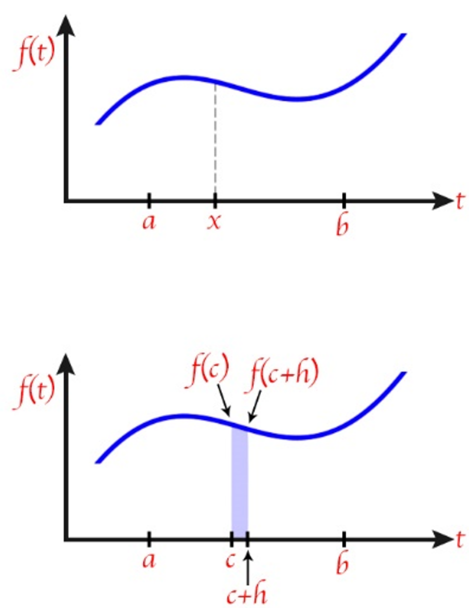


Figure 5: Integration

Assume $F(x)$ is differentiable in (a, b). Let Δx be very small ("small enough") then

$$F(c+\Delta x)-F(c)\approx f(c)\cdot\Delta x$$

so

$$f(c)\approx\frac{F(c+\Delta x)-F(c)}{\Delta x}$$

When $\Delta x\to 0$

$$\left.\frac{dF(x)}{dx}\right|_{\text{at }c}=\lim_{\Delta x\to 0}\frac{F(c+\Delta x)-F(c)}{\Delta x}=f(c)$$

So: for $a<x<b$

$$\frac{d}{dx}\left.\left(\frac{dF(x)}{dx}\right)\right|_{\text{at }c}=\frac{d}{dx}\int_a^x f(t)dt=f(c)$$

If $f(x)$ is continuous in $[a,b]$ then for any $x\in(a,b)$:

$$\frac{dF(x)}{dx}=\frac{d}{dx}\int_a^x f(t)dt=f(x)$$

Calculating the area

The area may be calculated/approximated using upper limit of lower bound, lower limit of upper bounds or both as shown in Figure 6.

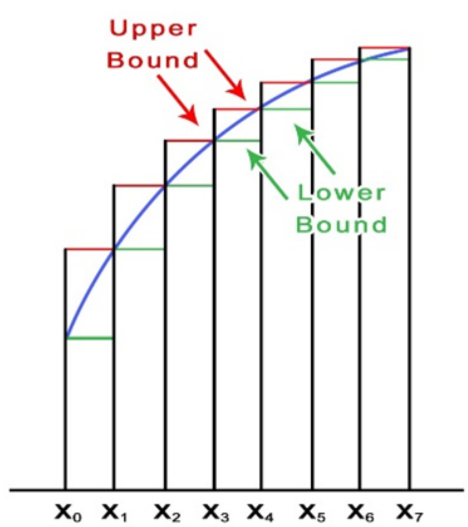


Figure 6: Calculating the area under the graph

3.2 FTOC Part 1 - Indefinite Integral

Part 1 establishes the relationship between differentiation and integration.

If $f(x)$ is continuous over an interval $[a, b]$, and the function $F(x)$ is defined by

$$F(x) = \int_a^x f(t)\, dt,$$

then $F'(x) = f(x)$ over (a, b).

The theorem not only establishes a correlation between integration and differentiation but also ensures that every integrable function possesses an antiderivative. To be more specific, it affirms that any continuous function inherently possesses an antiderivative.

Visually (Figure 7) we can state that the derivative of the area under the graph $F(x)$ which is a function of x is the function $f(x)$.

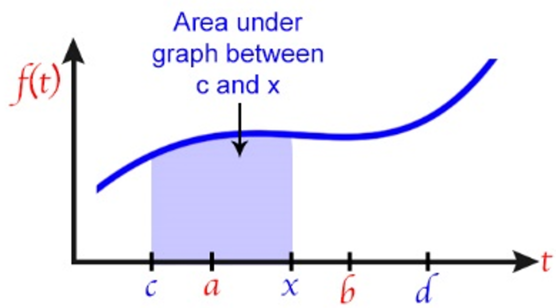


Figure 7: Visualizing the relationship between the area under the graph and its derivative

It is important to note that the relationship between $F(x)$ and $f(x)$ is not unique: When we add a constant to the anti-derivative function $F(x)$, and then take the derivative we still get the same $f(x)$, as illustrated in Figure 8.

3.3 FTOC Part 2 - Definite Integral

The fundamental theorem of calculus part 2 simplifies the evaluation of definite integrals by stating that if an antiderivative for the integrand is identified, the definite integral can be calculated by evaluating the antiderivative at the interval's endpoints and subtracting the results. It mathematically states

If f is continuous over the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

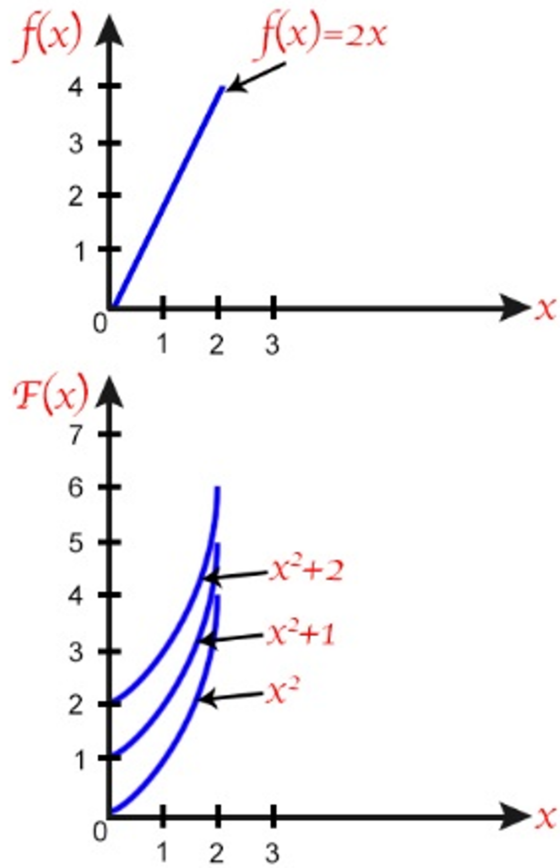


Figure 8: On the relationship between the anti-derivative function $F(x)$ and its derivative $f(x)$

Despite its apparent simplicity, this method proves effective in computing the entire area of under a curve region. Note that the inclusion of the constant term in the antiderivative is unnecessary, simply because the difference $F(b) - F(a)$ cancels it out. It almost seems too simple that the area of an entire curved region can be calculated by just evaluating an antiderivative at the first and last endpoints of an interval.

