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An Exploration of How Students Make Use of Hands-on Models to Learn Statics Concepts

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Abstract

This paper describes the results from an ongoing project where hands-on models and associated activities are integrated throughout an undergraduate statics course with the goal of deepening students' conceptual understanding, scaffolding spatial skills, and therefore developing representational competence with foundational concepts such as vectors, forces, moments, and free-body diagrams. Representational competence refers to the fluency with which a subject expert can move between different representations of a concept (e.g. mathematical, symbolic, graphical, 2D vs. 3D, pictorial) as appropriate for communication, reasoning, and problem solving.

This study sought to identify the characteristics of modeling activities that make them effective for all learners. Student volunteers engaged in individual interviews in which they solved problems that included 2D diagrams, 3D models, and worked calculations. Participating students had prior experience with the models and related activity sheets earlier in the course. Data was collected at the end of the quarter and the activities emphasized conceptual understanding. Thematic analysis was used to develop codes and identify themes in students' use of the models as it relates to developing representational competence.

Students used the models in a variety of ways. They wrote directly on the models, touched and gestured with the model, adjusted components, and observed the model from multiple orientations. They added new elements and deconstructed the models to *feel* the force or imagine how measurements would be impacted if one parameter was changed while all others held constant. In interviews students made connections to previous courses as well as previous activities and experiences with the models. In addition to using the 3D models, participants also used more than one representation (e.g. symbolic or 2D diagram) to solve problems and communicate thinking.

While the use of models and manipulatives is commonplace in mechanics instruction, this work seeks to provide more nuanced information about how students use these learning aids to develop and reinforce their own understanding of key concepts. The authors hope these findings will be useful for others interested in designing and refining hands-on mechanics activities toward specific learning goals.

Introduction

In engineering statics courses students work with contexts and concepts from a broad range of real-world applications. While there are a variety of formulae and procedural ideas to be learned and understood, it is also important to develop conceptual understanding of key course ideas. Active learning supports students' performance in the STEM disciplines as compared to traditional lecture [1], particularly for students from historically marginalized groups [2], and can support the development of conceptual understanding. One way to do this is to use 3D models of common statics concepts, such as vectors, forces, and moments to support students in understanding the relationships of different variables and parameters [3] [4].

This study is part of a larger project that, over the course of several years, has included designing 3D printed models and developing activities that support conceptual understanding of key content. Over several years of implementation in both face-to-face and remote learning, this project has offered insight into how 3D models can support students' learning. In prior work we described students' feedback about this curriculum when included as a series of in-class activities in a flipped classroom implementation. Students consistently reported the activities were useful for developing their understanding of concepts such as 3D vector operations, moments, and support reactions [5] [6]. Our implementation of the curriculum as a series of group learning activities however, made it impossible to disaggregate the elements of the activities that made the models effective. Classroom observations, focus groups, and student feedback all pointed to the usefulness of the models as a communication aid, both for student-to-student and student-to-instructor interaction. Pre and post course data has been collected over time to measure the impact of the models, including during the COVID-19 pandemic when the models and activities were adapted to be take-home materials. Students showed gains primarily on the topics of 3D vectors, moments, and rigid body equilibrium and line up reasonably well with areas where there are analogous differences in student feedback. While the sample size is small, the result that students with the models performed as well or better on nearly every assessment measure makes a compelling case that the models have benefits beyond facilitating communication [7].

Visualization of real-world concepts that are often presented in symbolic or pictorial ways can be challenging for students. Students use spatial skills to interpret representations and communicate their thinking. Using and choosing different representations when problem solving in statics is connected to both conceptual and procedural understanding. Representational competence, a construct introduced in the context of chemistry education [8], is the fluency with which a subject expert can move between different representations of a concept (e.g. mathematical, symbolic, graphical, 2D vs. 3D, pictorial) as appropriate for communication, reasoning, and problem solving. This construct has been used in science education research as an indicator of expertise [9] and is seen as connected to students' conceptual understanding [10].

In this paper, we share the ways in which statics students use 3D models, 2D diagrams, and symbolic representations to make sense of spherical angles used to indicate the direction of a vector in 3D space. Asking students to provide their reasoning aloud can offer insight into the ways in which different models can support conceptual understanding [11]. During the interviews, statics students solved and discussed their thinking using a variety of representations (mathematical, symbolic, pictorial, and concrete). This study adds to the empirical research on the role of representations and tools used in learning and practice in engineering, an area of need in the field [12].

