

Teaching the Concept of Tipping in Statics: Pedagogy, Practical Examples, and Potential Activities

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Abstract

The concept of tipping and stability are closely related and have several practical implications and applications. For example, tipping is the reason behind several accidents and recalls and the resulting injuries and deaths. For instance, 200+ fatalities were caused by furniture tipping between January 2000 and April 2022. Improper loading can also cause the tipping of airplanes, cranes, and other products, resulting in significant financial consequences. On the other hand, some devices use instability for their functioning.

While this concept is essential, it is usually given limited attention in traditional statics books and lectures. It is generally discussed as just a short section on the equilibrium of a rigid body. The current treatment of this topic in statics books needs to be revised. Thus, this paper recommends a more in-depth examination of this topic and provides insights into the pedagogy, potential activities, and practical examples to help faculty better integrate the concept of tipping. The experimental results highlight the benefits of integrating the footprint concept in the tipping module.

1. Introduction

The Consumer Product Safety Commission (CPSC) report [1] estimates an average of 22,000 injuries treated by emergency departments due to product instability between 2018 and 2020. Tipping of any device (furniture, TV, material handling equipment, etc.) can be expensive, deadly, and damaging to the corporate brand. Furthermore, it may result in product recalls and attract litigation. Thus, tipping is an essential concept with profound implications if not addressed correctly.

Engineers deal with tipping during all phases of a product lifecycle. For instance, during the design phase, designers consider the imposed loads to synthesize an appropriate form to minimize tipping. During product use, they may restrict the loading and monitor the reactions at the supports to mitigate the risk of tipping. Engineers also deal with the practical issue of stabilizing (or avoiding tipping) in the field.

In most engineering curricula, the concept of tipping is covered in statics. Widely used statics textbooks rely on calculating the moment about a tipping axis (or a point) to determine the tipping condition. The tipping module is a minor section of the equilibrium of a rigid body chapter and is discussed with an analytical mindset. A change in the pedagogy, coupled with illustrative examples, hands-on activities, and objective assessment, can bring engineering relevance and nurture students' development of intuition, qualitative reasoning, and design skills. Trained in such pedagogy, the next generation of engineers can create safe and efficient designs. To this end, this provides the concept of footprint and related materials to help future engineers.

2. Literature Review

Douglas et al. [2] categorized the problems in statics textbooks using Jonassen's typology [3]. Their findings revealed that most problems were algorithmic, with a few being rule-based or story problems. Although these problems helped students strengthen their well-defined problem-solving abilities, they did not equip them with the skills required to handle ill-defined problems. Consequently, the authors suggested including ill-structured problems in the textbook. In addition, they stressed the significance of training students to solve such problems, stating that it would enhance their ability to think like engineers.

Condoor et al. [4] discuss students' perception of statics as an extension of physics and emphasize the need to make statics relevant to engineers. To this end, they recommend specific strategies, which include purpose-driven learning, fostering qualitative reasoning, quantitative problem-solving skills, design and research experiences, and integrating digital tools.

In their article, Wodin-Schwartz et al. [5] detail the Hands On Wednesdays (HOW) program at Worcester Polytechnic Institute (WPI), which involves students working in small teams and engaging in various tasks, such as measuring reaction forces and analyzing them against theoretical calculations. According to their findings, the experiential activities facilitate a deeper understanding of critical principles and bolster the students' self-confidence.

Edgar et al. [6] present a set of laboratory activities using the Static Stability Factor (SSF), a concept commonly employed in vehicle design to establish the circumstances under which a vehicle will roll over (tipping) or spinout (slipping). The static stability factor uses the track width and the location of the center of gravity to determine the stability. They found the SSF concept easy to comprehend for first-year students.