The region of the area we just calculated is depicted in Figure 9.

Also, the area between point "a" and point "b" can be obtained by subtracting the area up to point "a" from the area up to point "b" (Figure 10).

The area under the graph between point "c" and point "b" is also the sum of the area under the graph between points "c" and "a" and the area under the graph between points "a" and "b". (Figure 11).

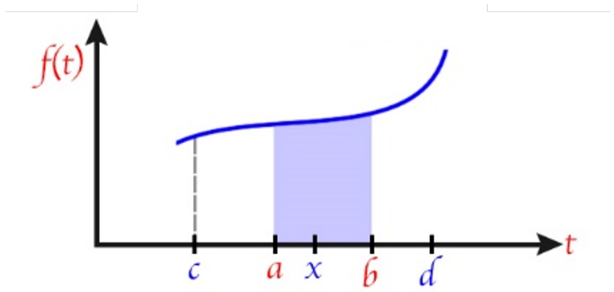


Figure 9: The area under a curve can be obtained by subtracting the anti-derivative values at the first and last endpoints of an interval (between “a” and “b”)

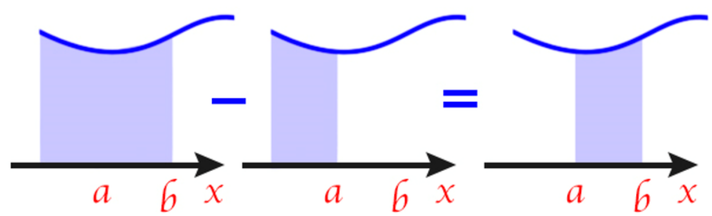


Figure 10: Obtaining the area of the graph between point “a” and point ”b”

3.4 FTOC Part 1 - Real Life Examples

Part 1 establishes the relationship between differentiation and integration.

Example 1: Power and Energy (Figure 12)

Energy is the anti-derivative of power.

DC motor: angle and angular velocity. (Figure 13)

Note that the angle θ (in radians) is the anti-derivative of the angular velocity ω (in radians per second). Also note that the motion of the motor is due to input voltage V_a.

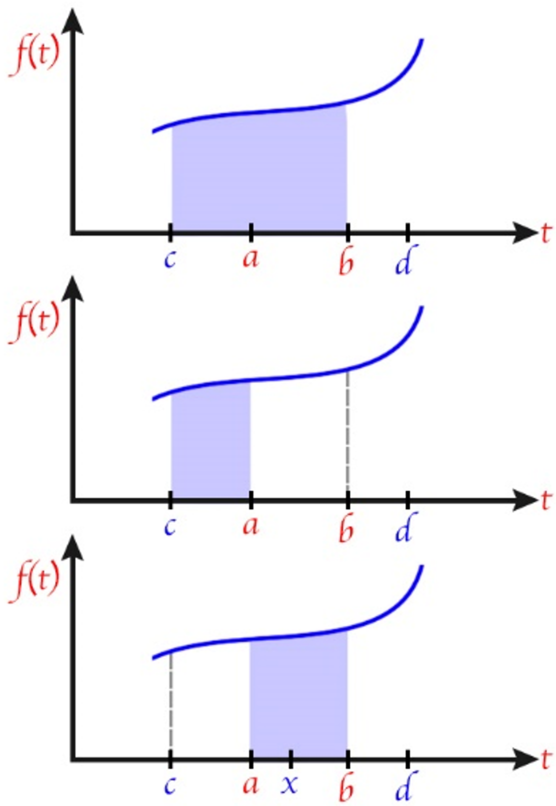


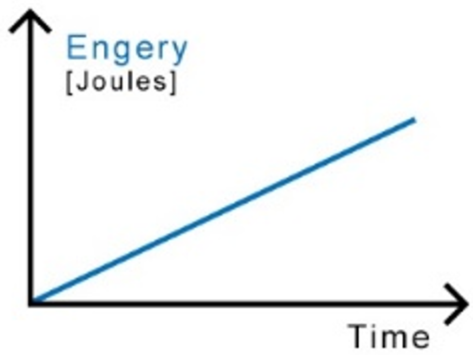
Figure 11: Summing up areas

3.5 FTOC Part 2 - Real Life Examples

Part 2 of the fundamental theorem of calculus stands as a pivotal theorem in calculus. Developed over hundreds of years, it provided scientists with essential tools for understanding various phenomena. Its impact is evident in astronomy, enabling the determination of distances in space, in everyday financial computations and engineering analyses, and analyzing three-dimensional motion.

Example 1: Speedometer and distance

You may calculate the traveled distance on a straight highway by multiplying the speed as obtained by speedometer at any moment by an infinitesimally small time interval and then add them together (which is a long process of integration). But you can also find it simply by subtracting the initial location from the final location without integration. Refer to Figure 14.



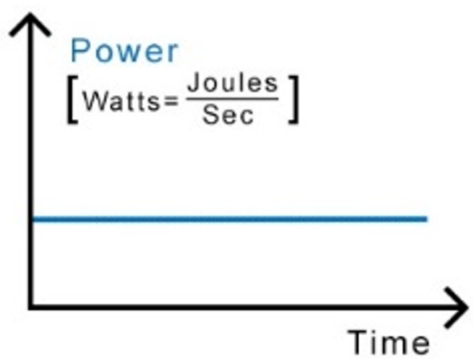


Figure 12: On the relationship between power and energy. Note that the energy is the anti-derivative of power.