Methodology

This study used semi-structured interviews with statics students in their last week of a 10-week quarter. Ten students engaged in hour-long interviews that included reflective prompts about their experiences in the statics course and STEM more broadly, as well as a series of three tasks focused on spherical angles, coordinate direction angles, and angles between 3D vectors. In each task the relevant calculations were provided, and the prompts asked students to consider how one value would be affected when another parameter was increased (e.g. when considering spherical angles, as a increases, does $\mathbf{r}_{DB}$ increase, decrease, or remain unchanged? See Figure 3.). During the recorded interview, students worked on one task, wrote their thinking, and then shared their reasoning aloud with the researcher, communicating the ways in which they reached their answers. The purpose of this structure and the decision to include the relevant calculations was to learn more about how students made sense of the conceptual ideas that underlie each task and the representations (calculations, 2D figures, and 3D models) they used to answer the questions and communicate their reasoning. Offering the calculations allowed for students to consider conceptual ideas and, if relevant and important to them, use the calculations to make sense of the problems. A 3D model (i.e. concrete representation) of the image provided in the Problem Statement (Figure 1) was given as well.

Problem Statement

The diagram to the right is like many you work with in Statics. The L-shaped post models a common structure used to support traffic signals and large road signs. Note the orientation of the coordinate system with origin at point A and how dimensional information is given both as dimension annotations and point coordinates.

Table 1. Numerical values for system parameters.

parameter	value	parameter	value
a	3.0 in	x_D	-5.0 in
b	7.0 in	y_D	-1.5 in
x_C	-4.0 in	z_D	4.0 in
y_C	4.0 in		

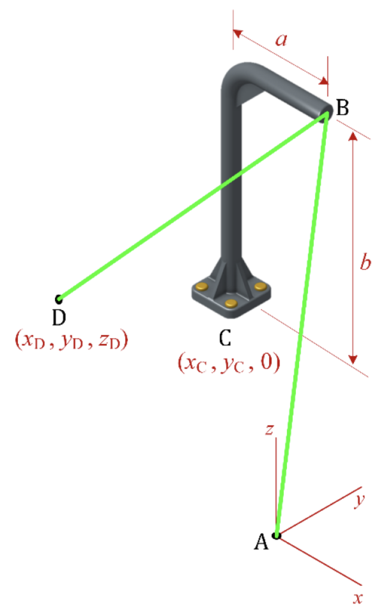


Figure 1. Problem Statement for the three tasks given during the interviews

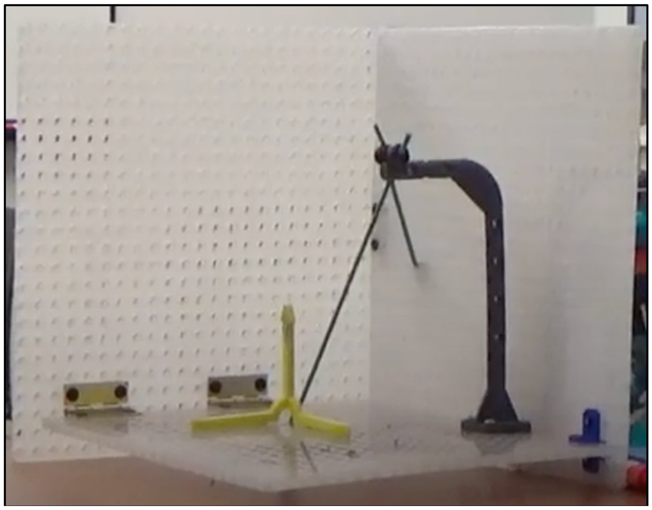


Figure 2. 3D Model used throughout the activities

Providing the worked calculations allowed for opportunities to see if students considered symbolic and numeric representations or used them to support sense-making of changing a parameter. Students were given time to respond to the task individually and then share their thinking with the researcher for each of the three tasks. Our analysis focuses on students' interactions with the 3D model and other representations for Task 1 only (Figure 3). This task was focused on spherical angles. All the models were provided; students did not need to create the 3D model (Figure 2). This allowed for them to choose the ways in which they engaged with the problems.

