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Work in Progress: Integrating Basic Stress Analysis Concepts into Statics

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Abstract

The paper describes how basic stress analysis concepts can be integrated into a sophomore-level engineering statics course using pinned frames. The course covers pin connections and supports, which are typically separate from a statics course, through a hoist frame project. The project focuses on the concepts of pin shear stresses, member pull-out shear stress, bearing stresses, and axial uniform normal stress in a centrically loaded member. The pin connection is examined as both a tight fit and a loose fit, and a general structured procedure is provided to conduct the analysis. All statics and stress equations are formulated symbolically, allowing for the repetitive analysis and design of similar structures using an engineering tool. The hoist frame project is also utilized in a follow-up mechanics of materials course, where advanced stress topics are introduced just-in-time throughout the term and include design. A qualitative assessment by students was carried out at the end of the course to provide guidance for the instructors in the future.

Introduction

Engineering design, defined by ABET [1], "is a process of devising a system, component, or process to meet desired needs and specifications within constraints. It is an iterative, creative, decision-making process in which the basic sciences, mathematics, and engineering sciences are applied to convert resources into solutions. Engineering design involves identifying opportunities, developing requirements, performing analysis and synthesis, generating multiple solutions, evaluating solutions against requirements, considering risks, and making trade-offs to obtain a high-quality solution under the given circumstances. For illustrative purposes only, examples of possible constraints include accessibility, aesthetics, codes, constructability, cost, ergonomics, extensibility, functionality, interoperability, legal considerations, maintainability, manufacturability, marketability, policy, regulations, schedule, standards, sustainability, or usability."

The authors believe that statics textbooks make it challenging to carry out an "iterative, creative, decision-making process in which the basic sciences, mathematics, and engineering sciences are applied to convert resources into solutions." This paper proposes a problem solution with a symbolic formulation approach to overcome this challenge to perform solution verification and parametric design studies in statics. Other statics textbooks do not consider using a total symbolic approach to formulate all problems.

The proposed method suggests retaining all variables and equations in their symbolic form without any algebraic manipulation. This approach enables students to concentrate on the fundamental physics of the problem rather than on the algebraic manipulation needed to isolate the required solution variable(s). The authors recommend using a commercial program equation solver for solving the equations, except for the most straightforward problems, which should be verified. This method allows for a natural extension to design, as all equations are in symbolic form and can be entered into modern engineering tools for validation and repetitive analysis. By incorporating a computer equation solver with the raw symbolic equations, the method enhances engineering productivity, reduces the chance of algebraic errors, and enables easy design applications.

In this paper, we will discuss a frame structure that is usually taught in engineering statics courses [2] – [6]. A frame is composed of interconnected members that are connected with pin joints where at least one member is a multi-force member (i.e., three or more forces act on it). Pin joints allow for the transfer of forces but not moments. Although the loads may not necessarily be applied at the joints, frames are designed to support external loads and are generally stationary, fully constrained structures. Only frames that are considered statically determinate both externally and internally are included in statics.

The paper discusses integrating fundamental stress analysis into a statics course for a statically determinate pinned frame structure. Stress analysis concepts in statics is not new but a symbolic formulation of the problem is. The analysis process involves creating a free-body diagram of the entire structure, determining support forces using equilibrium equations, and creating separate free-body diagrams for each member. Joint member forces are then determined using the support forces and equilibrium equations. The paper also covers pin connections and supports not typically included in statics courses. Stress analysis is limited to pin shear stress, member pull-out shear stress, connection bearing stress, and normal stress in axially loaded members. The authors use a structured symbolic approach to perform statics and stress analyses, which is helpful in design. This approach differs from those used in widely used statics textbooks [2] – [6] since we formulate all problems symbolically that allows an easy extension to complex design problems. Engineering design is considered in the statics book [6].

General Structured Procedure to Solve Statics and Mechanics of Materials Problems

A general structured procedure is presented to solve statics and mechanics of materials problems. The students must follow the appropriate steps listed below for every in-class lecture, homework problem, and design project they solve based on reference [7] and [8].

1. *Model.* The success of any analysis is dependent on the validity and appropriateness of the model to idealize the physical problem to predict and analyze its behavior, whether centric axial loading, torsion, bending, or a combination of the above. Assumptions and limitations also need to be stated in this step. This step is only explicitly emphasized in [6] and [16]. All dimensions and forces are defined symbolically.

2. *Free-Body Diagrams.* This step is where all the free-body diagrams initially thought to be required for the solution are drawn. The free-body diagrams include the complete structure and/or parts of the structure. Importantly, all dimensions and loads, even known ones, are symbolically defined.

3. *Equilibrium Equations.* The equilibrium equations for each free-body diagram required for a solution are written. All equations are formulated symbolically. There is no attempt made at this point to isolate the unknown variables. However, we must examine every term in each equation for dimensional homogeneity.

4. *Deformation Equations.* The deformation formulas are written for each part of a structure based on the Model in Step 1 using the method of segments [8]. All equations are formulated symbolically, and there is no algebraic manipulation. Therefore, we must examine every term in each equation for dimensional homogeneity.

5. *Compatibility and Boundary Conditions.* One or more compatibility equations are written in symbolic form to relate the displacements. A compatibility diagram is used when appropriate to develop the compatibility equations. All equations are formulated symbolically, and there is no algebraic manipulation. We must examine every term in each equation for dimensional homogeneity. Although compatibility equations are commonly written for indeterminate problems, the authors emphasize their use for determinate problems just as in the mechanics of materials textbooks [9] – [11].

6. *Complementary and Supporting Formulas.* Steps 1 through 5 are sufficient to solve the (primary) variables force and displacement in a structure's problem. Step 6 includes complementary formulas for other (secondary) variables such as stress and strain, variables which may govern the maximum allowable in-service values of force and displacement but do not affect the governing equilibrium or deformation equations. Supporting formulas might be required to supply variable values in the material law equations and complementary formulas; formulas such as area, the moment of inertia, centroid location of a cross-section, volume, etc.

   The complementary and supporting formulas are written symbolically and are necessary to develop a complete analysis. The complementary formulas involve solution-governing variables such as stress, strain, and stiffness. For example, supporting formulas and complementary formulas may be required to define variables in Steps 3 through 5 completely. These formulas include a cross-sectional area, polar moment of inertia, centroid location, the moment of inertia, section modulus, effective length, the radius of gyration, etc.

7. *Solve.* The independent equations developed in Steps 3 through 6 solve the problem. All equations in Steps 3 through 6 require retaining all variables and equations in their symbolic form without any algebraic manipulation and entering them into a modern engineering tool. The students compare the number of independent equations and the number of unknowns. The authors emphasize that the student should only proceed once the number of unknowns equals the number of independent equations. The known variables are then entered into a modern engineering tool and solved.

   The solution may be obtained by hand, which generally requires algebraic manipulation. Alternatively, the solution of any number of linear or non-linear equations can be achieved with a modern engineering tool. With the intelligent application of verification (Step 8), the computer program is a much more reliable calculation device than a calculator. The students are allowed to select the modern engineering tool of their choice, and this might include PTC Mathcad® [12], MATLAB® [13], and TKSolver [14].

8. *Verify.* This critical step critiques the answer and is discussed in-depth in the next section. The paper [15] focuses on educating students to question, test, and verify problem solutions for mechanics of materials problems.

Statics problems require only Steps 1, 2, 3, 7, and 8, and mechanics of materials and machine design problems require Steps 1 to 8. Since basic stress analysis will be carried out in this paper, only Steps 1-3 and 6-8 will be used. However, statics can only be applied to a statically determinate problem. For example, the proposed process can solve statically indeterminate problems (internally and externally) when Steps 1 to 8 are used.

A structured problem-solving approach is used in statics book [6] with the following steps: Road Map, Modeling, Governing Equations, Computation, and Discussion and Verification. Furthermore, the statics [3] and mechanics of materials [16] textbooks use the SMART problem-solving methodology, i.e., Strategy, Modeling, Analysis, and Reflect and Think. Both are like the approach used in this paper. A significant difference is that this paper formulates all equations symbolically, and then the unknowns are solved. Step 8 is also considered in the mechanics of materials textbook [9]. The authors are unaware of other structured problem-solving methods like those used in this paper in [6] and [16] in statics, mechanics of materials, and machine design textbooks.

Pedagogically, the step-by-step solution format allows students to build a structure in their minds of how to approach and solve a problem efficiently. This step-by-step procedure will help students build logic, promote analytical thinking, provide an accurate physical understanding of the subject, and, hopefully, extend the same disciplined process to other courses.

Step 8 Verify: Question and Test to Verify the Answers

Step 8, Verify, may be new to most students, but it is critical! As a professional, one must be prepared to guarantee their solution. Attempts at verifying the solution may take many forms, and although it may not yield absolute proof in some cases, it improves the confidence level. For example, verification might involve comparing with a hand calculation, comparing solutions to similar problems, examining limiting and obvious solution cases, and comparing with experimental data.

One of our educational goals is to convince students of the wisdom to question and test solutions to verify their 'answers.' We do this by integrating verification into the general structured solution procedure. Verification is new to almost all undergraduate students, but it is critical and must be formally incorporated into the solution process! The power of our proposed use of the computer equation solver rests in the ability to quickly and easily run many cases to verify the problem solution. Once an answer has been confirmed, the computer model becomes essential for parametric studies and design.

Using Modern Engineering Tools for Design

Much of the problem-solving in formal academic courses involves analyzing a single modeled system; calculations, typically, are performed once. However, the engineer involved in the design is often confronted with many calculations of a repetitive nature. For example, an initial concept of a structure, the "early design stage," requires initial sizing calculations. Refinement requires more calculations. Resizing to use standard commercially available parts requires more calculations, with the added complexity of swapping some variables from known to unknown. Any analysis tool that reduces the boredom of this process and simultaneously reduces the risk of calculation error should be investigated.

There are three relatively popular engineering equation-solving tools available to both professionals and students: PTC Mathcad® [12], MATLAB® [13], and TKSolver [14]. A computer equation-solving program is a more reliable calculation device than a calculator. All three programs have technical computation problem-solving features that are too numerous to discuss. For this course's requirements, each can be used as a basic scientific calculator to solve linear and non-linear equations and display results in graphical and/or tabular form. However, the authors do not emphasize or recommend any computer equation solver.

