2023 Annual Conference & Exposition
Baltimore Convention Center, MD | June 25 - 28, 2023
The Harbor of Engineering Education for 130 Years
ASEE
Paper ID #37793

An Approach to Understanding Problem Solving Using Multiple Solution Methods

Mr. Hao Li, Massachusetts Institute of Technology

Hao Li is currently a PhD student studying Mechanical Engineering at MIT. He earned his Bachelor's degree from Rice University.

Dr. Anette Hosoi, Massachusetts Institute of Technology

Anette (Peko) Hosoi is Associate Dean of Engineering and the Neil and Jane Pappalardo Professor of Mechanical Engineering, at MIT. She received her PhD in Physics from the University of Chicago and went on to become an NSF Postdoctoral Fellow in the MIT Department of Mathematics and at the Courant Institute, NYU. She is a leader in the study of the hydrodynamics of thin fluid films and in the nonlinear physical interaction of viscous fluids and deformable interfaces. Her work spans multiple disciplines including physics, biology and applied mathematics, and is being used, in collaboration with Schlumberger-Doll Research, Bluefin Robotics, and Boston Dynamics to guide the engineering design of robotic crawlers and other mechanisms.

Prof. Hosoi is an exceptional, innovative teacher and an inspiring mentor for women in engineering. She was awarded the Bose Award for Excellence in Teaching, and a MacVicar Fellowship. She is a recipient of the 3M Innovation Award and has held the Doherty Chair in Ocean Utilization at MIT. She is a Radcliffe Institute Fellow and a Fellow of the American Physical Society. Her research interests include fluid mechanics, bioinspired design and locomotion, with a focus on optimization of crawling gastropods, digging bivalves, swimming microorganisms and soft robotics. Prof. Hosoi is also an avid mountain biker and her passion for sports has led her to create MIT Sports Lab, a program that is designed to build an interconnected community of faculty, students, industry partners, alums and athletes who are dedicated to applying their technical expertise to advance the state-of-the-art in sports.

An Approach to Understanding Problem Solving using Multiple Solution Methods

Hao Li (WL11@mit.edu) and **A. E. Hosoi** (peko@mit.edu)
Department of Mechanical Engineering
Massachusetts Institute of Technology

Background

One of the key challenges of Engineering Education is developing students' ability to navigate and solve moderately- or ill-structured problems with multiple solution paths. Existing theoretical and conceptual frameworks can provide a basis for understanding this challenge. The framework of self-regulated learning can be applied to problem solving. In self-regulated learning, the problem solver (or learner) first plans, sets goals, and lays out strategies. Then, they implement these strategies. Finally, the problem solver reflects on their performance [1] [2]. For ill-structured problems where the solution path is not immediately obvious, the systematic approach of self-regulated learning can help students navigate the possible difficulties and dead ends. If a solution method does not work out, the problem solver can reflect on this and try a different approach.

The Model of Domain Learning is another conceptual framework that can be applied to problem solving. The goal is to understand how novices build expertise and become experts [3] [4]. In this framework, the learner progress through three stages. In the first stage, Acclimation, the learner has little knowledge of a field, and the knowledge is unstructured. For example, a novice problem solver may use the first approach that comes to mind when solving a problem, never changing approach if their attempt fails. In the second stage, Competence, the learner has begun to understand the key principles of the field and can accomplish basic tasks easily. For example, an intermediate problem solver may attempt to solve a problem using a standard approach, get stuck, and then switch to a simpler approach to obtain an answer. In the third stage, Proficiency, the learner has accumulated large stores of organized knowledge that they can use to efficiently accomplish a wide variety of tasks. For example, an expert problem solver may use a back-of-the-envelope calculation to first estimate the solution to a complex problem before investing time in a more precise method.

Problems range from closed (well-defined) to open (ill-defined) along a spectrum [5]. We are interested in problems that have a well-defined solution but multiple paths to reach that solution. These types of problems are valuable because the experience learned by solving them can be transferred to new situations [6]. The existence of a well-defined solution makes the analysis of problem solving activity potentially tractable compared to problems with no well-defined solution, yet the possibility of multiple methods gives ample opportunities to find optimal pathways, unlike simple textbook exercises with one solution path.

Previous works have analyzed the role of estimation in engineering education. When solving open-ended problems, the problem solver is often faced with a range of approaches. On one end of the range are low complexity methods, such as simple estimation. These methods can be implemented in a short amount of time and usually result in an approximate answer. On the other end of the range are high complexity methods, such as detailed analysis. These methods require much more time to implement, but can result in a very accurate or exact answer if implemented correctly. Previous work focused on assessing student's ability to perform simple estimation. It was found that students often had difficulty making basic estimates [7]. Furthermore, engineering classes overwhelmingly emphasized detailed analysis over estimation [8]. It was observed that students were unwilling to make rough estimates before and after performing Finite Element Analysis, often trusting the computer simulations without reservation [9]. Furthermore, these deficiencies in estimation ability were observed from undergraduate fourth-year students [7] [9].

