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Introducing Omnifinites and the Arithmetic Errorless Infinity Calculator

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Abstract

Typically, in applied mathematics, only real and complex numbers are considered and utilized in mathematical models, computations, and analyzes. Nonfinite numbers, such as infinitesimals and infinites, are most often omitted from these studies. The purpose of this paper is to present a new number system called omnifinites that is a modification of the hyperreals, which includes all finite and nonfinite numbers, and will help make these types of studies possible. All arithmetic for omnifinite numbers is fully defined, which overcomes the challenges of less robust number systems such as the reals, hyperreals or surreals, which are plagued with mathematical related errors such as division by zero and indeterminant forms, which have no solutions. In addition, unlike the reals and hyperreals which are open systems where there is no largest number, the omnifinite number system is closed. Two new numbers are introduced. These new numbers are absolute infinity, ⧞, and negative absolute infinity, $^{-}$⧞, and they are the largest positive number and largest negative number, respectively. Also, infinity, ∞, which is the natural countable infinity, may now be fully arithmetically manipulated like other finite numbers, resulting in numbers such as 0.3∞, $^{-}7\infty$, ∞^2, $\infty^{0.4}$, and so forth.

As an application of the new number system presented herein, referred to as omnifinites, an arithmetic errorless infinity calculator has been created and developed as a computer, web-based software tool. To the authors' knowledge, this is the first of its kind, an arithmetic errorless computational program. The arithmetic errorless infinity calculator was presented for review to a group of engineering seniors majoring in civil, mechanical, and electrical engineering, who used the calculator hands-on and completed a survey. Results show that the new number system, which is a modification of the hyperreals, may be created and developed. This new system was shown to be reasonable and logical based on the perspective of forty-two (42) senior engineering students surveyed. The resulting arithmetic errorless infinity calculator allows for numerical inputs and outputs to be finite, nonfinite, or a combination. Results show that this web-based calculator software tool was thought by engineering senior students surveyed to be intuitive to learn, easy to use, and logical in its results. This web-based software tool will be shown at the presentation and attendees may use this arithmetic errorless infinity calculator in real time.

Introduction to Omnifinite Numbers

This paper introduces the concept of omnifinite numbers, which is a new but similar system of numbers as compared to hyperreals. For clarity, the appendix herein provides some definitions for terms presented and discussed in this work. A detailed discussion regarding the properties of omnifinite numbers is beyond the scope of this work and is not needed for understanding the basics of these numbers and how the web-based, errorless calculator software tool works and functions. In general, omnifinites are a more robust and complete number system than other number system commonly used today such as the reals, hyperreals, or surreals. As mentioned, omnifinite numbers are similar to hyperreals in that numbers may be finite or nonfinite. Nonfinite numbers may be infinitesimal and/or infinite. Thus, omnifinites may be small, nonreal numbers; real numbers; large, nonreal numbers; or a combination of these number types. In addition, omnifinites may be complex or have complex parts that are finite and/or nonfinite. A brief discussion and overview of some of the important aspects and principles of omnifinites is presented in this section. The next section will present some of the properties of omnifinite numbers to help better understand how zero and differing infinities and infinitesimals work within the context of mathematical arithmetic. Of importance throughout this paper is basic fundamental arithmetic, which is what led to the development of the web-based errorless infinity calculator software tool.

For this work in developing and creating the errorless calculator, the authors introduce a new set of numbers referred to as omnifinites. Generally, in mathematics, a number is an arithmetic value used to represent a quantity. This definition implicitly implies a concept of size as well as order, but not explicitly. For omnifinites, this definition is used as well. For the Greeks, in defining and describing a number, the concept of "The part of a number is less than the whole." was foundational [1-3]. This concept was the governing principle of all numbers and for the Greeks, this meant all real numbers, finite numbers, according to Euclid (~305 BC - ~270 BC) [1]. Omnifinite numbers as well as hyperreals, which include all finite and all nonfinite numbers, share this fundamental principle regardless of the size of the number. This idea, that the Greek concept of numbers is equally applicable to finite and nonfinite numbers alike, is a significant change and departure from the well accepted work of transfinite numbers by Prof. Dr. Georg Cantor (1845-1918), who developed them more than a century ago [4,5]. However, the mathematical opinions and beliefs of the authors, regarding the behavior and properties of omnifinite numbers which are for the most part consistent with the hyperreals, are consistent with those of Prof. Dr. Leonhard Euler (1707-1783). He believed that infinitesimals and infinities "exist and behave like finite numbers" [6]. Who is right, Euler or Cantor? Only one can be mathematical truth, the other an abstraction of thought.

Omnifinites, $\mathbb{O}$, may be easily compared to other more familiar number systems like the reals, $\mathbb{R}$, and the hyperreals, $^*\mathbb{R}$, as shown in Figure 1. The line representing the reals is dashed in Figure 1 since all infinitesimals have been removed or compacted out of the number system. Unlike the reals and hyperreals, omnifinites are a closed system of numbers. The number line systems for the reals and hyperreals, as shown in Figure 1, have arrows at both ends of their respective line segment indicating that they continue in that direction indefinitely. This is fundamentally different than omnifinite number systems, which start with a number and end at another number. There are no graphical arrows associated with omnifinite number systems. Thus, omnifinite number systems are closed and not open, since all associated numbers, such as finite and nonfinite numbers, are present. The two ends of the omnifinite number system are numbers, which represent absolute infinity, ∞̶, and negative absolute infinity, ⁻∞̶, respectively. For omnifinites, as shown in Figure 1, infinites and infinitesimals are expressed using the infinity symbol, ∞. This infinity denotes the smallest infinity or natural countable infinity in an effort to be consistent with transfinite numbers as established by Prof. Dr. Georg Cantor.

The real numbers are an open system of numbers. This means that there is a smallest nonnegative, real number, zero, but no largest positive real number nor largest negative real number. This explains the arrowheads graphically on the real number line. The hyperreals are similar. They have no largest positive infinite number nor largest negative infinite number [7]. Omnifinites are different. They have a largest positive infinite number and a largest negative infinite number. These numbers, like zero, are special and have properties that no other finite nor nonfinite number possesses. In addition, these absolute numbers may not be increased nor decreased, respectively. The whole numbered reals extend finitely to nearly infinity, but infinity is generally deemed in mathematics to be a concept of boundlessness and not a number, since it is nonfinite. For omnifinites and hyperreals, nonfinite numbers may be infinites or infinitesimals, as shown in Figure 1. However, there are some differences between omnifinites and hyperreals. With omnifinites, division by zero is fully allowed and defined, which is not permitted with hyperreals, nor the reals [8]. This will be discussed in more detail in the next section. For the hyperreals, infinites and infinitesimals are specified by using omega, ω, and epsilon, ε, notational symbols, respectively [8]. For simplicity, omnifinites use the original lemniscate notational symbol, ∞, as established by Prof. Dr. John Wallis (1616-1703) for use in denoting nonfinite numbers such as infinities and infinitesimals [9]. All finite numbers have a hidden multiplicative term of ∞^0, which equals one. Nonfinite numbers, that are infinites, have a term, ∞^n, where n is greater than zero. Nonfinite numbers, that are infinitesimals, have the same term, where n is less than zero.