The introduction of modern equation-solving tools into this course is meant to accomplish the following:

1. Motivate writing equations in symbolic form.

2. Save time by providing a rapid solution of (many) simultaneous equations.

3. Reduce computation errors.

4. Provide a mathematical model of a problem that yields a fast and accurate means for a parametric study after appropriate testing for validity.

5. Provide a model for gaining a better understanding of the physics of the problem.

6. Stimulate interest and develop proficiency in the design process.

Hoist Frame Project

Problem Statement

The hoist frame structure shown in Figure 1 is assembled with smooth (frictionless) pins and is symmetric about the XY plane. It is used to hoist an object of weight W. The top view shows the complexity of the frame. A double shear connection is at joints A, C, and D, and a quadruple shear connection is at joint B.

A company manufactures this structure in several models. Each model has the same configuration with a different maximum lifting capacity and overall dimensions. The basic design is offered in custom sizes and lifting capacities. We ask the students to develop all equations symbolically to analyze any existing model based on statics and limited stress concepts. This project will focus on one model, and the students must carry out the statics and fundamental stress analysis. The follow-up mechanics of materials course allows the students to do a complete stress analysis of existing models and/or custom design a hoist for the customer.

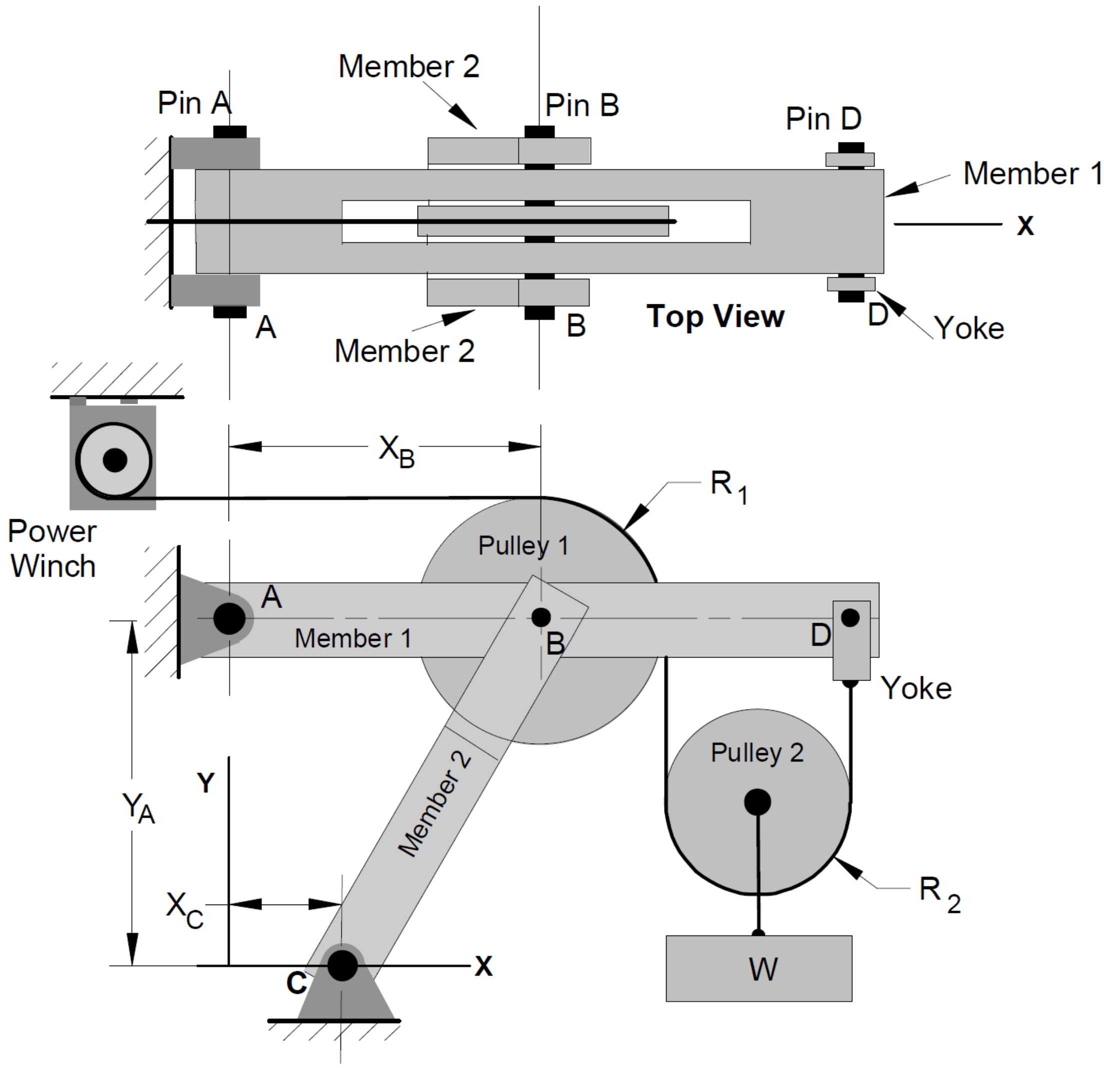


Figure 1. Hoist frame structure configuration and loading.

We selected this project since it is complicated and lengthy compared to traditional statics homework problems. This project duration is six weeks from beginning to end. The length is due to the number and types of free-body diagrams, equilibrium equations, stress concepts, and computer implementation. This project is divided into Phase 1 Equilibrium Analysis and Phase 2 Stress Analysis. After two weeks, the students first submit the hand analysis for Phases 1 and 2. The assignment is graded (50% of the final grade) and returned after one week. The implementation of Phases 1 and 2 in an engineering tool is completed over three weeks and then submitted (50% of final grade).

The students are introduced to the symbolic approach in class and through homework assignments. Each homework assignment required at least one problem to be solved using a modern engineering tool of the student's choice. This experience was beneficial for the students in carrying out the project. Based on experience, we have found that coding the equations, debugging, and solving them using an engineering tool can be challenging for the students, especially for this project since it was much more complex than the homework problems. The hoist frame project is also utilized in a follow-up mechanics of materials course, where advanced stress topics are introduced just-in-time throughout the term. Furthermore, the mechanics of materials hoist project also includes design. Therefore, learning the development of symbolic equations in statics provides a stronger foundation for the mechanics of materials course.

The specification for one hoist model offered by the company in Figure 1 are as follows:

$$W = 1{,}000\ lbf$$

$$X_B = 4\ ft; \quad X_C = 1\ ft; \quad Y_A = 4\ ft$$

$$R_1 = 2\ ft; \quad R_2 = 1.5\ ft$$

which are needed for the equilibrium analysis.

The pin and member dimensions for the hoist model provided by the company in Figures 6 and 7 are as follows:

$$d_A = 0.375 \text{ in}; \quad d_B = 1.25 \text{ in}; \quad d_C = 0.75 \text{ in}$$

$$tA = 3.00 \text{ in}; \quad tB_1 = 0.375 \text{ in}; \quad h_{1\text{-int}} = 2.0 \text{ in}$$

$$tC = 0.75 \text{ in}; \quad tB_2 = 0.50 \text{ in}; \quad h_{2\text{-int}} = 2.0 \text{ in}$$

Solution Phases for the Project

The project is divided into two solution phases as follows:

- *Phase 1 - Equilibrium Analysis.* Phase 1 is purely a force analysis of the hoist structure. Given the general configuration of the hoist structure, we will determine the force exerted on and by each component of the hoist structure when lifting the load. In other words, the complete force analysis will evaluate the joint member forces, pin forces, and internal pin forces required to calculate the stresses in Phase II.

- *Phase 2 - Stress Analysis.* The stress analysis is carried out using the forces from the equilibrium analysis in Phase I. The stress analysis is limited to pin shear stress, member pull-out shear stress, connection bearing stress, and normal stress in axially loaded members. The pin connection is considered a tight fit with uniform bearing stress and a loose fit with non-uniform bearing stress.

We will now summarize Phases I and II, and the detailed solution process can be found in Appendices A and B, respectively.

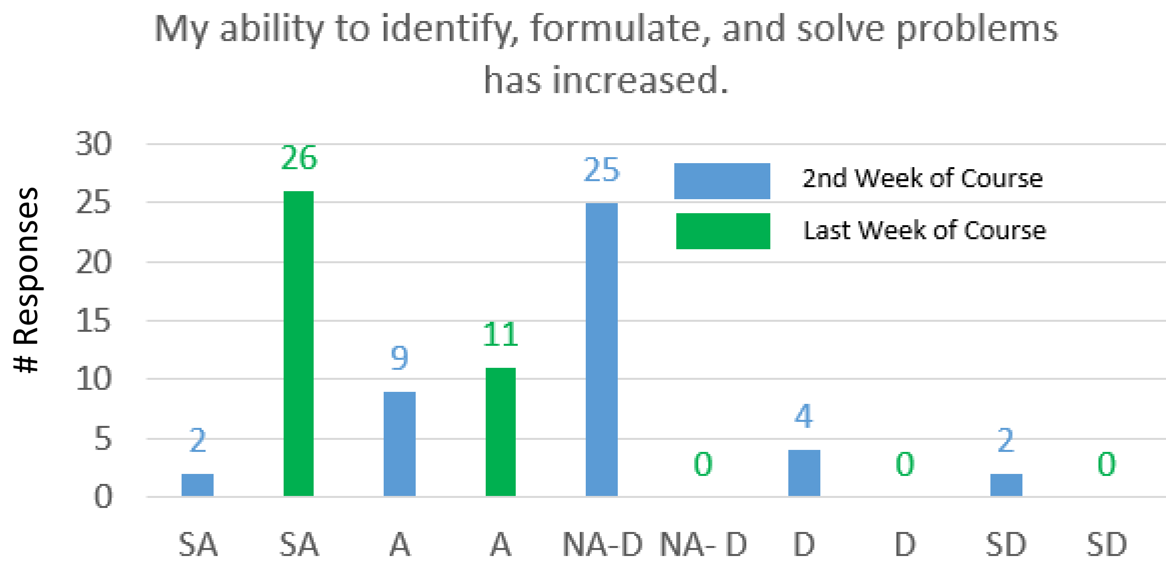
Qualitative Assessment

A qualitative assessment was carried out by using student surveys. We carried out a qualitative assessment for this work in progress since it was more straightforward than a quantitative assessment for this work in progress.

