# Storage and Interaction Diagrams: Extending the Diagrammatic Framework of Kinetic and Free-Body Diagrams to other Conservation and Accounting Principles

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## Abstract

After defining a system for analysis, a rigorous process is taught to students in their Statics and Dynamics courses on how to draw proper kinetic, free-body, and impulse-momentum diagrams. While numerous techniques and mnemonics have been mentioned in literature, any experienced instructor can tell a correct free-body diagram apart from an incorrect one. Unfortunately, this is not the case when considering scalar properties such as mass, energy, exergy, and entropy. Different fluid mechanics and thermodynamics texts have treated the diagrammatic representation of these properties either very poorly, or in the case of the latter two, not at all. In this paper, the concept of the storage and interaction diagrams is introduced as a graphical tool to represent the aforementioned scalar properties. The storage and interaction diagrams combine the conservation and accounting of extensive properties with a template similar to the kinetic, free-body, and impulse-momentum diagrams. Three examples are provided to show the application of this general diagrammatic approach to different types of problems that involve the change in multiple properties. The impact of incorporating storage and interaction diagrams when introducing conservation and accounting principles involving scalar properties is assessed through the evaluation of student performance on exams and student feedback. A comparison of two cohorts of students suggests that emphasizing drawing storage and interaction diagrams may help reduce the ramp-up time that most students need to get acclimated to the conservation and accounting principles problem-solving framework.

## Introduction and Motivation

It is not an exaggeration to state that the primary learning objective in any Statics course is for students to learn how to select a system and draw a proper free-body diagram (FBD), which provides a visual representation of the *external* forces and couples that act on the system. In fact, there have been numerous publications on different mnemonics[1,2] and techniques[3,4] that instructors have used to improve their students' ability to draw FBDs. Once students are introduced to systems in motion in Dynamics, the kinetic diagram (KD)[5,6] is introduced. This diagram depicts the motion of the object through the mass times acceleration $ma$ and mass moment of inertia times angular acceleration $J_O\alpha$ terms. An example of a generic KD-FBD pair is shown in Figure 1. A less discussed, but equally important set of diagrams that are also introduced in Dynamics involve the impulse-momentum principle. These diagrams are typically
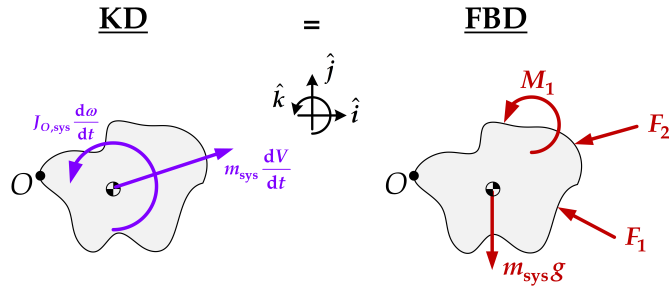
**Figure 1**  KD-FBD pair of a generic object moving and rotating in space

introduced to show the change in momentum and the impulses involved during impact. An example of a generic "Final-Initial-During" impulse-momentum trio is shown in Figure 2.
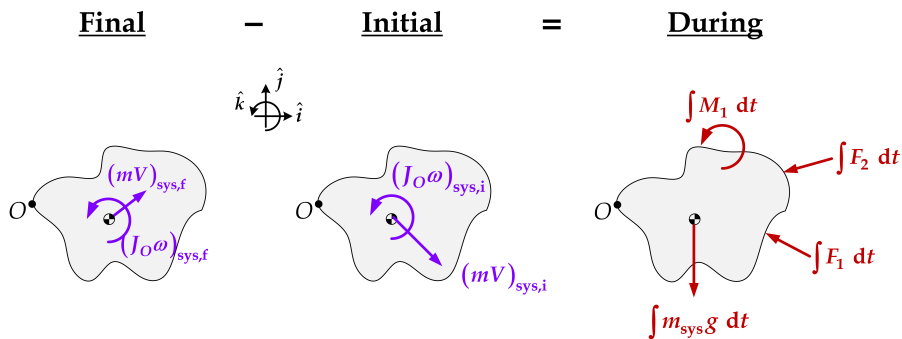


**Figure 2**  Final-Initial-During trio of diagrams of a generic object moving and rotating in space

One reason for the emphasis that instructors place on selecting correct systems and drawing proper diagrams in Statics and Dynamics is because these diagrams allow for the student to first visualize what is happening to the system and subsequently use the diagrams to simplify and solve the equations. While the KD-FBD pair and Final-Initial-During trio of diagrams are well-established and universally accepted in teaching Statics and Dynamics courses, there is less formality, if any, to drawing diagrams involving properties that are conserved and need to be accounted for in courses such as Fluid Mechanics and Thermodynamics. These include mass, energy, entropy, and exergy.[1]

The diagrammatic treatment of the aforementioned properties varies in the five fluid mechanics and three thermodynamics textbooks reviewed by the author. In terms of drawing a proper free-body diagram, especially when dealing with open systems that have mass inflow and outflow, both Çengel and Cimbala[7] and Gerhart, Gerhart, and Hochstein[8] do an excellent job of clearly identifying the system using dashed lines and showing terms that have units of force such as the force due to pressure $PA$ or mass flow rate $\dot{m}V$ on their FBDs. Fox and Mitchell[9] and White and Xue[10] clearly identify their system, while Hibbeler[11] does not, but all three texts included

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[1]Exergy is not an independent property, unlike mass, linear and angular momentum, energy, and entropy (at non-relativistic speeds); it is defined by combining energy and entropy together. It is considered with the other five properties in this paper because of its significance in thermodynamics.

pressure and velocity terms on their FBDs which clearly do not have units of force. With regards to the scalar property of mass, only Çengel and Cimbala[7] and Gerhart, Gerhart, and Hochstein[8] showed separate diagrams of mass flow rates entering and exiting the systems. The energy diagrams were found to be lacking in all eight texts reviewed, which was especially glaring in the thermodynamics texts that primarily deal with conservation of energy. Both Moran, Shapiro, Boettner, and Bailey[12] and Çengel, Boles, and Kanoğlu[13] clearly identify their system using dashed lines, but they do not draw a separate diagram from what is shown in the problem statement. Most of the diagrams that were reviewed showed some heat transfer rate and power terms, but not all of them, and in instances, only pressures and temperatures were shown on diagrams, not terms with units of power. Bejan[14] is an older textbook which does an excellent job highlighting conservation and account principles, but is limited in terms of its diagrams. None of the eight texts reviewed attempted a diagram of any kind for entropy and exergy.

The limitations of existing textbooks in the thermal-fluid sciences in providing students with consistent diagrammatic support to solve engineering problems has motivated the exploration of a straightforward unified approach to drawing these diagrams. With this in mind, the idea of using a generalized accounting approach[15,16] to draw *storage* and *interaction* diagrams is explored in this paper.

**Representation of Principles through Storage and Interaction Diagrams**

Consider a generic object that is moving and rotating in space and is subjected to forces and couples (moments) prior to colliding with a wall. We can consider two perspectives with respect to time when studying the behavior of this object:

- If we are interested in understanding how this object moves in space *at every instant in time* before it collides with the wall, we would consider a *rate form* analysis of the conservation of linear momentum principle (better known as Newton's second law). This is visualized through the KD-FBD pair of diagrams shown in Figure 1.

- If we are only interested in looking at the properties of the object at *any two instances in time*, we would consider the *finite-time* form of conservation of linear momentum (better known as impulse-momentum principle). It is important to note that there is nothing new or special about the finite-time form; it is simply the result of integrating the conservation principle in rate form over time. The finite-time form of conservation of linear momentum is represented by the Final-Initial-During trio of diagrams shown in Figure 2.

Due to the widespread acceptance of both perspectives and their respective sets of diagrams when it comes to linear and angular momentum, perhaps the simplest approach to creating guidelines for drawing diagrams of other properties such as mass, energy, entropy, and exergy would be to essentially follow a similar set of rules.

In general, any property that needs to be "accounted" for during a process would lend itself well to be represented visually. A summary of such properties, their definition, and the courses that they are encountered in presented in Table 1. Before proceeding though, it is important to establish the generalized accounting principle and define some nomenclature that will be used throughout the rest of this work.