Part 1. Spherical Angles

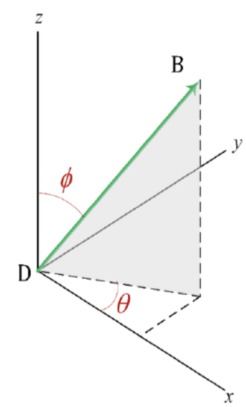
The vectors $\vec{\mathbf{r}}_{AB}$ and $\vec{\mathbf{r}}_{DB}$ can be represented in Cartesian components as:

$$\vec{\mathbf{r}}_{AB} = (x_C + a)\hat{\boldsymbol{\imath}} + y_C\hat{\boldsymbol{\jmath}} + b\hat{\boldsymbol{k}} = -1.0\hat{\boldsymbol{\imath}} + 4.0\hat{\boldsymbol{\jmath}} + 7.0\hat{\boldsymbol{k}} \text{ (inches)},$$

and

$$\vec{\mathbf{r}}_{DB} = (x_C + a - x_D)\hat{\boldsymbol{\imath}} + (y_C - y_D)\hat{\boldsymbol{\jmath}} + (b - z_D)\hat{\boldsymbol{k}} = 4.0\hat{\boldsymbol{\imath}} + 5.5\hat{\boldsymbol{\jmath}} + 3.0\hat{\boldsymbol{k}} \text{ (inches)}.$$

This representation has both magnitude and direction information embedded in the notation. Sometimes we need to separate out the direction and magnitude. One way to do that is by specifying the direction with angles in a spherical coordinate system as shown in the figure at right.



The angles ϕ and θ can be computed from the components of $\vec{\mathbf{r}}_{DB}$ as follows:

$$\theta = \tan^{-1}\left(\frac{r_{DBy}}{r_{DBx}}\right) = \tan^{-1}\left(\frac{5.5}{4.0}\right) = 54.0^\circ$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{r_{DBx}^2 + r_{DBy}^2}}{r_{DBz}}\right) = \tan^{-1}\left(\frac{\sqrt{4.0^2 + 5.5^2}}{3.0}\right) = 66.2^\circ$$

Task 1. Move the XYZ AXES MANIPULATIVE to point D to help visualize these spherical angles. Complete the following table to describe how the magnitude and direction of $\vec{r}_{DB}$ would change when the indicated parameter increases (↑). Use I for increase, D for decrease, and U for unchanged. In all cases, assume the other dimensions keep their original values.

Dimension	r_{DB}	θ	ϕ	Notes on reasoning
a ↑				
y_D ↑				

Figure 3. Task 1 focused on spherical angles

The prompts used during the semi-structured interview sections focused on problem solving included the following:

- *Please share how you decided whether the values increased, decreased, or remained unchanged.*
- *Which representations did you find helpful in making sense of the problem situation?*
- *I saw you (describes something student did), can you tell me more about why you did that?*