The course enrollment was 44, and statics is typically taken in the later part of the first year or early part of the second year (common) in mechanical engineering. This course was offered in the spring and consisted of 90% second-year students.

The survey was distributed to the students during the third and last weeks of the course (before the final exam) with a participation rate of 95% (42 responses) and 85% (37 responses), respectively. Five questions were asked using a 5-point Likert scale, and one was open-ended.

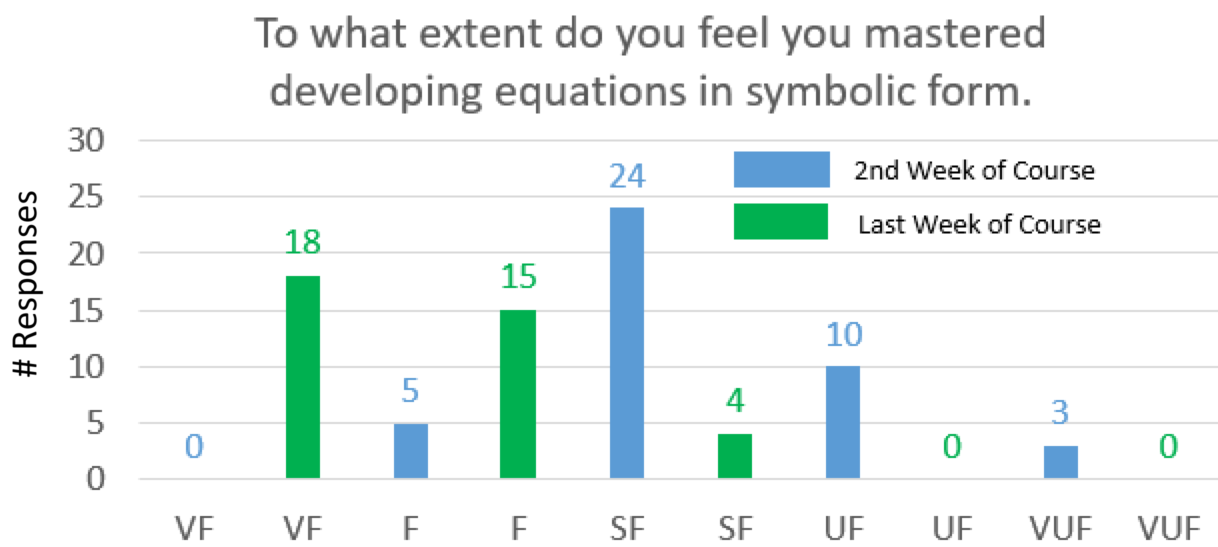
Bar Chart 1 asked, “My ability to identify, formulate, and solve problems has increased,” using a 5-point Likert scale agreement statement. The survey shows a significant increase in the student’s ability to identify, formulate, and solve problems at the second week and the last week of the course. Strongly Agree (SA) and Agree (A) is 100% at the end of the course versus 26% at the start of the course.



Bar Chart 1. My ability to identify, formulate, and solve problems has increased.

SA = Strongly Agree = 5; A = Agree = 4; NA-D = Neither Agree or Disagree = 3;
D = Disagree = 2; SD = Strongly Disagree = 1

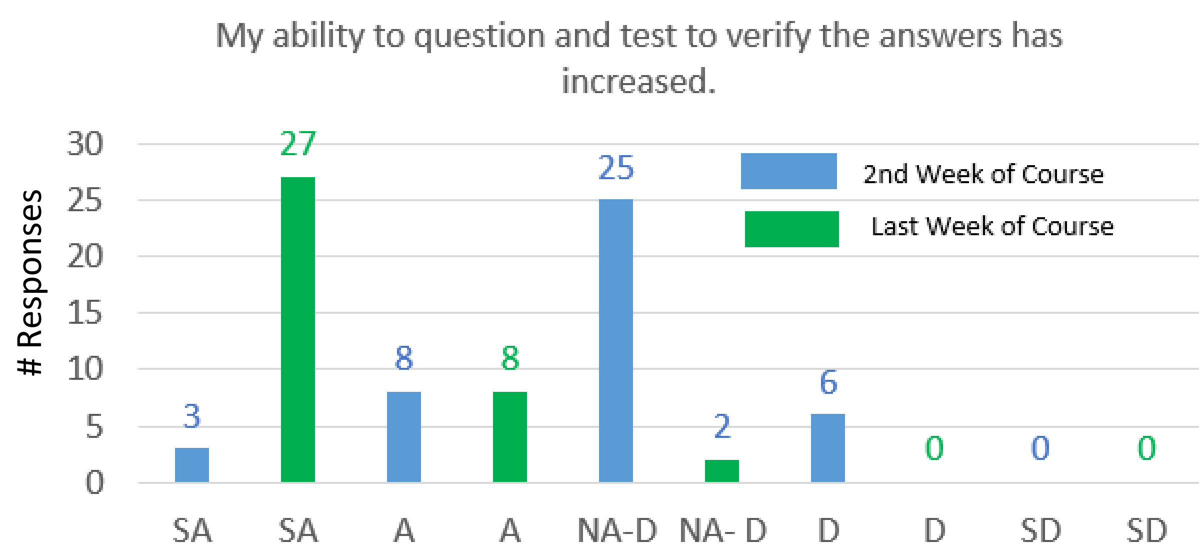
Bar Chart 2 asked, “To what extent do you feel you mastered developing equations in symbolic form” using a 5-point Likert scale familiarity statement. The students felt that they were significantly more familiar with mastering the development of equations in symbolism at the end of the course. Strongly Agree (SA) and Agree (A) is 89% at the end of the course versus 12% at the start of the course.



Bar Chart 2. To what extent do you feel you mastered developing equations in symbolic form.

VF = Very Familiar = 5; F = Familiar = 4; SF = Somewhat Familiar = 3;
UF = Unfamiliar = 2; VUF = Very Unfamiliar = 1

Bar Chart 3 asked, "My ability to question and test to verify the answers has increased" using a 5-point Likert scale agreement statement. The students felt that they significantly increased their ability to question and test to verify the answers at the end of the course. Strongly Agree (SA) and Agree (A) is 89% at the end of the course versus 12% at the start of the course.



Bar Chart 3. My ability to question and test to verify the answers has increased.

SA = Strongly Agree = 5; A = Agree = 4; NA-D = Neither Agree or Disagree = 3;
D = Disagree = 2; SD = Strongly Disagree = 1

The students were asked the following open-ended question only during the last week of the course: *Please provide feedback on your experience with using the structured procedure to solve statics problem using an engineering tool.* Fifteen students responded as follows:

1. I really enjoyed learning how to use TKsolver to solve a complex problem and it will be very valuable when I take future courses and do my senior design project.
2. I have some friends who have taken stress analysis and machine design. Some were introduced to the structure procedure in stress analysis and others in machine design. They all wish that they were introduced to the procedure earlier since they find it very valuable when doing design.
3. I found the structured procedure when solving small problems to be a pain. But later I realized that it was a good learning experience since it prepared me to solve bigger and complex problems.
4. The procedure was valuable. I enjoyed using Mathcad to solve the problems.
5. I did not like the structed procedure and felt it was a waste of my time.
6. I was able to use a modified version of the structured procedure in another course and found tksolver a great tool.
7. The instructor provided an easy way to learn how to apply the structure method using mathcad. I plan to use this approach and mathcad in future courses.
8. I found at first the method hard to understand, but once I used the method to solve many problems, the method was not that difficult. I practically found learning a new computer tool valuable for future courses.
9. I felt like I learned a lot in this course.
10. The professor provided a great experience to learn a new structured procedure and engineering tool that will be useful for years to come.
11. Good learning experience.
12. I now understand the power of TKSolver by taking this course and using it to solve problems. Excellent experience.
13. I am looking forward to using what I learned in this course to solve more difficult problems and apply it to design problems.
14. I felt like this was a valuable learning experience and I will be able to use in the future.
15. The structured procedure was a lot of work to learn, however, once I grasped it, I found the procedure and software Mathcad to be very valuable experience and useful in the future.

The response rate to the open-ended question was 41% (15 of 37 responded). The overall response was very positive, except for one student who felt the structured procedure was a waste of time. Overall, the students perceived that they learned a lot from this experience and will be able to use the structured procedure and software in the future.

We plan to conduct a complete qualitative and quantitative evaluation of student learning and development in the future. In particular, assessing and evaluating student work throughout the course.

Conclusion

This paper presents a general structured procedure to solve statics problems that includes simple stress concepts using a symbolic formulation approach to perform solution verification. The significant difference between the structured procedures used in [6] and [16] is that our equations are all formulated symbolically and then solved for the unknowns. Teaching the student to model a general physical problem with the fundamental equations written in symbolic form, with no variable values specified, helps the student concentrate more fully on the fundamental principles taught in the course. Introducing the modern engineering tool to solve the equations removes the necessary manipulation of the equations to isolate the dependent variables. Training the student to examine and test the answer becomes one essential goal in our course. Using the symbolic approach, with mastery of a computer equation solver and the discipline to insist on verification, should be a significant asset in preparing students to model complex problems for analysis and design. Qualitative feedback from the students indicated that their ability to identify, formulate, and solve problems and question and test to verify the answers has increased. In future work, the authors plan to conduct a quantitative assessment to determine student learning and development.

References

[1] *Criteria for Accrediting Engineering Programs, 2024-2025.* Baltimore, MD: ABET, [Online]. Available: https://www.abet.org/accreditation/accreditation-criteria/criteria-for-accrediting-engineering-programs-2024-2025/. [Accessed March 22, 2024].

[2] A.M. Bedford and W. Fowler, *Engineering Mechanics Statics*, Fifth Edition, Upper Saddle River, NJ: Pearson Prentice Hall, 2008.

[3] F.P. Beer, E.R. Johnston, D.F. Mazurek, P.J. Cornell, and B.P. Self, *Vector Mechanics for Engineers: Statics and Dynamics.* Twelfth Edition, New York, NY: McGraw Hill Education, 2019.