**Table 1** Properties that can be accounted for, their definition, and course(s) in which they primarily appear — $E_k$, $E_p$, $U_{\text{sys}}$, $E_{\text{other}}$ are kinetic, potential, internal, and other sources of energy in the system; $s$ is entropy per unit mass; $T_0$ and $P_0$ are the dead state (thermodynamic term) temperature and pressure.

| Property | Mathematical definition | Definition in words | Course(s) |
|---|---|---|---|
| Mass | $m_{\text{sys}} = \rho \forall_{\text{sys}}$ | density by volume | Fluids, Thermo |
| Linear momentum | $\vec{\mathbb{P}}_{\text{sys}} = m_{\text{sys}} \vec{V}_{\text{sys}}$ | mass by velocity | Statics, Dynamics, Fluids |
| Angular momentum | $\vec{\mathbb{L}}_{O,\text{sys}} = \vec{r} \times \vec{\mathbb{P}}_{\text{sys}}$ | moment of linear momentum | Statics, Dynamics, Fluids |
| Energy | $E_{\text{sys}} = E_k + E_p + U_{\text{sys}} + E_{\text{other}}$ | sum of all forms of energy in system | Dynamics, Fluids, Thermo |
| Entropy | $S_{\text{sys}} = m_{\text{sys}} s$ | measure of degree of disorder in system | Thermo |
| Exergy | $A_{\text{sys}} = E_{\text{sys}} - T_0 S_{\text{sys}} + P_0 \forall_{\text{sys}}$ | maximum amount of useful work produced by system relative to the environment | Thermo |

Consider a generic extensive property as the system undergoes a process. The amount of the property that is *stored* in the system will equal to the net amount that enters (what comes in minus what leaves) plus the net amount that is *generated* within the system (what is created minus what is destroyed). If the property that enters or exits the system involves mass, it is *transport*ed; if it does not, it is *transfer*red. A property that is neither generated nor destroyed is conserved.

Consider the KD-FBD pair in Figure 1 once more. The terms depicted in the kinetic diagram represent the rate of change of linear and angular momentum in the object (storage term), while the terms shown in the free-body diagram change the linear and angular momentum of the system without changing its mass (transfer terms). This particular system is closed (no mass enters or exits); if mass had been entering or exiting the system, the transport terms would appear on the FBD as well. Similarly, when considering the Final-Initial-During trio of diagrams in Figure 2, the final and initial momentum diagrams depict the storage term, while the impulse diagram shows the transfer terms.

Based on this framework, a general rule for diagrams of properties other than linear and angular momentum can be developed. When considering the rate form representation of a property in a

**Table 2**  Conservation and accounting of properties for open and closed systems (rate form)

| Property | Storage | = | Transfer | + | Transport | + | Generation |
|---|---|---|---|---|---|---|---|
| Mass | $\dfrac{dm_{sys}}{dt}$ | $=$ | $0$ | $+$ | $\sum\limits_{in}\dot{m}-\sum\limits_{out}\dot{m}$ | $+$ | $0$ |
| Linear momentum | $\dfrac{d\vec{\mathbb{P}}_{sys}}{dt}$ | $=$ | $\sum\limits_{ext}\vec{F}$ | $+$ | $\sum\limits_{in}\dot{m}\vec{V}-\sum\limits_{out}\dot{m}\vec{V}$ | $+$ | $\vec{0}$ |
| Angular momentum | $\dfrac{d\vec{\mathbb{L}}_{O,sys}}{dt}$ | $=$ | $\sum\limits_{ext}\vec{M}_O$ | $+$ | $\sum\limits_{in}\vec{r}\times\dot{m}\vec{V}-\sum\limits_{out}\vec{r}\times\dot{m}\vec{V}$ | $+$ | $\vec{0}$ |
| Energy | $\dfrac{dE_{sys}}{dt}$ | $=$ | $\dot{Q}_{in,net}+\dot{W}_{in,net}$ | $+$ | $\sum\limits_{in}\dot{m}e-\sum\limits_{out}\dot{m}e$ | $+$ | $0$ |
| Entropy | $\dfrac{dS_{sys}}{dt}$ | $=$ | $\sum\limits_{in}\dfrac{\dot{Q}}{T_b}$ | $+$ | $\sum\limits_{in}\dot{m}s-\sum\limits_{out}\dot{m}s$ | $+$ | $\dot{S}_{gen}$ |
| Exergy | $\dfrac{dA_{sys}}{dt}$ | $=$ | $\sum\dot{A}_{q,in}-\dot{A}_{w,out,useful}$ | $+$ | $\sum\limits_{in}\dot{m}a_f-\sum\limits_{out}\dot{m}a_f$ | $+$ | $-\dot{A}_{des}$ |

system, one diagram will solely show the storage term, which will be labeled as the *storage diagram* or SD, while the other diagram will depict the transfer, transport, and generation terms, which will be called the *interaction diagram* or ID. The conservation and accounting expressions of the properties listed in Table 1 is tabulated in Table 2 for the rate form analysis and the respective SD-ID pair for each property is shown in Figure 3.
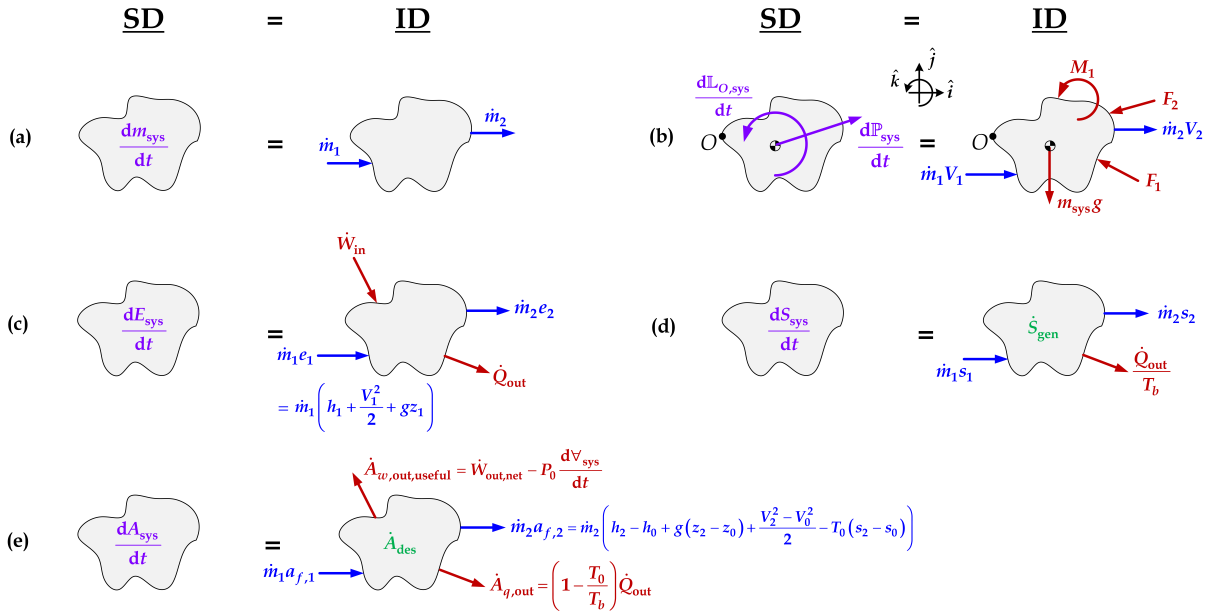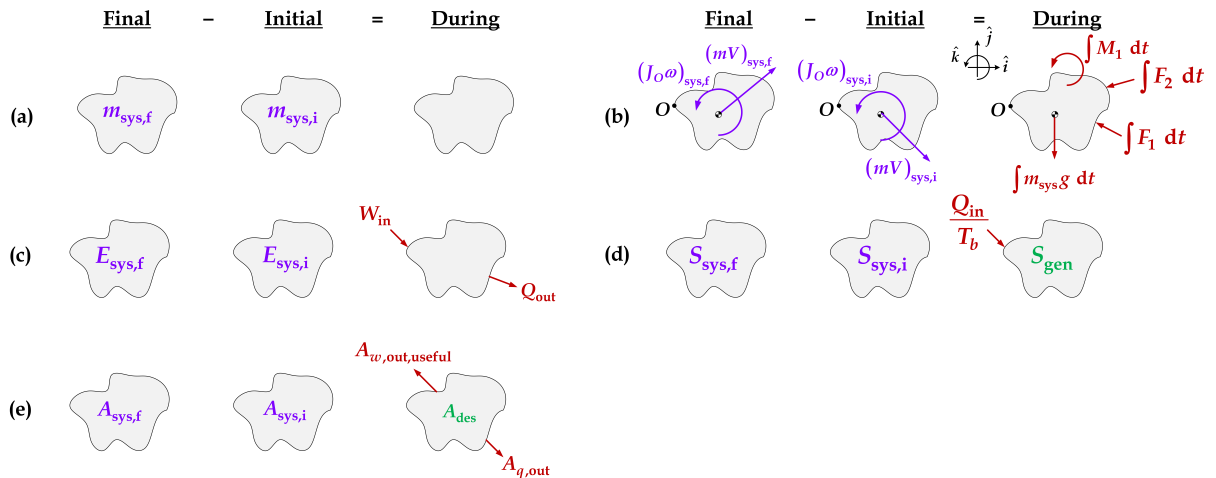