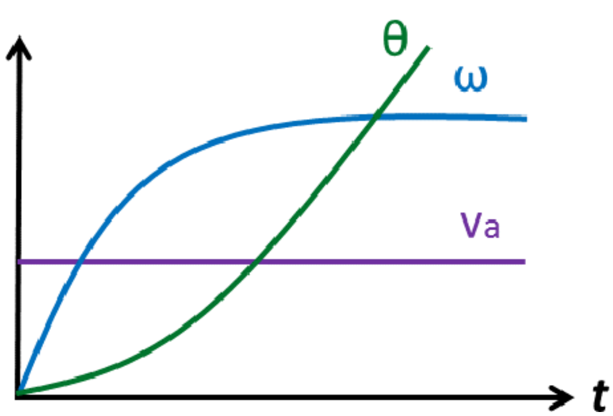


Figure 13: The relationship between angular velocity and angle of a dc motor.

Example 2: Flying from Miami, Florida to Chicago, Illinois

You may fly from one airport to another using multiple routes and multiple flight legs. However, the distance from the two cities does not change regardless of your

Figure 14: Using speedometer to calculate the total distance traveled on straight highway

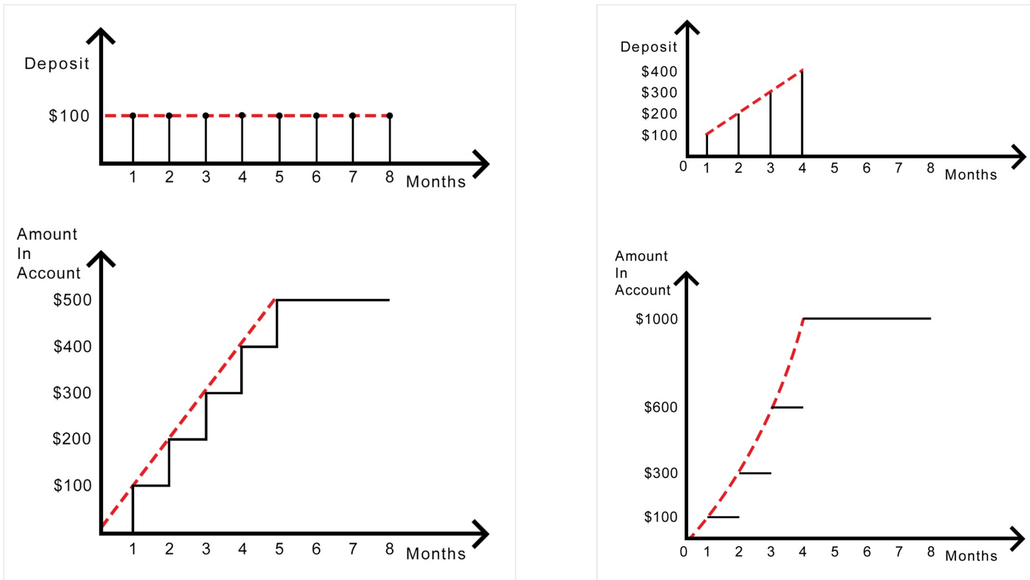
journey. Figure 15.

Figure 15: Shortest distance between Miami to Chicago.
Source: https://www.usgs.gov/media/images/general-reference-printable-map

Example 3: Bank account

In January 2020 you decided to start to deposit a monthly amount of $100 in your bank account. If you want to find how much you saved between August 2020 and February 2024, all you need to do is to subtract the February 2024 amount from the

August 2020 amount to get the answer. You do not need to add all the monthly deposits. The same calculation apply even if your deposits change from one month to another. (Figure 16.)



(a) (b)

Figure 16: Bank deposits and their accumulation over time: (a) constant monthly amount, (b) linearly changing monthly amount.

3.6 Mathematical Functions: Visual Examples

The following images show the values of the antiderivative of four different values of x: $x = 0, x = 0.5, x = 1$, and an arbitrary x (Figure 17.)

The following visualize a function and its antiderivative. (Figure 18.)

(a) Shows the accumulated area (antiderivative) from up to $x = 2$.

(b) Shows the accumulated area for all x including beyond $x = 2$. Note that the antiderivative graph is flat after $x = 2$ due to the 0 value of the original function.

(c) Shows that the original function can be obtained by taking the derivative of the antiderivative function using the slope, i.e., rise/run.

Figure 19 is another example similar to the previous example, except the original function is linear.