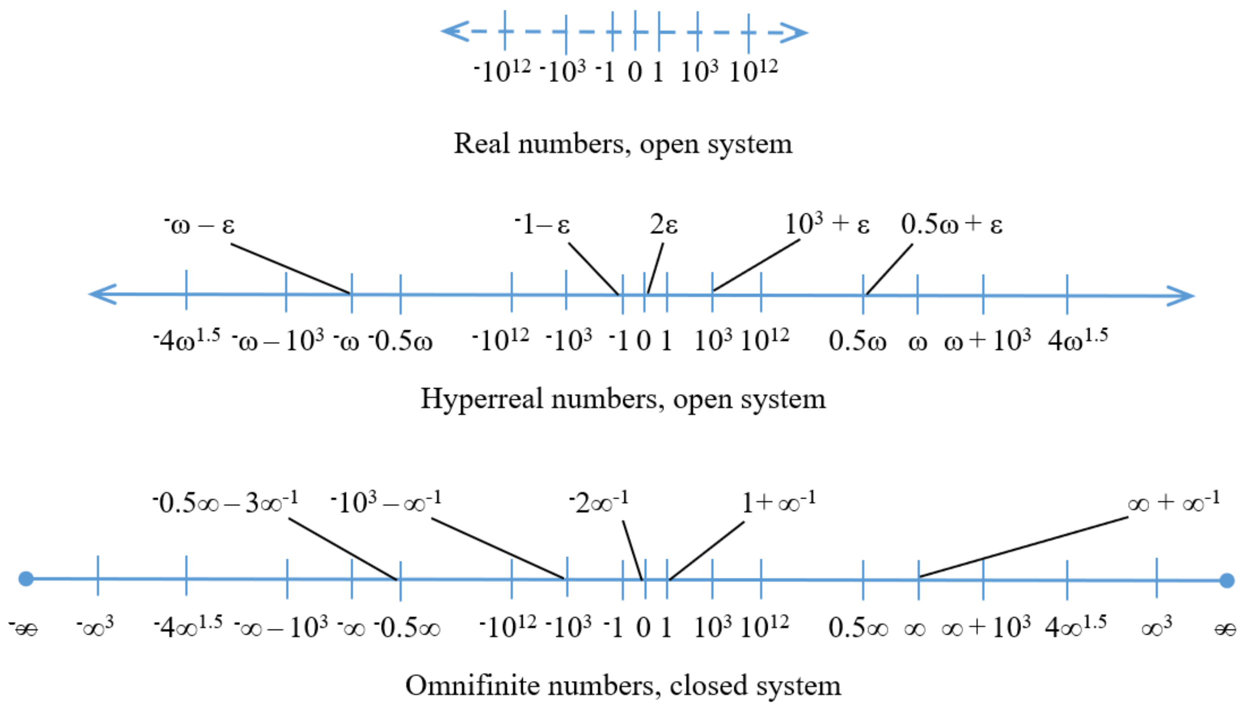

Figure 1. Common number systems in mathematics and an omnifinite number system

In general, nearly all numbers are well behaved quantities and act as expected. This is true of finite and nonfinite numbers. While most numbers behave alike in terms of operational properties, there are numbers which have unique or special properties that no other numbers possess. For the reals, there is only one special number, and this is zero [10,11]. Infinity, ∞, is not a real number and as stated previously is considered a concept in mathematics and not a number. However, in many instances in mathematics, infinity, ∞, is used as an actual number. In these cases, infinity and negative infinity, ∞ and ¯∞, like zero, have special properties that no other numbers have. For example, these special numbers, ∞, ¯∞, and 0, when added to themselves do not change in value, such as ∞ + ∞, ¯∞ + ¯∞, and 0 + 0, respectively. This property is not true for any other real or hyperreal number.

For omnifinites, infinities like ¯3∞ and $5\infty^2$ behave similarly to any other nonfinite number as they are just another infinite number in an ocean of infinities and may be increased or decreased depending on the mathematical arithmetic operation being performed. In addition, the inverse of omnifinite infinity, ∞, or 1/∞ is simply ∞^{-1}. However, absolute infinities, ⧞ and ¯⧞, are different. Positive absolute infinity, ⧞, is the infinity of all infinities. Thus, this number has special properties. This number may not be arithmetically increased nor exceeded by another number. For example, ⧞ + ⧞ = ⧞ and ⧞ × 5 = ⧞. However, this number may be decreased or reduced. For example, ⧞ – ⧞ = 0 and ⧞ – 4 – 2∞ = ⧞ – 2∞ – 4. Likewise, negative absolute infinity, ¯⧞, may not be reduced. For example, ¯⧞ + ¯⧞ = ¯⧞ and ¯⧞ × 5 = ¯⧞. However, this number may be increased. For example, ¯⧞ – ¯⧞ = 0 and ¯⧞ + 2 + ∞ = ¯⧞ + ∞ + 2. These special numbers will be discussed in more detail in the next section.

Infinities, Number Format, and Properties

For this new web-based calculator software tool, which includes reals and nonfinite numbers, the mathematical lemniscate notational symbol, ∞, behaves differently than is traditionally used. Again, infinity, ∞, is not regarded as a number but as a mathematical concept of boundlessness. For infinity, ∞, to be used as an actual number within an omnifinite number system, the mathematical definition must have specificity. For omnifinites, infinity, ∞, is the natural countable infinity, meaning 1, 2, 3, …, ∞. Omnifinite infinity may be compared with traditional infinity as shown in Table 1.

Table 1. Examples of omnifinite infinity compared to traditional infinity

Omnifinite infinity	**Traditional infinity**
∞ + ∞ = 2∞	∞ + ∞ = ∞
¯∞ + ¯∞ = ¯2∞	¯∞ + ¯∞ = ¯∞
$\infty \times \infty = \infty^2$	∞ × ∞ = ∞
¯∞ × ¯∞ = ∞^2	¯∞ × ¯∞ = ∞

Absolute infinity, ⧞, is the largest of all infinities and cannot be increased nor exceeded in value. Arithmetically, it acts in many cases identical to traditional infinity. However, there are differences. These similarities and differences are shown in Table 2.

The errorless calculator outputs an omnifinite number that is arranged in order by the size of the number part type. There are three (3) number types including infinites, finites, and infinitesimals. Infinites and infinitesimals may have multiple parts. Finites only have one part unless it is complex. Positive followed by negative infinites in order of their absolute value in size come first. This is followed by finites in the same manner and then infinitesimals similarly. For example, Table 3 shows examples of the output ordering of omnifinite numbers based on the arithmetic of the specified number inputs.

