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Student Perceptions on the Effectiveness of Incorporating Numerical Computations into an Engineering Linear Algebra Course

Dr. Meiqin Li, University of Virginia

Dr. Li obtained her Ph.D. in Applied Mathematics from Texas A&M University-College Station in 2017. Dr. Li holds a strong interest in STEM education. For example, she is interest in integrating technologies into classrooms to bolster student success, creating an inclusive and diverse learning environment, and fostering student confidence by redeveloping course curricula and assessment methods, etc. Beyond this, her research intertwines numerical computation, optimization, nonlinear analysis, and data science.

Dr. Jessica Taggart, University of Virginia

Jessica Taggart is an Assistant Director in the Center for Teaching Excellence at the University of Virginia, where she leads Scholarship of Teaching and Learning (SoTL) programming and supports curriculum (re)design efforts.

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Introduction

Linear algebra techniques have gained increasing significance and utility across diverse fields, including engineering, data science, computer science, and statistics. Despite its computational nature, the subject's topics often delve into abstract and conceptual realms. Recognizing the acknowledged challenges and obstacles associated with learning linear algebra [1], [2], [3], a plethora of teaching practices, strategies, and resources have been explored to address the difficulties encountered by students in grasping these abstract concepts. In this paper, we explore the potential, from the student perspective, of one possible strategy: incorporating the use of MATLAB into an engineering Linear Algebra course.

Many efforts have aimed to make the study of linear algebra more accessible, engaging, and conducive to effective learning outcomes. Researchers have also emphasized the significant advantages of incorporating technology into the teaching of linear algebra, such as through application projects developed using Jupyter Notebooks with Python [4]. Doing so can substantially improve students' comprehension and practical application of the subject [5]. Integrating various technologies into the linear algebra curriculum can augment the learning experience [6], [7], [8], [9], [10], [11], [12]. By leveraging computers for intricate problem-solving, students can attain a deeper understanding of linear algebra and effectively tackle more complex tasks. This approach fosters interactive learning, encouraging active participation and engagement. Consequently, the incorporation of both hardware and software in linear algebra instruction has become indispensable, as it not only improves problem-solving skills but also prepares students for real-world applications. This pedagogical approach not only enhances the overall effectiveness of learning, but also aligns with the evolving demands of the contemporary job market.

While literature exists on integrating technology into linear algebra classes[4], [6], [7], [8], [9], [10], [11], [12], most studies have focused on isolated aspects, used technology partially, or lacked a specific focus on engineering students. Additionally, thorough investigations into students' perceptions have been lacking. The present study begins to address these gaps in the literature through an exploration of students’ perceptions on the numerical components incorporated into a redesigned linear algebra course for undergraduate engineering students.

At the University of Virginia, the Center of Applied Mathematics (APMA) within the School of Engineering & Applied Science (SEAS) offers a variety of math courses to both graduate and undergraduate students. Almost all students in APMA courses are pursuing engineering degrees. To prepare students for the job market after graduation, APMA courses often include practical projects to better meet their needs.

Take, for instance, APMA 3080 - Linear Algebra. In the past, this course followed a traditional theoretical approach without much connection to real-world applications. However, it has undergone a transformation to become more relevant and dynamic. The course was tailor-made for engineering students and now includes four practical components, incorporating MATLAB as the primary tool for numerical computations. These numerical components have been integrated extensively throughout the course, from daily lectures to homework and projects. We will refer to this course as Redesigned Linear Algebra (RDLA) hereafter; for more information about the course, see [13] (RDLA here is CALM in [13]).

Class Format
RDLA incorporates four essential numerical components:

1. Students engage in active learning by solving worksheet problems in MATLAB ("solve WS"), addressing complex issues collaboratively during in-class group sessions.
2. Students write MATLAB codes for fundamental linear algebra concepts (“code core concepts”), offering a distinct perspective and deepening understanding.
3. Students complete five substantial application projects (“solve application”), illustrating the practical and intriguing nature of linear algebra.
4. Instructors use MATLAB live scripts to visualize abstract concepts (“visualization”), creating an interactive learning environment.

Alongside these components, students have weekly homework through Webwork, with the option to use MATLAB for answer validation. This holistic approach ensures a comprehensive and hands-on learning experience, specifically designed for engineering students.

Solve WS. In the "solve WS" component, students engaged with a worksheet distributed at the start of each class. The worksheet comprised several problems that required the utilization of MATLAB, while students practiced the remaining questions manually. MATLAB problems within the worksheet encompassed computations, visualizations, and the application of prewritten functions provided by instructors or built-in functions to solve linear algebra problems. In the 50-minute class setting, students typically devoted 15-25 minutes to working on these worksheet problems. During group work, one or two trained undergraduate teaching assistants (UTAs) [14], [15], along with the instructor, circulated the classroom to offer support and address any questions raised by students. Subsequently, all completed worksheets were submitted as PDF files via Gradescope and assessed by UTAs. For MATLAB problems, students were required to attach their codes alongside answers or graphs in the PDF submissions.

Code Core Concepts. In the “code core concepts” component, students create functions in MATLAB based on core linear algebra concepts, for example, one task for students is writing a function such that it can be used to determine the number of solutions for any given linear system. This component is implemented in MATLAB Grader [16], an automated grading tool that offers instant feedback to students, helping them identify and rectify any mistakes in their code promptly.

Solve Application. In the “solve application” component, students see practical examples showcasing the real-world applications of linear algebra. This not only deepens their comprehension but also fuel their motivation to master the subject. Students complete five application projects: traffic problems using linear systems, encrypting and decoding problems based on linear transformations and matrix operations, steady-heat flow problems by applying matrix factorizations like LU factorization, Markov Chain for solving page rank problems based on eigenvector/eigenvalues, and image compression problems through SVD decompositions. This segment is implemented in MATLAB Grader as well, facilitating automated grading for efficiency and convenience.

Visualization. In the "visualization" component, instructors apply the prepared MATLAB live scripts before class to visually represent abstract concepts during class, enabling students to tangibly comprehend theoretical ideas by “seeing what they are”. For instance, a prewritten MATLAB live script is utilized to visually illustrate concepts like "linear combination," "span," and "linear transformation." Visualizing these theoretical aspects not only captivates students' attention more effectively but also enhances learning efficiency [17]. This approach leverages the brain's natural inclination to grasp concepts more swiftly when connected with visual representations.