Given the lack of emphasis on estimation in the curriculum and the observed shortcomings in students' estimation ability, we sought to answer the question: How would a student select from a range of low complexity to high complexity methods if given the freedom to choose, and how would this choice affect their problem solving outcome? In this paper, we address this question in two ways. First, we conducted an experiment with student participants to give an illustration of the range of possible solution methods and problem solving outcomes. Second, we formulated a model to capture the underlying behaviors of problem solving with multiple solution methods. Incorporating the results from the experiment and model, we then give recommendations for problem solving instruction.

Method

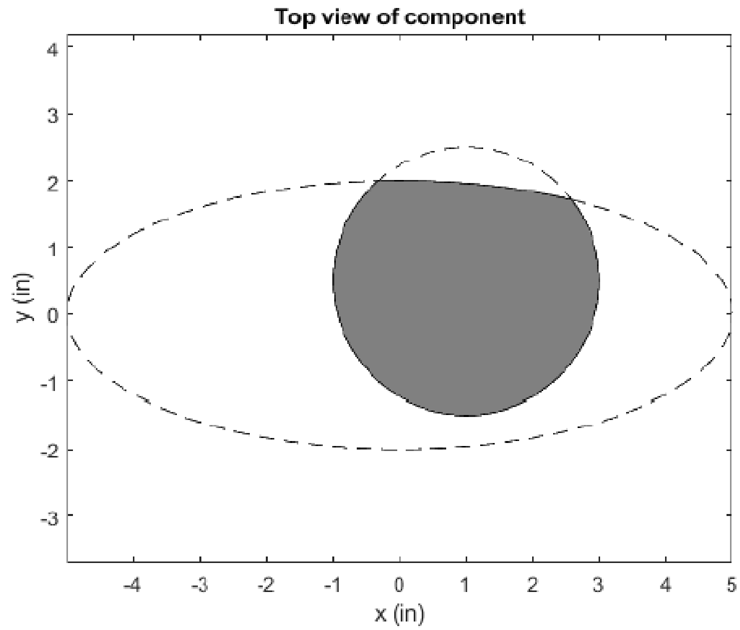
Participants were recruited for an experiment. The subject population consisted of 72 undergraduates and graduate students of the authors' institution, an engineering-focused private university on the U.S. East Coast. The objective of recruitment was to maximize the number of participants, so participants were not limited to students in one department; the diversity in students' disciplines may potentially result in a larger variety of solution methods.

Multiple recruitment methods were used. Subjects were recruited from the enrollees of Introduction to Engineering Computation (a second-year course in Mechanical Engineering), through announcements made to student organizations, and from flyers posted in the Author's institution. Additionally, participants were allowed to refer fellow students via snowball sampling, and some subjects were recruited informally.

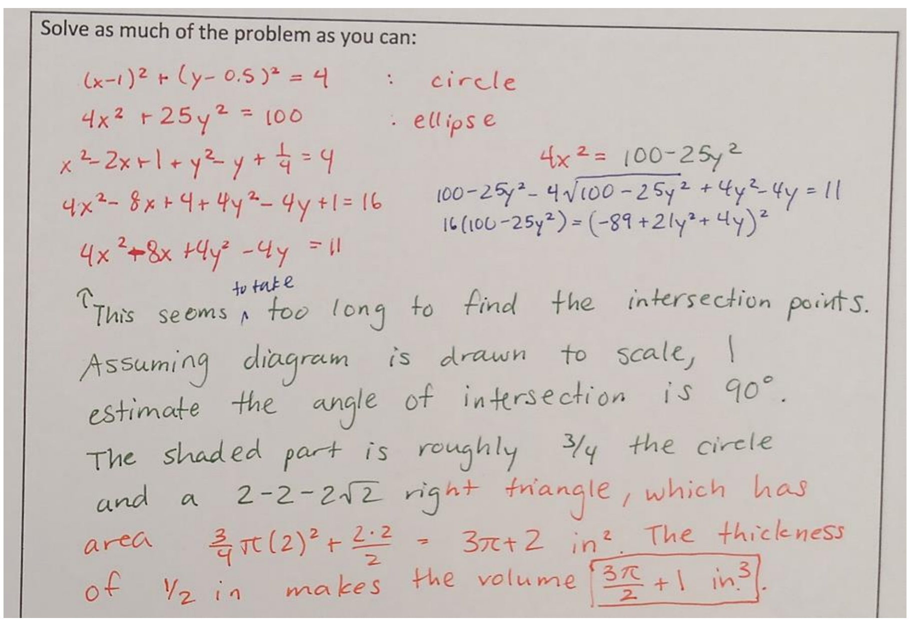
The participants in this study were given the Volume Problem (see Figure 1), which consisted of two sections. The first section asked students to "How would you start solving this problem?" and was five minutes long. The second section asked students to "Solve as much of the problem as you can" and was ten minutes long. The participants submitted their answers on paper answer sheets. Every two minutes, the participant's pen color was switched so that work can be identified within time intervals. Two minute intervals were found to be optimal for data gathering. A shorter pen switching interval would incur a large distraction overhead, while a longer pen switching interval would give less precise data. Additionally, the participant was able to access a computer connected to the internet. There were no restrictions on what tools they could use, but they could not consult other people.