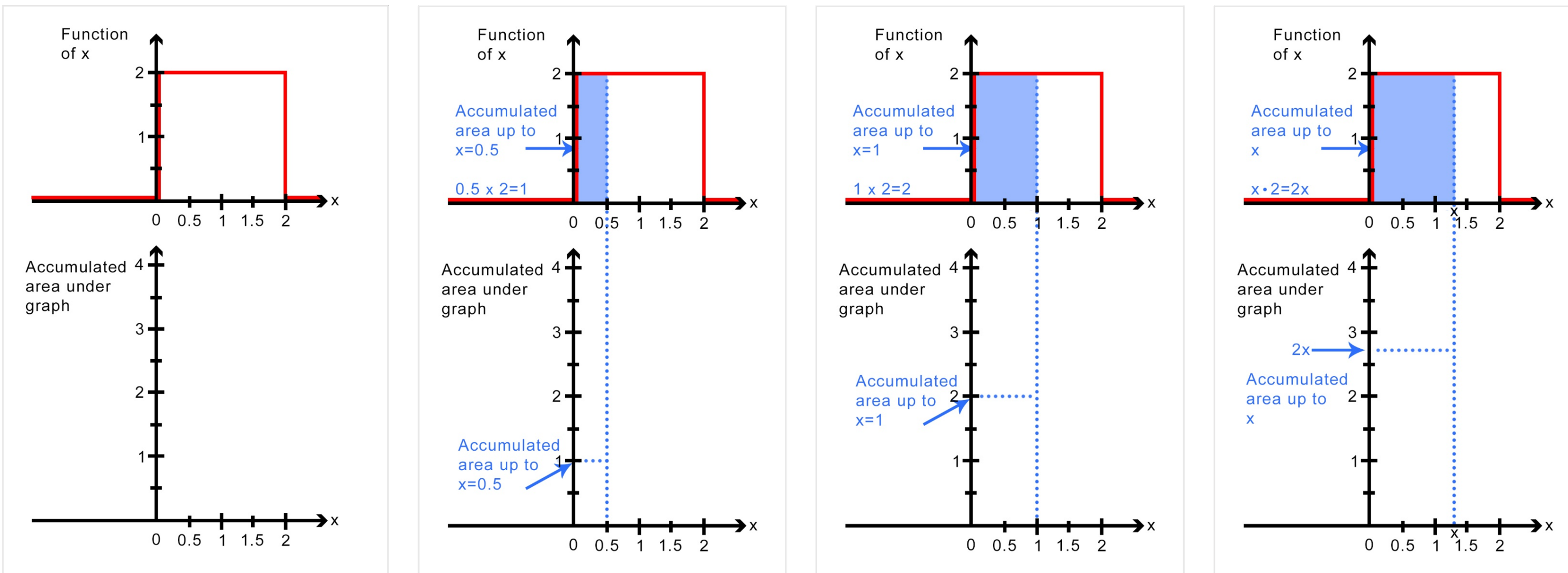


Figure 17: Values of the antiderivative

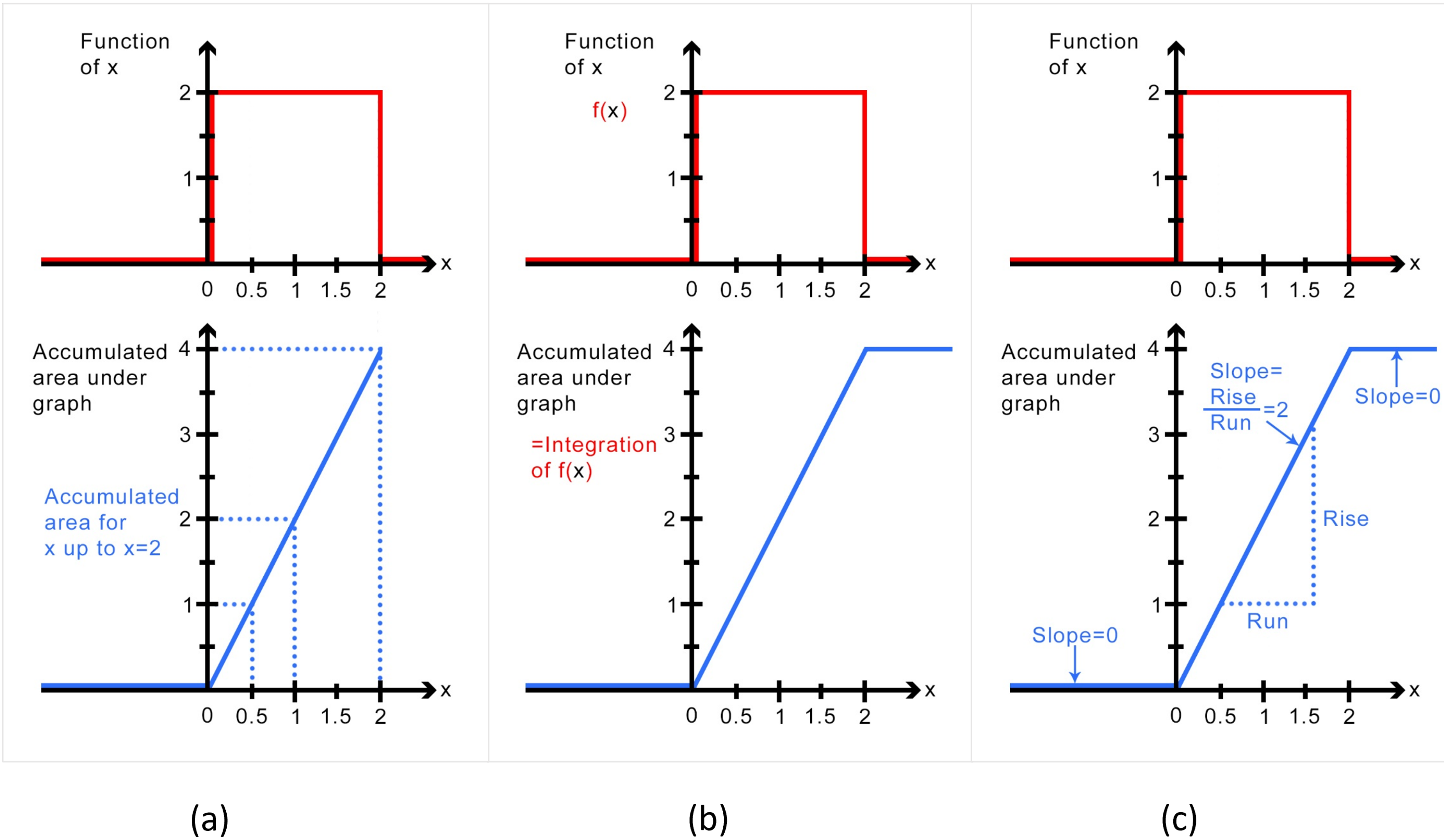
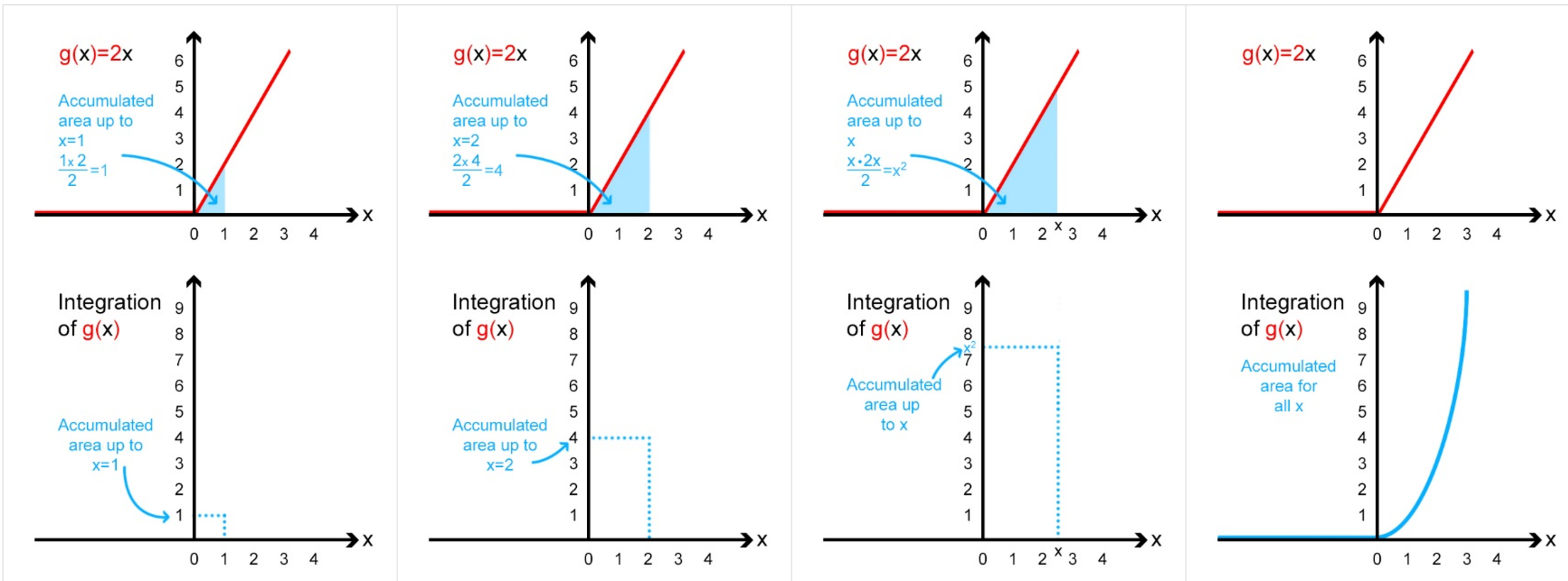