**Figure 3**  Rate form storage and interaction diagrams for (a) mass, (b) linear and angular momentum, (c) energy, (d) entropy, and (e) exergy for a generic system

**Table 3**  Conservation and accounting of properties for closed system (finite-time form)

| Property | Final | − | Initial (storage) | = | During (transfer) | + | During (generation) |
|---|---|---|---|---|---|---|---|
| Mass | $m_{\text{sys,f}}$ | − | $m_{\text{sys,i}}$ | = | $0$ | + | $0$ |
| Linear momentum | $\vec{\mathbb{P}}_{\text{sys,f}}$ | − | $\vec{\mathbb{P}}_{\text{sys,i}}$ | = | $\sum_{\text{ext}} \int \vec{F}\, dt$ | + | $\vec{0}$ |
| Angular momentum | $\vec{\mathbb{L}}_{O,\text{sys,f}}$ | − | $\vec{\mathbb{L}}_{O,\text{sys,i}}$ | = | $\sum_{\text{ext}} \int \vec{M}_O\, dt$ | + | $\vec{0}$ |
| Energy | $E_{\text{sys,f}}$ | − | $E_{\text{sys,i}}$ | = | $Q_{\text{in,net}} + W_{\text{in,net}}$ | + | $0$ |
| Entropy | $S_{\text{sys,f}}$ | − | $S_{\text{sys,i}}$ | = | $\sum_{\text{in}} \frac{Q}{T_b}$ | + | $S_{\text{gen}}$ |
| Exergy | $A_{\text{sys,f}}$ | − | $A_{\text{sys,i}}$ | = | $\sum A_{q,\text{in}} - A_{w,\text{out,useful}}$ | + | $-A_{\text{des}}$ |

For a finite-time form analysis, the final and initial value of a property will be shown in the Final-Initial diagrams (storage term), while the During diagram will include the transfer and generation terms. While it is possible to consider the transport terms in a finite-time form analysis, it is common to use a rate form approach for such problems. Expressions for the conservation and accounting of properties in the finite-time form analysis of closed systems is available in Table 3 and the interaction diagram for each property is shown in Figure 4.

In the next section, three examples are provided to demonstrate the application of interaction diagrams in solving problems that involve more than just conservation of linear and angular momentum.



**Figure 4**  Finite-time form storage and interaction diagrams for (a) mass, (b) linear and angular momentum, (c) energy, (d) entropy, and (e) exergy for a generic closed system

# Examples

**EXAMPLE 1: RATE FORM ANALYSIS OF DIFFERENT CLOSED SYSTEMS** An electric motor of mass $m$, specific heat $c$, and surface area $A_s$ is suddenly turned on and used to set a rack-and-pinion pair of gears in motion as shown in Figure 5. The motor receives electric energy at a known rate $\dot{W}_{\text{elec}}(t)$, loses heat to the ambient environment via convection (heat transfer coefficient $h_{\text{conv}}$ and ambient temperature $T_\infty$), and supplies torque $\tau(t)$ (an unknown) to a rigid massless shaft that is connected to the pinion. The internal circuitry of the motor is known, which means that $\dot{W}_{\text{shaft}} = f\left(\dot{W}_{\text{elec}}\right)$ is a known relationship. The pinion, with mass moment of inertia $J_O$ and radius $r$, is in perfect contact with the rack, which has a mass $m_r$. The stiffness of the spring connected to the rack is $k$. The dead state temperature is $T_0$. Set up but do not solve the equations that can be used to determine the following unknowns:

(a) the temperature of the motor, $T_m(t)$;

(b) the rotational degree of freedom, $\theta(t)$, of the pinion;

(c) the translational degree of freedom, $x(t)$, of the rack;

(d) the rate of entropy generation in the motor, $\dot{S}_{\text{gen}}$, and

(e) the rate of exergy destruction in the motor, $\dot{A}_{\text{des}}$.
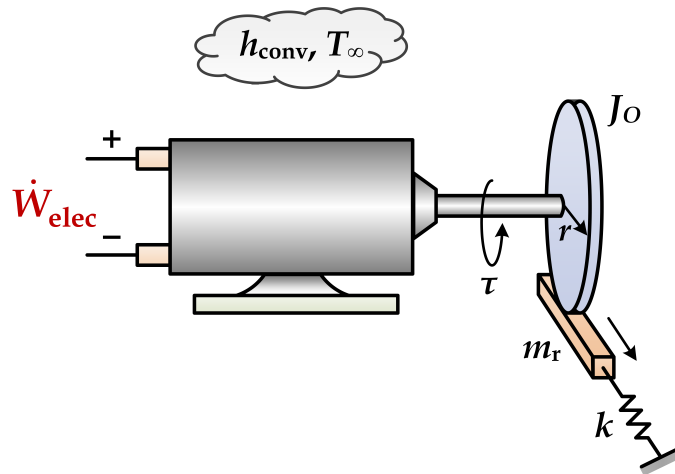


**Figure 5**   Example 1 - Electric motor connected to a rack-and-pinion pair of gears

**EXAMPLE 1 SOLUTION** The SD-ID pair for Parts (a)-(e) are shown in Figure 6. Since the problem involves rates of change properties, the rate form formulation will be considered.

(a) The system is the motor for this part. The energy SD-ID for this system is shown in Figure 8(a). Conservation of energy yields

$$\frac{\mathrm{d}E_{\text{sys}}}{\mathrm{d}t} = \dot{Q}_{\text{in,net}} + \dot{W}_{\text{in,net}} + \sum_{\text{in}} \dot{m}e - \cancel{\sum_{\text{out}} \dot{m}e}^{\,0}$$
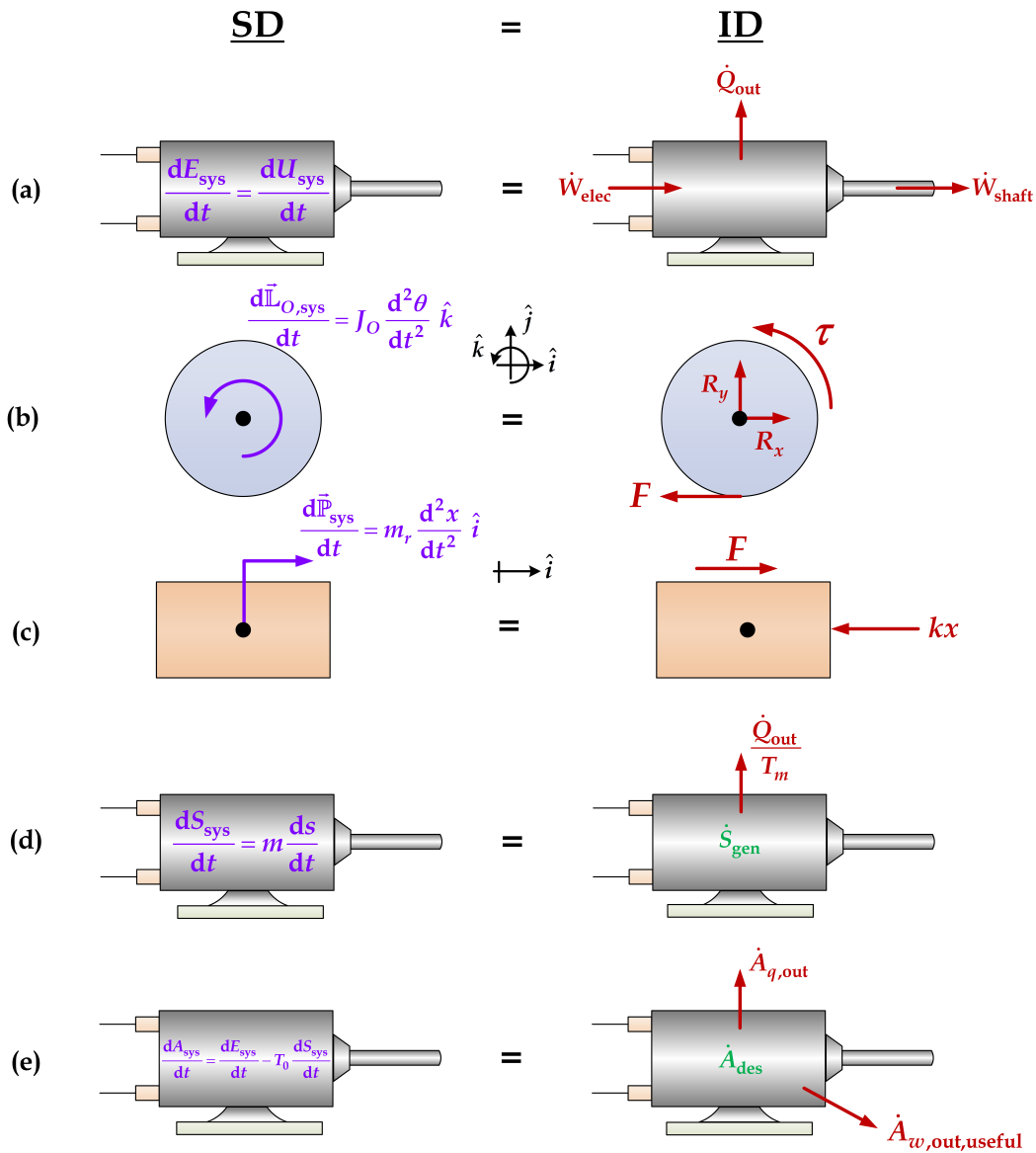
$$\cancel{\frac{\mathrm{d}E_k}{\mathrm{d}t}}^{0} + \cancel{\frac{\mathrm{d}E_p}{\mathrm{d}t}}^{0} + \frac{\mathrm{d}U_{\text{sys}}}{\mathrm{d}t} = -\dot{Q}_{\text{out}} + \dot{W}_{\text{elec}} - \dot{W}_{\text{shaft}} \tag{1}$$