The problem solving task was to “estimate the volume of the component.” This task was chosen such that participants are likely to consider both low complexity and high complexity methods. While it is possible to obtain an exact solution, most students are unlikely to do so within the time constraints.

A circular steel disc of radius 2 in and thickness ½ in will be turned into a crankshaft component. For the first cut, the center of the disc is placed at the location (x, y) = (1, 0.5)in on a mill. The cutting head will trace a toolpath such that all material outside of the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ will be removed.



Your task is to estimate the volume of the component after this cut **and justify your answer with reasoning**. You can use any resources you want, except talking to someone else.


Solve as much of the problem as you can:

$(x-1)^2 + (y-0.5)^2 = 4$: circle

$4x^2 + 25y^2 = 100$: ellipse

$x^2 - 2x + 1 + y^2 - y + \frac{1}{4} = 4$

$4x^2 - 8x + 4 + 4y^2 - 4y + 1 = 16$

$4x^2 + 8x + 4y^2 - 4y = 11$

$4x^2 = 100 - 25y^2$

$100 - 25y^2 - 4\sqrt{100 - 25y^2} + 4y^2 - 4y = 11$

$16(100 - 25y^2) = (-89 + 21y^2 + 4y)^2$

This seems to take too long to find the intersection points. Assuming diagram is drawn to scale, I estimate the angle of intersection is 90°. The shaded part is roughly 3/4 the circle and a 2-2-2$\sqrt{2}$ right triangle, which has area $\frac{3}{4}\pi(2)^2 + \frac{2 \cdot 2}{2} = 3\pi + 2$ in². The thickness of ½ in makes the volume $\boxed{\frac{3\pi}{2} + 1 \text{ in}^3}$.

Figure 1. Top: Volume Problem. Students were given five minutes to brainstorm how to start this problem. Then they were given ten minutes to implement their solution. The results from the ten-minute section were analyzed. Bottom: example of student work from the ten-minute section.

Experimental Results

For the Volume Problem, the results of the ten-minute section were analyzed. Any answer within 10% of exact was considered correct. 26 of 72 (36%) participants solved the problem correctly. The participants' results were first analyzed by school year. Students of different years performed somewhat similarly (See Table 1). The second year students did not perform as well as average, but the result was within the margin of error. Overall, a strong trend was not observed.

School year	Number correct	Total number	Fraction correct (with 95% confidence intervals)
First year	9	19	0.47 [0.27, 0.68]
Second year	2	11	0.18 [0.05, 0.48]
Third year	6	15	0.40 [0.20, 0.64]
Fourth year	4	11	0.36 [0.15, 0.64]
Graduate student	5	16	0.31 [0.14, 0.56]
Overall	26	72	0.36 [0.26, 0.48]

Table 1. Volume Problem results by school year. Clear differences between students in different years were not observed.

Participants' solution methods were placed into six categories: Visual Estimation, Geometric Approximation, CAD, Monte Carlo, Integral, and Other. With the exception of "Other", these categories are in approximate order of implementation complexity from simplest to most complex (see Table 2). They are also in approximate order of accuracy from least precise to most precise. It was found that a large fraction of students used Geometric Approximation (23 of 72), a method of intermediate complexity, and Integral (28 of 72), a method of higher complexity. Less common approaches included Visual Estimation (6 of 72), CAD (5 of 72), and Monte Carlo (4 of 72).

From the different pen colors of students' work, we extracted the amount of time they spent on each solution method to within one minute. The solve time depended on which method was used. On average, Visual Estimation was associated with the lowest solve time. Geometric Approximation, CAD, and Monte Carlo were associated with moderate solve time, while Integral was associated with the longest solve time. The fraction of students who correctly solved the problem also depended on which method was used. Integration was associated with the lowest fraction correct. Geometric Approximation and Monte Carlo were associated with a moderate fraction correct, while Visual Estimation and CAD were associated with the highest fraction correct. In general, shorter methods were associated with a higher fraction correct.

Students who correctly solved the problem using a given method tended to use less time on that method than students who did not correctly solve the problem. It was observed that students who did not solve the problem correctly tended to work until the time limit expired, while students who solved the problem correctly sometimes finished their work before the time limit was reached.

Method	Number of students using method	Fraction correct (with 95% confidence intervals)	Solve time (min)	Solve time (correct solution only)
Visual Estimation	6	0.83 [0.44, 0.97]	3.83	3.40
Geometric Approximation	23	0.39 [0.22, 0.59]	7.78	6.33
CAD	5	0.80 [0.38, 0.96]	7.80	7.25
Monte Carlo	4	0.50 [0.15, 0.85]	8.25	6.50
Integral	28	0.21 [0.10, 0.40]	9.53	9.00

Table 2. Accuracy and corresponding solve times. Fraction correct is the fraction of participants who obtained a correct solution with a given method. Solve time is the time spent on the method.