According to Ha and Fang [7], spatial abilities are crucial for learning engineering mechanics, yet often overlooked by engineering educators. They suggest encouraging sketching to enhance spatial skills instead of solely relying on figures from problems. Mueller et al. [8] also highlight the significance of graphic statics for nurturing design intuition. Johnson-Glauch and Herman [9] describe the use of heuristics by engineering students in drawing shear force and bending moment diagrams. Novices tend to focus on surface features, while experts identify deep features.

Sullivan et al. [10] found that incorporating an art module helped students improve their ability to solve three-dimensional statics problems. However, Litzinger et al. [11] discovered that spatial reasoning did not necessarily predict exam performance requiring sketching free-body diagrams. They also noted differences in the quality of diagrams and the use of self-explanation strategies between strong and weak students. To promote self-explanation, they recommend an intervention strategy. Finally, Sadowski and Jankowski [12] stress the importance of graphical methods in promoting structural intuition and visualizing forces.

Steif and Dollar [13] criticize traditional statics instruction and suggest a progressive approach to developing topics and concepts. Steif and Dantzler [14] introduce a validated Statics Concept Inventory (SCI) instrument – a set of multiple choice questions - to assess students'

understanding of statics concepts and evaluate progress, allowing for corrective action to be taken. Faculty teaching follow-on courses can also use this tool for prerequisite knowledge assessment. Gross and Dinehart [15] analyzed the errors made by thousands of students on quiz and examination problems. They found that most errors were non-conceptual, indicating that students better understood the material than their solutions showed. The literature review highlights the role of effective pedagogy and activities in nurturing engineering judgment, qualitative reasoning abilities, design skills, and self-confidence.

3. Pedagogy

3.1. Traditional method – The concept of tipping moment calculation

The concept of tipping moment calculation uses a point or an axis about which the rigid body will rotate when the force is applied. For determining the impending tipping condition, it uses the equilibrium equation.

Let us consider a table shown in Fig. 1(a), whose free-body diagram is shown in Fig. 1(b). Then, the moment equilibrium equations for the free-body diagram is:

$$\sum M_{\text{about point B}} = R_A \times 2a - W \times a + F \times b = 0$$

At tipping, the magnitude of R_A is equal to zero. Substituting the magnitude of R_A , we get

$$W = Fb/a$$

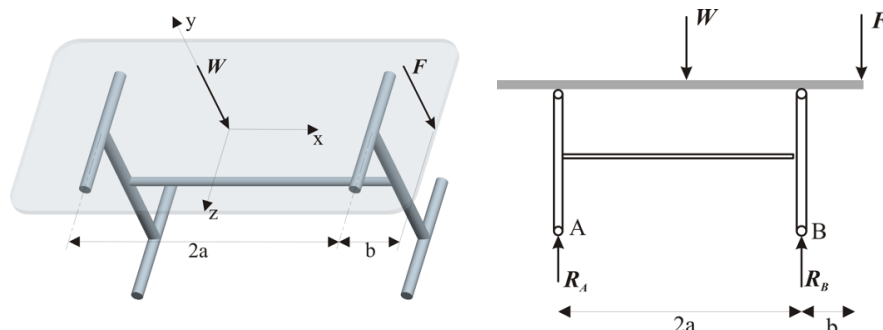


Fig. 1. (a) A table with a concentrated force acting on one end. (b) The free-body diagram.

Most statics textbooks discuss this method. While it is used but not highlighted, the reaction R_A is equal to zero is also helpful to experimentally determine the impending tipping condition by measuring the reaction force at the supports using force sensors. For instance, material handling equipment like cranes uses this strategy to detect impending tipping.

3.2. The Method of Footprint

This method of determining the tipping condition uses the concept of footprint. A footprint is *the minimum area by a closed curve with nonnegative (positive or zero) curvature at any point on the curve encompassing the base*. Another way to think about the footprint is that any line connecting two points of contact should lie entirely within the footprint.

While the definition is complex, the method for determining the footprint is quite simple. *Imagine an elastic band stretched to encompass the object at its base. The area enclosed by the band indicates the footprint.* Therefore, Fig. 2 shows the footprint for a table.