The terms in (1) can be expanded as follows:

$$\frac{\mathrm{d}U_{\text{sys}}}{\mathrm{d}t} = mc\frac{\mathrm{d}T_m}{\mathrm{d}t} \tag{2}$$

$$\dot{Q}_{\text{out}} = h_{\text{conv}}A_s\left(T_m(t) - T_\infty\right) \tag{3}$$

$$\dot{W}_{\text{shaft}} = \tau(t)\frac{\mathrm{d}\theta}{\mathrm{d}t} \tag{4}$$



**Figure 6** Example 1 storage and interaction diagram pairs involving (a) energy of the motor, (b) angular momentum of the pinion, (c) linear momentum of the rack, (d) entropy of the motor, and (e) exergy of the motor.

Substituting (2), (3), and (4) into (1) results in

$$mc\frac{\mathrm{d}T_m}{\mathrm{d}t} = \dot{W}_{\text{elec}} - h_{\text{conv}}A_s\left(T_m(t) - T_\infty\right) - \tau(t)\frac{\mathrm{d}\theta}{\mathrm{d}t} \tag{5}$$

(b) For this part, the system is the pinion. The SD-ID pair for angular momentum (also known as the KD-FBD pair!) for this system are shown in Figure 6(b). Conservation of angular momentum results in

$$\frac{\mathrm{d}\vec{\mathbb{L}}_{O,\text{sys}}}{\mathrm{d}t} = \sum_{\text{ext}} \vec{M}_O + \sum_{\text{in}} \vec{r} \times \dot{m}\vec{V} - \underbrace{\sum_{\text{out}} \vec{r} \times \dot{m}\vec{V}}_{\phantom{x}}{}^{\,0}$$

In scalar form,

$$\hat{k}: \qquad\qquad J_O\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \tau - rF \tag{6}$$

(c) The system is selected as the rack in this case. The linear momentum SD-ID diagrams are shown in Figure 6(c). Conservation of linear momentum yields

$$\frac{\mathrm{d}\vec{\mathbb{P}}_{\text{sys}}}{\mathrm{d}t} = \sum_{\text{ext}} \vec{F} + \sum_{\text{in}} \dot{m}\vec{V} - \underbrace{\sum_{\text{out}} \dot{m}\vec{V}}_{\phantom{x}}{}^{\,0}$$

Resolving the vector relation in the $\hat{i}$ coordinate leads to

$$\hat{i}: \qquad\qquad m_r\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = F - kx \tag{7}$$

It is important to note that the degrees of freedom in (6) and (7) are related through the following kinematic relation:

$$x = r\theta \tag{8}$$

(d) For this case, the system is the motor. The entropy SD-ID pair for this system is shown in Figure 6(d). Accounting of entropy results in

$$\frac{\mathrm{d}S_{\text{sys}}}{\mathrm{d}t} = \sum_{\text{in}} \frac{\dot{Q}}{T_b} + \sum_{\text{in}} \dot{m}s - \underbrace{\sum_{\text{out}} \dot{m}s}_{\phantom{x}}{}^{\,0} + \dot{S}_{\text{gen}}$$

$$\dot{S}_{\text{gen}} = \frac{\mathrm{d}S_{\text{sys}}}{\mathrm{d}t} + \frac{\dot{Q}_{\text{out}}}{T_m} = \frac{\mathrm{d}S_{\text{sys}}}{\mathrm{d}t} + \frac{h_{\text{conv}}A_s\left(T_m - T_\infty\right)}{T_m} \tag{9}$$

The specific entropy of an incompressible substance (the motor) is solely dependent on temperature. Therefore,

$$\frac{\mathrm{d}S_{\text{sys}}}{\mathrm{d}t} = m\frac{\mathrm{d}s}{\mathrm{d}t} = mc\frac{\mathrm{d}\left(\ln\left(T_m\right)\right)}{\mathrm{d}t} \tag{10}$$

Substituting (10) into (9) leads to

$$\dot{S}_{\text{gen}} = mc\frac{\mathrm{d}\left(\ln\left(T_m\right)\right)}{\mathrm{d}t} + \frac{h_{\text{conv}}A_s\left(T_m - T_\infty\right)}{T_m} \tag{11}$$

(e) The exergy SD-ID pair for the motor is shown in Figure 6(e). Accounting of exergy results in

$$\frac{\mathrm{d}A_{\text{sys}}}{\mathrm{d}t} = -\dot{A}_{q,\text{out}} - \dot{A}_{w,\text{out,useful}} + \sum_{\text{in}} \dot{m}a_f - \cancel{\sum_{\text{out}} \dot{m}a_f}^{0} - \dot{A}_{\text{des}}$$

$$\dot{A}_{\text{des}} = -\frac{\mathrm{d}A_{\text{sys}}}{\mathrm{d}t} - \dot{A}_{q,\text{out}} - \dot{A}_{w,\text{out,useful}} \tag{12}$$

From the definition of exergy transfer rate due to heat transfer and work along with the time rate of change of the exergy in the system,

$$\dot{A}_{q,\text{out}} = \left(1 - \frac{T_0}{T_m}\right)\dot{Q}_{\text{out}} = \left(1 - \frac{T_0}{T_m}\right)h_{\text{conv}}A_s\left(T_m(t) - T_\infty\right) \tag{13}$$

$$\dot{A}_{w,\text{out,useful}} = \dot{W}_{\text{out,net}} - P_0\cancel{\frac{\mathrm{d}V_{\text{sys}}}{\mathrm{d}t}}^{0} = \tau(t)\frac{\mathrm{d}\theta}{\mathrm{d}t} - \dot{W}_{\text{elec}} \tag{14}$$

$$\frac{\mathrm{d}A_{\text{sys}}}{\mathrm{d}t} = \frac{\mathrm{d}E_{\text{sys}}}{\mathrm{d}t} - T_0\frac{\mathrm{d}S_{\text{sys}}}{\mathrm{d}t} + P_0\cancel{\frac{\mathrm{d}V_{\text{sys}}}{\mathrm{d}t}}^{0} = mc\left[\frac{\mathrm{d}T_m}{\mathrm{d}t} - T_0\frac{\mathrm{d}\left(\ln\left(T_m\right)\right)}{\mathrm{d}t}\right] \tag{15}$$

Substituting (13), (14), and (15) into (12) and simplifying leads to

$$\dot{A}_{\text{des}} = T_0\left[mc\frac{\mathrm{d}\left(\ln\left(T_m\right)\right)}{\mathrm{d}t} + \frac{h_{\text{conv}}A_s\left(T_m(t) - T_\infty\right)}{T_m}\right]$$

$$+ \cancel{\left[\dot{W}_{\text{elec}} - mc\frac{\mathrm{d}T_m}{\mathrm{d}t} - h_{\text{conv}}A_s\left(T_m(t) - T_\infty\right) - \tau(t)\frac{\mathrm{d}\theta}{\mathrm{d}t}\right]}^{0} = T_0\dot{S}_{\text{gen}} \tag{16}$$

There are 7 unknowns involved in this problem: $T_m(t)$, $\tau(t)$, $\theta(t)$, $x(t)$, $F(t)$, $\dot{S}_{\text{gen}}(t)$, and $\dot{A}_{\text{des}}(t)$. The 7 equations needed to solve for these unknowns can be summarized as follows:

$$\tau(t)\frac{\mathrm{d}\theta}{\mathrm{d}t} = f\left(\dot{W}_{\text{elec}}\right) \text{ (given in problem statement)}$$

$$mc\frac{\mathrm{d}T_m}{\mathrm{d}t} = \dot{W}_{\text{elec}} - h_{\text{conv}}A_s\left(T_m(t) - T_\infty\right) - \tau(t)\frac{\mathrm{d}\theta}{\mathrm{d}t}$$

$$J_O\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \tau - rF$$

$$m_r\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = F - kx$$

$$x = r\theta \text{ (kinematics)}$$

$$\dot{S}_{\text{gen}} = mc\frac{\mathrm{d}\left(\ln\left(T_m\right)\right)}{\mathrm{d}t} + \frac{h_{\text{conv}}A_s\left(T_m - T_\infty\right)}{T_m}$$

$$\dot{A}_{\text{des}} = T_0\dot{S}_{\text{gen}}$$

This example demonstrates the utility of including the time-dependent storage term in its own diagram. Due to the vector nature of linear and angular momentum, the importance of the kinetic diagram is more pronounced compared to the storage diagram for scalar properties such as energy, entropy, and exergy, but depicting the storage term associated with each scalar property provides a visual check on the equations for the students and allows them to always maintain the same general framework for drawing diagrams. A problem involving the rate form formulation of conservation and accounting principles under steady-state conditions will be considered next.