Considering that some students may be new to MATLAB and the focus of this course is linear algebra instead of coding, we offer four types of resources to support their mastery of the four numerical components: (i) Instructors provide prewritten packages for direct use and established templates in MATLAB Grader for easy student guidance; (ii) Optional, ungraded MATLAB preparation sheets were provided to familiarize students with relevant commands used in the course; (iii) Most tasks were set up in MATLAB Grader, ensuring students received immediate feedback to aid their progress; and (iv) Introductory video tutorials [18] are available to help students set up MATLAB.

Research Questions

When the paper [13] gauged students’ perceptions of RDLA, primarily focusing on the application projects, the predominant response from students was positive. Students emphasized that the inclusion of projects afforded them opportunities to witness the practical application of linear algebra in real-life scenarios.

The current investigation delves deeper into students' perceptions regarding the use of MATLAB as a tool and the effectiveness of integrating all four numerical computational components into a linear algebra course. This study aims to center student perspectives rather than solely relying on expert viewpoints. More specifically, our key research questions (RQ) are:

1. How, if at all, do students' perceptions of MATLAB as a program evolve over the course of the semester? Do these perceptions vary based on factors such as academic major, gender, or race/ethnicity?
2. How, if at all, do students' perceptions of MATLAB as a tool for learning linear algebra evolve over the course of the semester? Do these perceptions vary based on academic major, gender, or race/ethnicity?
3. How do students perceive different numerical computational components in this course, along with the accompanying support resources? Is there any relationship between students' perceptions across different components?

Study Design

Context

APMA 3080-Linear Algebra caters specifically to engineering students, setting it apart from the general linear algebra classes typically offered by the Department of Mathematics for math majors or other disciplines within the School of Arts & Sciences. Notably, programming and application projects play pivotal roles in the APMA 3080 curriculum. The student body for this course predominantly consists of individuals from the School of Engineering, occasionally welcoming a few participants from other academic areas.

Each semester, approximately 200-300 students enroll in four to six sections of APMA 3080, taught by different instructors. The course coordinator, who is the first author of this paper, ensures seamless coordination among all sections, with a shared set of teaching materials and assessments, including lecture notes, homework, worksheets, application projects, and exams.

Participants

This IRB-approved study took place during the fall of 2023, involving the recruitment of undergraduate students enrolled in APMA 3080 - Linear Algebra. All students who took APMA 3080 in the fall of 2023 were invited to complete two surveys: a pre-survey at the beginning of the semester and a post-survey at the end. The pre-survey included demographic questions related to gender, race, major, and previous experience with programming.

A total of 211 students were enrolled in APMA 3080 during the fall of 2023, spread across five different sections taught by three instructors. Of these, 137 students agreed to participate in the research study, and approximately 128 (93%) submitted either the pre-survey, post-survey, or both, with some completing the surveys partially. Consequently, the number of participants in this study may vary based on the research questions.

Among the survey participants, representing 11 different majors, 54% were from Computer Science-BS (CS(BS)), 16% from Systems Engineering (Sys Eg), and a very small numbers from Biomedical Engineering (BME), Chemical Engineering (Chem Eg), Civil Engineering (Civil Eg), Computer Engineering (Comp Eg), Computer Science-BA (CS(BA)), Electrical Engineering (EE), Materials Science Engineering (MSE), and Mechanical Engineering (Mech Eg). The participant breakdown included 71% male, 27% female, and 2% identifying as 'else’. Majority participants were non-URM (underrepresented minorities). URM participants included those who identified as Black or Hispanic, or any combination thereof, while all other students were categorized as non-URM. All participants had some prior programming experience in languages such as Python, Java, C++, or MATLAB before joining the class.

Data Resources and Collection

In this study, two surveys were utilized as the primary resources: a pre-survey administered at the beginning of the semester and a post-survey administered at the semester's conclusion. Both surveys had three identical sets of questions, comprising 3, 6, and 4 Likert scale sub-questions per set, respectively. These questions aimed to assess students' perceptions of MATLAB as an independent programming language and how these perceptions evolved over the semester, addressing the first research question. Both surveys also featured an identical set of questions, comprising 4 Likert scale sub-questions, to explore students' perceptions of MATLAB as a tool for learning Linear Algebra and how these perceptions evolved throughout the semester, addressing the second research question.

To investigate the third research question, the post-survey included additional questions not presented in the pre-survey. These new inquiries sought students' opinions on the usefulness of various numerical components and resources, supplemented by open-ended questions to capture additional insights. The questions were carefully crafted by APMA 3080 instructors and underwent scrutiny by a science education and survey design expert.

Surveys were distributed to students via Qualtrics, ensuring meticulous attention to privacy and confidentiality. Measures were implemented to prevent the identification of individual students when reporting data individually or by student demographics, employing techniques such as pseudonyms and withholding identifiable information. The management of collected data adheres strictly to the university's policy.

Analysis Tools

The collected data from participants were analyzed using various statistical tools, including barplots, linear regression, correlation tests, independent t-tests, paired t-tests, one-way ANOVA, and the Kruskal-Wallis test. Prior to applying these tests, we assessed the assumptions to ensure the most robust conclusions possible.

For instance, we examined factors such as normality and sample size before employing tests like one-way ANOVA or paired t-tests. In cases where the data did not approximate normality or the sample size was insufficient ($n<30$), non-parametric tests were employed, for example, the Kruskal-Wallis test served as an alternative to ANOVA, and the Wilcoxon test acted as an alternative to the paired t-test. The results are presented collectively for quantitative and categorized qualitative data, while qualitative data is reported individually.

Data Analysis and Results

Students' Perceptions of MATLAB as a Program (RQ1)

Utilizing MATLAB as a tool in engineering education has been studied [19], [20], and some researchers adapted it in linear algebra class as well [10], [11]. However, little literature investigated students’ perceptions of MATLAB as program. In this section, based on the redesigned course RDLA [13], we mainly focus on the first research question: How, if at all, do students' perceptions of MATLAB as an independent program evolve over the course of the semester? Do these perceptions vary based on factors such as academic major, gender, race/ethnicity?

During the first class of the semester, we administered a pre-survey comprising three sets of Likert scale questions. The same three sets of questions were included in the post-survey delivered during the last class of the semester. After collecting the data, we focused on participants who completed identical questions in both the pre-survey and post-survey.

Subsequently, we conducted descriptive analyses and employed the paired t-test for sample sizes exceeding 30. The results can be found in Table 1, Table 2, and Table 3.