As mentioned previously, zero is a special number. The signage of zero is neither positive nor negative, and it is the only number with such a property. As such, in terms of omnifinites, specifically positive and negative absolute infinity, zero may be expressed mathematically by either of the following two (2) equations.

$$0 = \not\infty^{-1} = 1 \div \not\infty = {}^{-}1 \div {}^{-}\not\infty$$

$$0 = {}^{-}\not\infty^{-1} = {}^{-}1 \div \not\infty = 1 \div {}^{-}\not\infty$$

Table 2. Examples of omnifinite absolute infinity compared to traditional infinity

Absolute infinity	**Traditional infinity**
$\not\infty + \not\infty = \not\infty$	$\infty + \infty = \infty$
$^{-}\not\infty + \not\infty = 0$	$^{-}\infty + \infty = \text{NA}$
$^{-}\not\infty + {}^{-}\not\infty = {}^{-}\not\infty$	$^{-}\infty + {}^{-}\infty = {}^{-}\infty$
$\not\infty - \not\infty = 0$	$\infty - \infty = \text{NA}$
$^{-}\not\infty - \not\infty = {}^{-}\not\infty$	$^{-}\infty - \infty = {}^{-}\infty$
$^{-}\not\infty - {}^{-}\not\infty = 0$	$^{-}\infty - {}^{-}\infty = \text{NA}$
$\not\infty \times \not\infty = \not\infty$	$\infty \times \infty = \infty$
$^{-}\not\infty \times \not\infty = {}^{-}\not\infty$	$^{-}\infty \times \infty = {}^{-}\infty$
$^{-}\not\infty \times {}^{-}\not\infty = \not\infty$	$^{-}\infty \times {}^{-}\infty = \infty$
$\not\infty \div \not\infty = 1$	$\infty \div \infty = \text{NA}$
$^{-}\not\infty \div \not\infty = {}^{-}1$	$^{-}\infty \div \infty = \text{NA}$
$^{-}\not\infty \div {}^{-}\not\infty = 1$	$^{-}\infty \div {}^{-}\infty = \text{NA}$

Note, NA means not allowed. The expression is indeterminant [12].

Table 3. Ordering of omnifinite number output based on inputs using the errorless calculator

Input	**Output**
$^{-}\infty^{-1} + 22\infty^{2} + 78$	$22\infty^{2} + 78 - \infty^{-1}$
$13 + 3\infty^{-2} - 7 + 9\infty - 4\infty^{-1}$	$9\infty + 6 - 4\infty^{-1} + 3\infty^{-2}$
$\infty^{-1} + 7 + 5\infty + 2.2\infty^{-1} - 12\infty - 9$	$^{-}7\infty - 2 + 3.2\infty^{-1}$
$141 + 5\infty^{-3} - 4\infty - 61 + 11\infty + 8\infty^{4}$	$8\infty^{4} + 7\infty + 80 + 5\infty^{-3}$

When zero is multiplied by another finite or real number or by a nonfinite number such as an infinitesimal or infinity other than positive or negative absolute infinity, the result is itself, zero.

For omnifinites, division by zero is fully defined and not undefined. Any nonzero number divided by zero is positive or negative absolute infinity, where the signage of the resultant is based on the sign of the number in the numerator. In addition, zero divided by itself is also fully defined as follows.

$$0 \div 0 = 1$$

The largest of all infinites, (positive) absolute infinity, ∞̶, and the large negative infinity, negative absolute infinity, $^{-}$∞̶, are special numbers, like zero, and have similar properties as stated previously. The signage of absolute infinity may be positive or negative. For omnifinites, positive and negative absolute infinities in terms of zero may be expressed mathematically by the following equations, respectively.

$$∞̶ = 0^{-1} = 1 \div 0$$

$$^{-}∞̶ = {}^{-}1 \div 0$$

When positive or negative absolute infinity is multiplied by another finite or real number other than zero or by a nonfinite number such as an infinitesimal or an infinite, the magnitude of the resultant is equal to the absolute value of itself, and the resulting signage is dependent on the sign of the two numbers as is the customary practice involving multiplication of two numbers.

With omnifinites, all arithmetic computations have actual numerical solutions. This includes solutions to arithmetic computations which lead to errors such as division by zero and indeterminant forms. These errors resulted due to the nature of the finite limitations of real numbers being an open and not a closed number system. The hyperreals extend the real number system beyond finite numbers and include the infinitesimals and infinities. However, for consistency purposes, this system was developed and calibrated to be an open system, thereby retaining all similar such mathematical errors. Similarly, the surreal numbers do not allow for division by zero nor indeterminant number forms. The new web-based arithmetic errorless calculator uses number inputs based on an omnifinite number system, which is a closed system. As such, arithmetic errors of any kind are completely eliminated, so long as the mathematical expression entered is logical. For example, 4/3, 0/0, 0^0, 4^$^{-}$3, 7/0, and so forth are all logical mathematical expressions. Illogical expressions are not mathematical such as 8+^7, $^{-}$9 +÷ 8, 3÷×5, and so forth. The web-based arithmetic errorless calculator allows numbers to be input arithmetically resulting in an omnifinite number as output. The mathematical procedures and properties for evaluating omnifinite arithmetic are primarily consistent with traditional arithmetic. The following section provides a mathematical example showing the differences of using an omnifinite number system in comparison to a finite number system.

Comparative Mathematical Example using Omnifinites

Detailed examples are beyond the scope of this work. Omnifinites, which are numbers that may only be used in closed number systems, were developed by engineers (authors of this work and the list of persons included in the acknowledgement) for engineers and for people in all applied areas of study including but not limited to science, technology and medicine. The purpose of omnifinites is to develop a robust, applied numerical system that allows for the inclusion of models, systems, and equations of such to be developed which require and go beyond using real and complex numbers allowing for nonfinite numbers to be considered, used, and mathematically manipulated as needed. While this is a lofty goal, this will take time to nurture and mature properly. This work serves as a first and necessary step in the journey ahead.

As a simple mathematical example where omnifinites may be used, consider integrating the following function, $y(x) = 3 + x^3$, from 5/∞ to 2∞. Infinity is not a number; it is a concept. However, in mathematics, infinity is frequently used like a number such as in integration, limits, summation, and so forth. If 5/∞ were considered to be a number, it would not be a finite number. As such, 5/∞ is generally considered to be undefined, not a real number. However, this could be defined within the mathematical structure of other number systems like the hyperreals but using differing notation such as 5/ω. In addition, 2∞ is paradoxical since ∞ + ∞ = ∞. How could you even get to 2∞ in the first place? Despite these complications and challenges, we shall press forward to try and gain some insight and understanding into the nature of the resulting number for the area under the curve. Taking the approach of a typical university-based calculus course taken by engineers and students majoring in applied field of study, the function could be integrated as follows.

$$\int_{\frac{5}{\infty}}^{2\infty} (6+x^3)\,dx = \lim_{t\to\infty}\int_{\frac{5}{t}}^{2t} (6+x^3)\,dx = \lim_{t\to\infty}\left(6x+\frac{x^4}{4}\right]_{\frac{5}{\infty}}^{2\infty}$$

$$= \lim_{t\to\infty}\left[\left(6\times 2t+\frac{(2t)^4}{4}\right)-\left(6\times\frac{5}{t}+\frac{\left(5/t\right)^4}{4}\right)\right]$$

$$= \lim_{t\to\infty}\left[12t+4t^4-\frac{30}{t}-156.25t^{-4}\right]=\infty$$

Thus, this improper integral diverges, and the solution is ∞. Graphically, this may be shown as the area under the curve as shown in Figure 2. Note, this figure is not drawn to scale.