**EXAMPLE 2: RATE FORM ANALYSIS OF THE SAME OPEN SYSTEM OPERATING AT STEADY STATE**
A turbojet engine is fixed to a test stand as shown in Figure 7. Air enters and exits the test stand under *steady* operation. The mass flow rate $\dot{m}$, inlet velocity $V_1$, exit velocity $V_2$, the weight of the engine and the air inside $m_{sys}g$, the pressure and temperature at the inlet ($P_1$ and $T_1$) and at the outlet ($P_2$ and $T_2$), and the boundary temperature at which heat is added to the air $T_b$ are known. Model air as an ideal gas with constant specific heat $c_p$ and gas constant $R$. The dead state temperature is $T_0$. Assume that all the power produced in the turbine is used to power the compressor. Determine:

(a) the relation between inlet and outlet mass flow rates;

(b) the reaction forces and couple at the support;

(c) the heat transfer rate to the air in the engine;

(d) the rate of entropy generation in the engine, and

(e) the rate of exergy destruction in the engine.
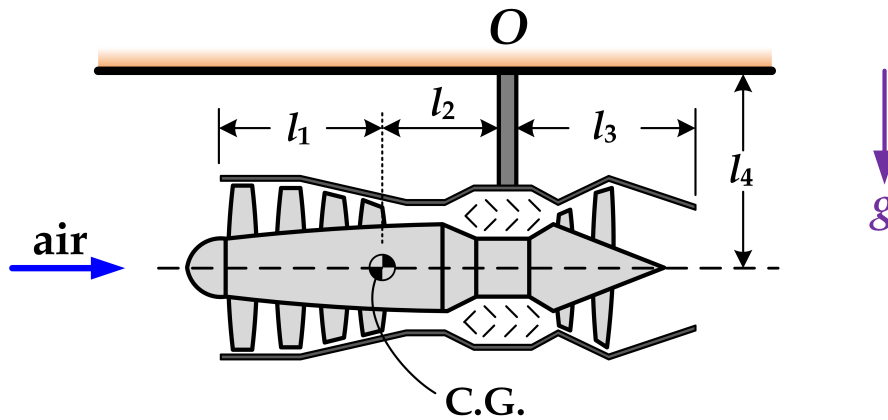


**Figure 7**  Example 2 - Turbojet engine test stand

**EXAMPLE 2 SOLUTION**  The proper storage and interaction diagram pairs for Parts (a)-(e) are shown in Figure 8. Since the engine is operating under steady-state conditions, the time derivatives are zero for every property. The system for every part of this problem is the turbojet engine, the air inside it, and the attaching rod.
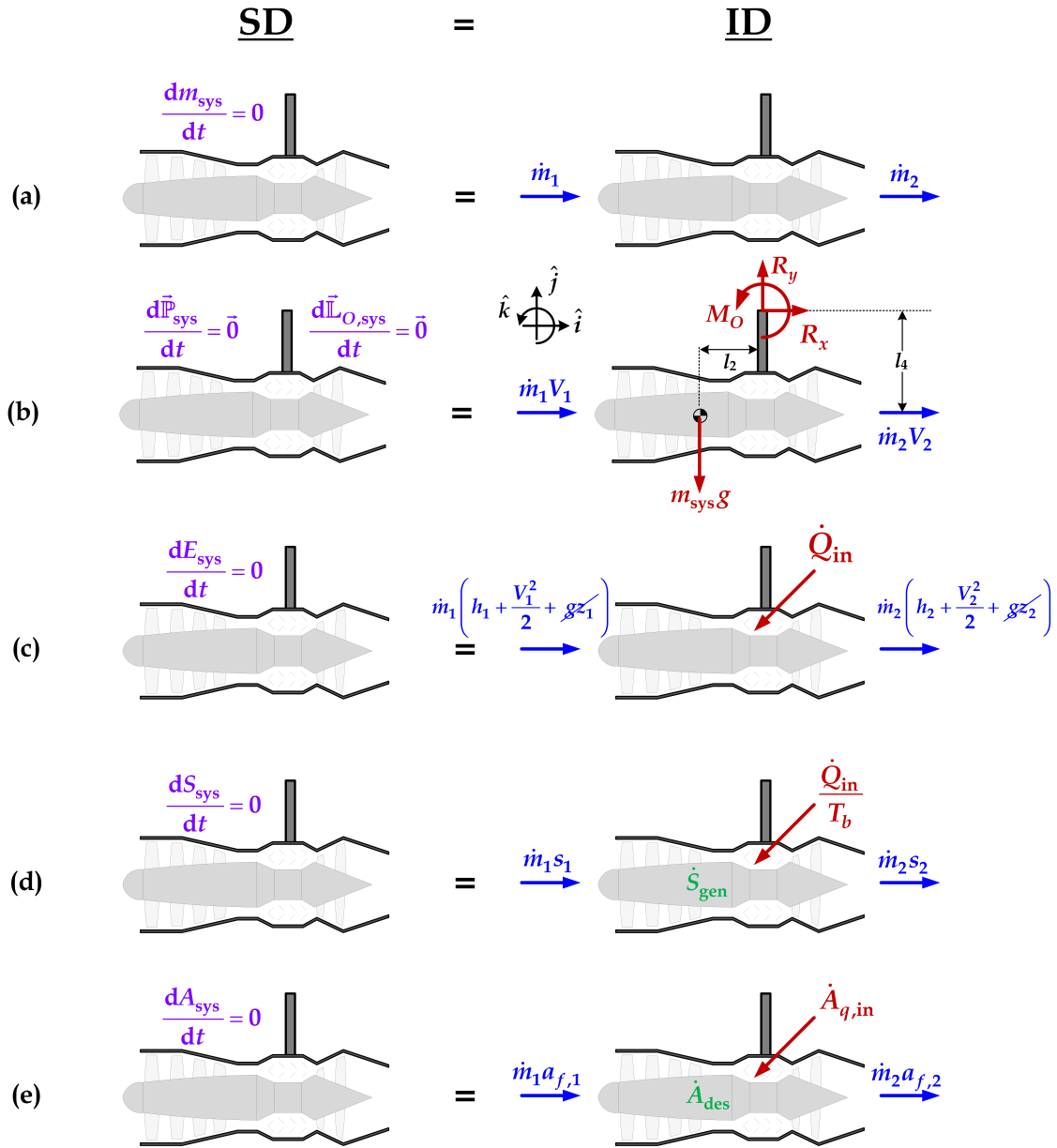
**Figure 8** Example 2 (a) mass, (b) linear and angular momentum, (c) energy, (d) entropy, and (e) exergy storage and interaction diagram pairs

(a) The mass SD-ID pair for this system is shown in Figure 8(a). Conservation of mass yields

$$\frac{\mathrm{d}m_{\mathrm{sys}}}{\mathrm{d}t} \overset{\text{S.S.}}{=} \sum_{\mathrm{in}} \dot{m} - \sum_{\mathrm{out}} \dot{m}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m} \tag{17}$$

(b) The linear and angular momentum SD-ID pair (once again, this is just a KD-FBD pair) for this system are shown in Figure 8(b). Conservation of linear momentum results in

$$\frac{\mathrm{d}\vec{\mathbb{P}}_{\text{sys}}}{\mathrm{d}t}^{\text{S.S.}} = \sum_{\text{ext}} \vec{F} + \sum_{\text{in}} \dot{m}\vec{V} - \sum_{\text{out}} \dot{m}\vec{V}$$