Of the 29 participants who obtained an answer, 26 (90%) obtained the correct answer. Of the 46 students who did not obtain the correct answer, 43 (93%) did not obtain an answer at all. The majority of incorrect solutions were unsuccessful because the problem solver didn't finish, not because they finished but obtained the incorrect answer.

Additionally, we analyzed the number of methods used by the participants (See Table 3). It was found that few students used two or more solution methods (8) compared to the number of students who used one method (64). Of seven participants who switched methods exactly once, four obtained correct answers, an accuracy of 57% compared to the overall accuracy of 36%. Additionally, one participant switched methods twice and obtained the correct answer. Even though this data is not statistically significant, it may suggest the existence of an optimal number of solution methods for maximizing the probability of correctly solving this problem.

Number methods	Number correct	Total number	Fraction correct (with 95% confidence intervals)
1	21	64	0.33 [0.23, 0.45]
2	4	7	0.57 [0.25, 0.84]
3	1	1	1.00 [0.21, 1.00]

Table 3. Number of methods used vs. solution outcome. Note that a higher fraction correct was associated with more solution methods.

We were able to recruit N = 72 participants for this study. This was enough to obtain a qualitative understanding of the relationship between problem solving performance and school year, type of method used, and number of methods used. However, it was uncertain whether further recruitment will yield additional insights for the effort required. To better investigate the underlying characteristics of problem solving with multiple methods, we formulate a mathematical model.

Modeling

A model was formulated to capture the essential behaviors of problem solvers with multiple methods. The model has the following basic elements:

- A time limit t_f, representing how much time the student has to solve the problem. This is a positive integer (no fractional quantities are allowed).
- Solve times t_i, representing how long it takes to solve the problem with method $i = 1, 2, 3, \ldots, n$. These solve times are also positive integers. A problem solver solves the problem with method i if they stay t_i consecutive timesteps on method i.
- Starting probabilities $0 \leq \beta_i \leq 1$, representing the probability that the initial solution method chosen is method i. Note that the sum of all starting probabilities is one, i.e. $\beta_1 + \beta_2 + \cdots + \beta_n = 1$.
- A switching probability $0 \leq \alpha \leq 1$, representing the probability that the problem solver switches methods at each time step. The probability the problem solver does not switch methods (at each timestep) is $1 - \alpha$.
- A total solve probability P_{solve}, representing the total probability the problem is solved.

The model makes the following assumptions about the problem solving process:

- The problem has a well-defined solution, and can be solved with more than one solution method. We do not account for problems without a well-defined solution, or problems with only one solution method.
- The time limit is known to the problem solver, so they can make decisions about how to allocate their time. This is consistent with the experiment.
- Once the student finishes a solution method, they have solved the problem. Within this work, we do not consider the case where the student finished the solution but made a mistake. (In our experiment, students who were unsuccessful tended run out of time instead of making a mistake.)
- If the student switches methods, they lose all progress on their current method.
- If the problem solver switches methods, and there is more than one other method available, the probability of switching to each of the other methods will be equal. For example, if there are n total methods, the probability of switching from any given method to another at each timestep will be $\frac{\alpha}{n-1}$.

The following example shows how the total solve probability P_{solve} is calculated as function of switching probability α.

Consider a problem with time limit $t_f = 4$ and two solution methods with solve times $t_1 = 2$ and $t_2 = 6$. Let the probability of starting on each method be $\frac{1}{2}$. Because the problem solver can work on either method 1 or method 2 on each of the four time steps, the following $2^4 = 16$ sequences of methods are possible:

1111, 1112, 1121, 1122, 1211, 1212, 1221, 1222, 2111, 2112, 2121, 2122, 2211, 2212, 2221, 2222

Let B be the number of method transitions in the sequence (e.g. going from method 1 to 2 or vice versa). For each sequence, the probability of the sequence occurring is the product of $\frac{1}{2}$ (the probability of starting on the either method), α^B (associated with the B method transitions), and $(1-\alpha)^{t_f-1-B}$ (associated with the $t_f - 1 - B$ instances of staying on the same method):

$$P_S = \frac{1}{2}\alpha^B(1-\alpha)^{t_f-1-B}$$

A sequence solves the problem if there are $t_1 = 2$ consecutive ones in the sequence. Method 2 cannot be used to solve the problem because the solve time is greater than the time limit. We can then summarize sequence probabilities and whether the sequence solves the problem:

Sequence	Solves problem?	Probability of the sequence
1111	Yes	$(1-\alpha)^3/2$
1112	Yes	$\alpha(1-\alpha)^2/2$
1121	Yes	$\alpha^2(1-\alpha)/2$
1122	Yes	$\alpha(1-\alpha)^2/2$
1211	Yes	$\alpha^2(1-\alpha)/2$
1212	No	$\alpha^3/2$
1221	No	$\alpha^2(1-\alpha)/2$
1222	No	$\alpha(1-\alpha)^2/2$
2111	Yes	$\alpha(1-\alpha)^2/2$
2112	Yes	$\alpha^2(1-\alpha)/2$
2121	No	$\alpha^3/2$
2122	No	$\alpha^2(1-\alpha)/2$
2211	Yes	$\alpha(1-\alpha)^2/2$
2212	No	$\alpha^2(1-\alpha)/2$
2221	No	$\alpha(1-\alpha)^2/2$
2222	No	$(1-\alpha)^3/2$

For all sequences that solve the problem, we add up their probabilities to obtain the total solve probability. We ignore sequences that don't solve the problem. Thus

$$P_{solve}(\alpha) = \frac{1}{2}(1-\alpha)^3 + 4 \cdot \frac{1}{2}\alpha(1-\alpha)^2 + 3 \cdot \frac{1}{2}\alpha^2(1-\alpha)$$
$$= \frac{1}{2}(1+\alpha-2\alpha^2)$$

Checking the graph of $P_{solve}(\alpha)$ (see Figure 2), we see that the maximum solve probability occurs at where $P'_{solve}(\alpha) = 0$, or $\alpha = \frac{1}{4}, P_{solve} = \frac{9}{16}$.

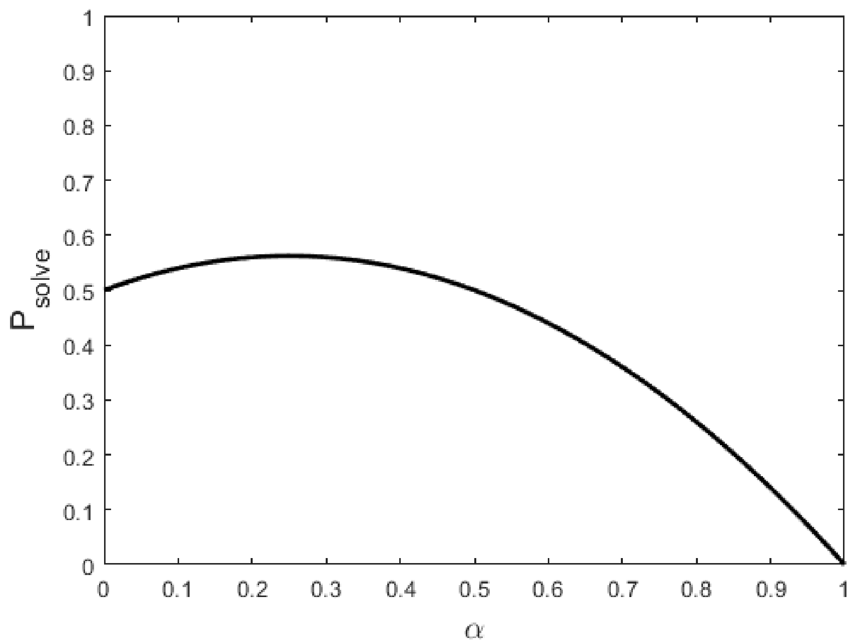


Figure 2. Graph of the solve probability $P_{solve}(\alpha)$. Note that at $\alpha = 0$, P_{solve} is $\frac{1}{2}$. This corresponds to the no-switch case, where the problem solver stays on the first method chosen. As α is increased, P_{solve} reaches a maximum of $\frac{9}{16}$ at $\alpha = \frac{1}{4}$. Further increasing α will lead to a decrease in P_{solve} until the solve probability reaches zero at $\alpha = 1$.

For the case with two solution methods, it was found that whenever $t_2 > t_f$ and $t_1 \leq \left\lfloor \frac{t_f}{2} \right\rfloor$ (where the $\lfloor \cdot \rfloor$ notation means to round down), there exists a maximum for P_{solve} at a nonzero switching probability $\alpha > 0$. That is, switching methods can help if the solve time of the shorter method is sufficiently low (see Figure 3). This fact can be proved mathematically, but the proof is outside the scope of this paper.