[4] R.C. Hibbeler, *Engineering Mechanics: Statics*, Fifteenth Edition, United Kingdom: Pearson, 2021.

[5] J.L. Meriam, L.G. Kraige, and J.N. Bolton, *Engineering Mechanics Statics*, Ninth Edition, Hoboken, NJ: John Wiley & Sons, Inc., 2018.

[6] M.E. Plesha, G.L. Gray, R.J. Witt, and F. Costanzo, *Engineering Mechanics: Statics*, Third Edition, New York, NY: McGraw-Hill Education, 2023.

[7] J.J. Rencis and H.T. Grandin, "Mechanics of Materials: an Introductory Course with Integration of Theory, Analysis, Verification and Design," in *Proceedings of the 2005 American Society for Engineering Education Annual Conference & Exposition,* Portland, OR, USA, June 12-15, 2005. [Online]. Available: https://peer.asee.org/mechanics-of-materials-an-introductory-course-with-integration-of-theory-analysis-verification-and-design. [Accessed March 22, 2024].

[8] H.T. Grandin and J.J. Rencis, "A New Approach to Solve Beam Deflection Problems using the Method of Segments," in *Proceedings of the 2006 American Society for Engineering Education Annual Conference & Exposition,* Chicago, IL, USA, June 18-21, 2006. [Online]. Available: https://peer.asee.org/a-new-approach-to-solve-beam-deflection-problems-using-the-method-of-segments. [Accessed March 22, 2024].

[9] R.R. Craig, *Mechanics of Materials.* Second Edition, New York, NY: John Wiley & Sons, 2000.
[10] S.H. Crandall, N.C. Dahl, and T.L. Lardner, *An Introduction to the Mechanics of Solids.* Second Edition, New York, NY: McGraw-Hill Book Company, 1978.
[11] I.H. Shames and J.M. Pitarresi, *Introduction to Solid Mechanics.* Third Edition, Upper Saddle River, NJ: Prentice Hall, 2000.
[12] *PTC Mathcad®*, Boston, MA: PTC Corporate Headquarters, http://www.mathsoft.com/en. [Accessed March 22, 2024]
[13] *MATLAB®*, Natick: MA: The MathWorks, Inc., http://www.mathworks.com/. [Accessed March 22, 2024]
[14] *TK Solver*, Loves Park, IL: Universal Technical Systems Inc., http://www.uts.com/. [Accessed March 22, 2024]
[15] J.J. Rencis and H.T. Grandin, "Educating Students to Question, Test and Verify Problem Solutions," in *Proceedings of the 2004 American Society for Engineering Education Annual Conference & Exposition,* Salt Lake City, UT, USA, June 20-23, 2004. [Online]. Available: https://peer.asee.org/educating-students-to-question-test-and-verify-problem-solutions. [Accessed March 22, 2024].
[16] F.P. Beer, E.R. Johnston, J.T. DeWolf, and D.F. Mazurek, *Mechanics of Materials.* Eight Edition, New York, NY: McGraw Hill Education, 2020.

Appendix A: Phase 1 - Equilibrium Analysis

We will carry out Phase 1 in two parts. The first part establishes the two-dimensional free-body diagrams for the members and pins and then solves for the forces using the equilibrium equations. The second part establishes the three-dimensional free-body diagram for each pin and then solves for the pin forces. We have separated Phase 1 into two parts to reduce the complexity of the problem.

<u>Part 1 – Two-dimensional Member and Pin Analyses</u>

The steps to carry out the analysis of Part 1 of Phase 1 are as follows:

1-1. ***Model.*** The hoist structure's weight is negligible compared to the load W as shown in Figure 1. The structure and the loading are symmetric about a common plane. Members 1 and 2 are uniform and straight, and the pin centers are on the centroidal axis of each member. We also assume all cables in the structure are rigid. The pins will be analyzed for direct shear only, and friction is negligible. It is assumed that all cables support tension and no compression.

1-2. ***Free-Body Diagrams.*** We will first focus on developing the two-dimensional free-body diagrams for the members and pins. To solve for all the forces on each member and pin, free-body diagrams of Members 1 and 2 and each pin are required. The coplanar free-body diagrams are shown in Figure 2. The foundation support reaction forces have a force label which begins with letter R, the second letter defines the pin. Thus, RA_X is the X component of the force that the foundation support exerts on pin A.

The free-body diagrams which we have decided to use are shown in Figure 2. Here we have removed pins A and B from Member 1 to show exactly the force the problem asks us to solve. FBD I involves 5 unknowns. Therefore, we require 5 independent equations to solve for all forces on Member 1. We can write only 3 independent equations for FBD I. The second free-body diagram is drawn, FBD II, was chosen to show the same force components B_X and B_Y between pin B and Member 1. If we sum moments about pin C in FBD II, we will have an additional independent equation without adding more unknowns. With the third free-body diagram, FBD III, we may write an equation to determine the cable tension. This gives us 5 independent equations with no additional unknowns; we can solve the problem.

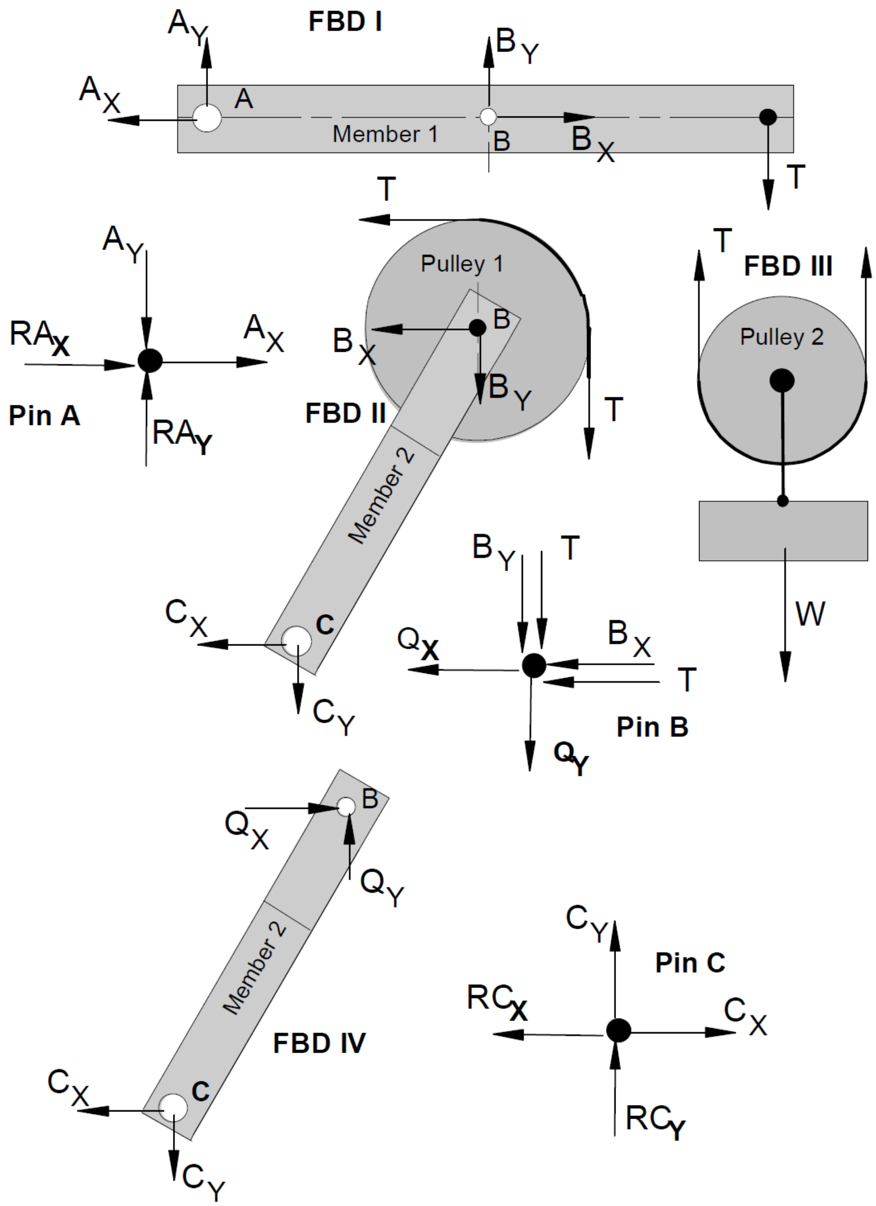


Figure 2. Two-dimensional free-body diagrams for the hoist frame structure.

1-3. ***Equilibrium Equations.*** From the free-body diagrams in Figure 2, we observe that there are 13 forces to be determined as a function of the known hoist dimensions (X_B, X_C, Y_A, R_1, R_2) and load (W) shown in Figure 1.

$$A_X,\ A_Y,\ B_X,\ B_Y,\ C_X,\ C_Y,\ T,\ Q_X,\ Q_Y,\ RA_X,\ RA_Y,\ RC_X \text{ and } RC_Y$$

This will require the 13 independent equations. We will employ the equation numbering, (Ph a.b), where 'a' represents the phase number and 'b' is the sequential equation number in the phase. This numbering makes it easier for the student to know what equations are associated with each phase when implemented into an engineering tool. The independent equilibrium equations are as follows:

$$\text{FBD III}, \sum F_Y \; : \; 2T = W \qquad \text{(Ph1.1)}$$
$$\text{FBD I}, \sum F_X \; : \; A_X = B_X \qquad \text{(Ph1.2)}$$
$$\text{FBD I}, \sum F_Y \; : \; A_Y + B_Y = T \qquad \text{(Ph1.3)}$$
$$\text{FBD I}, \sum M_A \; : \; B_Y(X_B) = T\,(X_B + R_1 + 2R_2) \qquad \text{(Ph1.4)}$$
$$\text{FBD II}, \sum M_C \; : \; T(Y_A + R_1) + B_X(Y_A) = B_Y(X_B - X_C) + T(R_1 + X_B - X_C) \qquad \text{(Ph1.5)}$$

One should note that the Equilibrium Equations (Ph1.1) through (Ph1.5) determine the pin forces on Members 1 and 2, in terms of the applied load and dimensions.