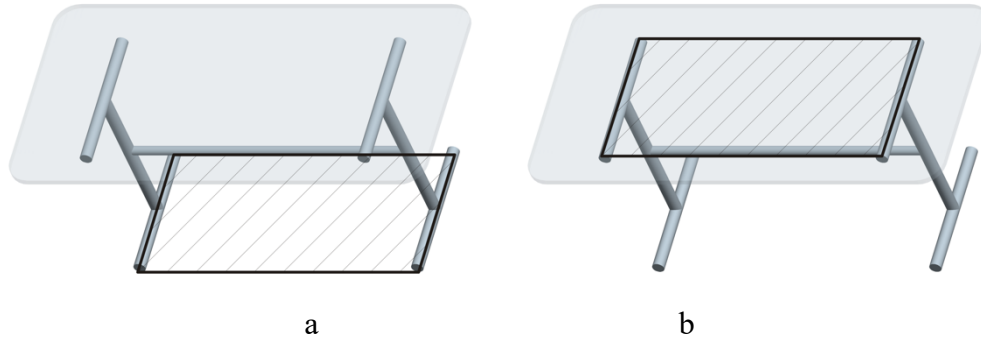


Fig. 2. The footprint assuming (a) the whole table is tipping (b) the glass top is tipping

The strategy for applying this concept is to find *the line of action of the equivalent load* and visualize its path with respect to the footprint. If the line of action of the equivalent load passes through the footprint, the body is stable. However, if the line of action is outside the footprint, the object will tip.

Let us look at the following problem – the Leaning Tower of Pisa has a diameter of 15.5 m, a height of 56 m, and weighs 15,000 metric tons. The tower leans at an angle of 5.50. Find the critical angle at which it will become unstable (from the rigid-body point of view).

If we approximate the tower as a cylinder with uniform density, the solution can easily be found using geometry, as shown in the figure below.

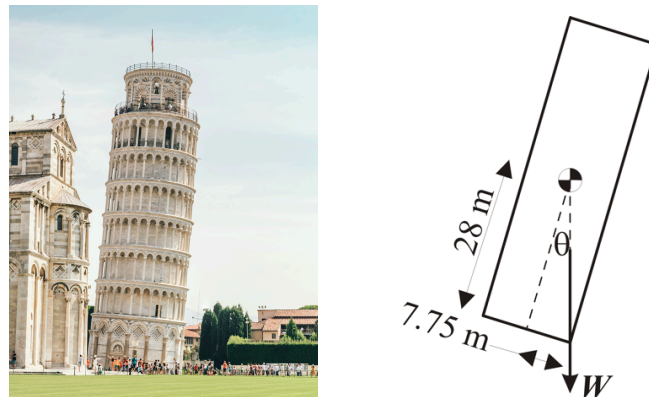


Fig. 3. The leaning tower of Pisa

3.3. Pedagogical Activities

To engage students, the suggested pedagogical activities include:

1. *Discussion of tipping and footprint concepts* – Reflective discussion can help students to understand the two approaches clearly, and when and how to apply them to their lives. These discussions can be constructive for students who need help with the retention and application of information.

2. *Demonstration* of tipping, even applying forces on the table in the classroom to using an elastic band to capture the geometry of the footprint, can help students to retain the concept.
3. *Design projects* foster the ability to formulate, ideate, experiment, and learn from concrete experiences. It can challenge students to stretch their abilities and empower students with different skill levels. Appendix I includes a sample design project based on the work of MacNamara [16].
4. Multiple choice questions (appendix II) can help in the quick quantitative assessment of the concepts.