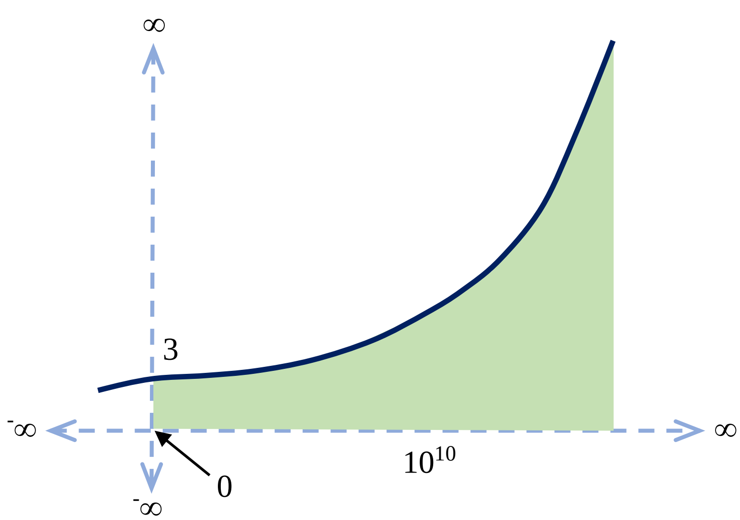

Figure 2. Graph of $y(x) = 3 + x^3$ with traditional *x* and *y* axes

Solving the problem using an omnifinite number system which allows for numbers that are infinitesimal and/or infinite, results in the following.

$$\int_{\frac{5}{\infty}}^{2\infty}(6+x^3)\,dx = \left(6x+\frac{x^4}{4}\right]_{\frac{5}{\infty}}^{2\infty} = \left[\left(6\times 2\infty+\frac{(2\infty)^4}{4}\right)-\left(6\times\frac{5}{\infty}+\frac{\left(5/\infty\right)^4}{4}\right)\right]$$

$$=\left[12\infty+4\infty^4-\frac{30}{\infty}-156.25\infty^{-4}\right]=4\infty^4+12\infty-30\infty^{-1}-156.25\infty^{-4}$$

Graphically, this may be shown as the area under the curve as shown in Figure 3. Note, this figure is not drawn to scale. The graphs of Figures 2 and 3 are different. The latter is a closed number system, so the axes will differ having lines with starting and ending points as opposed to axes with dotted lines having arrows at both ends.

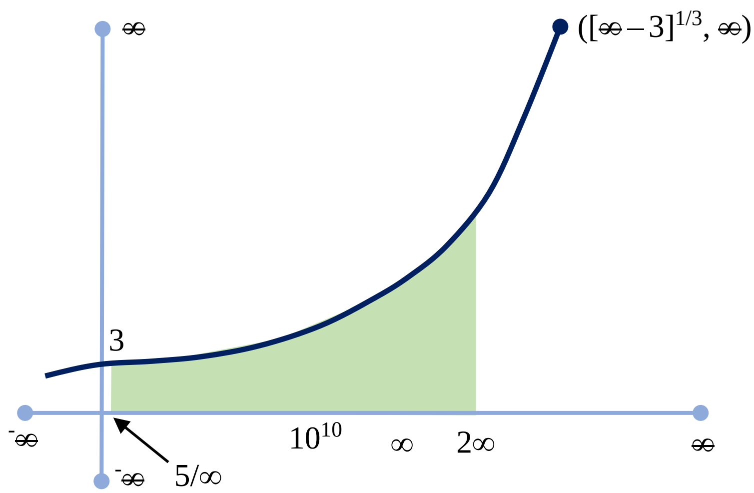

Figure 3. Graph of $y(x) = 3 + x^3$ with omnifinite x and y axes

The solution obtained from the first method, which was ∞ involving limits, is reasonable and correct based on the fundamental logic of the number system used. Likewise, the solution obtained from the second method, which was $4\infty^4 + 12\infty - 30\infty^{-1} - 156.25\infty^{-4}$, is reasonable and correct based on the fundamental logic for that system. However, the result of the first method is not particularly helpful especially if we had to repeat the above exercise for a different function with the same boundaries and compare them only to obtain the same exact solution of ∞. One could potentially infer incorrectly that both are fundamentally the same, such as if the exercise were repeated with an entirely different function of $y(x) = 9 + 2x^5$ and integrated over the same interval. This is not a fair comparison. The system in the first method is not robust enough to adequately describe nonfinite results compared to the other system used in the second method.

Using a different known number system for the first method, such as the hyperreals, can assist and help bring greater mathematical clarity. However, other number systems, including the hyperreals or transfinite ordinal numbers, are typically not taught as required course topics in engineering education or in other applied areas of study other than upper-level graduate mathematics course work. Thus, models, computations, and analyzes and ultimately these solutions are all confined and limited to finite, real and/or complex, numbers. Better solutions to challenging problems are possible if our number system, that is used in engineering education and in applied fields of study, was more robust and allowed for finite as well as nonfinite numbers. The omnifinite number system and the newly developed arithmetic errorless infinity calculator were created and developed to lower this barrier leading to another world of future mathematical possibilities.

Introducing World's First Arithmetic Errorless Infinity Calculator

This computer web-based software tool functions as a simple calculator and cannot produce an arithmetic error, hence the name and title of this work. All arithmetic errors, including mathematical forms which result in undefined expressions such as division by zero or are of indeterminant form like zero divided by itself, have been removed by defining the underlining arithmetic solution within the structure of a closed number system like omnifinites. Other mathematical errors are possible such as rounding or exceeding the limits of inputs or outputs but not errors of an arithmetic nature. Figure 4 shows the proposed new errorless calculator.

This calculator appears to be simple and ordinary but includes two new button inputs, ∞ and ⧞, that are typically not featured on basic calculators. The new web-based software tool is programmed to act as a calculator as shown in the schematic diagram in Figure 5. During software processing, the web-based software first scans each input expression looking for division by zero and indeterminant forms. Once located, solutions are specified for these expressions, which would typically result in an error. The calculator then follows order of operations and completes the arithmetic, specifying an omnifinite number as the output solution.