Set 1: Please indicate the ease with which you can do the following in MATLAB:

Table 1: Student's perceptions of their ability to use MATLAB (n=115)

	Presurvey Mean	Postsurvey Mean	*t-value*	*p-value*
Remember instructions and coding styles.	4.10	4.53**	-2.4957	.014
Know what code to use for the output I want.	3.97	4.37 **	-2.5894	.01087
Debug a program when the result is not what I want.	3.88	4.25 **	-2.2205	.02836

** *significant, $p < .05$.*
1-with a lot of difficulty, 2-with difficulty, 3- with some difficulty, 4- neutral, 5-somewhat easily, 6-easily, 7-very easily

In this specific set of questions, 115 students provided responses to all sub-questions in both the pre-survey and post-survey. Descriptive analysis, as presented in Table 1, indicated an increase in the average Likert scale. Simultaneously, the paired t-tests, reflected by small p-values, revealed significant improvements in students' perceptions of MATLAB. Specifically, by the end of the course after the incorporation of MATLAB, students found it significantly easier to remember instructions and coding styles, select the correct codes for desired outputs, and debug codes.

Set 2: Please indicate how overwhelmed you feel about the following:

Table 2: Student's perception of feeling overwhelmed using MATLAB (n=115)

	Presurvey Mean	Postsurvey Mean	*t-value*	*p-value*
Programming in general.	1.74	1.76	-0.25715	.79
Programming in MATLAB.	2.37	2.20*	1.743	.08
Solving a mathematical problem in general.	1.56	1.57	-0.22552	.82
Solving a mathematical problem using MATLAB.	2.32	2.04 ***	3.0228	.003
Designing and creating a tutorial.	2.57	2.37 **	1.9576	.05
Designing and creating a tutorial in MATLAB	3.13	2.82 ***	3.0653	.002

*** *significant, $p < .005$,* ** *significant, $p < .05$,* * *significant, $p < .1$*
1-Not at all overwhelmed, 2- Slightly overwhelmed, 3- Moderately overwhelmed, 4- Very overwhelmed, 5- Extremely overwhelmed

A total of 115 students completed all sub-questions for this set in both the pre-survey and post-survey. Analysis from the paired t-tests revealed that, by the semester's end, students experienced a significant increase in their comfort level with programming in MATLAB, solving mathematical problems using MATLAB, and creating tutorials in MATLAB.

However, there was no notable difference in students' comfort with general programming. This could be attributed to the fact that participants had prior programming experience, having taken programming courses before joining the class. Their existing proficiency in languages like Python, Java, or C++ was evident, as indicated by the low average Likert scale in both the pre-survey and post-survey (1.74 vs. 1.76). The introduction of MATLAB to their programming repertoire did not significantly alter their comfort levels. Similarly, there was no significant difference in students' comfort with solving mathematical problems in general, as reflected in the low average Likert scale in both the pre-survey and post-survey (1.56 vs. 1.57). Participants expressed a consistent comfort level in solving mathematical problems, whether with or without the use of MATLAB.

Set 3: Please indicate your agreement with the following statements about the usefulness of MATLAB as a general language:

Table 3: Student's perceptions on the usefulness of MATLAB as a general language

	n	Presurvey Mean	Postsurvey Mean	*t-value*	*p-value*	Postsurvey % somewhat agree+ agree+ strongly agree	Postsurvey % somewhat disagree+ disagree+ strongly disagree
MATLAB will be useful in my future coursework	112	4.78	4.37 ***	2.6217	.0099	59%	28%
I can see myself doing a project in the future that utilizes MATLAB	112	4.60	4.26 **	2.0853	.0393	55%	32%
My future career will likely include work with MATLAB	113	3.85	3.58 *	1.7186	.0885	32%	49%
The experience in MATLAB will help me master a different program (e.g., Python, R, Java, etc.)	113	4.49	4.12 **	2.3801	.019	53%	32%

*** *significant,* $p < .01$, ** *significant,* $p < .05$, * *significant,* $p < .1$

1-strongly disagree, 2-disagree, 3- somewhat disagree, 4- neutral, 5-somewhat agree, 6-agree, 7-strongly agree

Different number of students completed sub-questions for this set in both pre-survey and post-survey, and the specific counts are detailed in Table 3. The small p-values from paired t-tests indicated that, by the end of the semester, students perceived MATLAB as significantly less useful as a general programming language. While the potential reasons for this perception are intricate and not explored here, they will be discussed later in the dedicated discussion section.

Despite the decline in averages, approximately half of the students still believed that MATLAB would be beneficial in their coursework, project management, and aiding the learning of other programming languages. The average Likert scales for these perceptions were 4.37, 4.26, and 4.12, respectively, in the post-survey. Importantly, this proportion significantly exceeded the number of those who held a contrary view (somewhat disagree + disagree + strongly disagree), as depicted in Table3.

For this set of questions, we conducted a more detailed examination of students' perceptions regarding the usefulness of MATLAB for their future careers, specifically addressing the third sub-question. To analyze these responses, we disaggregated the post-survey data by major, gender, and race/ethnicity. Notably, we did not disaggregate the responses based on prior programming experience, as all participants had coding experience before enrolling in this course. We employed one-way ANOVA when assumptions of normality or a large sample size were met and utilized the Kruskal-Wallis Test when such assumptions were not fulfilled.

When we looked at students from different majors, the Kruskal-Wallis test showed a p-value of .03, indicating a significant difference among majors. To dig deeper and find out which majors differed, we used Dunn's test along with a barplot (Figure 1). The results highlighted those students majoring in Mechanical Engineering (Mech Eg) and Computer Science BA (CS(BA)), Mechanical Engineering and Computer Science BS (CS(BS)), Biomedical Engineering (BME) and CS(BA), Mechanical Engineering and Civil Engineering (Civil Eg), as well as Biomedical Engineering (BME) and Civil Engineering, had significantly different opinions about how useful MATLAB is for their future careers (Figure 1).