3.4. Practical Examples

While the examples used in textbooks point to the negative implications, one can use tipping creatively to solve problems in real life. An example is a tipping bucket rain gauge, which measures the intensity of the precipitation. It consists of a cylindrical vessel (see Fig. 4). A funnel at the top of the vessel gathers and routes rainwater onto a tipping mechanism. The mechanism includes two buckets balanced like a seesaw or a balance. The mechanism tips when a small amount of rainwater falls into one of the buckets. In the process, it empties one bucket and exposes the second one to rainwater. The tipping motion also triggers an electronic signal often recorded and transmitted to the remote weather center. The rate of tipping corresponds to the precipitation's intensity and helps to characterize the precipitation. Although the tipping bucket rain gauge is not entirely accurate due to its sensitivity to few drops, it is a useful tool for measuring precipitation intensity. It creatively utilizes the phenomenon of tipping for its functioning.

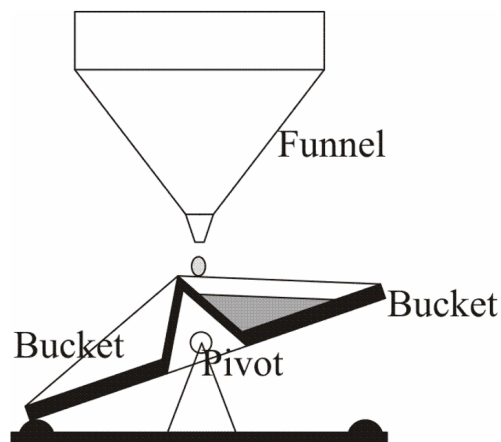


Fig. 4. A schematic of a tipping bucket rain gauge

Tail tipping occurs when an airplane tips over the rear wheels due to an increased load in the aft section. Fig. 5 shows a small aircraft before and after tipping due to the weight of a person sitting on its tail. Note that the tail tipping occurs about the rear wheels. It can occur even in large cargo aircraft, particularly while loading. An image search on the web for "tail tipping aircraft" can reveal a fantastic collage of tipped planes. When the aircraft approaches the tipping condition, the normal force acting on the nose wheel is greatly reduced. As a result, an aircraft close to tail tipping loses the traction force needed for steering during the take-off or landing. Thus, the tipping phenomenon plays an undesirable role in this example.



Fig. 5. A schematic of a tipping bucket rain gauge

4. Experimental Design

The experimental method is aimed at answering the following two research questions:

1. Will exposure to the footprint concept significantly improve the ability to identify the tipping correctly?
2. Does the footprint method help to address tipping-related issues?

The first question addresses the ease of identifying the footprint correctly. The second question determines whether it improves the performance of the problem solver.

The undergraduate aerospace, civil, and mechanical engineering students ($N = 31$) participated in the study. Initially, the participants were randomly divided into two groups and assigned two tasks. The entire session uses a pre-test and post-test design with a between-groups factor, as shown in Fig. 6. This experimental design aims to measure the participants' performance before and after (pre- and post-test), discussing the new method (intervention strategy – the footprint method). Note that although the two groups are relatively homogenous and the two tasks relatively similar, it may lead to different results. Hence, the tasks were switched between the groups to reduce variations in experimental conditions.