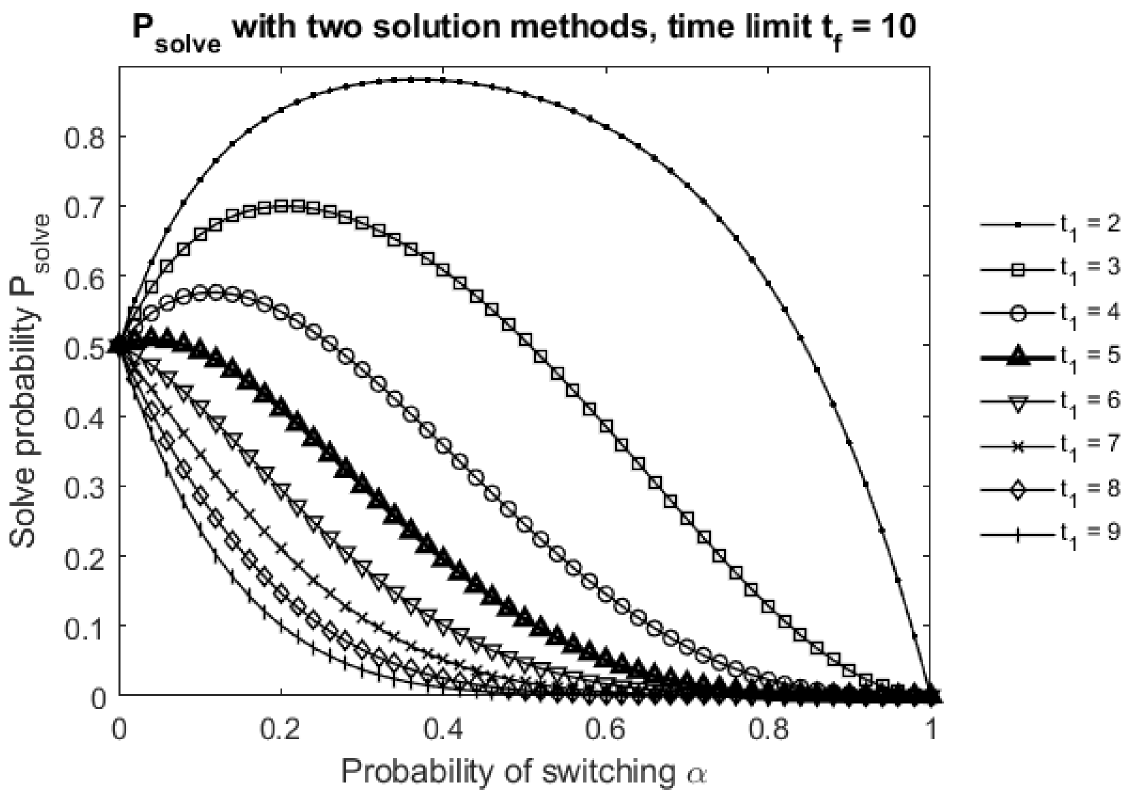


Figure 3. Plot of P_{solve} as a function of α. Here, $t_f = 10$ and $t_2 > t_f$. Whenever $t_1 \leq 5$, the maximum for P_{solve} occurs at a nonzero α. For $t_1 > 5$, the P_{solve} maximum occurs at $\alpha = 0$.

Additionally, the effect of the starting probability can be analyzed. In Figure 3, the starting probabilities were set to a default $\beta_1 = \beta_2 = \frac{1}{2}$. In Figure 4, the starting probability on the shorter method β_1 was varied. It was found that the higher the value of β_1, the higher the solve probability P_{solve} for all values of α. The greatest improvement in P_{solve} occurred for small values of α. Furthermore, at higher values of β_1, there is less need to switch methods, since the problem solver is more likely to start on the shorter (more desirable) method.