The equations to determine the components C_X and C_Y are:

$$\text{FBD II}, \sum F_X \; : \; C_X + B_X + T = 0 \qquad \text{(Ph1.6)}$$
$$\text{FBD II}, \sum F_Y \; : \; C_Y + B_Y + T = 0 \qquad \text{(Ph1.7)}$$

In addition, we can determine the support reaction forces on Pins A and C, and the contact force components between Member 2 and pin B:

$$\text{FBD Pin A}, \sum F_X \; : \; A_X + RA_X = 0 \qquad \text{(Ph1.8)}$$
$$\sum F_Y \; : \; RA_Y = A_Y \qquad \text{(Ph1.9)}$$
$$\text{FBD Pin B}, \sum F_X \; : \; Q_X + B_X + T = 0 \qquad \text{(Ph1.10)}$$
$$\sum F_Y \; : \; Q_Y + B_Y + T = 0 \qquad \text{(Ph1.11)}$$
$$\text{FBD Pin C}, \sum F_X \; : \; C_X = RC_X \qquad \text{(Ph1.12)}$$
$$\sum F_Y \; : \; C_Y + RC_Y = 0 \qquad \text{(Ph1.13)}$$

1-4. ***Deformation Equations.*** Not applicable.

1-5. ***Compatibility and Boundary Conditions.*** Not applicable.

1-6. ***Complementary and Supporting Formulas.*** Not applicable.

1-7. ***Solve.*** Solving the 13 equations for the 13 unknowns yields:

$$A_X = 718.75\,lbf;\; A_Y = -625.00\,lbf;\; B_X = 718.75\,lbf;\; B_Y = 1125.00\,lbf;$$

$$C_X = -1218.75\,lbf;\; C_Y = -1625.00\,lbf;\; T = 500.00\,lbf;$$

$$Q_X = -1218.75\,lbf;\; Q_Y = -1625.00\,lbf;\; RA_X = -718.75\,lbf;$$

$$RA_Y = -625.00\, lbf;\ RC_X = -1218.75\, lbf;\ RC_Y = 1625.00\, lbf$$

These equations can be easily solved by hand or can be entered into the engineering tool in the symbolic form as shown in Equations (Ph1.1) through (Ph1.13) and the known values inputted.

1-8. ***Verify.*** The students use their hand solution to verify the engineering tool solution. Part 2 will also discuss how limiting cases can be used for verification.

Part 2 – Three-dimensional Pin Analyses

The steps to carry out the analysis of Part 2 of Phase 1 are as follows:

2-1. ***Model.*** The pins in Figure 2 were shown as two-dimensional to be consistent with the frame members and to simplify the equilibrium analysis. In reality, pins should be modeled as three-dimensional to reflect the member forces transferred at the connection. As stated in Part 1, the structure and the loading are symmetric about a common plane. Members 1 and 2 are uniform and straight, and the pin centers are on the centroidal axis of each member. The pins will be analyzed for direct shear only, and friction is negligible.

2-2. ***Free-Body Diagrams.*** In this problem, we have three different pins, each with specific regions of concern and two force components on each region cross section as shown in Figure 3.

To obtain the maximum interior shear force in a pin, we must understand the construction of the pin joint, and we must determine the axial variation of the internal pin shear force. In Figure 3, each of the three pins, A, B, and C, have been drawn with the pin joint interior forces, in component form, applied by the structure member and frame foundation support. The structure is symmetrical about the XY plane, resulting in equal distribution of the pin forces on each side of the plane of symmetry. Also shown is the exposed cross section of each pin with shear force components acting on the interior cross sections of concern. For pins A and C, only one cross section is of concern. Pin B, however, must be analyzed in two different regions, region DE and region CD.

In this problem, we have three different pins, each with specific regions of concern and two force components on each region cross section. If the analysis is to be programmed, we must have a notation scheme that identifies each shear force. We use the following notion:

$$F_{sAB_X}^{(P)}$$

which defines the shear force, F_S in the X direction in the region between A and B of pin (P).

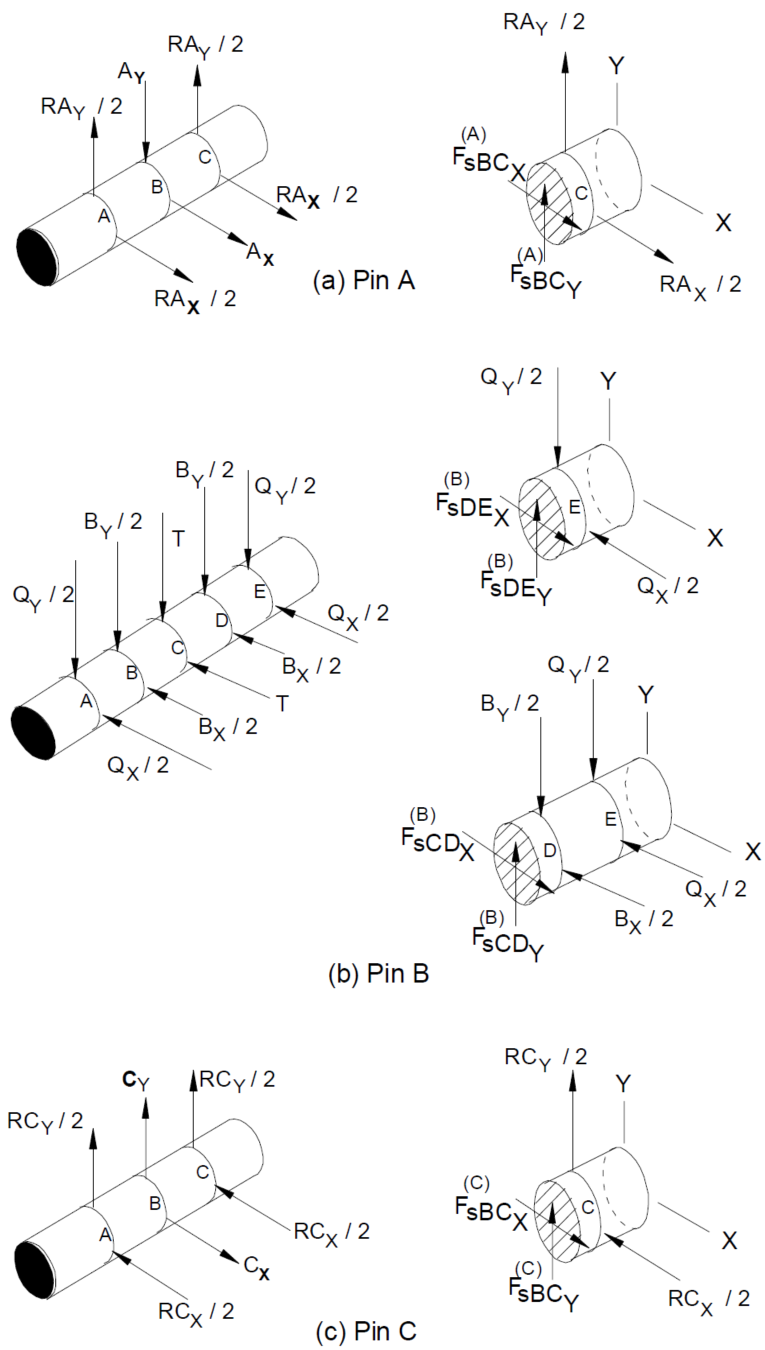


Figure 3. Three-dimensional pin free-body diagrams and cross-sections of maximum shear force.

2-3. ***Equilibrium Equations.*** The forces on the free-body diagrams shown on the right-hand side of Figure 3 are known based on the previous equilibrium analysis in Part1. The free-body diagram on the right-hand side in Figure 3 is used to determine the unknown internal shear forces. The pins on the right-hand side free-body diagrams are cut where the maximum internal shear forces occur. One should note that due to symmetry, only one side of the pin is considered. Therefore, we have the following 8 unknown internal shear force components on the pin cross-sections:

$$F^{(A)}_{sBC_X},\ F^{(A)}_{sBC_Y},\ F^{(B)}_{sDE_X},\ F^{(B)}_{sDE_Y},\ F^{(B)}_{sCD_X},\ F^{(B)}_{sCD_Y},\ F^{(C)}_{sBC_X} \text{ and } F^{(C)}_{sBC_Y}$$

From the sectioned pin free-body diagrams in Figure 3, the following 8 independent equilibrium equations are obtained for the internal shear force components in each pin.

- **Pin A Internal Shear Force.**

$$\text{FBD Pin A,} \sum F_X \quad : \quad F^{(A)}_{sBC_X} + \frac{RA_X}{2} = 0 \qquad \text{(Ph1.14)}$$

$$\sum F_Y \quad : \quad F^{(A)}_{sBC_Y} + \frac{RA_Y}{2} = 0 \qquad \text{(Ph1.15)}$$

The resultant maximum shear force acting on the pin cross section must be determined, and it is obtained by the vector sum of the orthogonal components. For pin A:

$$F^{(A)}_s = \sqrt{\left(F^{(A)}_{sBC_X}\right)^2 + \left(F^{(A)}_{sBC_Y}\right)^2} \qquad \text{(Ph1.16)}$$

which is always a positive quantity.

- **Pin B Internal Shear Force.** The shear force in pin B must be investigated in two different regions to identify the maximum shear force.