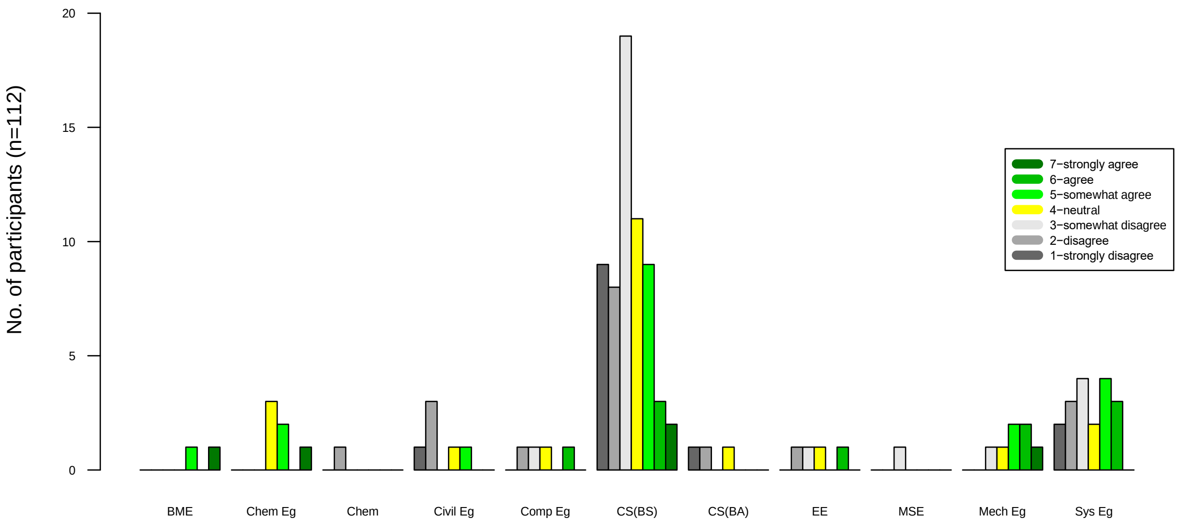

Figure 1: Students' perception on the usefulness of MATLAB for their future careers based on majors.
"My future career will likely include work with MATLAB "
1-strongly disagree, 2-disagree, 3- somewhat disagree, 4- neutral, 5-somewhat agree, 6-agree, 7-strongly agree

While the statistical tests supported the findings mentioned earlier, it's important to exercise caution due to variations in sample sizes across majors. A significant majority of participants (61 out of 112, or 55%) were from Computer Science, with only a handful from other majors. To address this, we grouped CS-BA and CS-BS together as "CS" and combined all other majors into a group labeled "others." This reclassification allowed us to reanalyze the data with a more balanced representation.

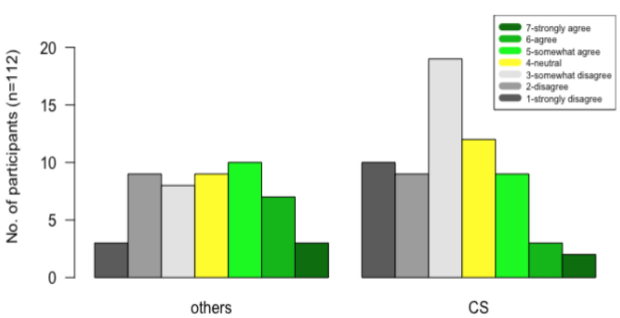

Figure 2

Conducting an independent t-test between "CS" and "others" yielded a non-significant difference, with t = -0.88749 and p = .3982. The number of students disagreeing (somewhat disagree + disagree + strongly disagree) surpassed those agreeing (somewhat agree + agree + strongly agree) (Figure 2), which indicated the relatively low averages of 3.85 and 3.58 for the sub-question "My future career will likely include work with MATLAB" in this set.

We conducted further analysis using either one-way ANOVA, Kruskal-Wallis test, or independent t-test to explore the data based on gender and race. However, no significant differences were observed in relation to these factors (gender: p=.35; race: p=.599).

When looking at the responses from the post-survey for the question "Please explain why if you don't believe MATLAB will be useful to you in the future", some typical responses were:

"already used other programming languages and all things can be done in MATLAB could be done by the other programming as well"

"won't pursue a career that require MATLAB"

"the syntax in MATLAB is not as good as others"

"other programming language such as Python and C++ are used in industry instead of MATLAB."

While it might not be surprising that more students don't perceive MATLAB as useful in their careers than those do, undertaking this analysis provides valuable data evidence to support this conclusion.

Students' Perceptions of MATLAB as a Tool for Learning Linear Algebra (RQ2)

In this section, we mainly focus on the second research question: How, if at all, do students' perceptions of MATLAB as a tool for learning Linear Algebra evolve over the course of the semester? Do these perceptions vary based on factors such as academic major, gender, race/ethnicity?

We incorporated the same set of four 7-point Likert scale questions in both the pre-survey and post-survey (Table 4), subsequently comparing responses through paired t-tests. On one hand, the consistently high averages in the pre-survey and the relatively high averages in the post-survey suggested a generally positive outlook among students regarding the usefulness of MATLAB as a tool for learning linear algebra (Table 4). By the semester's end, majority students still concurred (somewhat agree + agree + strongly agree) that MATLAB was helpful in the course (Figure 3). On the other hand, the small p-values from comparing the pre-survey and post-survey via paired t-tests indicated that students were significantly more optimistic about the utility of MATLAB in managing linear algebra projects, mastering linear algebra concepts, and overall effectiveness before engaging in real tasks with MATLAB.

Table 4: Student's perceptions on the usefulness of MATLAB as a tool to learn linear algebra.

	Presurvey Mean	Postsurvey Mean	*t-value*	*p-value*	Postsurvey % somewhat agree+ agree+ strongly agree
MATLAB will help me manage linear algebra projects	5.26	4.98**	2.007	.04717	71%
MATLAB will help me answer linear algebra questions that cannot be solved by hand	5.91	5.95	-0.247	.8053	91%
MATLAB will help me master linear algebra concepts	5.23	4.61***	4.022	.0001	60%
MATLAB will make learning linear algebra more efficient overall	5.29	4.96*	1.783	.07732	66%

*** *significant, $p < .001$,* ** *significant, $p < .05$,* * *significant, $p < .1$*

"Please indicate your agreement with the statements about the usefulness of MATLAB in a linear algebra class:"
1-strongly disagree, 2-disagree, 3- somewhat disagree, 4- neutral, 5-somewhat agree, 6-agree, 7-strongly agree