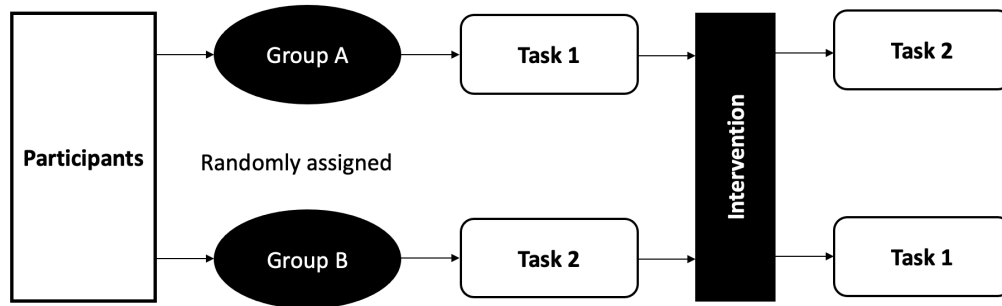




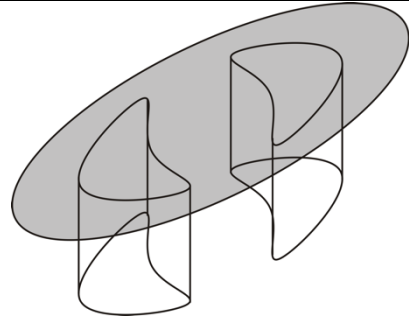
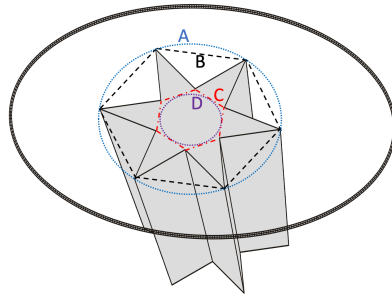


Fig. 6. Schematic of the experimental design.


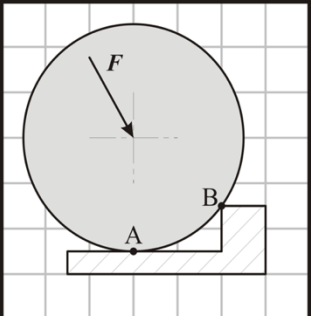
In each task, the participants answered two multiple-choice and one open-ended question. In task 1, the participants select the footprint for the elliptical tabletop, the condition for the tipping of aircraft, and describe three strategies to avoid the tipping of a crane. In task 2, they chose the footprint of a tabletop supported by a star-shaped base, the condition for a car wheel to go over a curb, and three strategies for avoiding the tipping of a dresser.

The two tasks included the following questions to determine how well the participants could identify the footprint.

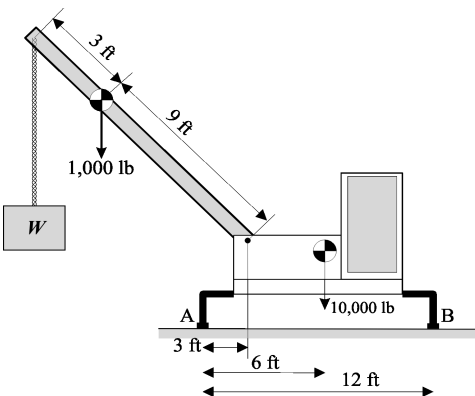

<p>Task 1</p> <p>1a. The footprint for the glass tabletop is resting on two supports is:</p> <p>A. </p> <p>B. </p> <p>C. </p> <p>D. </p>	
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<p>Task 2</p> <p>2a. A star-shaped base supports a glass tabletop. The footprint is:</p> <p>A. Blue dotted big circle A</p> <p>B. Black dashed hexagon B</p> <p>C. Red small hexagon C</p> <p>D. Purple dotted circle D</p> <p>E. The star shape</p>	
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The participants identified the conditions under which an object will tip in each task.

<p>TASK 1: 1b. The aircraft will tip if:</p> <ul style="list-style-type: none"> A. The aircraft is heavily loaded B. One side of the aircraft is heavily loaded C. The front is heavily loaded D. The back of the aircraft is heavily loaded E. None of the above – The plane can't tip 	<p>TASK 2: 2b. The car wheel hits the curb. The wheel can go over the curb, if</p> <ul style="list-style-type: none"> A. The magnitude of force F is large in comparison to the weight of the car B. The magnitude of force F is small in comparison to the weight of the car C. The magnitude of force F is equal to the weight of the car D. None of the above – the wheel cannot go over the curb. 
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Finally, to test the participants' abilities to identify strategies to stabilize structures, the two tasks include one open-ended question/task. The questions were:

<p>List three strategies to avoid tipping.</p>	
<p>Task 1 1c.</p> 	<p>Task 2 2c.</p> 

The study occurred in the statics classroom, and each session lasted 5 minutes. During the intervention, the participants learned about the footprint concept for about 15 minutes. The tasks were switched after the intervention.