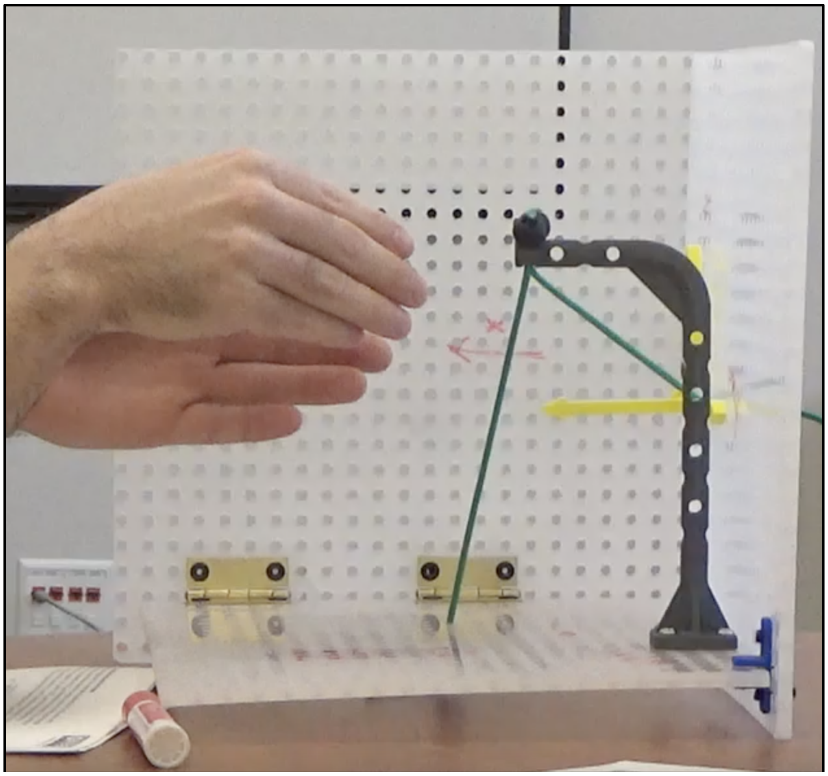


Figure 4. Image of student interacting with the 3D model

Analysis

Thematic analysis of the videos was used [13]. After all the interviews had been completed, each was viewed several times and general observations about the tools students used while solving problems were noted. Initial codes were developed, and multiple researchers coded selected sections of student interviews to discuss clarity of codes and consistency in coding. Revisions to the coding scheme were made based on the coding discussions and all interviews were coded to ensure reliability of coding. These discussions promoted reflexivity and dialogue among the research team, ultimately leading to the development of new codes (e.g. codes related to communication) [14] [15]. After refining the coding scheme, reliability of coding was 98%. Disagreements were discussed and resolved, resulting in 100% agreement in coding. More details about the development of the codes is provided in the next section.

Development of Codes

After all 10 of the recorded interviews had been completed, a thematic analysis was conducted through multiple views of each video. Throughout the videos, key moments were identified as times where students were actively engaging with or describing their thinking around the tasks. This meant that times when students were reading the directions or writing their responses on the activity sheet were not considered key moments. After repeated viewings and analytical descriptions of the key moments in videos were compiled, themes were developed. Students broadly engaged with the problems using either the 3D model, 2D diagrams provided in the tasks, or worked calculations included in the activity sheets. Within each of the models, there was variety and similarity in the ways students used the different models. For each code, representative examples and quotes from the analytical notes were included to support consistent coding.

Table 1. Ways Students Engaged with and Communicated Thinking about Statics Tasks

Representation Used	Code	Description, examples
Uses 3D SMK	**M-W** *Writes* on the 3D model	While engaging in problem solving, uses a dry erase marker to label or make other marks on the model
	M-G Uses *gestures* or touches with the 3D model	As a student considers changing a parameter, they use their hands (or a pencil or other object) to gesture how the model would change. Counts holes in the coordinate plane panels Puts hand or finger on part of the model, but does not change any of parts.
	M-A Physically changes or *adjusts* a component of the model	When considering a problem, the student makes adjustments or changes to the model by taking it apart in some way Student removes a cord lock and moves a green cord to show a changing parameter
	M-S *Stares* at the model, does not engage in another way	Looks at the model for an extended period of time (more than 10 seconds), but do not engage in other ways with it. Staring quietly… "I think y_D would remain unchanged"
	M-O *Observes/analyzes* the model from multiple vantage points	Moves physical orientation of body (standing up, tilting head, etc) to observe the model while problem solving.
	C-M *Communicates* reasoning using the 3D model	Justifies thinking about the parameters and relationships by interacting with the 3D model. "Just imagine *a* getting really long, like all the way out here, you can see how this would impact the angle α in this situation."
Uses Diagram	**D-W** *Writes* on the 2D diagram	While engaging in problem solving, writes on the given diagrams Adds labels, sketches or extends a line
	D-G *Gestures* while engaging with the 2D diagram	Student uses their hands (or a pencil or other object) to gesture how changing a parameter would impact the other values in the model. Points at the origin and then runs their finger along part of the diagram. Holds hand against a green line in the diagram and tilts their hand to show changing a parameter.
	C-D *Communicates* reasoning using the 2D diagram	Justifies thinking about the parameters and relationships by using 2D diagram. "You can see how B is going off in a direction that makes me think about the x-, y-, and z-axes"
Uses Calculations or Symbolic Reasoning	**S-C** *Calculates* values using calculator	Uses the symbolic representation to make sense of their answers and chooses to calculate a value. "If this increased to, say 10, then I can see that this value will decrease" (uses a calculator to confirm this)
	S-E Uses mathematical relationships to make sense of relationships between parameter changes	Without calculating, the student considers how changing a value in a calculation would impact the value of another parameter. "I can see that if this value gets less negative, then the other value will increase" Underlines a part of the formula and circles a value in the calculation, then writing an arrow points upwards.
	C-S *Communicates* reasoning using the analytical work (equations)	Justifies thinking about the parameters and relationships by connecting to the given calculations or known mathematical relationships. "If you think about how changing this value would impact the angle, you can see that it would increase, even if you take the square root of the value"