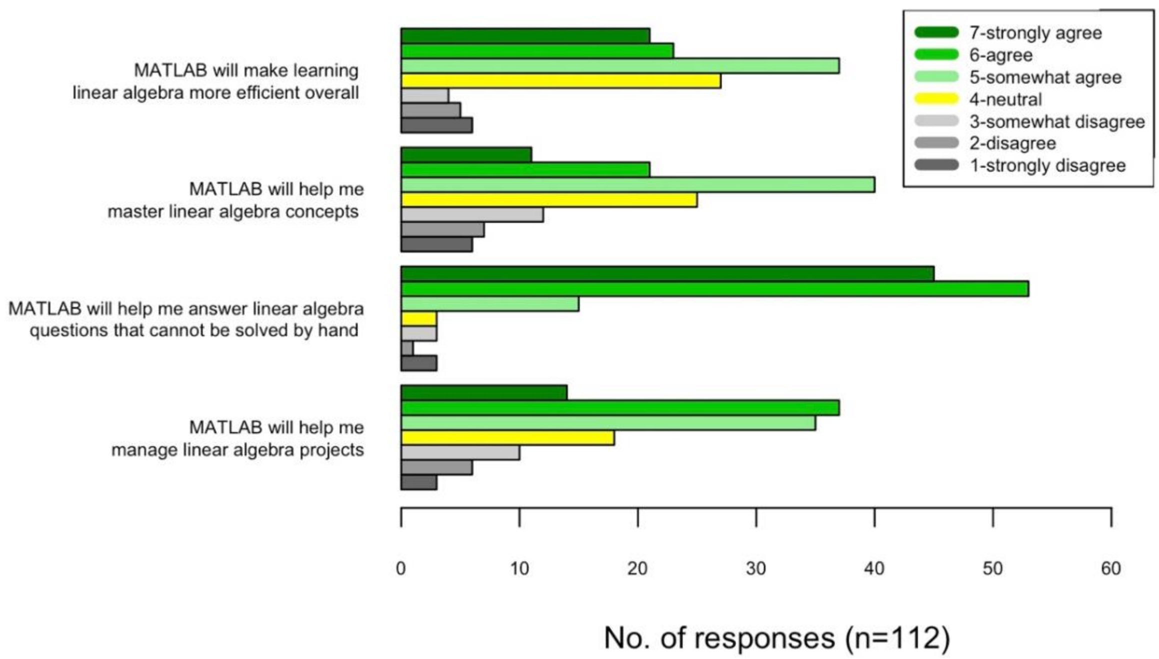

Figure 3: Post survey data of students' perceptions on the usefulness of MATLAB as a tool to learn Linear Algebra

In assessing the overall efficacy of MATLAB in the course, namely, the last sub-question in Table 4, we disaggregated the data by gender, major, and race/ethnicity and found no no significant differences among different groups based on these factors (gender: $p=.1461$, major: $p=.491$, race: $p=.323$).

Students' Perceptions on Different Numerical Components (RQ3-P1)

In this section, we focus on the third research question: How do students perceive different numerical computational components incorporated into an engineering linear algebra course using MATLAB, along with the accompanying support resources? Is there any relationship in students' perceptions across different components? This section is subdivided into three parts: (1) quantitative assessments of perceptions on four numerical components, (2) correlations of perceptions among different components, and (3) qualitative perspectives on them.

Quantitative assessments of perceptions of four numerical components

As previously mentioned, the present version of APMA 3080-Linear Algebra (RDLA) integrates four distinct numerical components utilizing MATLAB [13]. These components include solving worksheet problems in MATLAB ("solve WS"), writing MATLAB codes for core linear algebra concepts ("code core concepts"), solving application projects in MATLAB ("solve application"), and visualizing abstract concepts in MATLAB during the professor's lectures ("visualization").

In the post survey, students were asked to rank these four numerical components according to their "preference" (how much students liked them), their "capability" (how capable students managed them), and their perceived "helpfulness" (how helpful these components were to learn linear algebra), ranging from most (1) to least (4). The number of students varied for each question, and the descriptive analysis (Table 5) unveiled that solving worksheet problems in MATLAB was the component students favored the most, found most manageable, and considered most helpful for their learning of linear algebra, with averages of 1.84, 1.37, and 2.09, respectively. Subsequent statistical tests were applied to draw statistically convincing conclusions.

Table 5: Student's average ranks of different numerical components on "preference", "capability" and "helpfulness"

	"preference" (n=120)	"capability" (n=119)	"helpfulness" (n=116)
"solve WS"	1.84	1.37	2.09
"code core concepts"	2.55	2.72	2.45
"solve application"	2.8	2.96	3.01
"visualization"	2.81	2.95	2.45

1-most, 4-least

The results of the one-way ANOVA tests indicated that there was a significant difference in "preference", "capability" and "helpfulness" for the four components, as evidenced by the small p-values (p=0s) (Table 5). The Tukey HSD post hoc tests further revealed that students significantly preferred and were more capable in the "solve WS" component compared to others, with no significant differences for the remaining three components. While in terms of "helpfulness", the "solve WS" component was perceived as significantly more helpful than the others, and "code core concepts" and "visualization" were also perceived as significantly more helpful than "solve application" (Table 6).

Table 6: Tukey HSD for students' perceptions of the "helpfulness" of different components

Mean Difference of Ranks	diff	p adj
visualization – solve WS	.36*	.06
code core concepts- solve WS	.35*	.06
solve application - solve WS	.91***	0
code core concepts -visualization	0	1
solve application - visualization	.56***	.0005
solve application - code core concepts	.56***	.0005

********* *significant, $p < .001$,* ******* *significant, $p < .1$*

"Please rank these components in order of how helpful they were to your learning of linear algebra (from most to least):"
1-most, 4- least

It is noteworthy that in the paper [13], prominent students expressed that solving application projects helped them apply linear algebra skills in real-world scenarios, and they were pleased to have this component included in the course. However, the statistical analysis conducted in this study indicated that the solving application projects component was perceived as least helpful in learning linear algebra compared to the other components. Such findings present an intriguing topic for discussion, which will be further explored in the discussion section.

Correlations of perceptions among different components

Correlation tests were examined for each component to identify any relationships between students' rankings of "preference", "capability", and "helpfulness". A total of 113 students provided rankings for all components in terms of "preference", "capability", and "helpfulness" (Table 7-10).