    - **In region DE:**

$$\text{FBD Pin B,} \sum F_X \quad : \quad F^{(B)}_{sDE_X} = \frac{Q_X}{2} \qquad \text{(Ph1.17)}$$

$$\sum F_Y \quad : \quad F^{(B)}_{sDE_Y} = \frac{Q_Y}{2} \qquad \text{(Ph1.18)}$$

The resultant force on the cross section is:

$$F^{(B)}_{sDE} = \sqrt{\left(F^{(B)}_{sDE_X}\right)^2 + \left(F^{(B)}_{sDE_Y}\right)^2} \qquad \text{(Ph1.19)}$$

    - **In region CD:**

$$\text{FBD Pin B,} \sum F_X \quad : \quad F^{(B)}_{sCD_X} = \frac{Q_X}{2} + \frac{B_X}{2} \qquad \text{(Ph1.20)}$$

$$\sum F_Y \quad : \quad F^{(B)}_{sCD_Y} = \frac{Q_Y}{2} + \frac{B_Y}{2} \qquad \text{(Ph1.21)}$$

The resultant force on the cross section is:

$$F_{sCD}^{(B)} = \sqrt{\left(F_{sCD_X}^{(B)}\right)^2 + \left(F_{sCD_Y}^{(B)}\right)^2} \qquad \text{(Ph1.22)}$$

The maximum shear force in the pin B is the maximum vector sum of the components at the two cross sections and may be determined using a common equation solver programming statement such as:

$$F_s^{(B)} = \max(F_{sDE}^{(B)}, F_{sCD}^{(B)}) \qquad \text{(Ph1.23)}$$

- **Pin C Internal Shear Force.**

$$\text{FBD Pin C, } \sum F_X \; : \; F_{sBC_X}^{(C)} + \frac{RC_X}{2} = 0 \qquad \text{(Ph1.24)}$$

$$\sum F_Y \; : \; F_{sBC_Y}^{(C)} + \frac{RC_Y}{2} = 0 \qquad \text{(Ph1.25)}$$

The resultant force on the cross section is:

$$F_s^{(C)} = \sqrt{\left(F_{sBC_X}^{(C)}\right)^2 + \left(F_{sBC_Y}^{(C)}\right)^2} \qquad \text{(Ph1.26)}$$

Equations (Ph1.14) through (Ph1.26) will determine the shear force components and the resultant maximum shear force on the pin cross section.

2-4. ***Deformation Formulas.*** Not applicable.

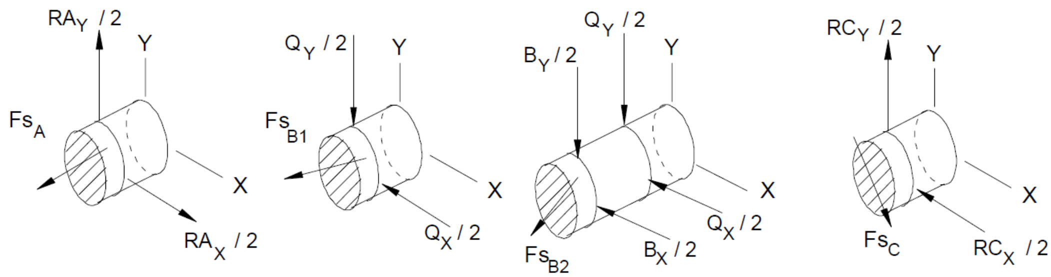
2-5. ***Compatibility and Boundary Conditions.*** Not applicable.

2-6. ***Complementary and Supporting Formulas.*** Not applicable.

2-7. ***Solve.*** Equations (Ph1.14) through (Ph1.26) contains 8 independent equilibrium equations and 5 equations to determine the resultant shear forces. Equations (Ph1.14) through (Ph1.26) can be solved by hand or using an engineering tool to determine the shear force components and the resultant maximum shear force on the pin cross section as follows:

Pin A:	$F_{sBC_X}^{(A)}$ = 359.37 lbf,	$F_{sBC_Y}^{(A)}$ = 312.50 lbf,	$F_s^{(A)} =$ 476.24 lbf
Pin B, Segment DE:	$F_{sDE_X}^{(B)}$ = 609.37 lbf,	$F_{sDE_Y}^{(B)}$ = -812.50 lbf,	$F_s^{(B)} = F_{sDE}^{(B)} =$ 1015.63 lbf
Pin B, Segment CD:	$F_{sCD_X}^{(B)}$ = -250.00 lbf,	$F_{sCD_Y}^{(B)}$ = 250.00 lbf,	$F_{sCD}^{(B)} =$ 353.55 lbf
Pin C:	$F_{sBC_X}^{(C)}$ = 609.37 lbf,	$F_{sBC_Y}^{(C)}$ = -812.50 lbf,	$F_s^{(C)} =$ 1015.63 lbf

The maximum resultant shear force for Pin B is in Segment DE. Figure 4 shows the four free-body diagrams used to determine the maximum shear force for the three pins.



(a) Pin A FBD (b) Pin B FBD Section 1 (c) Pin B FBD Section 2 (d) Pin C FBD

Figure 4. Pin free-body diagrams of maximum shear force.

2-8. ***Verify.*** Although we will substitute these 26 equations into an engineering equation solver program, verifying the computer's output is always necessary. One of the primary tests that we should run is a hand calculation. In this case, the 26 numbered equations can be solved by hand; the individual equations are elementary. Once the program has been tested to show that the results are correct, this model may be used for very convenient, rapid, and reliable recalculation of the forces for any changes in the dimensions and/or applied load.

Let's look at two limiting cases to verify just the equilibrium equations:

- $X_C = X_B$, $R_1 = 0 = R_2$. Member 2 is vertical and force T on Member 1 is at pin B. This would yield $B_X = 0$ and $B_Y = W/2$.

- $X_C = X_B$, $R_1 = 0$, $2R_2 = X_B$. Member 2 is vertical and force T is at end of Member 1. This would yield $B_X = 0$ and $B_Y = 2T$.

- Cable force T in compression. We assumed all cables in this model are rigid. A real cable can only be in tension. If we reverse the direction of the load W to upward in Figure 1, then the forces will change direction. We could check on whether the cable is in tension or compression and add a flag for the compression case.

The first student assignment consisted of carrying out a hand analysis of Phases 1 and 2 and they are required to demonstrate different ways to verify their analysis. After this assignment is submitted, we review how to verify the analysis.

Remark. In a typical design process of a prototype, it is not uncommon to have suggested, or required changes in the configuration or load capacity of the structure after much work has progressed in the design. This may mean that all calculations previously done have to be redone! To have the analysis programmed in a symbolic form means that any changes can be implemented very quickly. Also, if a product has a general configuration which can be custom tailored to a customer's specifications, the symbolic model provides a quick and accurate determination of the product's specific details.

Appendix B: Phase 2 – Stress Analysis

Using the results from the Phase 1 force analysis, Phase 2 will be divided into the following three parts:

- Part 1 – Maximum Shear Stress in the Pins;
- Part 2 – Maximum Bearing Stress at each Pin Joint, and;
- Part 3 – Maximum Axial Normal Stress in Centrically Loaded Member 2.

Step 6, Complementary and Supporting Formulas, is required to determine the stress in Parts 1 through 3. We will first develop the equations for Parts 1 through 3, and then solve for the stresses in Phase 2 using Step 7, Solve.

Part 1 – Maximum Shear Stress in the Pins

2-6. ***Complementary and Supporting Formulas.*** From the Phase 1 analysis, we determined the maximum shear force in each of the three pins A, B and C as shown in Figure 3. Maximum shear stress in pins A, B and C in terms of the diameters. The shear stress in each pin requires the cross-section diameter and the internal shear force. The unknown pin diameters will be represented by d_A, d_B and d_C, where the subscript identifies the pin location, and the maximum shear force in each pin, F_{s_A}, F_{s_B}, and F_{s_C}, respectively, ill have been determined in Phase 1. The shear stress in each pin is determined as follows:

$$\tau_A = \frac{4F_{sA}}{\pi d_A^2} \qquad \text{(Ph2.1)}$$

$$\tau_B = \frac{4F_{sB}}{\pi d_B^2} \qquad \text{(Ph2.2)}$$

$$\tau_C = \frac{4F_{sC}}{\pi d_C^2} \qquad \text{(Ph2.3)}$$

A test will be carried out to prove that the Member 2 pin forces are collinear with its pin centers, (since Member 2 is a two-force member). A comparison of slopes of the pin forces F_C and F_Q with the slope of Member 2 should be made as a test; the slopes must be equal. The slope of Member 2 can be obtained from geometry in Figure 1:

$$\theta_2 = \tan^{-1}\frac{Y_A}{X_B - X_C} \qquad \text{(Ph2.4)}$$

The slope of force F_C in Figure 5 is:

$$\theta_{F_C} = \tan^{-1}\frac{C_Y}{C_X} \qquad \text{(Ph2.5)}$$

The slope of force F_Q in Figure 5 is:

$$\theta_{F_Q} = \tan^{-1}\frac{Q_Y}{Q_X} \qquad \text{(Ph2.6)}$$

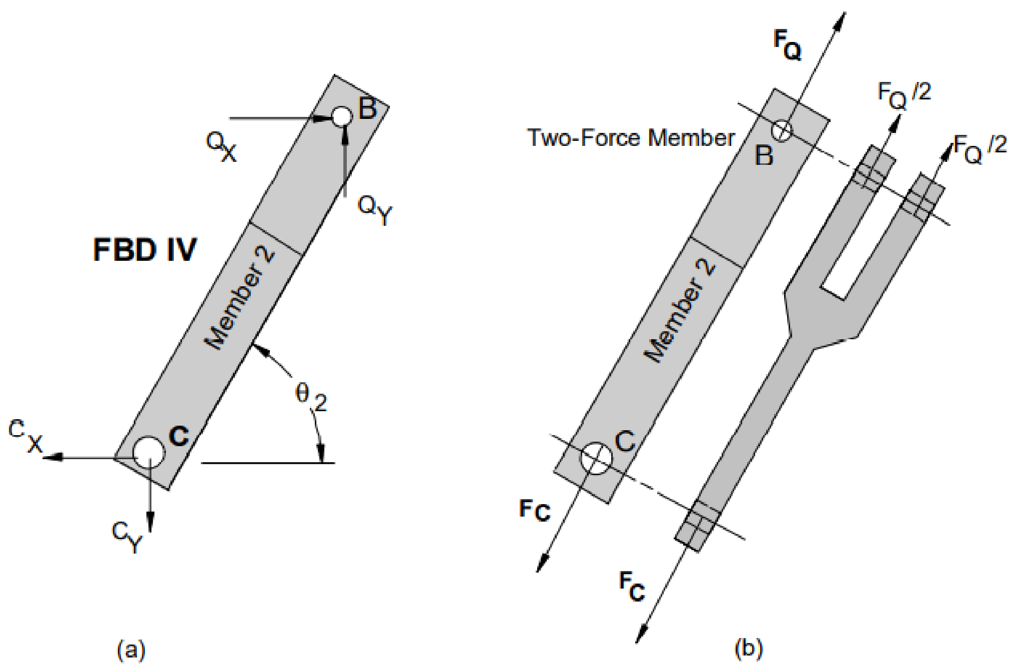


Figure 5. Member 2 free-body diagram.