Figure 18: Area under the graph



(a) (b) (c) (d)

Figure 19: Integration of a linear function.

4 Assessment

To assess the effectiveness of the new approach for learning FTOC we conducted an in-class anonymous questionnaire. 58 students responded. The results clearly show that understanding the concept of FTOC is either important or very important. Most students praised the visualization approach for teaching FTOC. Even though we have not used activities and exercises, students felt that more hands-on activities and in-class exercises could be very helpful. In general, they like traditional presentations, but not as excited as we thought about Powerpoint presentations. Surprisingly most of them prefer not to read texbooks. Overall, we feel that visualizations were very well-accepted and preferred by students as an additional way of learning. Refer to Table 1 for the percentage of very important and important activities as perceived by students. For more details please refer to the Appendix.

Percentage	**Activity**
69.0%	Understanding the concept of FTOC
75.9%	Visualizing the concept of FTOC
81.0%	Being introduced with the concept of FTOC through visual examples
67.2%	Being introduced with the concept of FTOC through hands-on activities
84.5%	Being engaged in class exercises to learn the concept of FTOC
52.6%	Learning using traditional presentations
46.6%	Learning using PowerPoint
34.5%	Learning the concept by reading book

Table 1: Percentage of very important and important

5 Conclusion

This paper contributes to the ongoing conversation on teaching the fundamental theorem of calculus (FTOC) by highlighting the importance of integrating intuitive and visual learning methods into traditional mathematical education. Our innovative approach aims to bridge the gap between conceptual understanding and practical techniques, enriching the comprehension of FTOC through creative teaching strategies.

Building on our exploration, we stress that the methods and examples presented here are not intended to replace conventional calculus textbooks but rather complement existing educational materials. Our objective is to make complex mathematical concepts, especially those related to FTOC, more accessible and understandable for students. This work, still in its developmental and assessment stages, provides a fresh perspective on teaching calculus and encourages educators to consider adopting these methods in their teaching contexts.

Our paper aligns with the goal of explaining FTOC using a visual and intuitive approach. By emphasizing the link between differentiation and integration, particularly through the exploration of inverse operations, we contribute to promoting a foundational understanding of FTOC.

As we review the concept of inverse functions and offer a historical overview of FTOC, our pivotal contribution lies in visualizing the theorem as a cornerstone in mathematical analysis. Through this exploration, we hope to provide readers with a comprehensive understanding of the foundational mathematical processes, inspiring further innovation in the teaching and learning of mathematics across various disciplines.