Table 7: Correlation table for the component of "solve WS" (n=113)

	"preference"	"capability"	"helpfulness"
"preference"	1.00		
"capability"	0.33 (.0004) ***	1.00	
"helpfulness"	0.39 (.000015) ***	.08 (.41)	1.00

**** significant, p < .001, * significant, p < .1,*
Correlation coefficients with their p values inside the parenthesis

Table 8: Correlation table for the component of "code core concepts" (n=113)

	"preference"	"capability"	"helpfulness"
"preference"	1.00		
"capability"	.28 (.0023) **	1.00	
"helpfulness"	.40 (.000013) ***	.22 (0.018) *	1.00

**** significant, p < .001; ** significant, p < .005; * significant, p < .05*
Correlation coefficients with their p values inside the parenthesis

Table 9: Correlation table for the component of "solve application" (n=113)

	"preference"	"capability"	"helpfulness"
"preference"	1.00 (		
"capability"	.58 (0) ***	1.00	
"helpfulness"	.28 (0.0031) **	.33 (0.0003) ***	1.00

**** significant, p < .001; ** significant, p < .005*
Correlation coefficients with their p values inside the parenthesis

Table 10: Correlation table for the component of "visualization" (n=113)

	"preference"	"capability"	"helpfulness"
"preference"	1.00		
"capability"	.57 (0) ***	1.00	
"helpfulness"	.53(0) ***	.47 (0) ***	1.00

**** significant, p < .001*
Correlation coefficients with their p values inside the parenthesis

Correlation tests for the numerical component "solve WS" showed significant positive relationships between "preference" and "capability", as well as between "preference" and "helpfulness". However, no significant relationship was found between "capability" and "helpfulness" (Table 7). Notably, the correlation coefficient between "helpfulness" and "capability" is nearly zero, which warrants further discussion later.

For all the other three numerical components, "code core concepts", "solve application", and "visualization"**,** correlation tests revealed significant positive relations at different significant levels between each pair of "preference", "capability" and "helpfulness". One possible interpretation can be students tended to perceive the component to be more helpful to learn linear algebra if they are more capable to manage it, and like it more consequently.

For "code core concepts", the correlation coefficient (.28) between "capability" and "preference" was smaller than the correlation coefficient (.4) between "helpfulness" and "preference." This suggests that the perceived helpfulness of these tasks in learning linear algebra more closely relates to students' preference for them compared to their own capability in managing such tasks.

On the other hand, for the "solve application" tasks, the correlation coefficient (.58) between "capability" and "preference" was larger than the correlation coefficient (.28) between "helpfulness" and "preference." This implies that how capable students feel in managing these tasks there is more closely related to their preference for them compared to the perceived helpfulness of these tasks in learning linear algebra.

Qualitative perceptions of the four numerical components

When students were asked the easiest part of the MATLAB components, the common responses included: solving worksheet problems in MATLAB, using built in functions (rref, null, inv etc.), using the provided code/functions provided by professors, applying simple syntax, checking work, speeding up computation, solving linear equations, performing linear algebra calculation, applying simple function to solve problems, and solving problems that couldn't solved by hands. Some others reflected that coding the core linear algebra concepts is the easiest part, since they deepened the understanding of the abstract concepts, while only one reflected that the project codes were fairly easy to understand and complete. Below are some common response examples:

"Solving the worksheet problems with given code is the easiest part. It is also easy to use MATLAB to verify answers."

"Solving the worksheet problems and using MATLAB to help visualize concepts better".

"The easiest part has been analyzing given code and then tweaking it to align to different problems. This helps in a bottom-up processing to better understand and visualize concepts."

"The easiest part was using MATLAB to check my work process when studying or doing work."

"Solving simpler problems at a bigger scale using MATLAB was the easiest part."

"Definitely applying MATLAB to the worksheets, as they helped me learn the most and were usually given in a couple of steps, so it was not difficult to comprehend what the code was doing."

"The easiest part of the MATLAB components is how versatile the program is and has a lot of good documentation for the user."

"The easiest part is the core concept questions and help to reinforce the concepts well."

"Because MATLAB is designed around matrix operations, simple calculations using MATLAB are easy to access and understand. For example, matrix multiplication, indexing, and functions are easy to understand with MATLAB."

When students were asked about the most challenging aspect of the MATLAB components, the predominant response was completing the projects. Various students expressed different difficulties, including debugging the codes for the projects, learning the syntax and applying it in the context of the projects, filling in the blank project codes, finding it conceptually challenging to understand the projects, writing scripts, dealing with slightly confusing instructions, and identifying the necessary functions for the projects. Additionally, a few students mentioned that visualizing the concepts posed the greatest challenge. Some common response examples:

"Writing code online for the MATLAB projects. Much of the time I knew the math that guided the question at hand but was not able to implement the coding."

"Completing lengthy coding projects for a deeper understanding was the hardest part of the MATLAB components."

"Using MATLAB for more abstract/complex applications. Having to first understand the concept and then having to figure out how to code it in MATLAB was difficult."

"The projects were a little difficult sometimes since I didn't know what functions to use all the time."

"The hardest part of the MATLAB component for me was visualizing abstract concepts."

"MATLAB's syntax is very, very different from most languages, making it more difficult to understand or remember."

"The syntax was quite challenging to learn at first

Students' Perceptions of Different Support Resources (RQ3-P2)

We offered four types of resources to assist students in mastering MATLAB tasks: (I) video tutorials from www.mathwork.org ("videos"),
(ii) MATLAB preparation sheets ("prep sheets"), (iii) MATLAB Grader ("Autograder"), and (iv) prewritten packages and templates in MATLAB Grader for direct use ("packages & templates"). In the post-survey, we asked students to rate the helpfulness of these provided resources on a scale of 1 (least helpful) to 5 (most helpful), with a rating of 0 indicating that they did not use the resource at all. Only a small number of students reported that the provided resources were not helpful at all, with the highest occurrence being for the "videos" resource, followed by the "prep sheets" resource (Figure 4, Table 11).

