Work in Progress: Promoting the Transfer of Math Skills to Engineering Statics

Dr. Alexander John De Rosa, University of Delaware

Alex De Rosa is an Associate Professor in the Department of Mechanical Engineering at the University of Delaware. Alex received his Ph.D. in Mechanical Engineering from The Pennsylvania State University in 2015 where he studied combustion instabilities in gas turbine engines and also served as a Graduate Teaching Fellow. His research focuses on improving the educational experience through the creation and promotion of new teaching tools and techniques. Alex is particularly interested in the areas of deeper learning and knowledge transfer, where he is working to help students better apply their knowledge and skills in new contexts, including in their future careers.

Dr. Teri Kristine Reed, University of Oklahoma

Teri K. Reed is the inaugural Director of the OU Polytechnic Institute and Professor and George Kaiser Family Foundation Chair at OU-Tulsa.

Angela E. Arndt

Work in Progress: Promoting the Transfer of Math Skills to Engineering Statics

Abstract
Students often face difficulties in transferring concepts, knowledge and skills between their courses. This difficulty is especially true of the fundamental mathematics and science courses that are often taught outside the major of the student and without engineering context. At the same time, graduating engineers are moving into an increasingly interdisciplinary workplace that values the ability to work broadly across a range of contexts. More work is needed to better prepare students to adapt their knowledge and skills to new situations and to demonstrate how the various courses and concepts within their curricula relate.

In this study, we ask students, teaching assistants and faculty to "think aloud" through their solution to an engineering statics problem that requires mathematical knowledge to be transferred in order to be solved. Two faculty, two teaching assistants and seven undergraduate students are interviewed as they think aloud through the problem. Interview transcripts and solutions to the statics problem are then examined for themes and patterns in responses in order to draw conclusions about the challenges different populations face in transferring knowledge and solving such problems.

Observations indicated that students could apply simple integration skills to find the area of a shape when given a curve describing its shape, but could not use integration to find the centroid. The participants did however recall being taught how to calculate centroids in the past and discussed a lack of usage of this skill causing their inability to recall it correctly. Student participants in general displayed simple approaches to problem solving based on reading the problem statement rather than following an engineering approach starting with governing equations. A potential barrier to problem solving success was identified in the varying symbols used by different research participants which could lead to a lack of understanding if these symbols are not clearly explained and defined in a classroom setting.

Future work will further examine these themes, as well as developing prompts and activities to promote knowledge transfer and problem solving success.

Introduction
Engineering graduates are moving into a workplace that is increasingly interdisciplinary and that requires the transfer of knowledge and skills across a broad range of contexts. Various professional interest groups such as the ABET, the National Academy of Engineering (NAE) and the American Society for Engineering Education (ASEE) have discussed the need for engineers of the future to be "T-shaped" professionals who possess a deep subject knowledge and the ability to apply that knowledge broadly [1-4] (the vertical of the T-shape refers to this depth of

knowledge and the horizontal refers to the ability to to transfer this knowledge to various applications).

A problem exists, however, in that engineering students often face difficulties in transferring knowledge and developing the connections that exist between concepts and courses in their program of study [5-7]. This deficiency is not often addressed by faculty teaching engineering courses, which are often taught in silos and where students typically do not learn the applications of the theory they learn, or how this work fits into the larger engineering context.

This deficit in student ability to link courses and concepts is often exacerbated by the fact that many fundamental STEM courses (typically mathematics, physics and chemistry) are taught by faculty from departments outside the major of the student. This lack of engineering context further hampers student ability to transfer learning from these courses and to see the connections between the concepts they learn in their fundamental courses and the engineering curriculum.

Given these reported difficulties in student ability to transfer knowledge, effort must be spent to develop activities to promote transfer and to develop the necessary connections between courses, concepts and applications that engineering professionals require in their careers. In this work-in-progress study, we present one aspect of a larger engineering research program that aims to develop such activities and promote knowledge transfer.