5. Results and Conclusions

Fig. 7 shows the outcome of questions 1a and 2a for pre- and post-tests. The results suggest that irrespective of the table shape, most participants responded correctly after the intervention. Interestingly, in the pre-test, all participants in Group 2 incorrectly identified the correct geometry for the star-shaped tabletop, perhaps due to the complex shape as well as a lack of knowledge of footprint. These results reveal the participants' mental model before the intervention – the literal interpretation of "footprint" as the ground surface area in direct contact with the supports. The results signify that the intervention helped to clarify/correct the footprint concept.

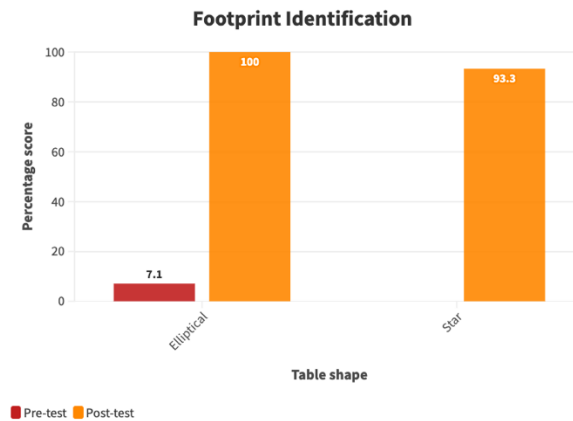


Fig. 7. Percentage score for identifying the footprint (For the star-shaped table, none of the participants identified the footprint correctly in the pre-test).

In the case of question 1b and 2b, which looks to identify the conditions under which an object will tip, the results are shown in Fig. 8. In both cases, more participants provided correct responses in the post-test (intervention). The percentage score for question 2b (car wheel) is less compared to question 1b, perhaps due to familiarity with the problem. However, the magnitude of increase in the score for pre- and post-test is almost the same. Hence, the results indicate that the footprint concept helped better identify the tipping conditions.

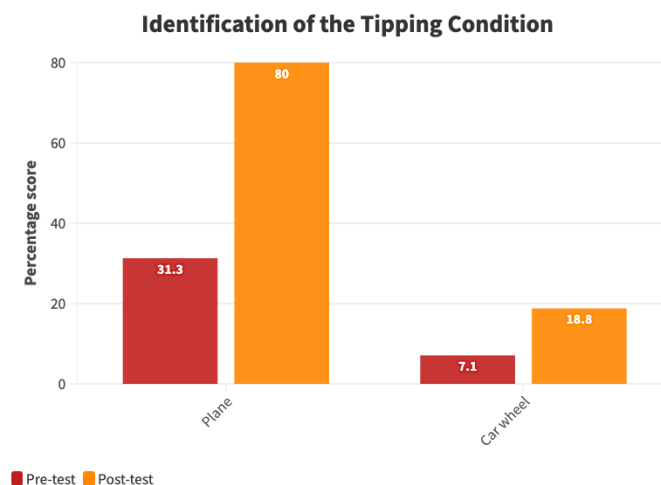


Fig. 8. Percentage score for identifying the footprint.

As mentioned in the previous section, questions 1c and 2c are open-ended, requiring higher-order cognitive skills. Each participant listed three strategies to deal with the scenarios. Table 1 shows the frequency response of the top three strategies. While tallying the frequencies, the scorers interpreted the intent. For instance, moving the location of the crane's feet is construed as increasing the footprint.

The results show a significant increase in the frequency of responses dealing with footprint after the intervention. For instance, the frequency of the footprint response for the crane tipping question more than tripled; perhaps the theory or examples inspired them to respond correctly. Further, in the post-test, including the footprint concept suggests that participants could implement it. Hence, one can see that the participants understood the concept and the importance of footprint.