Acknowledgements

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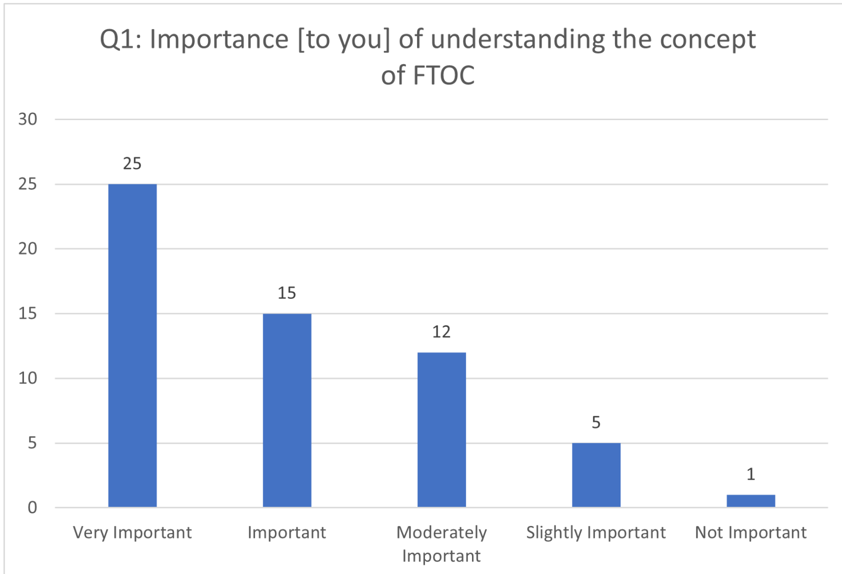
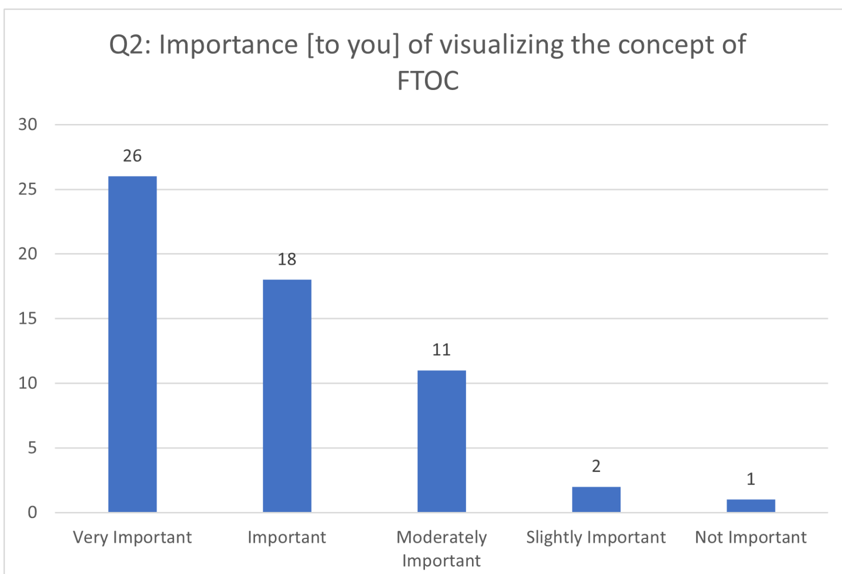
Appendix