Resolving the vector relation in the $\hat{i}$ and $\hat{j}$ coordinates leads to

$$\hat{i}: \qquad R_x = \dot{m}_2 V_2 - \dot{m}_1 V_1 = \dot{m}\left(V_2 - V_1\right) \tag{18}$$

$$\hat{j}: \qquad R_y = m_{\text{sys}}g \tag{19}$$

Conservation of angular momentum yields

$$\frac{\mathrm{d}\vec{\mathbb{L}}_{O,\text{sys}}}{\mathrm{d}t}^{\text{S.S.}} = \sum_{\text{ext}} \vec{M}_O + \sum_{\text{in}} \vec{r} \times \dot{m}\vec{V} - \sum_{\text{out}} \vec{r} \times \dot{m}\vec{V}$$

In scalar form,

$$\hat{k}: \qquad M_O = -l_2 m_{\text{sys}}g - l_4 \dot{m}_1 V_1 + l_4 \dot{m}_2 V_2 = -l_2 m_{\text{sys}}g + l_4 \dot{m}\left(V_2 - V_1\right) \tag{20}$$

(c) The energy SD-ID pair for this system is shown in Figure 8(c). Conservation of energy results in

$$\frac{\mathrm{d}E_{\text{sys}}}{\mathrm{d}t}^{\text{S.S.}} = \dot{Q}_{\text{in,net}} + \dot{W}_{\text{in,net}}^{0} + \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

$$\dot{Q}_{\text{in}} = \dot{m}_2\left(h_2 + \frac{V_2^2}{2}\right) - \dot{m}_1\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right) \tag{21}$$

From the ideal gas substance model,

$$h_2 - h_1 = c_p\left(T_2 - T_1\right) \tag{22}$$

Substituting (22) into (21) leads to

$$\dot{Q}_{\text{in}} = \dot{m}\left(c_p\left(T_2 - T_1\right) + \frac{V_2^2 - V_1^2}{2}\right) \tag{23}$$

(d) The entropy storage and interaction diagram pair for this system is shown in Figure 8(d). Accounting of entropy results in

$$\frac{\mathrm{d}S_{\text{sys}}}{\mathrm{d}t}^{\text{S.S.}} = \sum_{\text{in}} \frac{\dot{Q}}{T_b} + \sum_{\text{in}} \dot{m}s - \sum_{\text{out}} \dot{m}s + \dot{S}_{\text{gen}}$$

$$\dot{S}_{\text{gen}} = \dot{m}_2 s_2 - \dot{m}_1 s_1 - \frac{\dot{Q}_{\text{in}}}{T_b} = \dot{m}(s_2 - s_1) - \frac{\dot{m}\left(c_p\left(T_2 - T_1\right) + \frac{V_2^2 - V_1^2}{2}\right)}{T_b} \tag{24}$$

From the ideal gas substance model,

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \tag{25}$$

Substituting (25) into (24) leads to

$$\dot{S}_{\text{gen}} = \dot{m}\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) - \frac{c_p\left(T_2 - T_1\right) + \frac{V_2^2 - V_1^2}{2}}{T_b}\right] \tag{26}$$

(e) The exergy SD-ID pair for this system is shown in Figure 8(e). Accounting of exergy results in

$$\underbrace{\frac{\mathrm{d}A_{\text{sys}}}{\mathrm{d}t}}_{\text{S.S.}} = \dot{A}_{q,\text{in}} - \underbrace{\dot{A}_{w,\text{out,useful}}}_{0} + \sum_{\text{in}} \dot{m}a_f - \sum_{\text{out}} \dot{m}a_f - \dot{A}_{\text{des}}$$

$$\dot{A}_{\text{des}} = \dot{m}_1 a_{f,1} - \dot{m}_2 a_{f,2} + \dot{A}_{q,\text{in}} = \dot{A}_{q,\text{in}} - \dot{m}(a_{f,2} - a_{f,1}) \tag{27}$$

From the definition of exergy transfer rate due to heat transfer and the specific flow exergy,

$$\dot{A}_{q,\text{in}} = \left(1 - \frac{T_0}{T_b}\right)\dot{Q}_{\text{in}} \tag{28}$$

$$a_{f,2} - a_{f,1} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} - T_0\left(s_2 - s_1\right) \tag{29}$$

Substituting (22), (23), (25), (28), and (29) into (27) and simplifying leads to

$$\dot{A}_{\text{des}} = \dot{m}T_0\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) - \frac{c_p\left(T_2 - T_1\right) + \frac{V_2^2 - V_1^2}{2}}{T_b}\right] = T_0 \dot{S}_{\text{gen}} \tag{30}$$

The results can be summarized as follows:

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$
$$R_x = \dot{m}\left(V_2 - V_1\right) \rightarrow$$
$$R_y = m_{\text{sys}}g \uparrow$$
$$M_O = -l_2 m_{\text{sys}}g + l_4 \dot{m}\left(V_2 - V_1\right) \circlearrowleft$$
$$\dot{Q}_{\text{in}} = \dot{m}\left(c_p\left(T_2 - T_1\right) + \frac{V_2^2 - V_1^2}{2}\right)$$
$$\dot{S}_{\text{gen}} = \dot{m}\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) - \frac{c_p\left(T_2 - T_1\right) + \frac{V_2^2 - V_1^2}{2}}{T_b}\right]$$
$$\dot{A}_{\text{des}} = T_0 \dot{S}_{\text{gen}}$$

This example demonstrates that when a system is operating under steady conditions, the storage diagrams will all be empty. It is up to the individual instructor to decide whether they would like their students to draw a diagram knowing that it will be empty, but this is the author's preferred approach for the same reasons as in Example 1: it is an additional visual check on the equations and the same general framework for drawing diagrams is still maintained. Also, as has been demonstrated through both Examples 1 and 2, the rate of exergy destruction is directly proportional to the rate of entropy generation. This is because, as mentioned earlier, exergy is simply a linear combination of energy and entropy. Accounting of exergy has only been presented in Examples 1 and 2 for the sake of demonstrating how it follows the same general guideline as other extensive properties. If energy and entropy have already been considered, accounting of exergy will not provide any new information. Next, a problem involving the finite-time formulation of conservation and accounting principles will be considered.

**EXAMPLE 3: FINITE-TIME FORM ANALYSIS OF DIFFERENT CLOSED SYSTEMS** Block B of mass $m_B$ sits atop a cylinder of cross-sectional area $A_c$ which contains a gas as shown in Figure 9. Block A of mass $m_A$ is dropped vertically from rest at a height $z_1$ above Block B and collides with it in a perfectly inelastic manner. The two blocks compress the gas adiabatically from an initial height $l_i$ such that $P\forall^k = \text{constant}$, after impact. The initial pressure and temperature of the gas ($P_i$ and $T_i$) are known. Assume the gas is ideal with constant specific heat $c_v$, gas constant $R$, and specific heat ratio $k$. Neglect air resistance and assume the impact time between blocks A and B is very small. Also assume that the change in the kinetic and potential energy of the gas is negligible. Determine:

(a) the velocity of block A just before impact;

(b) the velocity of blocks A and B just after impact;

(c) an expression for the height of gas in the cylinder under maximum compression;

(d) the temperature of the gas when it is subjected to maximum compression, and

(e) the entropy generated in the gas when it is subjected to maximum compression.

**EXAMPLE 3 SOLUTION** The Final-Initial-During diagram trios for Parts (a)-(e) are shown in Figure 10. Since the beginning and end of processes is of interest, the finite-form formulation will be considered.

(a) The system for this part of the problem is block A. The energy Final-Initial-During diagram trio for this system is shown in Figure 10(a). Conservation of energy yields
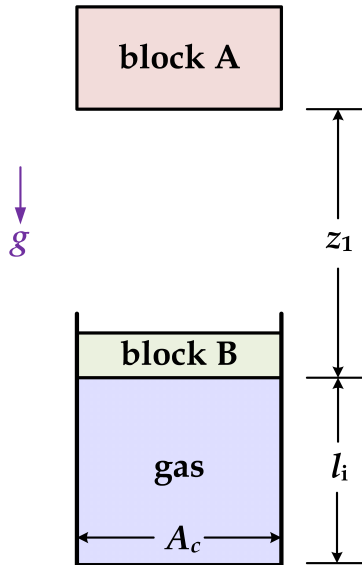
$$E_{k,2} - \cancel{E_{k,1}}^0 + \cancel{E_{p,2}}^0 - E_{p,1} + \cancel{U_2 - U_1}^0 = \cancel{Q_{\text{in,net}}}^0 + \cancel{W_{\text{in,net}}}^0$$
$$\frac{1}{2}m_A V_2^2 - m_A g z_1 = 0$$

Thus,

$$V_2 = \sqrt{2gz_1} \tag{31}$$

**Figure 9**  Example 3 - Perfectly inelastic collision of blocks leading to the compression of a gas

(b) The system for this part of the problem is blocks A and B. The linear momentum Final-Initial-During diagram trio (also known as the impulse-momentum diagram!) for this system is shown in Figure 10(b). Conservation of linear momentum results in

$$\vec{\mathbb{P}}_{\text{sys},3} - \vec{\mathbb{P}}_{\text{sys},2} = \sum_{\text{ext}} \int_{t_2}^{t_3} \vec{F}\,\mathrm{d}t \overset{\approx\,0}{}$$