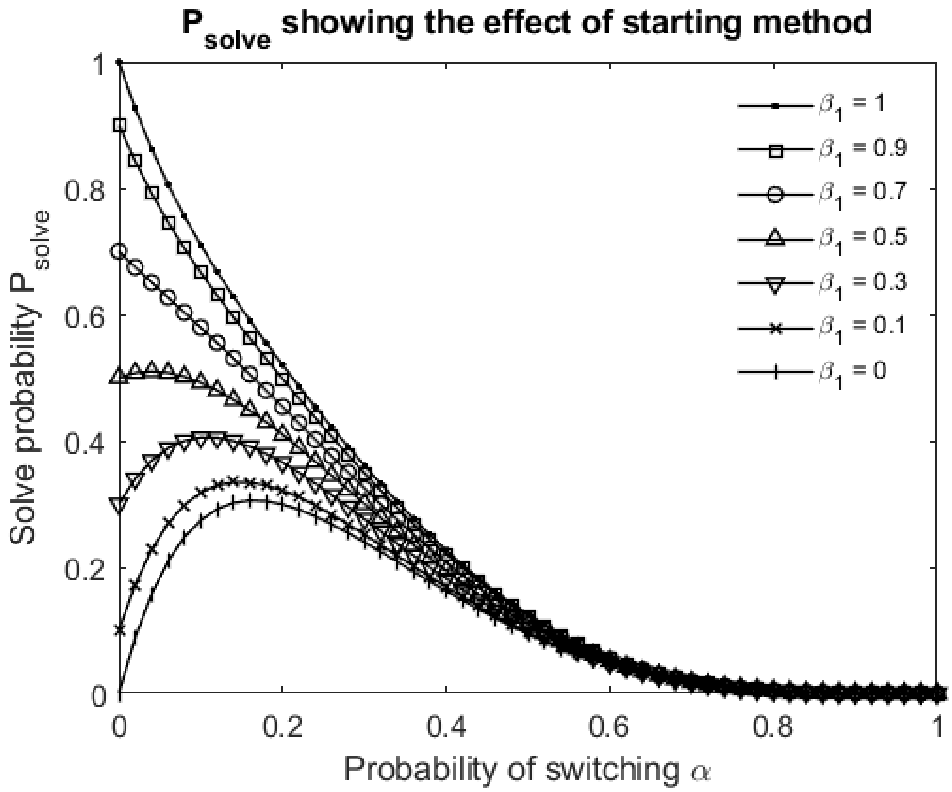


Figure 4. Plot of P_{solve} as a function of α with starting probability β_1 varied. Here, $t_1 = 5$, $t_f = 10$, and $t_2 > t_f$ Note that larger values of β_1 were associated with higher solve probability P_{solve} for all α. At the same time, the maximum P_{solve} is achieved with no switching for large values of $\beta_1 >$ 0.5. Switching maximizes P_{solve} for $\beta_1 \leq 0.5$.

Discussion

The model predictions and experimental data can be compared. In the experiment, there were a total of five participants who obtained the correct answer and more than one method. These five participants used various methods, but they obtained the correct answer only through two methods: Visual Estimation and Geometric Approximation.

Three of these five participants reached the correct answer through Visual Estimation; their average solve time was 4 minutes, compared to 3.4 minutes for all students who obtained the correct answers with Visual Estimation. The other two obtained the correct answer through Geometric Approximation; their average solve time was 2 minutes, compared to 6.3 minutes for all students who obtained correct answers with Geometric Approximation (see Table 1). These five students had an average solve time of 3.2 minutes, but the average solve time of all students who obtained the correct answer (through any method) was 6.5 minutes, approximately two times greater. This difference in solve times may imply that students who switched methods may be more adept at using problem solving strategy; they were not necessarily better at using a specific tool (such as Visual Estimation or Geometric Approximation). Instead, they were able to reduce their overall solve time by switching to a simpler method. In fact, all five of these participants spent five minutes or less on the method they used to successfully solve the problem. Given that the time limit is 10 minutes, this provides some corroboration for a key prediction of the model: that it would be beneficial to switch if there were methods with solve time of $\frac{t_f}{2}$ or less.

The experimental results and model predictions allow us to formulate the following recommendations for improving problem solving outcomes (see Figure 5).

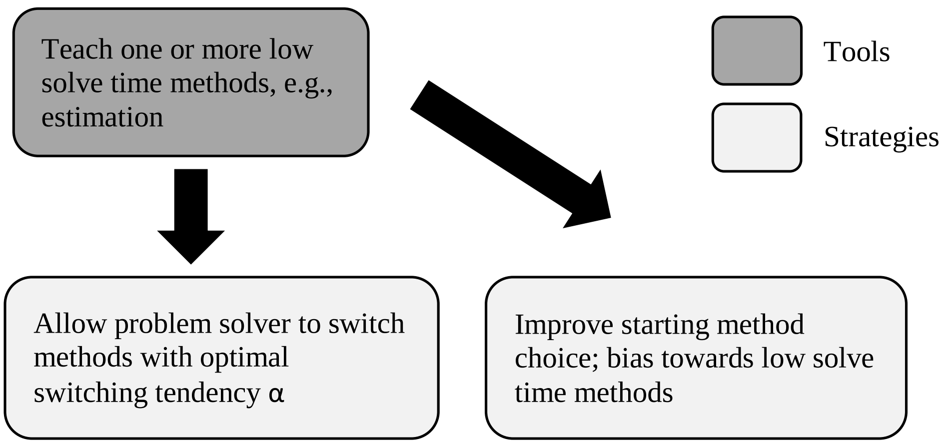


Figure 5. Tools and strategies to improve problem solving outcome with multiple solution method.

The first recommendation is to have students learn low solve time methods. One way to implement this is to instruct students to use estimation or approximation techniques in addition to traditional tools used for detailed analysis, thereby adding low complexity methods to students' toolbox. The experimental results suggest that students who use lower complexity methods are more likely to correctly solve the problem within the time limit. Additionally, the model suggests that adding low complexity methods will give students a chance to improve their P_{solve} by switching.

Once the student has low solve time methods in their toolbox, the next recommendation is to switch methods with the right frequency, that is, selecting an optimal α. To implement this, students can be taught that it is okay to switch their approach, if they see their current method as taking too long or unlikely to succeed. This strategy works by allowing the student to find a method that they are able to complete within the time limit, and can only improve P_{solve} if the student knows a sufficient amount of low solve time methods. By default, the model switches from method to method at random, without regard to method choice. Even so, it is still possible to improve P_{solve} if there are enough low complexity (low solve time) methods. This strategy is consistent with experimental results, which suggest that students who switch methods are more likely to correctly solve the problem.