Upon development of the initial coding scheme, video clips from students that utilized a variety of models were coded by two researchers to explore reliability of coding and allow for refinement of the coding schemes. For example, while the majority of key moments were coded consistently, there was the observation that some students used the models in unique ways while *communicating* their understanding to the researcher, thus codes for communicating (M-C, D-C, and S-C) were included and videos were recoded.

The analysis of the 10 hour-long interviews provided a wealth of information about the ways in which students engaged with tasks that emphasized conceptual understanding of spherical angles. In this paper, we focus on comparing and contrasting the ways in which students used different representations, specifically a 3D physical model, 2D diagram, and symbolic representations that included given formulae and worked calculations.

Visualization of Student Interactions
One way to understand and compare students' ways of reasoning about the tasks posed during these interviews is to compare the types of models and ways in which they engaged with them (e.g. gesturing). One consideration posed during initial coding included whether to do a *count* of instances or a measure of *elapsed time.* For example, as a student was drawing on a 2D model, we deemed it was more valuable to consider the amount of time they spent writing on the model versus trying to determine if within a sustained amount of time a student labeled two distinct ideas, which was open to interpretation. Thus, we considered the amount of elapsed time a student spent engaging in a consistent way with a model in our coding.

The visualizations of each students' engagement during the tasks supported comparison among students and highlights the unique ways they interacted with the provided representations to make sense of the given tasks. Students A and B (Figures 5 and 6, respectively) show that while working on Task 1, they both used the 3D model and 2D diagram. Differences between the duration of time spent using the models, the ways in which they used each type of representation, and the distribution of codes varied in potentially meaningful ways. For example, Student A wrote on, gestured, adjusted, and communicated their thinking using the 3D model for extended moments throughout the whole task. They also wrote on and gestured with the 2D diagram for brief moments. Student B briefly wrote on the model, then spent a noteworthy amount of time gesturing or touching both the 3D model and 2D diagram, using both of those representations to also communicate their reasoning.