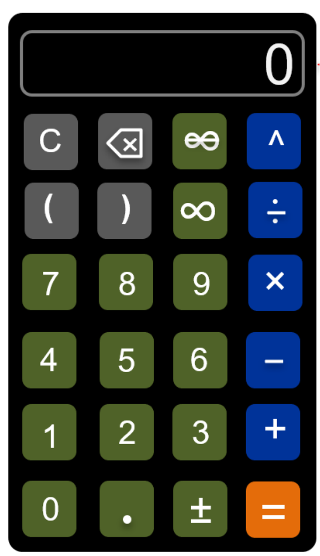

Figure 4. Basic errorless infinity calculator

This is a basic errorless infinity calculator. Basic implies the ability of the calculator to perform simple mathematical operations in the correct order based on the inputs. Operations for the basic errorless infinity calculator include addition, subtraction, multiplication, division, and exponentiation. In addition, parentheses may be input as part of the arithmetic ordering process as needed. Numbers may be input as positive or negative except for zero. Complex number inputs or complex outputs are not allowed as this is the basic errorless infinity calculator and not scientific. In addition, number inputs may be finite or nonfinite. Finite numbers are the reals as frequently discussed in mathematics such as 0, 1, e, π, 100, ${}^{-}1$, ${}^{-}2$, ${}^{-}5$, ${}^{-}109$, and so forth. Nonfinite numbers are infinities and infinitesimals. Infinite numbers may include $\infty^{0.2}$, $\infty^{0.5}$, 0.134∞, 0.5∞, ∞, 3∞, 11.7∞, ∞^{2}, $\infty^{3.2}$, ∞^{∞}, ${}^{-}\infty^{0.23}$, ${}^{-}\infty^{0.9}$, ${}^{-}0.338\infty$, ${}^{-}\infty$, ${}^{-}4\infty$, ${}^{-}9.9\infty$, ${}^{-}\infty^{4}$, ${}^{-}\infty^{7.1}$, ${}^{-}\infty^{2\infty}$, and so forth. Infinitesimal numbers may include $\infty^{-0.25}$, $\infty^{-0.5}$, ∞^{-1}, $7\infty^{-1}$, $\infty^{-2.1}$, $\infty^{-5.2}$, $\infty^{-\infty}$, ${}^{-}3\infty^{-0.37}$, ${}^{-}\infty^{-1}$, ${}^{-}\infty^{-3}$, ${}^{-}4\infty^{-5}$, ${}^{-}\infty^{-3.2\infty}$, and so forth.

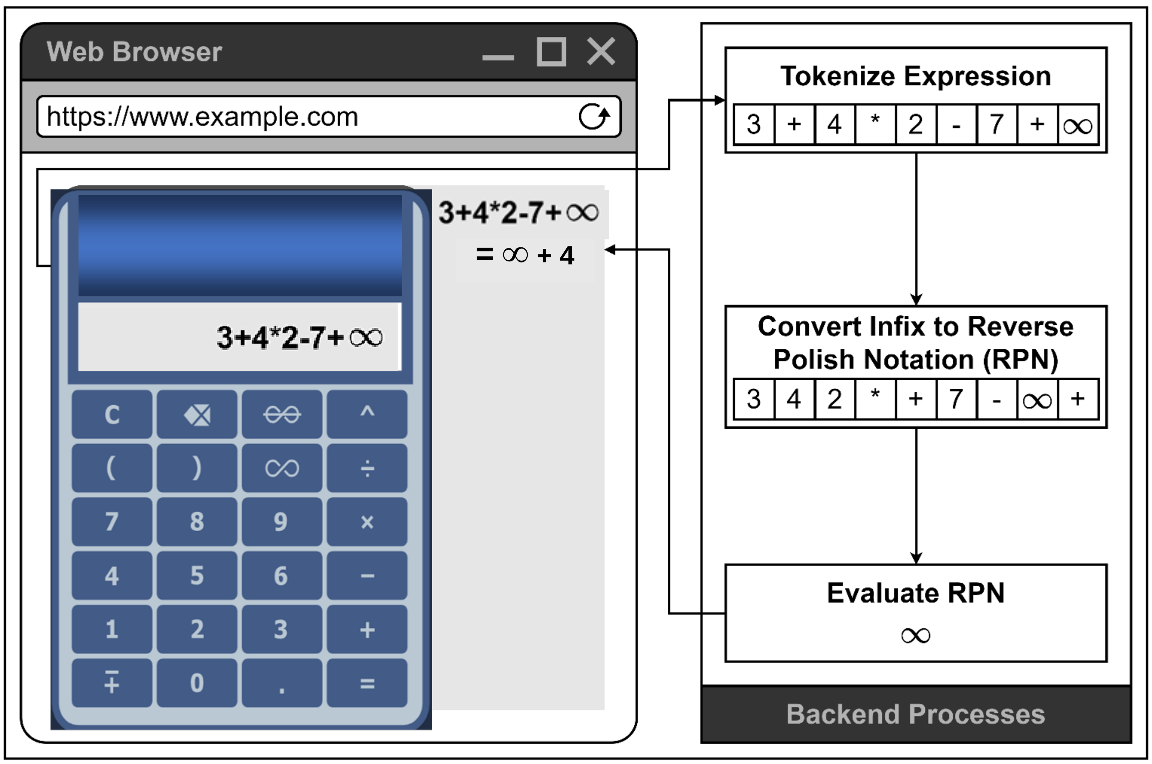

Figure 5. Schematic diagram showing the errorless infinity calculator performing computations

Errorless Infinity Calculator Examples

This section provides sample arithmetic computations with solutions, which show the intuitive and simplistic nature of using the errorless infinity calculator. Examples involving addition, subtraction, multiplication, division, exponentiation, and mixed operators are shown in Figures 6 thru 11, respectively. The arithmetic errorless infinity calculator is available for free to use at infinicoretechnologies.com.