The ability to use low solve time methods allows for another recommendation: start on less complex, lower solve time methods. It is a strategy consistent with how experts start problems, according to previous work by Li and Hosoi [10]. This strategy can be implemented by emphasizing to the student that they should first try a simple approach, and only proceed to a more detailed method if more accuracy is needed. This strategy requires the student to already know low solve time methods. Additionally, the model suggests that a better starting method improves P_{solve} regardless of whether the student switches methods or not. It has the biggest effect at low α; the better the starting method, the less need there is to switch methods.

These three recommendations should be deployed in a certain order for maximum effect (see Figure 5). Teaching low solve time methods should be implemented first. Doing so will give the problem solver a basis for deciding when to switch methods while problem solving, and which methods should be used first.

Conclusion

The experimental results show that students chose a variety of methods when solving a problem with multiple solution paths. Different method choices were associated with different outcomes; low complexity methods were associated with better problem solving outcomes than high complexity methods (Table 2). Additionally, most students who failed to solve the problem did so because they not finish their solution, not because they made a mistake. It was also found that most students only used one solution method. However, the students who used more than one solution method tended to have a higher rate of success than average, though the effect is not statistically significant (Table 3).

Building off these experimental results, the model shows that the existence of sufficiently short, low complexity methods (with solve times of approximately half the time limit or less) gives the problem solver an opportunity to improve their outcome by either switching methods. A lower solve time was associated with higher maximum solve probability and a higher optimal switching tendency α. Furthermore, the problem solver can improve their outcome by starting on the short method. A higher start probability on the shorter method was associated with higher maximum solve probability and a lower optimal switching tendency α. A better starting method reduces the need for a switching strategy.

The model and experimental results were then compared. It was found that students who used more than one method solved the problem with a low complexity method, and their solve time using this method was half the time limit or less. This confirms a key prediction of the model. From these results, we made recommendations for problem solving instruction. The first recommendation is to teach students low solve time methods. The second is to have students switch methods if they see their current method as taking too long or unlikely to succeed. The third is to have students start on less complex, lower solve time methods. These recommendations represent tools and strategies that a problem solver can use to improve their outcomes.

One way to implement these recommendations is to emphasize estimation. Previous work framed estimation as a standalone skill currently missing from the curriculum [7] [8]. However, the findings of this work suggest that estimation is a low complexity method that can supplement higher complexity methods along a spectrum of solution methods. The small time requirements and high probability of solve associated with lower complexity methods suggest that there may be concrete situations in which these can be incorporated into a class. An example of curriculum integration would be to ask a student to solve a problem two ways on an assignment or examination. For an assignment, this could potentially be helpful for increasing student awareness of multiple solution paths. For an examination, this could allow the instructor to better assess students' understanding.

Additionally, future case studies may seek to explicitly determine which subject areas in the curriculum it would make sense to deploy these lower complexity methods. For example, would there be opportunities include estimation or simple approximations in Fluid Dynamics, Statics, or Controls in mechanical engineering? If so, what are the benefits? If case studies can demonstrate effectiveness, it would show the value of estimation and approximation and motivate the inclusion of these skills into the engineering curriculum.

Acknowledgement

The authors would like to thank Prof. Warren Seering for his feedback on the ideas presented in this paper. Additionally, we would like to acknowledge the undergraduate and graduate students who participated in the study.

References

[1] B. Zimmerman, "Becoming a self-regulated learner: An overview," *Theory into practice,* vol. 41, no. 2, pp. 64-70, Spring 2002.

[2] "What is Self-Regulated Learning?," SERC at Carleton College, [Online]. Available: https://serc.carleton.edu/sage2yc/self_regulated/what.html. [Accessed 31 January 2023].

[3] P. Alexander, "Mapping the multidimensional nature of domain learning: The interplay of cognitive, motivational, and strategic forces," *Advances in motivation and achievement,* vol. 10, pp. 213-250, 1997.

[4] M. Svinicki, "A guidebook on conceptual frameworks for research in engineering education," *Rigorous Research in Engineering Education,* vol. 7, no. 13, pp. 1-53, 2010.

[5] A. Bahar and C. Maker, "Cognitive backgrounds of problem solving: A comparison of open-ended vs. closed mathematics problems," *Eurasia Journal of Mathematics, Science and Technology Education,* vol. 11, no. 6, pp. 1531-1546, 2015.

[6] R. Foshay and J. Kirkley, "Principles for teaching problem solving," PLATO Learning, Bloomington, MN, 1998.

[7] B. Linder, *Understanding estimation and its relation to engineering education,* Massachusetts Institute of Technology, 1999.

[8] S. Shakerin, "The art of estimation," *International Journal of Engineering Education,* vol. 22, no. 2, p. 273, 2006.

[9] D. Smith, J. D'Angelo, R. DaSilva and J. Sherwood, "An Experiment in Improving Student Estimation Skills," in *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, 1998.

[10] H. Li and A. E. Hosoi, "Starting Problems in Mechanical Engineering," in *IEEE Frontiers in Education Conference,* San Jose, CA, 2018.