Table 1: Frequency response of top three strategies

	Pre-test	Frequency	Post-test	Frequency
Dresser	Larger footprint	7	Increase footprint	11
	Reduce height	4	Reduce height	4
	Distribute weight evenly	4	Increase the weight of the dresser	4
Crane	Decrease the length of the crane arm	5	Increase the footprint	11
	Increase the weight of the base	3	Shorten the arm	9
	Increase the distance between the feet (or larger footprint)	3	Decrease the weight lifted	8

In short, the results indicate the utility of integrating the footprint concept into the tipping discussion.

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Appendix I - Sample Design Project

The project given to the class was based on the Assymmetric Equilibrium design competition of MacNamara [16], where students design structures that are in equilibrium but look like they are about to tip over. The requirements of the project are:

- Design an aesthetic, efficient, and economic structure inspired by equilibrium, the center of gravity, and tipping.
- Max allowable dimensions - 18 inches x 12 inches x 12 inches
- Can be 2D or 3D structures or machines – new or replicas
- Can be suspended from a frame or sitting on a plinth
- Can be made from any material.

The deliverables are:

Project Report - 60%

Report format

- The design concept (brief description – include inspiration and sketch)
- Design Calculations
- Determination of the center of gravity, forces and moments (theoretical)
- Determination equilibrium
- Picture of the final prototype
- Explanation of the difference between the prototype and calculations
- Reflection (what did you learn)

Physical prototype – 40% (Judges award points)

- Functional (works)
- Aesthetics

Examples of the student projects are shown in the figure below.

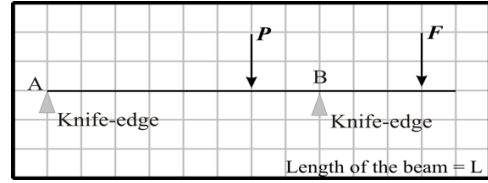


Fig. 9. Sample student work

Appendix II – Multiple-Choice Questions

1. At tipping, the following statement is false:

- A. R_A and R_B are equal
- B. $P = 1.5 F$
- C. R_A is zero and R_B is equal to $P+F$
- A. R_B is equal to $P+F$



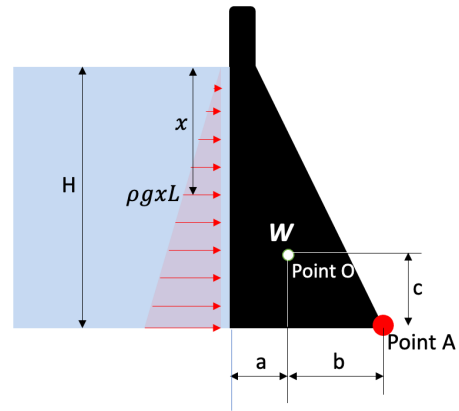
2. What is the maximum load that can be lifted without tipping:

- A. 0
- B. 3750 lb
- C. 7500 lb
- D. 15,000 lb



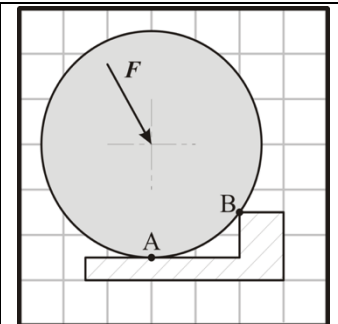
3. The hydrostatic force per unit length on the face of the dam at a depth x is given by $\rho g x L$ where ρ is the density, g is the acceleration due to gravity and L is the width of the dam. The weight of the dam is W and acts at point O. If the dam doesn't tip, then weight of the dam:

- A. $W \geq 0.5 \rho g H L \times H$
- B. $W \geq 0.5 \rho g H^2 L \times 0.333 H / a$
- C. $W \geq 0.5 \rho g H^2 L \times 0.333 H / b$
- D. $W \geq 0.5 \rho g H^2 L \times 0.333 H / c$