Note that since the impact time is assumed to be very small, the impulses involved in the problem are assumed to be negligible. Resolving the vector relation in the $\hat{j}$ coordinates leads to
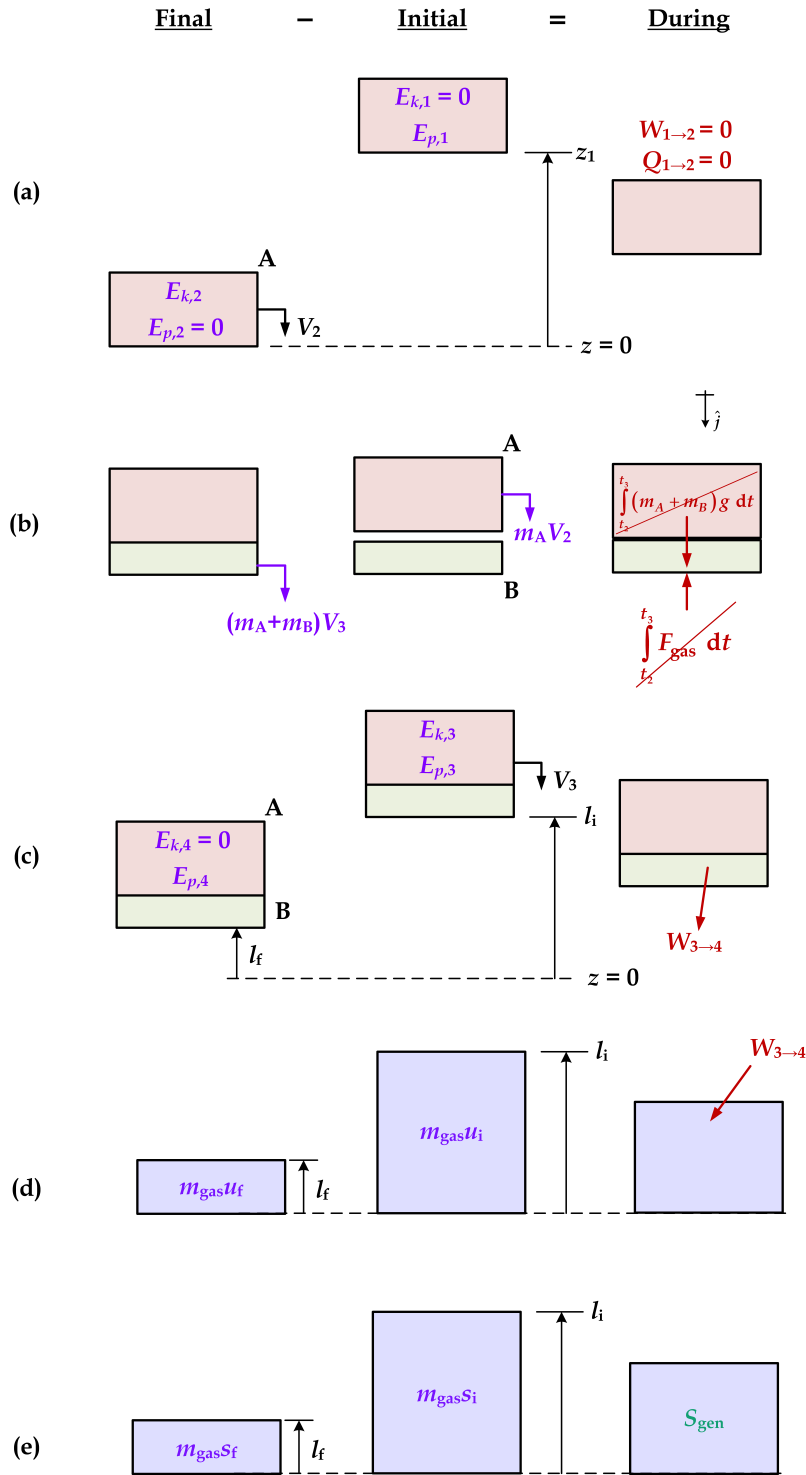
$$\hat{j}: \qquad V_3 = \frac{m_A}{m_A + m_B} V_2 = \frac{m_A}{m_A + m_B}\sqrt{2gz_1} \qquad (32)$$

(c) The system for this part is still blocks A and B. The energy Final-Initial-During diagram trio for this system is shown in Figure 10(c). Conservation of energy leads to

$$E_{k,4}^{\,0} - E_{k,3} + E_{p,4} - E_{p,3} + U_4^{\,0} - U_3^{\,0} = Q_{\text{in,net}}^{\,0} + W_{\text{in,net}}$$

$$-\frac{1}{2}\left(m_A + m_B\right)V_3^2 - \left(m_A + m_B\right)g\left(l_{\text{i}} - l_{\text{f}}\right) = -W_{3\to4}$$

The work out of blocks A and B is used to compress the gas. For a polytropic process $P\forall^k = \text{constant}$, this work can be expressed as

$$W_{3\to4} = \frac{P_{\text{f}}\forall_{\text{f}} - P_{\text{i}}\forall_{\text{i}}}{k-1} = \frac{P_{\text{i}}A_c l_{\text{i}}}{k-1}\left[\left(\frac{l_{\text{f}}}{l_{\text{i}}}\right)^{1-k} - 1\right] \qquad (33)$$

$E_{k,1} = 0$
$E_{p,1}$

$W_{1\to2} = 0$
$Q_{1\to2} = 0$

$z_1$

**(a)**

**A**

$E_{k,2}$
$E_{p,2} = 0$

$V_2$

$z = 0$

$\hat{j}$

**A**

**(b)**

$\int_{t_2}^{t_3}(m_A + m_B)g\,dt$

$m_A V_2$

$(m_A + m_B)V_3$

**B**

$\int_{t_2}^{t_3} F_{gas}\,dt$

$E_{k,3}$
$E_{p,3}$

$V_3$

$l_i$

**A**

**(c)**

$E_{k,4} = 0$
$E_{p,4}$

**B**

$l_f$

$W_{3\to4}$

$z = 0$

$l_i$

$W_{3\to4}$

**(d)**

$m_{gas}u_f$

$l_f$

$m_{gas}u_i$

$l_i$

**(e)**

$m_{gas}s_f$

$l_f$

$m_{gas}s_i$

$S_{gen}$

**Figure 10**   Example 3 Final-Initial-During diagram trio involving (a) energy of block A until before impact, (b) linear momentum of blocks A and B during impact, (c) energy of blocks A and B post impact until maximum compression, (d) energy of gas until maximum compression, and (e) entropy of gas until maximum compression.

Thus, a transcendental equation for $l_f$ is obtained as follows:

$$\frac{l_f}{l_i} + \frac{P_i A_c/(k-1)}{(m_A + m_B)\,g}\left(\frac{l_f}{l_i}\right)^{1-k} - \left[1 + \frac{P_i A_c/(k-1)}{(m_A + m_B)\,g} + \left(\frac{m_A}{m_A + m_B}\right)^2 \frac{z_1}{l_i}\right] = 0 \quad (34)$$

(d) The system for this part is the gas undergoing compression. The Final-Initial-During diagram trio for this system is shown in Figure 10(d). Conservation of energy yields that

$$\cancelto{0}{E_{k,f} - E_{k,i}} + \cancelto{0}{E_{p,f} - E_{p,i}} + U_f - U_i = \cancelto{0}{Q_{in,net}} + W_{in,net}$$

$$m_{gas}\left(u_f - u_i\right) = W_{3\to4} \quad (35)$$

From the ideal gas substance model,

$$m_{gas} = \frac{P_i \forall_i}{R T_i} = \frac{P_i A_c l_i}{R T_i} \quad (36)$$

$$u_f - u_i = c_v\left(T_f - T_i\right) \quad (37)$$

Substituting (36), (37), and (33) into (35) leads to

$$T_f = T_i\left(\frac{l_f}{l_i}\right)^{1-k} \quad (38)$$

(e) Similar to the previous part, the system is still the gas undergoing compression. The entropy Final-Initial-During diagram trio for this system is shown in Figure 10(e). Accounting of entropy requires that

$$S_f - S_i = \sum_{in} \cancelto{0}{\frac{Q}{T_b}} + S_{gen}$$

$$S_{gen} = m_{gas}\left(s_f - s_i\right) \quad (39)$$

From the ideal gas substance model,

$$s_f - s_i = c_v \ln\left(\frac{T_f}{T_i}\right) + R \ln\left(\frac{\forall_f}{\forall_i}\right) = c_v(1-k)\ln\left(\frac{l_f}{l_i}\right) + R \ln\left(\frac{l_f}{l_i}\right) = 0 \quad (40)$$