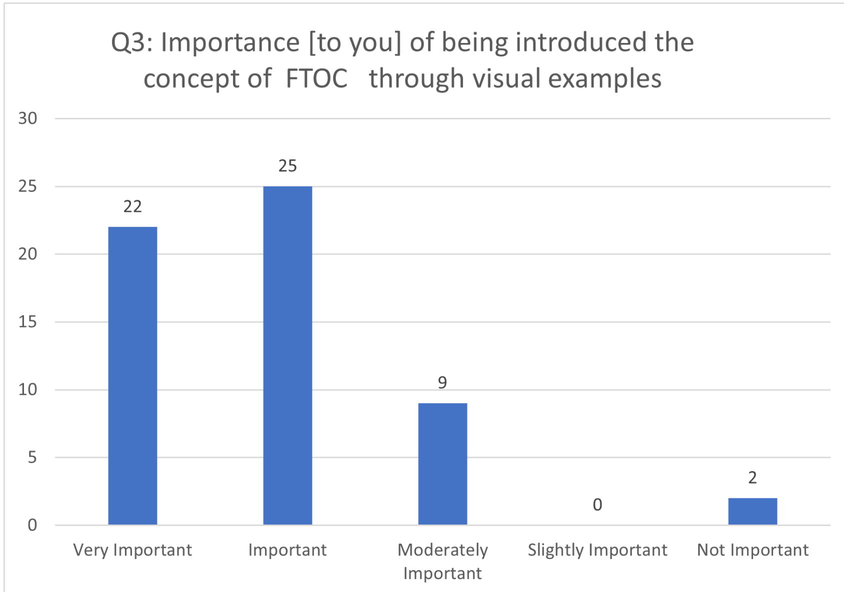
Q1



Q2

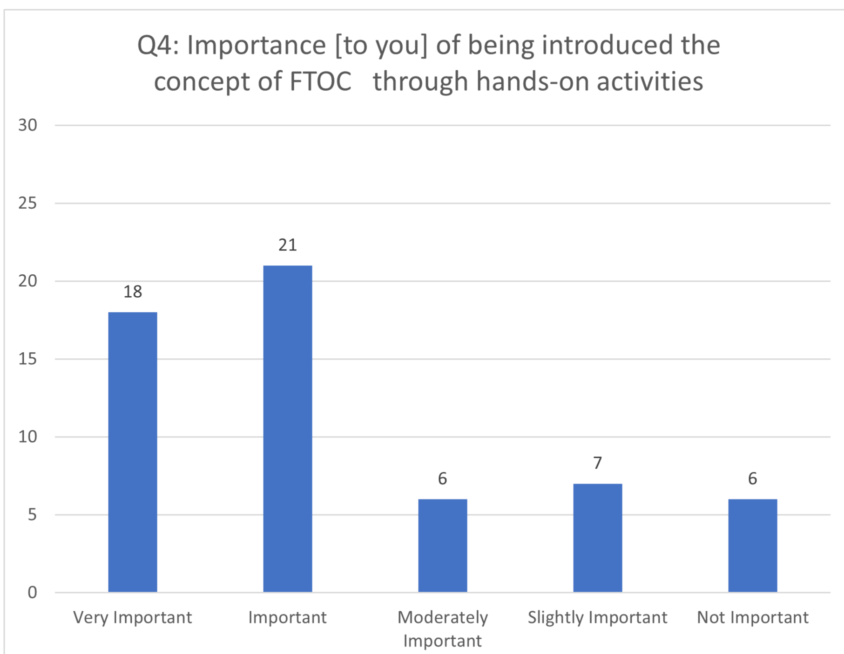


Q3



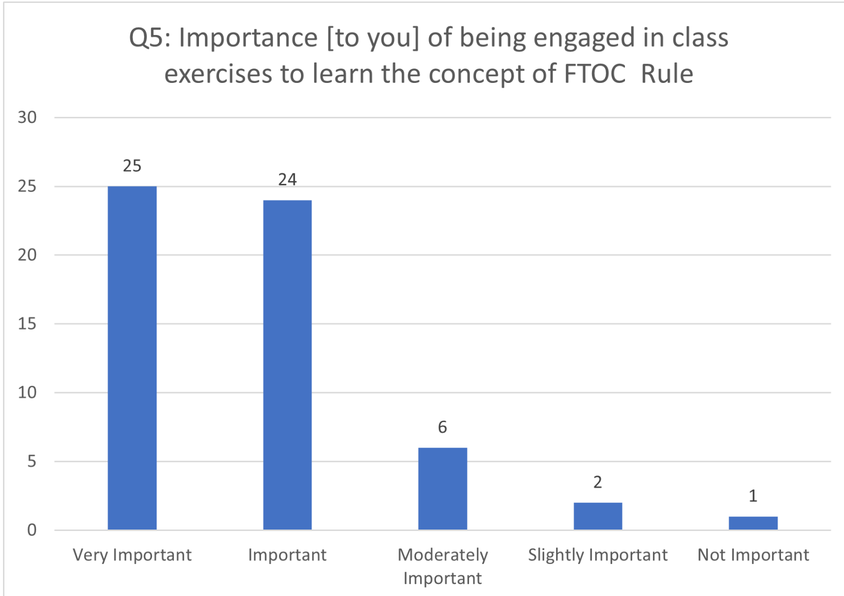
Q3: Importance [to you] of being introduced the concept of FTOC through visual examples
30
25
20
15
10
5
0
22
25
9
0
2
Very Important
Important
Moderately Important
Slightly Important
Not Important

Q4

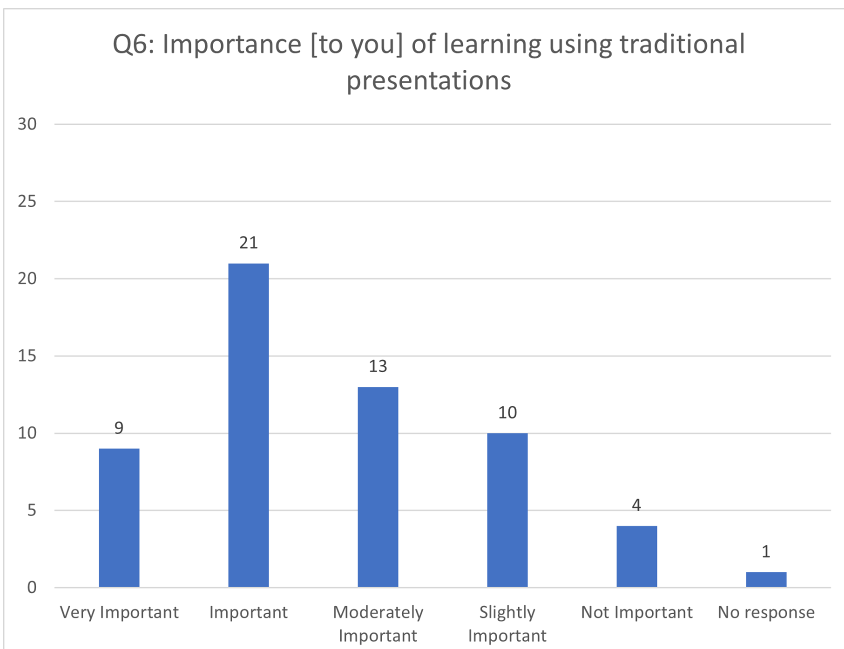


Q4: Importance [to you] of being introduced the concept of FTOC through hands-on activities
30
25
20
15
10
5
0
18
21
6
7
6
Very Important
Important
Moderately Important
Slightly Important
Not Important

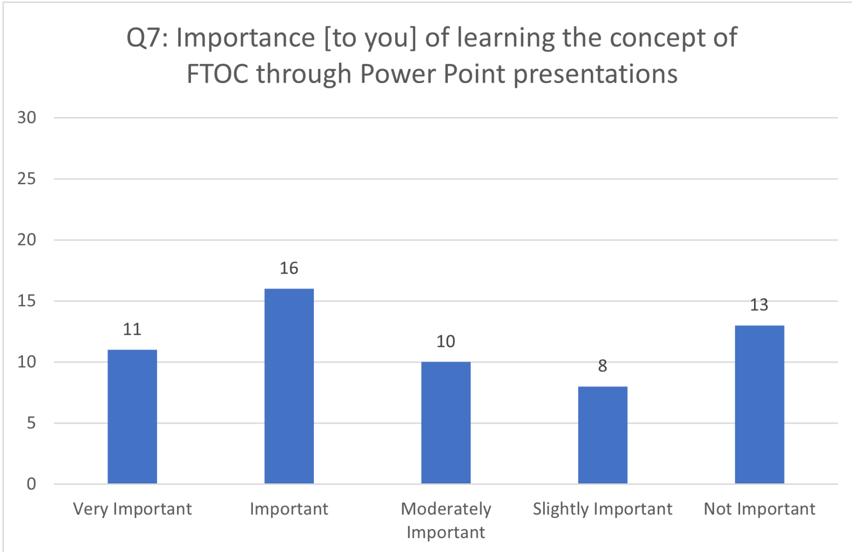
Q5


Q5: Importance [to you] of being engaged in class exercises to learn the concept of FTOC Rule
30
25
20
15
10
5
0
25
24
6
2
1
Very Important
Important
Moderately Important
Slightly Important
Not Important