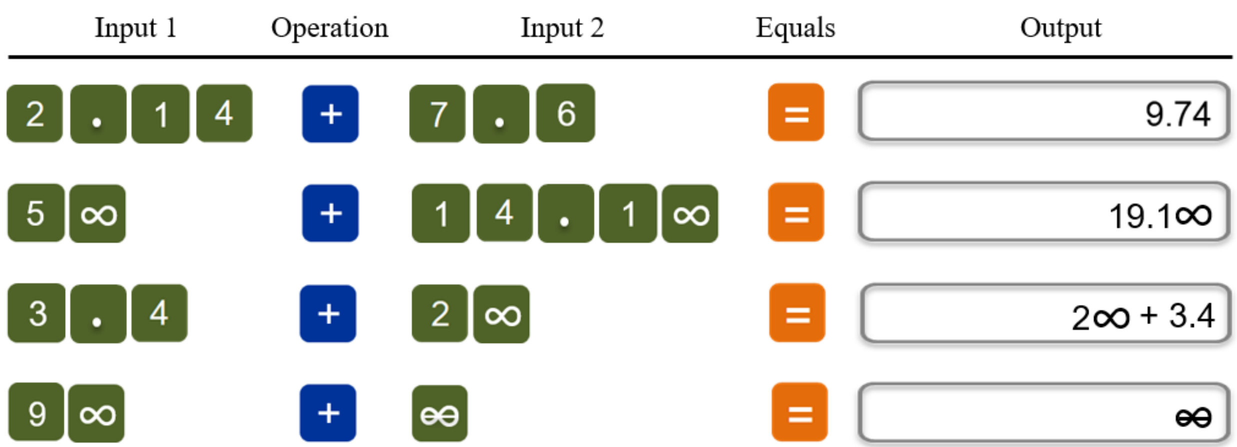

Figure 6. Errorless arithmetic with addition

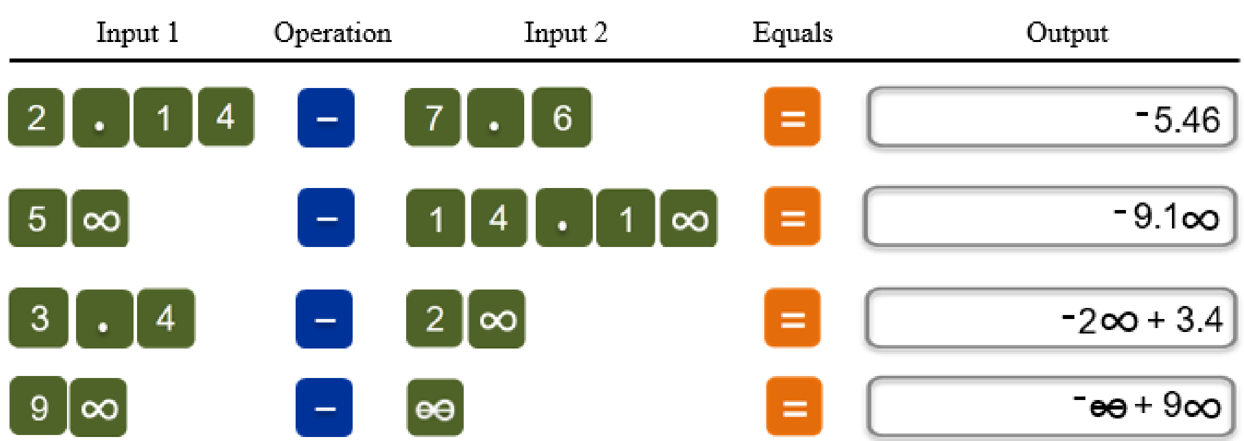

Figure 7. Errorless arithmetic with subtraction

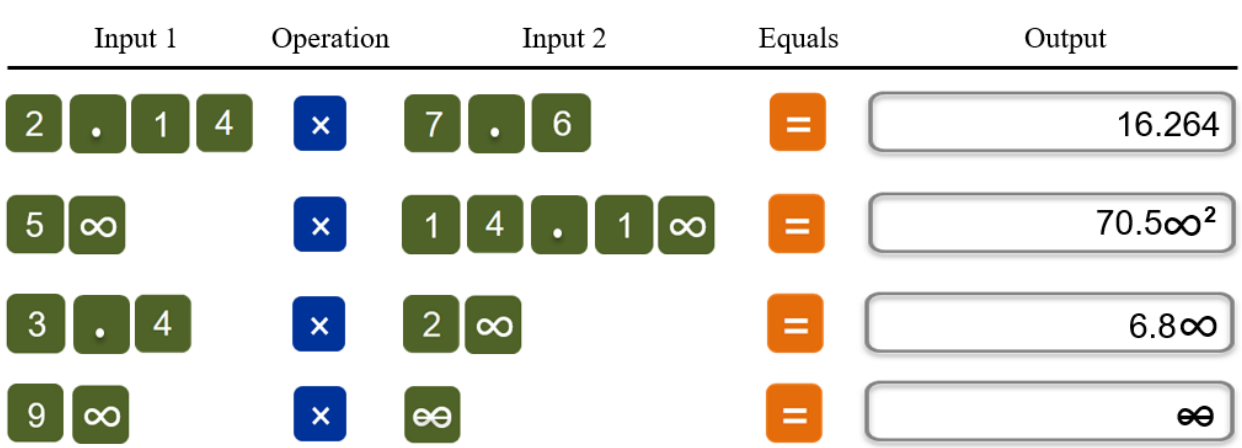

Figure 8. Errorless arithmetic with multiplication

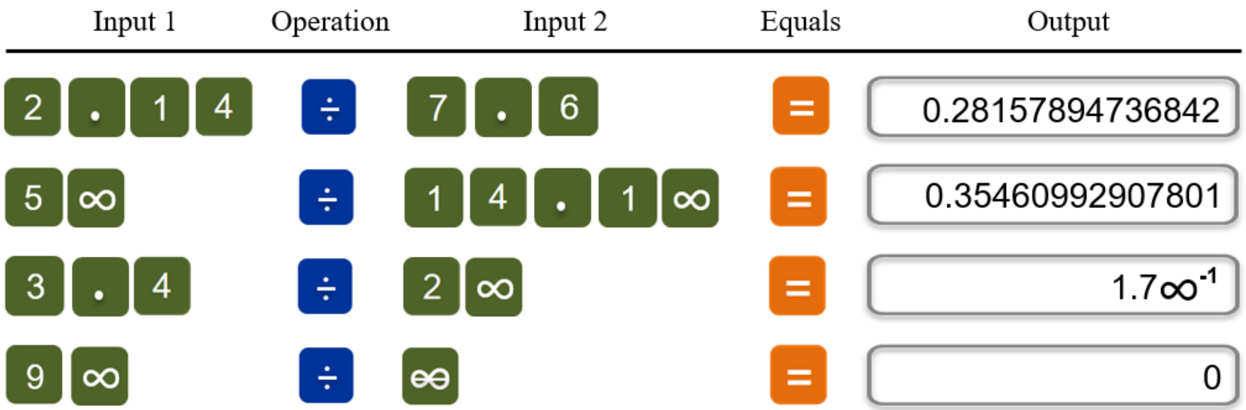

Figure 9. Errorless arithmetic with division

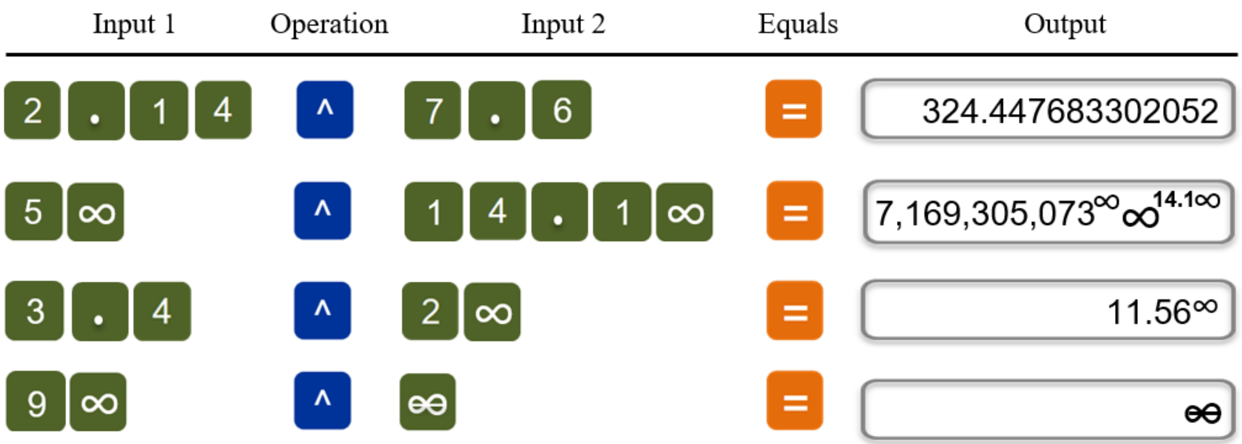

Figure 10. Errorless arithmetic with exponentiation

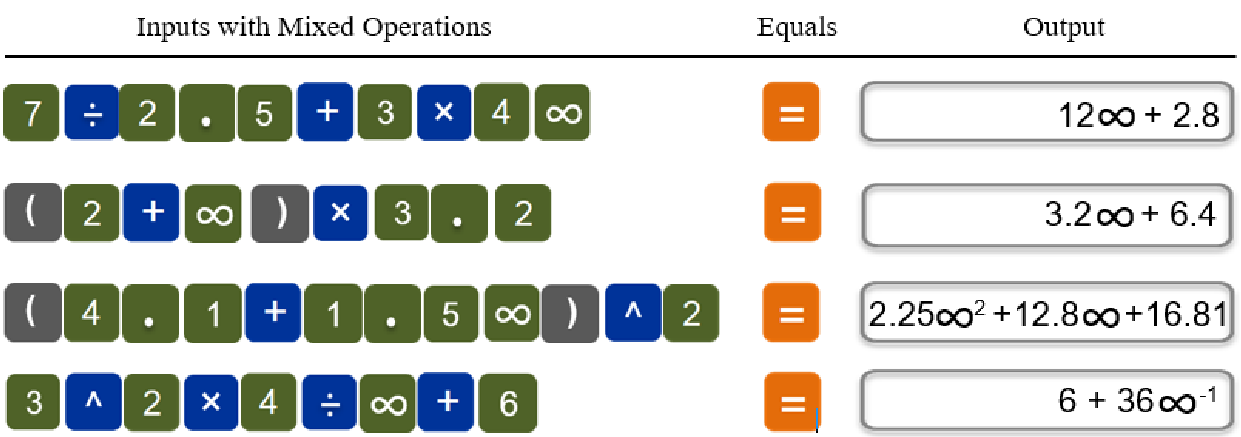

Figure 11. Errorless arithmetic with mixed operations

Engineering Senior Student Assessment and Evaluation

The arithmetic errorless infinity calculator was presented for review to a group of forty-two (42) engineering undergraduate seniors majoring in civil, mechanical, and electrical engineering, who used the calculator hands-on and completed a survey. Prior to completing the survey, the authors of this work gave the senior engineering students a twenty-minute PowerPoint presentation and an opportunity to use the calculator and ask questions. The survey consisted of forty (40) simple arithmetic computations to solve using the errorless infinity calculator that involved basic mathematical operations using finite and nonfinite numbers as inputs. A sample of the mathematical problems solved by the engineering senior students is shown in Figure 12.

$3 + \not\infty =$ __________	$8 - {}^{-}9 =$ __________	$\not\infty \times {}^{-}\not\infty =$ __________	$\infty \div {}^{-}3\infty =$ __________
$-\infty + {}^{-}7 =$ __________	$\infty^2 - \infty^2 =$ __________	$0 \times {}^{-}\infty =$ __________	$\infty^2 \div {}^{-}\infty^2 =$ __________
$\infty + {}^{-}8 =$ __________	$9 - {}^{-}2\infty =$ __________	$9 \times {}^{-}\infty =$ __________	${}^{-}\not\infty \div {}^{-}\not\infty =$ __________
$\not\infty + 4\infty =$ __________	$\not\infty - \not\infty =$ __________	$7\infty \times {}^{-}2\infty =$ __________	$8 \div {}^{-}\infty =$ __________

Figure 12. Sample of problems solved by senior students using the errorless infinity calculator