Part 2 – Maximum Bearing Stress at each Pin Joint

2-6. ***Complementary and Supporting Formulas.*** The bearing stresses in pins A, B, and C, and Members 1 and 2 in terms of the dimensions as indicated in Figures 6 and 7, respectively.

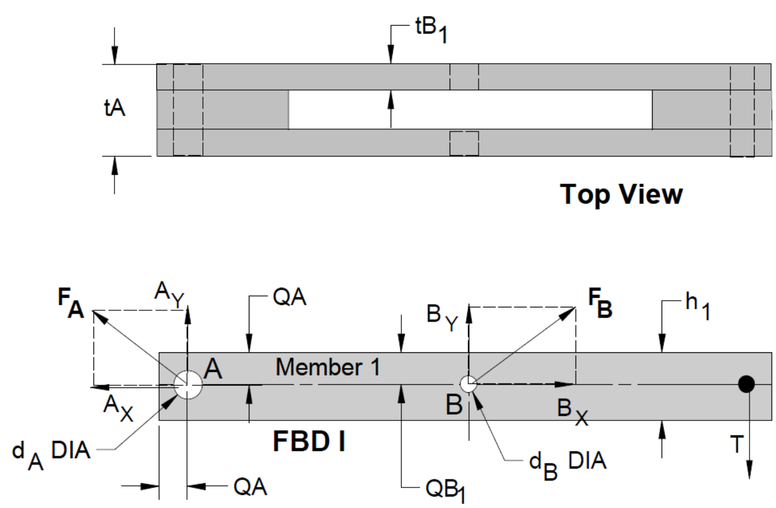


Figure 6. Member 1 dimensions and free-body diagram.

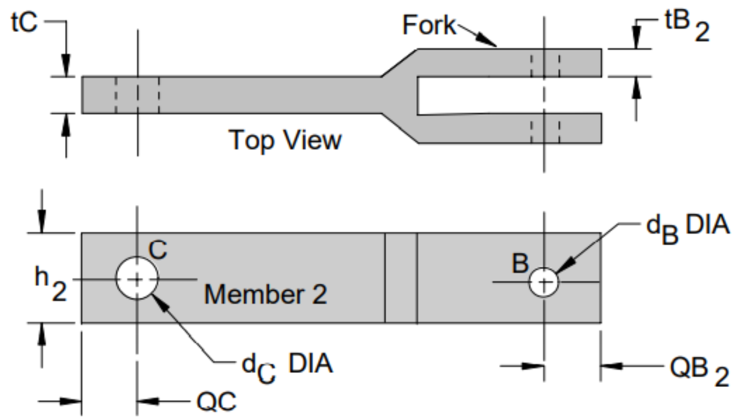


Figure 7. Member 2 dimensions.

The bearing stress between pin A and the beam, Member 1 requires the magnitude of the force between pin A and the Member 1 in Figure 6:

$$F_A = \sqrt{A_X^2 + A_Y^2} \tag{Ph2.7}$$

and the bearing stress for a Loose-Fitting Pin in Equation (C.3) is:

$$\sigma_{brg_A} = 1.3\frac{F_A}{d_A\,(tA)} \tag{Ph2.8}$$

We assumed a Loose-Fitting Pin versus a Tight-Fitting Pin since it is conservative, as discussed in Appendix C. In the course, we develop the bearing stress equations and discuss the difference between a Loose- and Tight-Fitting Pin and when to apply it in practice.

The bearing stress between pin B and the beam, Member 1 requires the magnitude of the force between pin B and Member 1 in Figure 6:

$$F_B = \sqrt{B_X^2 + B_Y^2} \tag{Ph2.9}$$

and the bearing stress for a Loose-Fitting Pin is:

$$\sigma_{brg_{B1}} = 1.3\frac{\frac{F_B}{2}}{d_B\,(tB_1)} \tag{Ph2.10}$$

The bearing stress between pin B and Member 2 requires the magnitude of the force between pin B and the Member 2 in Figure 7:

$$F_Q = \sqrt{Q_X^2 + Q_Y^2} \tag{Ph2.11}$$

and the bearing stress for a Loose-Fitting Pin is:

$$\sigma_{brg_{B2}} = 1.3\frac{\frac{F_Q}{2}}{d_B\,(tB_2)} \qquad \text{(Ph2.12)}$$

The bearing stress between pin C and Member 2 requires the magnitude of the force between pin C and the Member 2 in Figure 7:

$$F_C = \sqrt{C_X^2 + C_Y^2} \qquad \text{(Ph2.13)}$$

and the bearing stress for a Loose-Fitting Pin is:

$$\sigma_{brg_C} = 1.3\frac{F_C}{d_C\,(tC)} \qquad \text{(Ph2.14)}$$

Part 3 – Maximum Axial Normal Stress in Centrically Loaded Member 2

2-6. ***Complementary and Supporting Formulas.*** The cross-section normal stress in Member 1 resulting from the axial load and the normal stress in the two-force Member 2 in terms of the symbolic dimensions are shown in Figure 7.

The cross-section stresses require the dimension 'h' of Members 1 and 2, and the allowable minimum dimension is dependent on the pin diameters. We may specify a preliminary value of this dimension for each member, and this will be input to the calculation as variable h_{1_init} and h_{2_init}.

The location of the pins from the outer edges of Members 1 and 2 will be guided by the recommendation in Appendix D that there be at least one hole diameter between the edge of the bar or plate and the edge of the hole. Thus, for pin A and Member 1 from Equation (D.4),

$$QA = 1.5d_A \qquad \text{(Ph2.15)}$$

For the location of pin B in Member 1:

$$QB_1 = 1.5d_B \qquad \text{(Ph2.16)}$$

Given that h_{1_init} is a proposed initial dimension, the minimum 'h' dimension for the uniform cross section of Member 1 would be the maximum of the three conditions:

$$h_1 = \max(h_{1\_init}, 2\max(QA, QB_1)) \qquad \text{(Ph2.17)}$$

For the location of pins B and C in Member 2:

$$QB_2 = 1.5d_B \quad \text{(Ph2.18)}$$
$$QC = 1.5d_C \quad \text{(Ph2.19)}$$

Likewise, given that h_{2_init} is a proposed initial dimension, the minimum 'h' dimension for the cross-section of Member 2 would be:

$$h_2 = \max(h_{2\_init}, 2\max(QB_2, QC)) \quad \text{(Ph2.20)}$$

In Member 1, the axial component of the pin forces at A and B would result in uniform normal stress if the pin centers are located at the centroid of the member's cross-section. However, this stress is not the only stress on the cross-section because the pin force resultants are not collinear with the centroidal axis of the member. Therefore, we may calculate this contribution to the stress realizing that this is a combined loading case considered in mechanics of materials course.

Note, if the axial force component B_X on Member 1 is tensile, the maximum normal stress must be determined by calculating two pins and the section between points A and B.

$$\sigma_{1axialA} = \frac{B_X}{(h_1 - d_A)(tA)} \quad \text{(Ph2.21)}$$
$$\sigma_{1axialB} = \frac{\frac{B_X}{2}}{(h_1 - d_B)(tB_1)} \quad \text{(Ph2.22)}$$

If the force component B_X is positive (tension in section from A to B):

$$\sigma_{1axial} = \frac{\frac{B_X}{2}}{h_1(tB_1)} \quad \text{(Ph2.23b)}$$

The normal stress in the fork of Member 2 is determined from the forces F_Q and F_C, which may be tensile or compressive. The sense of the forces can be determined by the sign of either of its components, B_X, B_Y, or C_X, C_Y. If the force is tensile, ($B_X > 0$, etc.),

$$\sigma_{2axialB} = \frac{\frac{F_Q}{2}}{(h_2 - d_B)(tB_2)} \quad \text{(Ph2.24)}$$
$$\sigma_{2axialC} = \frac{F_C}{(h_2 - d_C)(tC)} \quad \text{(Ph2.25)}$$

$$\sigma_{2axial} = \max(\sigma_{2axialB}, \sigma_{2axialC}) \quad \text{(Ph2.26a)}$$

If the forces are compressive, ($B_X < 0$, etc.), the normal stress in the center section of Member 2 is:

$$\sigma_{2axial} = \frac{F_C}{h_2\,(tC)} \qquad \text{(Ph2.26b)}$$

Equations (Ph2.1) through (Ph2.26) can now be coded into a software program and the solution can be found in the next step.

2-7. ***Solve.*** We can now calculate the pin shear stresses, the bearing stresses, and the axial uniform normal stresses in the centrically loaded Member 2 for any input values of dimensions and load Solving Phase 2. There are 26 equations for the 26 unknown stresses and angles in Equations (Ph2.1) through (Ph2.26). The solution can be easily solved by hand or using an engineering tool yields:

$\tau_A =$	4,312 psi	$\tau_B =$	827.6 psi
$\tau_C =$	2,298.9 psi	$\theta_2 =$	53.13 deg
$\theta_{F_C} =$	53.13 deg	$\theta_{F_Q} =$	53.13 deg
$F_A =$	952.5 lbf	$\sigma_{brg_A} =$	1,100.6 psi
$F_B =$	1,335 lbf	$\sigma_{brg_{B1}} =$	1,851.2 psi
$F_Q =$	2,031.3 lbf	$\sigma_{brg_{B2}} =$	2,112.5 psi
$F_C =$	2,031.3 lbf	$\sigma_{brg_C} =$	4,694 psi
$QA =$	0.56 in	$QB_1 =$	1.88 in
$QB_2 =$	1.88 in	$QC =$	1.13 in
$h_1 =$	3.8 in	$h_2 =$	3.8 in
$\sigma_{1axialA} =$	71 psi	$\sigma_{1axialB} =$	383.3 psi
$\sigma_{1axial} =$	383.3 psi	$\sigma_{2axialB} =$	534.5 psi
$\sigma_{2axialC} =$	712.7 psi	$\sigma_{2axial} =$	-722 psi

where the solution is presented above is the order of Equations (Ph2.1) through (Ph2.26).

Appendix C: Pin in Hole Bearing Stress

Referring to Figure C.1(a) and (b), a pin of diameter d is bearing against the plate of thickness t with a resultant force of magnitude R. Our objective is to determine the magnitude of the compressive stress exerted by the pin surface on the hole wall and, likewise, the hole wall on the pin surface.