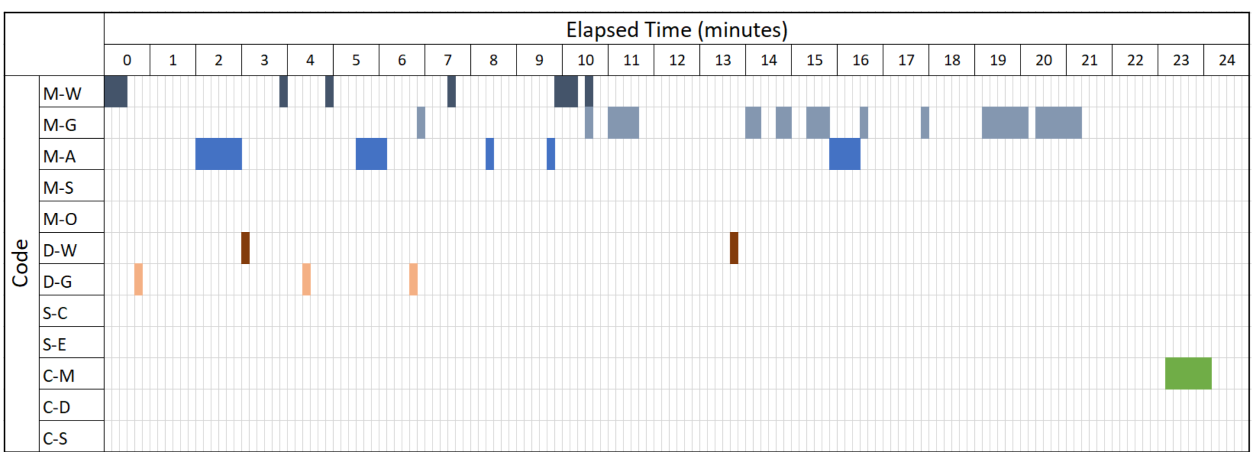


Figure 5. Visualization of Student A's engagement with models during Task 1

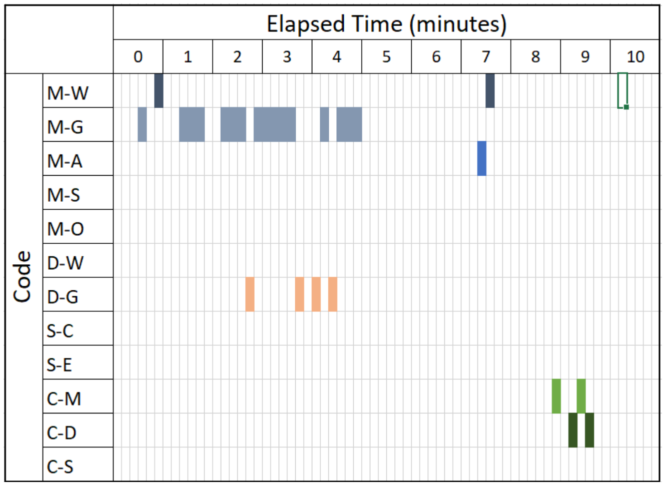


Figure 6. Visualization of Student B's engagement with models during Task 1

In contrast, Student C (Figure 7) used the provided calculations several times, employing their analytical and symbolic understanding to make sense of the task. In addition to using the formulae and calculations provided, when the student got stuck, they engaged with the model to support their understanding. Student C remarked,

> *For example, I know that if that* a *was increasing, it would influence the x-value of* r_{DB}*. That would make the top of the tangent or the inverse tangent for* θ *increase and would make the top of the inverse tangent for* ϕ *increase. I know that. But then I froze. I don't know what really happens when you increase the tangent. Does that make it go up or make it go down? So, then I started staring at the model.*

While they say they "stared" at the 3D model, in actuality the student was gesturing and touching the model. They counted units on the *y*-plane and gestured in a way that showed what *a* increasing would look like on the 3D model. They made connections between the symbolic and physical models.

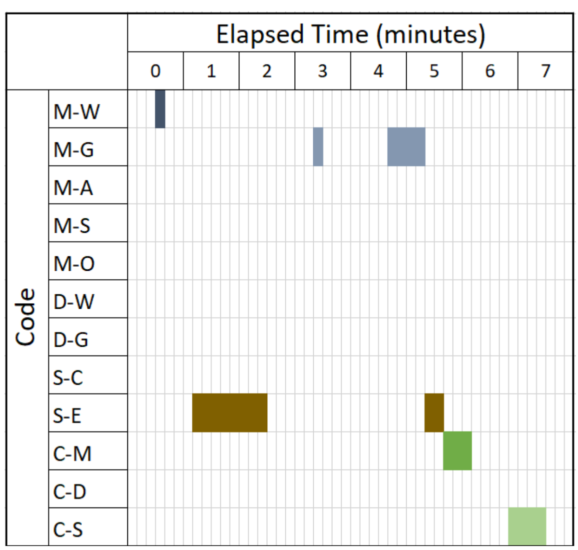


Figure 7. Visualization of Student C's engagement with models during Task 1

Student D used both the 3D model and 2D diagram to make sense of Task 1. For both representations, they used them in more than one way. Student D used both gestures and physical adjustment of the model during the interview. They both wrote on and gestured with the diagram as well. From the provided visualization (Figure 8), the ways in which they worked with different representations in different ways throughout the given time can be seen. When explaining their understanding and reasoning for their answers, they used the model as a tool for communication. This was consistent among all the students in this study.