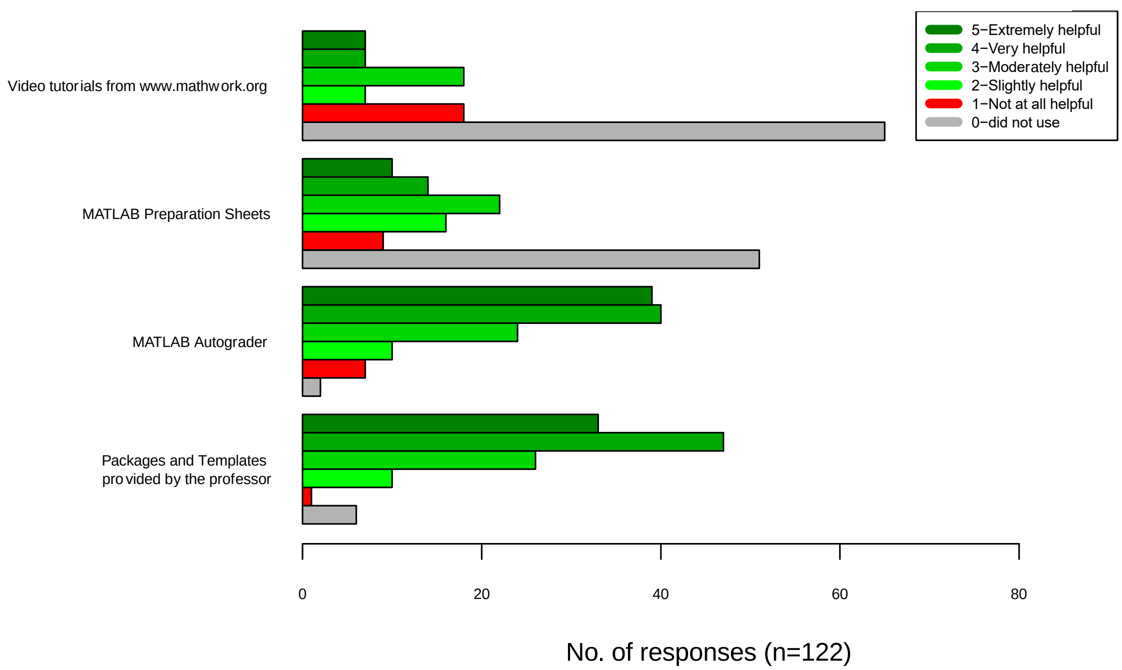

Figure 4: Students' perceptions on different types of support resources.

Table 11. Students' perceptions of different support resources (removed those who did not use the resources)

	Packages & Templates (n=117)	Autograder (n=120)	Prep Sheets (n=71)	Videos (n=57)
Mean	3.86	3.78	3	2.61

1-not at all helpful, 2-slightly helpful, 3- moderately helpful, 4- very helpful, 5-extremely helpful

The small p-value (p=0) obtained from a one-way ANOVA test, along with subsequent Tukey HSD post hoc test, indicated significant differences in the perceived helpfulness of different resources. Specifically, the packages & templates provided by instructors and the MATLAB Autograder were perceived to be significantly more helpful than MATLAB preparation sheets. And MATLAB preparation sheets were perceived to be significantly more helpful than the tutorial videos (Table 12).

Table 12: Tukey HSD of students' perceptions for "helpfulness" of different provided resources

Mean Difference	diff	p adj
prep Sheets -videos	.386	.24
Autograder-videos	1.169 ***	p < .00001
packages & templates-videos	1.249 ***	p < .00001
Autograder-prep sheets	.783 ***	p < .00005
packages & templates -prep sheets	.863 ***	p < .00001
packages & templates template-Autograder	.080	.95

*** *significant, p < .001*
"Please indicate how helpful of those resources were to your learning of linear algebra?"
1-not at all helpful, 2-slightly helpful, 3-moderately helpful, 4-very helpful, 5-extremely helpful

Students provided various reflections on the four resources. Regarding the packages & templates, they commonly mentioned that these resources guided them and provided functioning code examples that helped with understanding and problem-solving. The packages & templates were seen as valuable tools for solidifying concepts and providing a foundation to start from.

In terms of the Autograder, students highlighted its usefulness in providing feedback and allowing them to test their code effectively. They appreciated the ability to try different ideas without fear of making mistakes and found the Autograder's ability to identify errors and provide immediate feedback to be beneficial in confirming their understanding.

In relation to MATLAB preparation sheets, students mentioned that these resources helped them make the transition to using MATLAB for the first time. The sheets were seen as a helpful bridge between classroom learning and practical application.

Regarding the videos, students described them as a good introduction for those unfamiliar with MATLAB. While some found the tutorials useful in explaining various functions, others felt that they were too basic and did not provide the specific code necessary for the projects.

The students' reflections align with the intended goals of each resource. The video tutorials were primarily aimed at helping students who had little to no experience with MATLAB become comfortable with the software's working environment. Consequently, it is understandable that some students found these tutorials less helpful when it came to directly solving linear algebra problems, as that wasn't their main purpose.

Conclusion and Discussion

In this study, we conducted thorough investigations into students' perceptions of the incorporation of MATLAB into a linear algebra course for undergraduate engineers.

Regarding students' perceptions of MATLAB as an independent programming language and as a tool for learning linear algebra, our study revealed that students perceived a significant improvement in their abilities to work with MATLAB after its incorporation into the course. They also reported feeling significantly less overwhelmed when faced with tasks in MATLAB throughout the semester. Additionally, the majority of students believed that MATLAB was useful for their learning of linear algebra, enabling them to check their work and solve problems on a larger scale. However, they did not find it significantly helpful for their future careers.

Students' perceptions of MATLAB as a general programming language and as a tool for learning linear algebra became less optimistic by the end of the semester. The reasons for this shift in perception are likely complex. One possibility could be that participants had prior experience with other programming languages that they were more accustomed to and found more helpful when approaching tasks in MATLAB due to the familiarity with the structure and syntax. Another possible reason could be that some students had no prior experience with MATLAB, and despite having previous programming experience, they had to navigate and solve issues they encountered in MATLAB on their own, despite the availability of support resources. This lack of systematic teaching of MATLAB may have had a negative impact on students' perceptions.

Regarding students' perceptions of the four numerical components, students believed the "solve WS" component was the one they felt most capable in, found most helpful for learning linear algebra, and liked the most, but the "solve application" was the component students perceived to be least helpful for learning linear algebra.

Previously, students expressed that solving application projects in the course helped them apply linear algebra skills in real-world scenarios [13]. However, in this study, students perceived the solving application projects component as least helpful for learning linear algebra compared to other components. This may seem contradictory, but perhaps it can be attributed to shifted perspectives of "important" and "helpful" from students.

In the course, students studied additional linear algebra materials not covered by instructors and completed application projects outside of class using MATLAB Autograder. These projects aimed to broaden students' horizons by applying their linear algebra skills to real-life problems, rather than focusing solely on exam scores. Solving projects were much more challenging and time consuming than all the other components. Some students tend to perceive resources as helpful if they can positively impact their exam scores, and more helpful if they take less time while also bolstering exam scores. As a result, it was not necessarily surprising that some students felt that application projects were not as helpful for learning the subject.