The goal of this larger program is to develop an intervention that promotes knowledge transfer and helps make the links between a student's courses more explicit. The study is based on prior research [8,9] that observed student difficulties in applying mathematical concepts in an engineering context and which piloted a 3-stage intervention aimed at promoting the transfer of knowledge from mathematics to an engineering course. The piloted intervention was based on findings from the literature in mathematics that suggested revisiting fundamental mathematical concepts and reinforcing them throughout the curriculum [10]. Thought was also given to the fact that while students may understand the math, they might not see how it is applied in a given context, or the approximations and assumptions they are required to make to solve a certain problem [11,12]. An intervention such as the one to be developed here, was suggested (but not explored) in the literature as a potential tool for remedying these problems [13,14].

In this work-in-progress paper we present the results of an emergent think aloud interview protocol [15-17] that examines student ability to transfer knowledge and the barriers they face in doing so while solving a typical engineering problem. The justification for this study is that in developing an intervention to promote knowledge transfer, one must first identify the correct prior knowledge to activate and transfer, as well as develop some understanding of the barriers students face in doing so.

Methodology
In this study, a think aloud protocol is used to explore student ability to transfer knowledge and the challenges they face in doing so.

Think alouds are generally defined as a research methodology in which participants talk through their thinking as they complete some (typically) predetermined task [15]. Think alouds as an activity were developed from the work of Vygotsky [18] who discussed the idea of "inner speech" and used the concept of thinking aloud in an attempt to develop an understanding of an individual's thought processes. Given that it is sometimes impossible to put exact words to more abstract thoughts and cognitive processes however, it is important to recognize that think alouds are an imperfect attempt to reveal one's thinking. Best practices for think aloud research therefore involves participants thinking verbally through some activity, and triangulating the results with supplementary data such as surveys or questionnaires that can be used to add context to the resulting transcripts [15].

In the current study, research participants were asked to solve an engineering problem as their think aloud activity. Transcripts of each interview were collected as well as the written problem solutions each participant generated. Institutional data was also obtained to provide information about various participant demographics. The problem chosen for the participants to solve in their think aloud interview was a rigid body equilibrium problem, Problem 9-20 from Hibbeler [19], which concerns the determination of the forces acting on an irregular shaped plate and is typical of an engineering statics problem. Crucially, this problem requires the transfer of mathematical skills to solve as integration must be applied to find both the area of the plate and its centroid (the position at which its weight can be assumed to act). The problem text and a diagram are shown in Figure 1.

A steel plate (density 490 lb/ft^3) has a thickness of 0.5 in. and is supported by a pin at A and a rope at B. Determine the magnitude of the reaction forces at the pin and the tension in the rope.

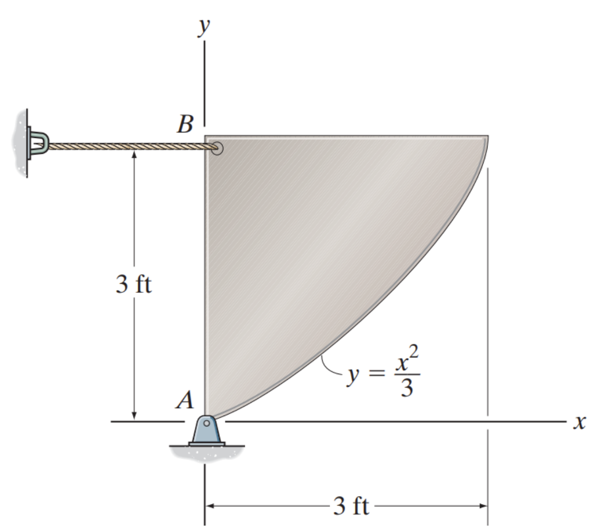

Figure 1: Engineering statics problem used in think aloud interviews [19].