After completing the survey, the engineering senior students were then given a series of statements where they selected one of five answer choices. The choices were “strongly agree”, “agree”, “neutral”, “disagree” and “strongly disagree”. These survey item statements included the following.

1. The errorless infinity calculator is simple to use.
2. Results from the errorless infinity calculator appear reasonable and logical.
3. The errorless infinity calculator seems intuitive to learn to use.
4. The errorless infinity calculator is complicated.
5. Results from the errorless infinity calculator are unreasonable (illogical).

The results of the survey are given in Figure 13. Results of the survey as shown in Figure 13 indicated that the vast majority of the senior engineering students believed that the web-based errorless calculator was simple to use, logical, and intuitive to learn based on survey items 1 thru 3. For these three survey items, the total number and percentage of senior engineering students choosing “strongly agree” or “agree” was 37 (88.1%), 35 (83.3%), and 34 (81.0%) for survey items 1, 2, and 3, respectively. The total number and percentage of senior engineering students choosing “neutral” was 5 (11.9%), 7 (14.3%), and 8 (19.0%) for survey items 1, 2, and 3, respectively. For survey items 1 thru 3, only one (1) senior engineering student chose “disagree”, and this was only on survey item, item 2. In addition, the vast majority of senior engineering students believed that the errorless calculator was not complicated nor illogical in terms of the solutions computed as per survey items 4 and 5 as shown in Figure 13. For these three survey items, the total number and percentage of senior engineering students choosing “strongly disagree” or “disagree” was 34 (81.0%) and 31 (73.8%) for survey items 4 and 5, respectively. The total number and percentage of senior engineering students choosing “neutral” was 7 (16.7%) and 8 (19.0%), for survey items 4 and 5, respectively. Only one (1) engineering senior student chose "agree" on survey item 4. Three (3) engineering students chose "agree" on survey item 5.

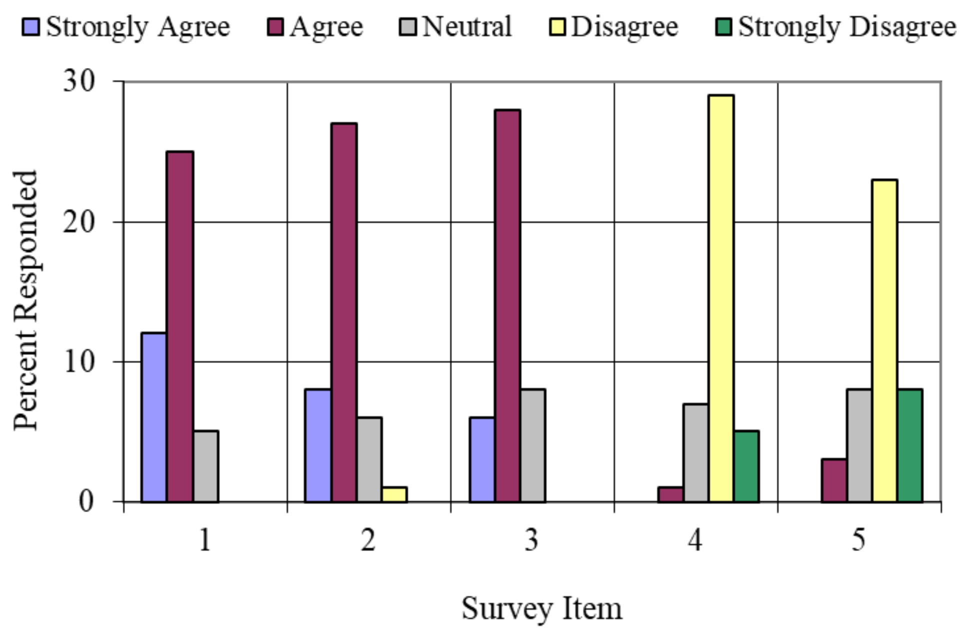

Figure 13. Survey results of senior students using the errorless infinity calculator