These findings highlight a divergence between instructors' and students' perspectives on the helpfulness of certain course materials, such as the application component. Instructors may believe that any material students appreciate should be beneficial for learning the course. However, from the students' viewpoint, they may consider the material as "crucial" to include, but not necessarily helpful for learning if it is not assessed in exams. Thus, instructors should be cautious when selecting course material, considering not only students' perceptions of its helpfulness for better grades but also a combination of many aspects such as the contribution to the course's horizon, the difficulty level, students' needs and ability, etc. They may also need to help students to see the connections between course content and approaches to learning, as well as the value of the approaches.

This study revealed positive relationships between students' "preference," "capability," and the perceived "helpfulness" of different components in learning linear algebra. However, it is not possible given the current study design to determine the cause-effect relationship between these factors. One possible interpretation is that students are more likely to perceive a component as helpful for learning linear algebra if they have a higher level of capability in managing it and subsequently develop a preference for it. On the other hand, it is also possible that the relationship works in other ways. For example, students might prefer a particular component, which leads them to focus on developing their capabilities in it and subsequently perceive it as helpful. Therefore, this study emphasizes the interplay between students' preferences, capabilities, and perceived helpfulness in understanding their perceptions of different components in the learning of linear algebra. Further research is needed to explore the underlying factors and mechanisms driving these relationships.

Students found the support resource provided to be helpful, with packages & templates and Autograder significant more helpful than MATLAB preparation sheets and videos. Many students did not use MARLAB preparation sheets and videos at all.

Based on the findings of this study, instructors may consider implementing certain strategies to enhance students' learning experience. These strategies could include providing a dedicated 1-credit course specifically focused on teaching MATLAB, providing more explicit guidelines for projects, or exploring the use of Python as an alternative to MATLAB.

However, it is important to acknowledge that this study was conducted with students from different sections, and their perceptions were examined as one big group rather than by individual sections. It is worth considering that different instructors may have a significant influence on students' perceptions of course materials, which was not specifically examined in this study. Therefore, future research could delve into the role of individual instructors and their impact on students' perceptions to gain a more comprehensive understanding of the factors influencing student experiences.

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References

[1] M. Allen and A. Kelley, “Challenges and innovations in teaching linear algebra,” in *ASEE Annual Conference and Exposition, Conference Proceedings*, 2008. doi: 10.18260/1-2--4405.

[2] S. Britton and J. Henderson, “Linear algebra revisited: An attempt to understand students’ conceptual difficulties,” *Int J Math Educ Sci Technol*, vol. 40, no. 7, 2009, doi: 10.1080/00207390903206114.

[3] J. M. Day and D. Kalman, “Teaching Linear Algebra: Issues and Resources,” *The College Mathematics Journal*, vol. 32, no. 3, 2001, doi: 10.2307/2687464.

[4] M. Silva *et al.*, “Innovating and modernizing a Linear Algebra class through teaching computational skills,” in *ASEE Annual Conference and Exposition, Conference Proceedings*, 2022.

[5] D. Carlson, C. R. Johnson, D. C. Lay, and A. D. Porter, “The Linear Algebra Curriculum Study Group Recommendations for the First Course in Linear Algebra,” *The College Mathematics Journal*, vol. 24, no. 1, 1993, doi: 10.2307/2686430.

[6] J. Klasa, “A few pedagogical designs in linear algebra with Cabri and Maple,” *Linear Algebra Appl*, vol. 432, no. 8, 2010, doi: 10.1016/j.laa.2009.08.039.

[7] N. Lohgheswary, M. H. Abdul Halim, Z. M. Nopiah, A. A. Aziz, and E. Zakaria, “Developing new lab base teaching approach for linear Algebra subject in engineering mathematics courses,” *Journal of Mechanical Engineering*, vol. 5, no. Specialissue3, 2018.

[8] M. Pecuch-Herrero, “Strategies and computer projects for teaching linear algebra,” *Int J Math Educ Sci Technol*, vol. 31, no. 2, 2000, doi: 10.1080/002073900287237.

[9] S. Se, B. Ashwini, A. Chandran, and K. P. Soman, “Computational thinking leads to computational learning: Flipped class room experiments in linear algebra,” in *ICIIECS 2015 - 2015 IEEE International Conference on Innovations in Information, Embedded and Communication Systems*, 2015. doi: 10.1109/ICIIECS.2015.7193021.

[10] P. M. Shankar, “A Matlab workbook on the pedagogy of generalized eigenvectors,” *Computer Applications in Engineering Education*, vol. 25, no. 3, 2017, doi: 10.1002/cae.21808.

[11] P. M. Shankar, “Pedagogy of solutions to a set of linear equations using a Matlab workbook,” *Computer Applications in Engineering Education*, vol. 25, no. 3, 2017, doi: 10.1002/cae.21803.

[12] C. Tang, “Computer-aided Linear Algebra Course on Jupyter-Python Notebook for Engineering Undergraduates,” in *Journal of Physics: Conference Series*, 2021. doi: 10.1088/1742-6596/1815/1/012004.

[13] M. Li, “Developing Active Learning of Linear Algebra in Engineering by Incorporating MATLAB and Autograder,” in *ASEE Annual Conference and Exposition, Conference Proceedings*, 2023.

[14] G. Guadagni and H. Ma, “A sustainable model to structurally improve outcomes in Math courses for Engineering students,” in *ASEE Annual Conference and Exposition, Conference Proceedings*, 2022.

[15] G. Guadagni, H. Ma, and L. Wheeler, “The benefit of training undergraduate teaching assistants,” in *ASEE Annual Conference and Exposition, Conference Proceedings*, 2018. doi: 10.18260/1-2--31079.

[16] “MATLAB Grader.” Accessed: Apr. 20, 2024. [Online]. Available: https://grader.mathworks.com/

[17] H. Dogan, “Mental Schemes of: Linear Algebra Visual Constructs,” 2018. doi: 10.1007/978-3-319-66811-6_10.

[18] “MathWorks”, Accessed: Apr. 20, 2024. [Online]. Available: https://www.mathworks.com/

[19] N. Nevaranta, P. Jaatinen, K. Grasbeck, and O. Pyrhonen, “Interactive learning material for control engineering education using matlab live scripts,” in *IEEE International Conference on Industrial Informatics (INDIN)*, 2019. doi: 10.1109/INDIN41052.2019.8972282.

[20] J. L. Vicéns, B. Zamora, and D. Ojados, “Improvement of the reflective learning in engineering education using MATLAB for problems solving,” *Computer Applications in Engineering Education*, vol. 24, no. 5, 2016, doi: 10.1002/cae.21748.