The think aloud interviews were structured around a conceptual framework of knowledge transfer, as shown in Fig.2, that breaks down the process of transferring knowledge and solving a problem into five steps that were used as a guide for prompting students to understand their thinking in each of these five stages. For example, when participants are arriving at the "Current Representation" of the problem, examination of their free-body diagrams and asking questions such as "*What type of problem do you think this is?*" helps to reveal more specific thought processes and their approach to the problem.

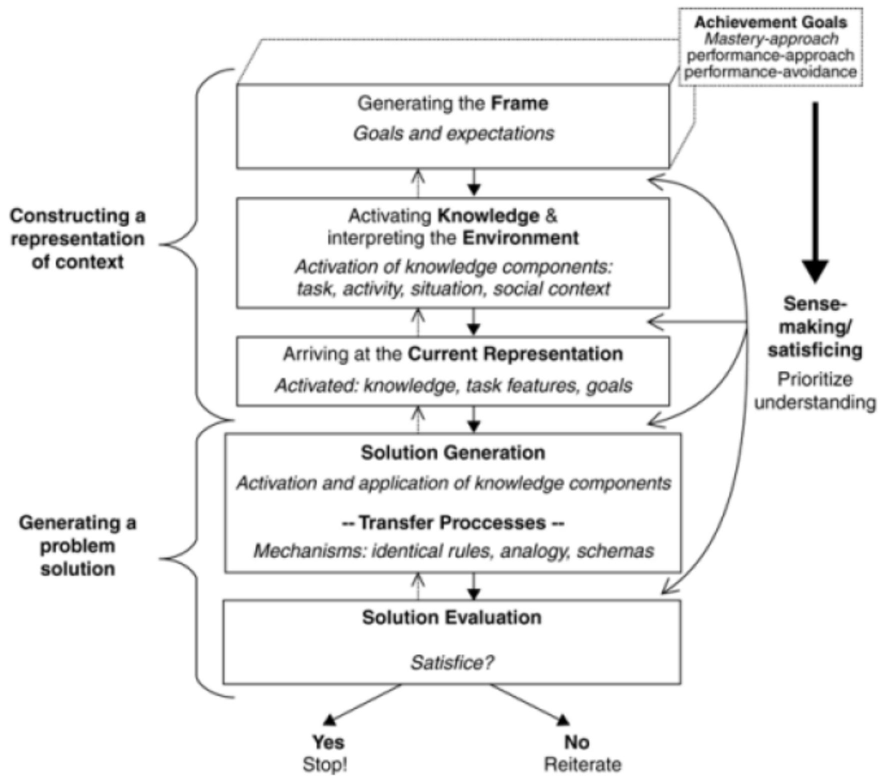

Figure 2: Five-stage sense making framework of knowledge transfer [20]

While the think aloud interviews did follow a structure guided by dimensions drawn from this conceptual framework, space was also given for the process to evolve naturally and for more emergent themes to be exposed. That is, a strict interview protocol was not adhered to for the think alouds and instead, the interviews were guided by the participant's approach to the problem. It is also important to note that the think aloud protocol evolved as interviews were conducted; as patterns of behavior or interesting approaches were employed by one or more participants, these behaviors were looked for and questioned in later interviews.

The research population consisted of 11 individuals. Subjects included two female faculty members, two female Teaching Assistants (TAs), and seven undergraduate students (three male and four female). Three of the undergraduates were from underrepresented minority (URM) groups. Six were sophomores and one was a junior. All are in the Mechanical Engineering program.

Data Analysis

The think aloud interviews were examined based on the stages of the chosen conceptual framework for knowledge transfer. These stages, their associated questions and observations from the think alouds, and the emergent themes and patterns in responses are identified in Table 1.

Table 1: Themes and patterns of responses in think aloud protocols.