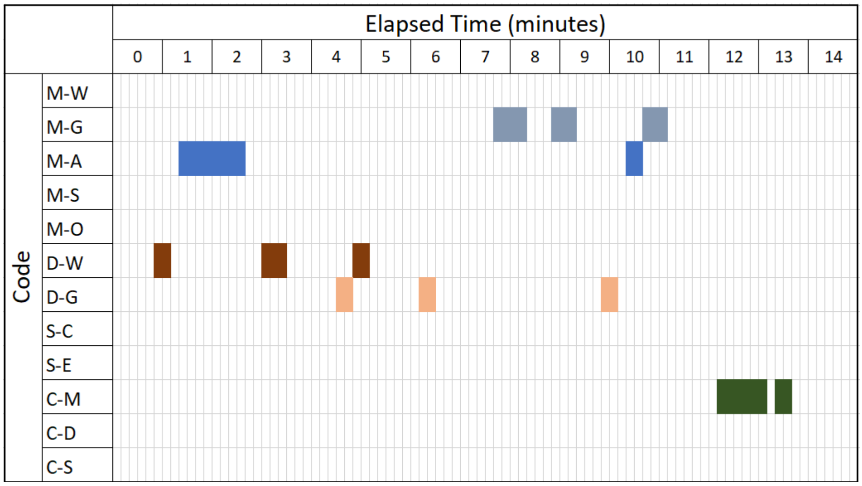


Figure 8. Visualization of Student D's engagement with models during Task 1

The visualizations (Figure 5-8) provide opportunities to see the similarities and differences between students as well as the amount of time students spent engaged with representations. These results demonstrate that learning aids with which students can interact with in a multitude of ways can be more helpful for larger numbers of students. Another way to compare the ways in which the students' engaged with the task is to look at the variety of representations and ways in which they used the representations (Table 2).

Table 2. Student Summary of Task Engagement

Student	Interactions during the problem solving									Communication		
	M-W	M-G	M-A	M-S	M-O	D-W	D-G	S-C	S-E	C-M	C-D	C-S
A	**X**	**X**	**X**			**X**	**X**			**X**		
B	**X**	**X**	**X**				**X**			**X**	**X**	
C	**X**	**X**							**X**	**X**		**X**
D		**X**	**X**			**X**	**X**			**X**		
E						**X**			**X**	**X**	**X**	
F		**X**		**X**		**X**	**X**			**X**		**X**
G	**X**	**X**	**X**		**X**	**X**	**X**		**X**	**X**	**X**	**X**
H		**X**	**X**	**X**					**X**	**X**		**X**
J		**X**				**X**		**X**	**X**	**X**	**X**	**X**
K		**X**		**X**		**X**	**X**			**X**	**X**	

See Table 1 above for the meanings of codes.

Among the 10 students, all used the model to communicate their reasoning. Nine of the 10 participants used two of more representations to communicate their reasoning. During the time they were working on the task, all 10 participants used more than one representation. Additionally, nine of the 10 participants used gesturing during the time they spent independently solving the task. One surprising finding, in light of the emphasis on symbolic understanding at the college level, is that only half of the students used the formulae and worked calculations during their problem solving process. Student E was an outlier in two ways – they did not use gesturing and they were the only person to comment that they didn't find the model helpful for solving this task.

Conclusion

Learning more about how students use and connect representations, such as 3D models, 2D diagrams, and given calculations can offer insight into what students find useful and how they make sense of statics concepts. Through individual interviews where students solved problems individually and then communicated their understanding and reasoning, we learned about the variety of ways students can engage with different representations, which offers insight into their developing representational competence.

Through analysis of individual problem-solving interviews, a set of codes was developed that provides insight into the ways students used 3D models, 2D diagrams, and symbolic representations to make sense of conceptual problems about spherical angles. Among the 10 students that participated in these interviews, there were notable similarities and differences in the ways they made sense of how increasing one parameter would impact other values related to important statics topics. All the students used multiple representations when solving and communicating their thinking about the tasks. The visualizations (Figures 5-8) provide a breakdown of the amount of time and type of representations used. By making connections between and showing flexibility with representations, students exhibited representational competence. The fact that all the students used the 3D model while explaining their thinking about how values would change when one parameter is increased provides evidence for the value that 3D models can offer in making sense of and communicating understanding. Considering that abstract, symbolic representations and understanding are often emphasized in statics, it is worth noting that only half the participants use the worked calculations or given formulae during their solution process or communication of their understanding.

This paper focused on developing a coding scheme and analyzing students' engagement with and communication around conceptual problems about spherical angles. These codes and analysis of students' thinking offer ways in which students use different representations in multiple ways to support their understanding of important statics concepts. Offering students multiple representations, specifically 3D models, can support students in making sense of and communicating their thinking. This can have implications for instruction, including offering 3D models and collaborative learning opportunities where students can use model to communicate with peers. Future work will include analysis of students' use of representations for other topics in statics. This will allow exploration into patterns among the ways students used 3D models, 2D diagrams, and symbolic representations across and within other topics.

Acknowledgment

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