Substituting (36) and (40) into (39) leads to

$$S_{gen} = 0 \quad (41)$$

The results can be summarized as follows:

$$V_2 = \sqrt{2gz_1}$$

$$V_3 = \frac{m_A}{m_A + m_B}\sqrt{2gz_1}$$

$$\frac{l_{\mathrm{f}}}{l_{\mathrm{i}}} + \frac{P_{\mathrm{i}}A_c/(k-1)}{(m_A + m_B)\,g}\left(\frac{l_{\mathrm{f}}}{l_{\mathrm{i}}}\right)^{1-k} - \left[1 + \frac{P_{\mathrm{i}}A_c/(k-1)}{(m_A + m_B)\,g} + \left(\frac{m_A}{m_A + m_B}\right)^2\frac{z_1}{l_{\mathrm{i}}}\right] = 0$$

$$T_{\mathrm{f}} = T_{\mathrm{i}}\left(\frac{l_{\mathrm{f}}}{l_{\mathrm{i}}}\right)^{1-k}$$

$$S_{\mathrm{gen}} = 0$$

In this example, multiple systems were considered to solve for the variables of interest. In each case, since the finite-time form formulation was applicable, the Final-Initial-During diagram trio was implemented, which, once again, reinforces the consistency in utilizing interaction diagrams to solve problems involving conservation and accounting principles. An error that is often encountered in class with problems that involve perfectly inelastic collisions such as this is that some students apply conservation of energy from when block A is released to the point of maximum compression of the two blocks without accounting for any additional heat transfer in the "During" diagram or a change in the internal energy of the system through the "Final-Initial" diagrams. In a perfectly inelastic collision, momentum is conserved, but kinetic energy is converted to other forms of energy such as sound and heat.

## Assessment

Perhaps the biggest impediment to assessing the usefulness of storage and interaction diagrams is requiring rigor and consistency in drawing these diagrams across a wide scope of classes, as outlined in Table 1, that are not necessarily prerequisites or co-requisites of one another. This may require a significant undertaking for the faculty who teach these courses to retain a consistent approach, especially if there are many of them involved. At the author's current institution, Rose-Hulman Institute of Technology, the Mechanical Engineering curriculum was redesigned in the 1990s to add a centerpiece course called Conservation and Accounting Principles, or ConApps for short, which the typical student would take in the Fall quarter of their sophomore after completing Statics and the Physics and Calculus sequences[15]. In this course, the general accounting principle is first introduced and applied to the mass, linear and angular momentum, energy, and entropy properties throughout the course. In subsequent quarters, students build on the principles they learn in ConApps in their Mechanical Systems (equivalent to Dynamics), Fluid Systems, Applications of Thermodynamics (where exergy is introduced), and Analysis and Design of Engineering Systems (equivalent to System Dynamics) courses. While there is complete agreement among the faculty in the department about the importance of the principles discussed in ConApps, there is less agreement on emphasizing the importance of diagrams for all principles, especially in subsequent classes. A survey of the faculty involved in teaching these classes indicated that only those who are involved in teaching Mechanical Systems emphasize

drawing both the storage and interaction diagrams in the form of the KD-FBD pair. All but one of the faculty teaching the other aforementioned courses only required students to draw interaction diagrams. One instructor did not require the drawing of any diagrams when discussing conservation of energy in the context of the Analysis and Design of Engineering Systems course.

The author has been teaching ConApps regularly since 2018. There is a strong emphasis in defining systems and drawing interaction diagrams for every application of a conservation and accounting principle by every course instructor. More recently, the author expanded the focus on diagram drawing in their sections of the course to include *both* storage and interaction diagrams as has been outlined in this paper. A comparison of two cohorts of students taught by the author will be provided. *Cohort A* consisted of 37 students who completed the term and were only taught to draw interaction diagrams. *Cohort B* consisted of 24 students who completed the term, but were taught and asked to draw storage and interaction diagrams when solving problems. The difference in the average grade point average (GPA) between the two cohorts was found to be statistically insignificant, which if used as an indicator of the "strength" of a class would mean that two cohorts may be assumed to have been fairly even at the beginning of the term. Both cohorts were given four midterm exams and a cumulative final. The midterm exams were different between the cohorts, but covered the same topics and were deemed to be similar in terms of difficulty. The final exam was nearly identical between the two cohorts. Cohort averages and standard deviations for these five exams are provided in Table 4. The two-tailed two-sample unequal variance p-value for the exam scores of the two cohorts is also provided.

**Table 4** Exam averages (standard deviation) comparison between *Cohort A* and *Cohort B*

| Exam | Principle covered | *Cohort A* | *Cohort B* | p-value |
|---|---|---|---|---|
| Exam 1 | Mass | 76.5% (14.6%) | 88.5% (5.8%) | $4.34 \times 10^{-5}$ |
| Exam 2 | Linear momentum | 80.3% (13.5%) | 81.4% (12.6%) | 0.748 |
| Exam 3 | Linear/angular momentum, energy (closed) | 83.6% (15.8%) | 89.2% (8.0%) | 0.070 |
| Exam 4 | Energy (open), mechanical energy balance | 85.9% (11.9%) | 87.5% (15.6%) | 0.673 |
| Final Exam | Cumulative | 80.4% (14.1%) | 80.3% (15.3%) | 0.927 |

The scores in Table 4 suggest that *Cohort B* outperformed *Cohort A* in a statistically significant manner on Exam 1 and marginally significantly on Exam 3. There was no significant difference between the two cohorts on the remaining three exams, particularly on the cumulative final exam. While it would be wise not to draw too many conclusions from a comparison of two relatively small cohorts, the scores suggest that emphasizing the storage and interaction diagrams at the

beginning of the term, and especially for conservation of mass, may help reduce the ramp-up time that most students need to get acclimated to the problem-solving framework that they encounter for the first time in ConApps. Feedback received from the students in *Cohort B* suggests while every student who commented on the diagrams found them to be helpful, there were a couple who thought that drawing storage diagrams in rate-form problems became tedious after a few weeks. A sample of some of this feedback is provided below:

*"Having taken this course before, I can say that the diagram structure made a huge difference in my learning. Having that set pattern was really helpful. In my opinion, it made me more understanding of the left hand/right hand sides of the equations, and what I could do with each."*

*"The formal emphasis of the diagrams really helped me conceptualize the motivations of what I was trying to solve within the problem sets. I think that it made using the conservation equations easier since the skill was transferred from understanding a bunch of symbols to understanding a physical representation of the system through the diagram."*

*"I thought the focus on storage diagrams at the start of each topic was a good inclusion. However, once you get multiple weeks in, they just start to become tedious. The finite-form diagrams are more useful in my opinion."*

**Conclusions and Future Work**

In this work, a general framework to visualizing the conservation and accounting of properties through storage and interaction diagrams in rate and finite-time form has been presented and its utility is demonstrated with three examples. While students are typically introduced to a rigorous treatment of the free-body diagram in Statics and the kinetic and impulse-momentum diagrams in Dynamics, there seems to be little agreement on diagrammatically expressing the conservation and accounting of scalar properties such as mass, energy, entropy, and exergy in other texts. The idea behind storage and interaction diagrams is to use the KD-FBD pair (in rate form analysis) and the Final-Initial-During diagram trio (in finite-time form analysis) for the conservation of linear and angular momentum as a template for scalar properties. As such, for rate form analysis, one diagram will represent the storage term (kinetic diagram), while the other will include transfer, transport, and generation terms (free-body diagram). Similarly, in finite-form analysis, the Final and Initial diagrams will represent the storage term while the transfer and generation terms will be depicted in the During diagram. As the three examples in this paper show, the consistency in representation of storage and interaction diagrams provides a useful visual tool for students to solve problems that may at first seem complicated in a straightforward manner.

While student performance and feedback indicate that drawing storage and interaction diagrams are a step in the positive direction, there is more to do. Both of the cohorts considered in Table 4 consist primarily of students who successfully completed the course in their first try. It would be valuable to apply this approach to the off-sequence sections of the ConApps course to see whether it would be useful in improving the performance of students who are repeating the course. There was one such student in *Cohort B* whose comments suggest that this diagram structure may be helpful. Furthermore, data suggests that student performance on exams later in

the term remains virtually the same irrespective of whether the storage diagram is introduced or not. More data points will be collected to verify this observation.

## Acknowledgment

The author would like to thank and acknowledge Dr. Don E. Richards, Professor Emeritus of Mechanical Engineering, and his colleagues for developing the framework for the conservation and accounting principles that have been outlined in this paper and implementing them at the Rose-Hulman Institute of Technology. The author would like to also acknowledge Dr. Calvin Lui, Associate Professor of Mechanical Engineering, for coining the term "interaction diagram".

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