Stage of Framework	**Guiding Questions**	**Themes & Patterns in Responses**
Generating the Frame	What type of problem is this? (i.e. do participants recognize the problem as a "rigid body equilibrium" or RBE?).	Participants generally did not distinguish this problem as anything other than a "statics problem". Only one student called this problem an RBE in addition to the faculty who both did so. The TAs from the engineering statics course were not able to identify this problem as an RBE.
	Do students recall the correct governing equations? (Sum of forces and moment balance).	The majority of participants (10/11) wrote the correct governing equations for this problem, though three students were notable in that they struggled conceptually with the problem and had issues understanding and setting up the problem in general. One (junior) student took the unique approach of writing their equations in vector form using *i,j,k* notation. When asked to explain this, the student discussed their love for dynamics and preference for this notation to keep things clean and in a single equation.
Activate Knowledge & Interpret the Environment	Do participants recognize the need to draw on their mathematical knowledge to find the area of the plate? (Integration).	Most participants (10 of 11) realized the need to integrate although most students inferred this from the fact that an equation for a curve (shape of plate) was given in the problem statement rather than determining this from their governing equations. This is perhaps an indication that most students are familiar with rote problem solving rather than using an engineering-thinking type approach.
	Do participants recognize the need to draw on their mathematical knowledge to find the centroid? (Integration).	Almost all participants (9 of 11) realized the need to find a location at which the weight acted and the need to draw on some mathematical skills to do so.
Current Representation	Can students draw the correct free body diagram?	Only one student failed to draw a correct free body diagram although several struggled in the process. These students who struggled with the diagram also struggled more generally with the problem and could not piece it together from a more conceptual standpoint. All (3) of these students who struggled had lower GPAs and scores in prior math courses compared to the other participants.

	What symbols and terminology do students use to represent the problem?	In the course of this study it was noticed that participants used different symbols to represent the density of the plate. Six of eleven subjects used rho(ρ), three used *D* and, perhaps most surprisingly, the course instructor who teaches the engineering statics course (that all of these students take) used gamma(γ). This emergent result was unexpected and is an important issue that will be factored into future work.
Solution Generation & Transfer Processes	Do students follow a logical approach to solving the problem? (i.e. typical "engineering problem solving method").	All of the students (except one course TA) preferred to calculate unknowns before writing down governing equations or understanding why they might need certain variables. For example, the lack of a given weight and the use of a density seemed to push students towards using the density for something even though they were, at that stage, unaware of what that something should be i.e. they had not written governing equations so did not know how the density would later be used. The faculty and the more senior TA examined in this work followed a much more routine, step-by-step approach to solving the problem.
	Can students integrate to find the area of the plate?	Ten of eleven participants were able to correctly determine the area of the plate although two students made minor errors in the process such as integrating the wrong area or making minor miscalculations that they later corrected. One student (URM, male, sophomore) took the unique approach of using a double integral to solve for the area of the plate. When questioned about this, the student referred to recently tutoring a friend in calculus so reverting to this method.
	Can students find the centroid of the plate?	Most students (7/9) recognized the need to find a centroid but could not do so. Interestingly,one student found a (non-correct) value close to the right answer by comparing the shape of the plate to known values for triangles and squares.
	Why did students have difficulty finding the centroid?	A common theme among student participants (6/9) was that they knew they needed to find the centroid but could not do so. Several students made reference to having been taught this process in prior courses but not being able to recall the method used. The two statics TAs interviewed in this process could not calculate the centroid. Both made reference to not using this skill in other courses and typically dealing with simple shapes (i.e. squares and rectangles) or using tabulated values of centroids. Neither had "prepped" for this topic in the statics course yet.

Solution Evaluation & Sense-Making	Do participants reflect on their work as they solve the problem?	Relatively few students reflected on any of their work as they progressed through the problem. Only two students were notable in their thinking through stages of the problem and whether they made sense, these students were recording musing to themselves on the consistency and correctness of their solution methods.
	Do participants evaluate the plausibility of their final solution?	Both faculty but only one student considered whether their solution made sense at the end of the problem. They did this by relating values used in the problem, their order of magnitude, and their eventual effect on the final numerical answer. Most students did not know how to relate their answer to other values that they could use to check their solution. The only (sophomore) student who performed this evaluation (and who got the closest to the correct answer) was the student with the highest cumulative GPA in this pool of student participants.