First, we will assume a *Sung-Fitting Pin* based on the following:

- Assume a snug fit of the pin in the hole to ensure that the pin contacts the hole wall surface over the complete semicircular arc.

- Assume that the compressive stress (pressure), p, of the hole surface against the pin surface (and the pin against the hole wall surface) is uniform over the arc of the contacting surfaces and acts normal to the contacting surfaces as shown in Figure C.1(d). We recognize this pressure as the compressive bearing stress on the pin and plate circular surfaces.

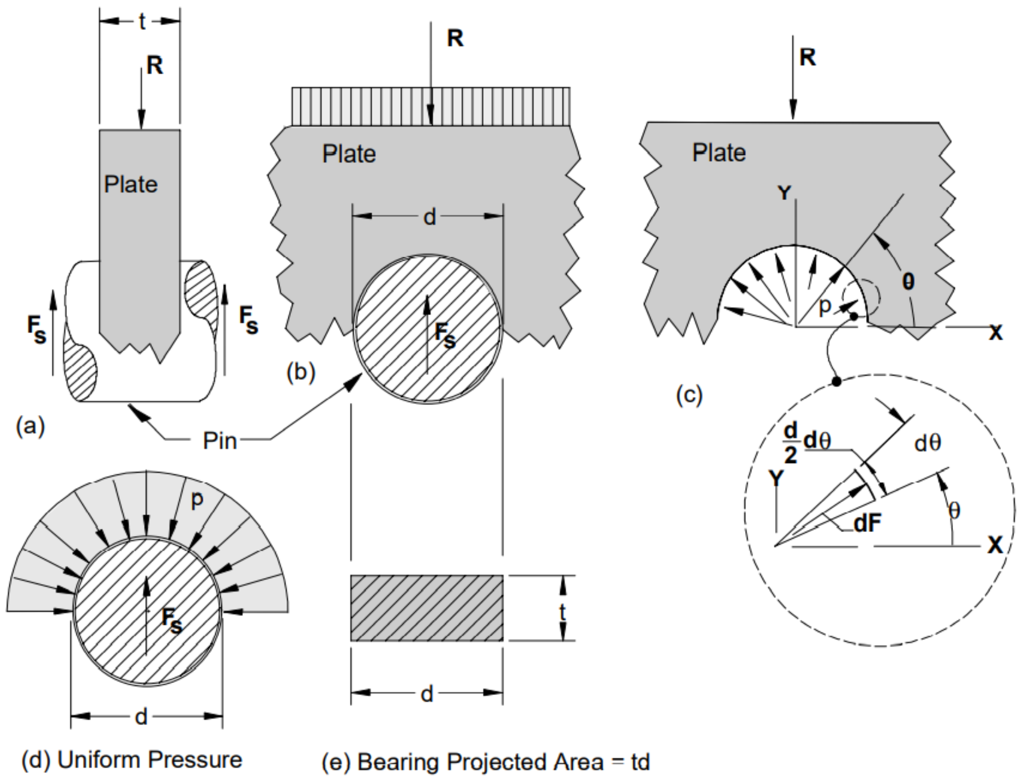


Figure C.1. Pressure distribution for a Snug-Fitting Pin.

From Figure C.1, one can determine the magnitude of the pressure, and the compressive bearing stress for a *Snug-Fitting Pin* as:

$$p = \sigma_{brg} = \frac{R}{td} \qquad \text{(C.1)}$$

Note that since the stress is force divided by an area, the area used for the contact stress in this pin connection is the pin diameter times the hole depth. This is the rectangular area shown in Figure C.1(e), commonly referred to as the bearing projected area.

Second, we will assume a *Loose-Fitting Pin* based on the following:

- Assume a close, but loose pin fit in the hole so that the pin does not make firm contact with the hole wall surface over the complete semicircular arc. This would be reasonable for a bearing application where relative rotational motion is expected.

- Assume that the pressure, p, of the pin against the hole wall surface, is non-uniform over the arc of the contacting surfaces. A sinusoidal function may easily describe this.

$$p = p_m \sin\theta \qquad \text{(C.2)}$$

where the pressure p is zero on both sides ($\theta = 0^o$, and 180^o), and p_m is the maximum pressure occurring at $\theta = 90^o$. This would be consistent with a loose pin fit in the hole. Figure C.2 illustrates the form of this pressure distribution.

The expression for the maximum pressure and maximum bearing compressive stress for a *Loose-Fitting Pin* is:

$$p_m = \sigma_{brg} = 1.27\frac{R}{td} \qquad \text{(C.3)}$$

A *Loose-Fitting Pin* can occur in the construction of the connection. A loose fit can also happen when a significant difference in the plate and pin material and the load is applied, e.g., one is steel, and the other is plastic.

The implication of all of this is that a conservative estimate of the bearing compressive stress on the pin or hole surface for a close-fitting pin should be somewhere between one and 1.3 times the resultant load divided by the projected area.

$$\frac{R}{td} \le \sigma_{brg} \le 1.3\frac{R}{td} \qquad \text{(C.4)}$$

The least conservative lower value, R/td, for a *Snug-Fitting Pin* is most used. We assume a *Loose-Fitting Pin* since is it conservative.

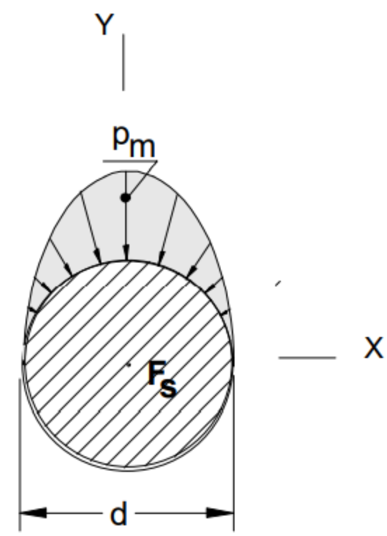


Figure C.2. Pressure distribution for a Loose-Fitting Pin.

Appendix D: Shear Tearout Stress in a Pin Connection

When a structural bar or a plate is connected to another object with a pin (which could be a bolt, a rivet, a straight or tapered shear pin, etc.), consideration must be given to the location of the pinhole relative to the edge of the plate. Figure. D.1 shows a bar (or plate) that is secured by means of a pin of diameter d, and the hole center is located at a distance Q from the edge of the bar. The bar is of thickness t. Due to the applied force F, consider the possibility of the pin shearing out the section of the bar to the left of the pin, as shown in Figure D.1(a). (This might be easier to imagine if the bar were made of wood with the grain was running parallel to its longitudinal axis.)

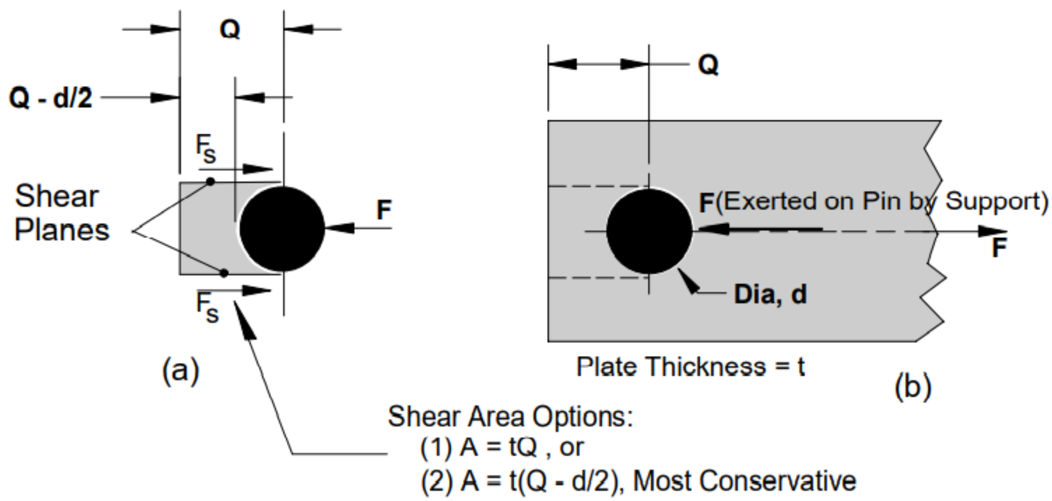


Figure D.1. Shear tearout forces in pin connection.

An estimate of average shear stress is very simple. From the free-body diagram of the sheared-out piece in Figure D.1(a), the shearing force on each of the two shear surfaces is

$$F_s = \frac{F}{2} \quad \text{(D.1)}$$

The area of each shear surface, however, is a matter of judgement. The full area of each surface would naturally cover the space between the hole center and the edge of the plate. However, if we assume that the small tail around the pin has minimal contribution to the strength of the sheared-out piece, then the conservative analyst would opt for using the area corresponding to the space between the edge of the hole and the edge of the bar. We will elect to use that area, so each shear area is

$$A_s = \left(Q - \frac{d}{2}\right) t \quad \text{(D.2)}$$

The uniform average shear tearout stress on this area is calculated using:

$$\tau_{to_{avg}} = \frac{F_s}{A_s} = \frac{F}{2\left(Q - \frac{d}{2}\right)t} \quad \text{(D.3)}$$

where F is the pin force with a line of action normal to the outer edge of the bar. It is assumed that the shear stress, $\tau_{to_{avg}}$ same on the top and bottom shear planes of Figure D.1.

As we will show the students in a follow-up mechanics of material course, the shear strength of most engineering structural materials is approximately half of the normal tensile or compressive strength. If we then say that the maximum shear tearout stress should be no larger than one-half of the low value of the predicted range of compressive bearing stress, we might get a reasonable approximation for the bar dimension, Q – d/2 the bar.

$$\begin{aligned} \tau_{to_{avg}} &= \frac{F}{2\left(Q - \frac{d}{2}\right)t} = \frac{1}{2}\sigma_{brg} = \frac{1}{2}\frac{F}{td} \\ Q - \frac{d}{2} &= d \\ Q &= 1.5d \end{aligned} \quad \text{(D.4)}$$

Thus, a reasonable recommendation, rule of thumb, would be to have the edge of the hole at least one hole diameter away from the outer edge of the bar or plate as shown in Figure D.1 by Q.