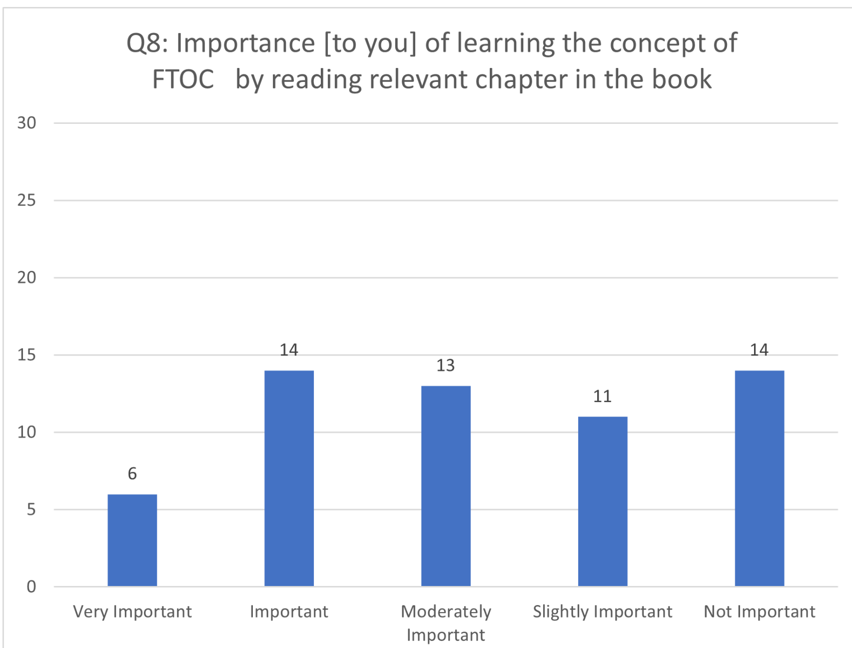
Q6


Q6: Importance [to you] of learning using traditional presentations
30
25
20
15
10
5
0
9
21
13
10
4
1
Very Important
Important
Moderately Important
Slightly Important
Not Important
No response

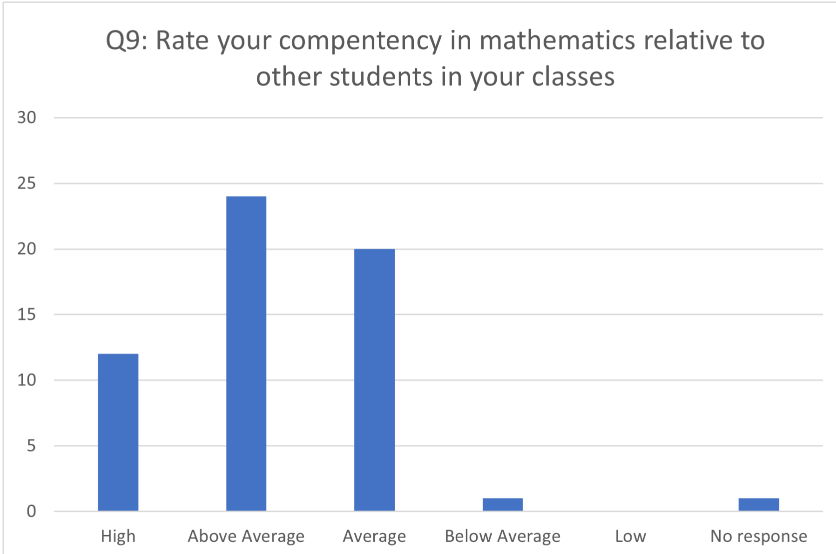
Q7


Q7: Importance [to you] of learning the concept of FTOC through Power Point presentations
30
25
20
15
10
5
0
11
16
10
8
13
Very Important
Important
Moderately Important
Slightly Important
Not Important

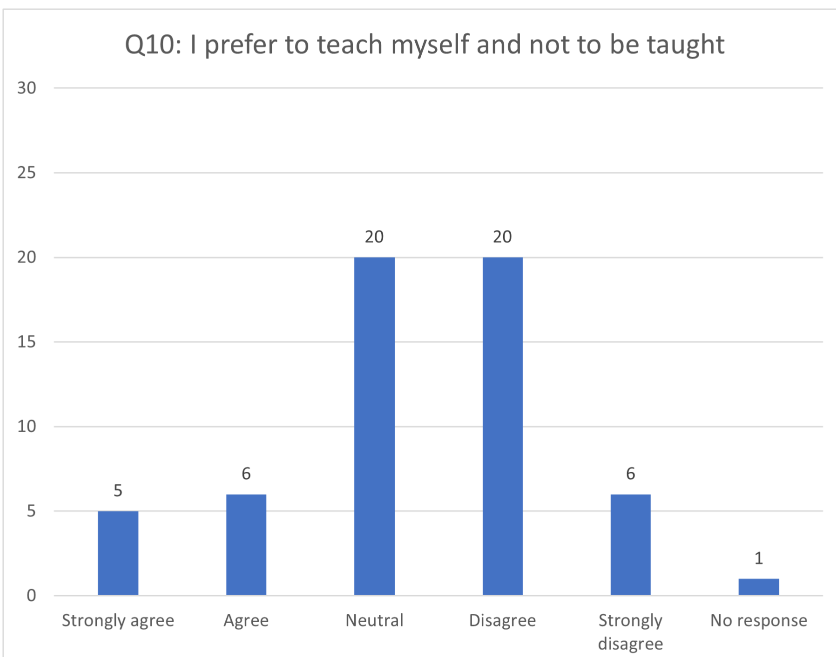
Q8


Q8: Importance [to you] of learning the concept of FTOC by reading relevant chapter in the book
30
25
20
15
10
5
0
6
14
13
11
14
Very Important
Important
Moderately Important
Slightly Important
Not Important

Q9


Q9: Rate your compentency in mathematics relative to other students in your classes
30
25
20
15
10
5
0
High
Above Average
Average
Below Average
Low
No response

Q10


Q10: I prefer to teach myself and not to be taught
30
25
20
15
10
5
0
5
6
20
20
6
1
Strongly agree
Agree
Neutral
Disagree
Strongly disagree
No response