Findings & Discussion

The primary findings of the think aloud interviews are summarized and discussed below based on the stage of the conceptual framework (Fig.2) they relate to:

1. Generating the frame (goals and expectations):
Students only recognized this as a "statics problem" rather than an RBE but do generally recall the correct governing equations of an RBE.

Given that participants arrived at the correct governing equations for this problem the question arises as to whether it is important for students to be able to distinguish this problem more specifically as a rigid body equilibrium (RBE). In this study, students were observed to take more cues as to the nature of the problem from the problem statement and diagram rather than starting from more fundamental principles such as basic governing equations or a free body diagram. A secondary example of this would be the handful of students who also realized the need to integrate from the fact that an equation for a curve was given in the problem rather than examining their equations first to see what variables were needed. Based on these observations, it would appear that the participating students in this study are still more used to rote problem solving than employing an engineering approach grounded in fundamental concepts.

2. Activating knowledge and interpreting the environment
Participants recognized the need to draw on their prior, mathematical knowledge to solve the problem.

Ten of eleven participants recognized the need to use mathematical skills to solve this problem although, as discussed above, this realization was initially arrived at through the parameters of the problem statement rather than from examination of governing equations. This is particularly true of the need to integrate to find the area of the plate. In contrast, students' most often recognized the need to find the centroid only when employing their governing (moment) equations and needing a distance variable. It is possible that this behavior results from the calculation of the centroid being a higher level concept and one that is not obviously necessary in terms of the problem solution unless one starts their solution from the governing equations. Again, this observation indicates that students are more comfortable working with the problem statement and visual figures first, before later moving into engineering thinking. Importantly, most students (6/9) knew they needed to solve for the centroid but could not remember how. In this manner they had interpreted the problem correctly.

3. Arriving at the current representation
All participants drew free-body diagrams that were largely correct. An emergent observation was the varying symbols used to represent the density of the plate.

Students were well drilled in drawing free body diagrams (FBD) and almost all of them chose to do this early in the solution of the problem. These free body diagrams were largely correct as only one student failed to draw a correct FBD. An emergent observation was that participants used varying symbols for density, indeed the faculty teacher of the statics course used γ(gamma), a symbol which no other participants employed (typically D or ρ). While to some extent the symbology used to solve the problem is unimportant as long as one tracks their variables correctly, the use of different symbols than students are used to can provide a barrier to learning, especially if these new symbols are not adequately explained [21]. Care needs to be taken to ensure that cognitive effort on the behalf of students is spent tackling the problems and concepts at hand rather than on deciphering the language of the problem.

4. Solution generation
Student participants preferred to find unknown values before writing governing equations and did not generally follow the "engineering method" of problem solving. Participants were able to correctly apply their mathematical knowledge to find the area of the plate but students could not recall how to find the centroid.

Again, student preference for finding values based on the problem statement and given values rather than those values needed to find unknowns in their equations was demonstrated in this stage. For example, many students decided to calculate the area of the plate before knowing how (or even if) it would be needed to solve the overarching problem. Clearly this is a potential problem as students could perform work that is unnecessary to solving the problem. This approach taken by the students is understandable given that most problems they encounter in engineering classes are presented with just enough information to solve the problem and no extraneous variables. This result suggests that students might need to be given more open-ended problems with misleading or otherwise non-useful information to correct this approach - if indeed we want them to solve problems based on first principles rather than by using the information they are given.