The authors believe that the results of the senior engineering student survey were very positive and encouraging. The results serve to help support and validate the errorless infinity calculator. However, one significant problem did occur while the students were completing the survey. This problem was that significant slow-down of the website occurred during computing the results, and this problem was reported to the student and faculty member who programmed the errorless infinity calculator to make the website interface. Another technical problem occurred during programming of the website for the errorless infinity calculator. Programming "bug" errors in the software occurred resulting in erroneous numerical solutions. As a result, a significant amount of verification and testing had to occur to ensure that nearly all results are consistent and logical as per the arithmetic of omnifinites.

Conclusions

Our ability to quantify the world around us is important, and this is not limited to just finite numbers. The new errorless infinity calculator is a software tool that allows for computations involving finite and nonfinite numbers. The calculator also eliminates all arithmetic errors of which there are an infinite number such as those resulting from undefined expressions such as division by zero as well as errors due to indeterminant arithmetic forms like zero divided by itself. Errorless computations require a mathematical numerically closed system, such as the omnifinite number system proposed and used herein by the authors and those listed in the acknowledgements. An open system, such as the reals, hyperreals or surreals, is not robust enough and will result in inconsistencies or paradoxes which cannot be arithmetically resolved. These errors result due to the inherent limitations in the fundamental logic of the number system itself. These errors are not of an arithmetic nature; they are fundamentally system errors. As a result, when such outputs occur, they must be blocked or flagged as an "error" since the arithmetic computation cannot be performed in these instances.

The new software tool, the arithmetic errorless infinity calculator as introduced herein and is believed by the authors of this work to be the world's first, has been found by senior engineering students to be relatively simple to use and logical. The omnifinite number system used by the software tool has been developed based on simplicity using the lemniscate notational symbol, ∞, for infinities as well as for infinitesimals. For omnifinites, infinity, ∞, is specified as natural countable infinity. As part of the omnifinite number system, two new numbers are introduced. These new numbers are absolute infinity, ~~∞~~, and negative absolute infinity, ¯~~∞~~. Like zero, these are special numbers. They are the most positive and negative infinities of all infinities, respectively. No computation can be performed that results in a number greater or lower than these, respectively. Currently, the authors are working to create and develop a scientific errorless infinity calculator. In the future, work will begin on creating and developing a graphing errorless infinity calculator.

Finally, the omnifinite number system shown herein includes all real numbers (finite numbers) as well as all nonfinite numbers such as infinites and infinitesimals. This is a closed number system. As a result, this software tool, the basic errorless infinity calculator, is capable of performing error free arithmetic computations, and this will serve in the future to help make and develop more comprehensive and robust studies using advanced mathematical models involving computations and analyses that use a complete spectrum of numbers. Improving our understanding of nonfinite numbers in the nature of mathematics, such as in applied fields of study, including but not limited to science, technology and medicine, will lead to greater understanding and innovation in these areas.

Appendix

This section provides some definitions for terms presented and discussed herein this work.

absolute infinity	refers to a known largest positive nonfinite number in a closed number system. Sometimes, this is referred to as positive absolute infinity. While this number can be reduced arithmetically, it cannot be increased or exceeded. This number is the infinity of all infinities. See negative absolute infinity.
closed number system	refers to an ordered numerical system that has both a known nonpositive and nonnegative number, zero, and a known largest positive number and largest negative number in addition to specified numbers between them. Arithmetic is fully defined for closed systems like omnifinites. See omnifinites.
finite number	refers to a real number that is nonfinite. Examples include ¯1009, ¯101, ¯4.431, ¯e, ¯1, 0, 1.06, e, π, 321.4, 1145.019, and so forth.

infinity	refers to a number often used in complete omnifinite systems that is natural countable infinity, ∞, meaning 1, 2, 3, ..., ∞, which are closed number systems. Generally, in mathematics, infinity is regarded as a concept and not a number even though often times is used as a number such as in integrals, limits, summation and so forth.
lemniscate notation	refers to number systems, such as omnifinites, which use the symbol for infinity, ∞, as originally specified by Prof. Dr. John Wallis for all nonfinite numbers. Sometimes this is referred to as lemniscate notational numbers or numbering. Infinitesimals are specified by a number multiplied by ∞ with a negative exponent, whereas infinities are specified by a number multiplied by ∞ with a positive exponent. Finites may be represented this way as well but ∞ has an exponent of zero.
negative absolute infinity	refers to a known largest negative nonfinite number in a closed number system. While this number can be increased arithmetically, it cannot be decreased. This number is the negative infinity of all negative infinities. See absolute infinity.
nonfinite number	refers to a nonreal number that may be an infinitesimal and/or an infinite. Examples of nonfinite numbers include $^-7.12\infty^{-5}$, $^-3.9\infty^{-1.2}$, $^-5\infty^{-1}$, $^-4.12\infty^{0.45}$, $\infty^{0.2}$, $\infty^{0.5}$, ∞, $2\infty^2$, $4.21\infty^{3.1}$, and so forth. Examples of mixed nonfinite numbers include $^-2.1\infty + 7\infty^{-5}$, $3.1\infty - 4.1\infty^{-0.4}$, $9\infty^2 - \infty^{1.4} + 5\infty^{-4}$, and so forth. Numbers may be nonimaginary and/or imaginary.
omnifinites	refers to an ordered closed number system that uses lemniscate notation for all nonfinite numbers, where ∞ refers to natural countable infinity, meaning 1, 2, 3, …, ∞, and all geometric objects have size. See closed number system. In a complete omnifinite closed number system, all numbers are present and may be mathematically manipulated. In a closed number system, a known number must be specified that represents the smallest possible magnitude of a number such as zero and known numbers, that are the largest positive and negative numbers, must also be specified.
open number system	refers to an ordered numerical system that has only a specified known nonpositive and nonnegative number, zero, and other numbers but no known or specified upper nor lower limiting number. Arithmetic is only partially but mostly defined. Errors will result in instances which cannot be eliminated due to the nature of the incomplete system. Examples of open number systems include all known number systems, except omnifinites,

	such as reals, complex numbers, ordinals, hyperreals, complex hyperreals, surreals, and so forth.
special number	refers to a number that has additional properties not shared by all other numbers including but not limited to additive self identity and multiplicative self magnitude identity properties. For a complete omnifinite number system, special numbers include zero as well as positive and negative absolute infinities.

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