The fact that students managed to solve for the area of the plate was surprising given the results of prior work [8,9], in which students struggled to apply relatively simple integrals in an engineering context. This observation is important as it speaks to the broader goal of the overarching research program of which this study is a small part–that of promoting knowledge transfer - in that one must know a-priori the correct knowledge that needs to be activated and transferred. Had these efforts focused on simple integration there would have been no benefit to the students as determining the centroid is where they struggled and needed help. Interestingly, one of the faculty participants in this study referred to the fact that they thought their students would struggle with both of the integration steps of this problem. The reasoning for this was that their class spent more time on conceptual elements of the problem rather than the mathematical skills needed to solve them.

The fact that students knew what they needed to do (find the centroid) but could not do it is an interesting observation that was followed up on during the think alouds. Many students reflected that they did not need to calculate centroids in many courses (lack of applications) or that other courses used CAD software or tabulated data to find centroids. An additional observation is that none of the participants who could not find the centroid made an attempt to find the value from first principles i.e. starting from a more mathematical or physical context. Again this is an indication of student unwillingness to move away from more rote problem solving approaches and to avoid thinking more critically about the fundamental nature of the problem at hand. Clearly a prior knowledge prompt or refresher of some kind is needed to enable students to calculate the centroid. When quizzed about this, three students said that seeing an example problem would help them in solving for the centroid while another three (including both TAs) thought that an equation on its own would be enough. Future work will examine the nature of this prompt to enable solving for the centroid and which methods for doing so are most effective.

5. Solution evaluation

Relatively few students displayed any unprompted reflective processes as they solved this problem. Both faculty and the one student with the highest GPA in the participant pool did reflect on their work and the potential accuracy of their solution.

One faculty member (during the first interview) used a method of relating the centroid to other, known shapes (triangles and squares) to check the accuracy of their solution and to provide an order of magnitude value for their eventual solution. This observation was important, not only in that it demonstrated a reflective evaluation process, but that it also altered the think aloud protocol moving forward as this kind of behavior was then looked for in future interviews. Observations of the faculty and TAs were especially useful in this regard, as they represent "expert" approaches to problem solving. As such, once behaviors such as that described above (solution evaluation via reflective processes) were made, both future interviews and past interview data were scanned for similar patterns or results. Based on this change, it was observed that only one other participant (student, highest GPA) employed reflective approaches to evaluate their solution in terms of the numerical value they found. This observation is again an indication of some of the metacognitive skills that need to be better developed in students.

Summary and Future Work

In this study, a think aloud protocol was used to explore student ability to transfer knowledge and the challenges they face in doing so. Research participants (n=11, mechanical engineers) were asked to solve a typical problem from an introductory statics course while verbalizing their thoughts during the activity. The problem chosen to be solved required mathematical skills (integration) to be transferred in order to successfully complete.

Students were successfully able to use integration to solve one part of the statics problem (an area), but were unable to apply integration to find the centroid of the object being analyzed. Common reasons given for this deficiency were the lack of applications of centroids being taught or used in intervening classes since learning about centroids in the math curriculum. Student participants (9/11) displayed approaches to problem solving typical of individuals who are used to solving rote, formulaic problems rather than employing what could be considered as an engineering approach grounded in fundamental equations and first principles.

Future research will build on this study by considering what methods of prior knowledge promotion and activation are most effective at enabling students to transfer their mathematical skills in order to solve the problem at hand. In the upcoming study, one of two prompts (an equation or a worked example relating to centroids and their calculation) will be provided in future think aloud interviews to determine which prompt improves student problem solving success.

Acknowledgements

Support for this work was provided by the National Science Foundation under Award No. 2301341. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. Research work was conducted under institutional IRB protocols, IRB#1965654.

References

1. ABET, “Criteria for Accrediting Engineering Programs, 2020 – 2021 | ABET,” ABET, 2021. https://www.abet.org/accreditation/accreditation-criteria/criteria-for-accrediting-engineering-programs-2020-2021/ (accessed Nov. 02, 2021).
2. N. A. of Engineering and N. A. of Engineering, The Engineer of 2020: Visions of Engineering in the New Century. Washington, DC: The National Academies Press, 2004. doi: 10.17226/10999.

3. ASEE, “Transforming Undergraduate Education in Engineering: Phase 1 Synthesizing and Integrating Industry Perspectives,” ASEE, 1, May 2013. Accessed: Oct. 27, 2021. [Online]. Available: https://tuee.asee.org/phase-i/report/
4. Y. Moghaddam, H. Demirkan, and J. Spohrer, T-Shaped Professionals: Adaptive Innovators. Hampton, NJ: Business Expert Press, 2018.
5. How People Learn: Brain, Mind, Experience, and School. Washington, DC: The National Academies Press, 1999. doi: 10.17226/6160.
6. J. D. Bransford and D. L. Schwartz, “Chapter 3: Rethinking Transfer: A Simple Proposal With Multiple Implications,” Rev. Res. Educ., vol. 24, no. 1, pp. 61–100, Jan. 1999, doi: 10.3102/0091732X024001061.
7. Barnett, S. M., & Ceci, S. J. (2002). When and where do we apply what we learn?: A taxonomy for far transfer. Psychological bulletin, 128(4), 612.
8. De Rosa, A. J., & Serbin, D., & Lee, S. (2019), Facilitating Cross-Course Connections & Knowledge Transfer between Engineering Thermodynamics and Mathematics (WIP) Paper presented at 2019 Fall Mid Atlantic States Conference, New York, New York. https://peer.asee.org/33805
9. De Rosa, A. J. (2020), Examining Knowledge Transfer Between Thermodynamics and Mathematics Paper presented at 2020 ASEE Virtual Annual Conference Content Access. 10.18260/1-2--34610
10. Orton, A. (1983). Students' understanding of integration. Educational Studies in Mathematics, 14(1), 1-18.
11. Rebello, N. S., Cui, L., Bennett, A. G., Zollman, D. A., & Ozimek, D. J. (2007). Transfer of learning in problem solving in the context of mathematics and physics. Learning to Solve Complex Scientific Problems, 223-246.
12. Schoenfeld, A. (1985). Mathematical Problem Solving. New York: Academic Press
13. Loverude, M. E., Kautz, C. H., & Heron, P. R. (2002). Student understanding of the first law of thermodynamics: Relating work to the adiabatic compression of an ideal gas. American Journal of Physics, 70(2), 137-148.
14. Meltzer, D. E. (2004). Investigation of students’ reasoning regarding heat, work, and the first law of thermodynamics in an introductory calculus-based general physics course. American Journal of Physics, 72(11), 1432-1446.
15. Charters, E. (2003). The use of think-aloud methods in qualitative research: An introduction to think-aloud methods. Brock Education Journal, 12(2).
16. Han, J., & Kelley, T. R. (2022). STEM Integration through shared practices: examining secondary science and engineering technology students’ concurrent think-aloud protocols. Journal of Engineering Design, 33(5), 343-365.
17. Murray, J. K., Studer, J. A., Daly, S. R., McKilligan, S., & Seifert, C. M. (2019). Design by taking perspectives: How engineers explore problems. Journal of Engineering Education, 108(2), 248-275.
18. Vygotsky, L. S. (2012). Thought and language. MIT press.
19. Hibbeler, R. C. (1997). Engineering Mechanics: Statics. 12th. Upper Saddle River, NJ: Pearson/Prentice-Hall.
20. T. J. Nokes-Malach and J. P. Mestre(2013), “Toward a Model of Transfer as Sense-Making,” Educ. Psychol., vol. 48, no. 3, pp. 184–207.
21. Samo, M. A. (2009). Students' Perceptions about the Symbols, Letters and Signs in Algebra and How Do These Affect Their Learning of Algebra: A Case Study in a Government Girls Secondary School Karachi. International Journal for Mathematics Teaching and Learning.