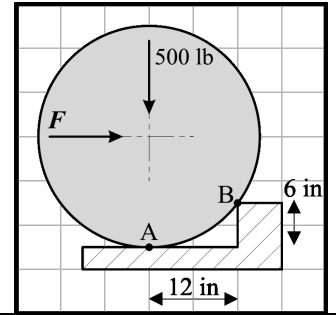
4. The car wheel hits the curb. The wheel can go over the curb, if

- A. The magnitude of force F is large in comparison to the weight of the car
- B. The magnitude of force F is small in comparison to the weight of the car
- C. The magnitude of force F is equal to the weight of the car
- D. None of the above – the wheel cannot go over the curb.



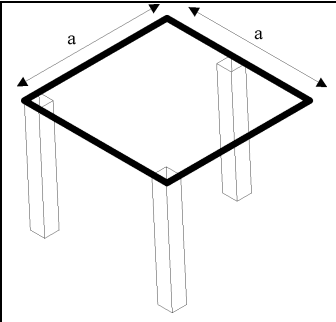
5. A car wheel hits a curb. The radius of the wheel is 15". The force F required to go over the curb is:

- A. 375 lb
- B. 500 lb
- C. 667 lb
- D. 1000 lb



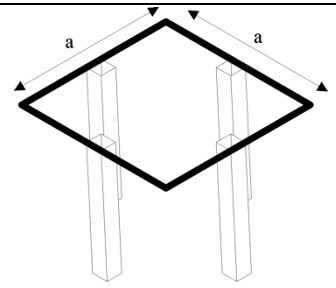
6. Determine the minimum load for tipping the table. The weight of the table is W .

- A. $W/3$
- B. $W/2$
- C. W
- D. $2W$
- E. None of the above



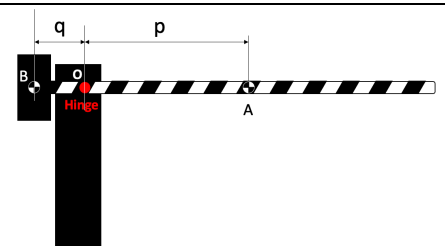
7. Determine the minimum load for tipping the table. The weight of the table is W .

- A. $W/3$
- B. $W/2$
- C. W
- D. $2W$
- E. None of the above



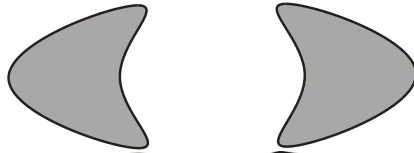
8. A gate for the parking garage is hinged at point O. The weight of the wooden bar from hinge O to the free end is 20 lb and its center of gravity is point A. For balance, if $p = 4$ ft and $q = 1$ ft, the counterweight must be:

- A. 5 lb
- B. 10 lb
- C. 20 lb
- D. 40 lb
- E. None of the above

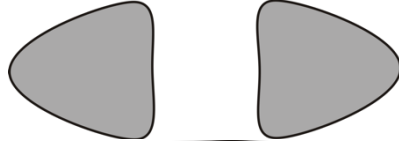


9. The footprint for the glass tabletop is resting on two supports is

A.



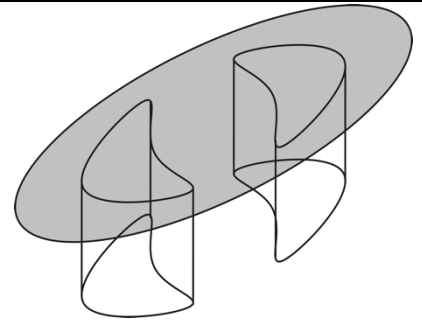
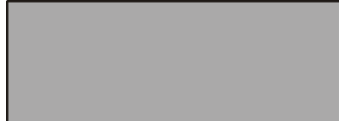
B.



C.

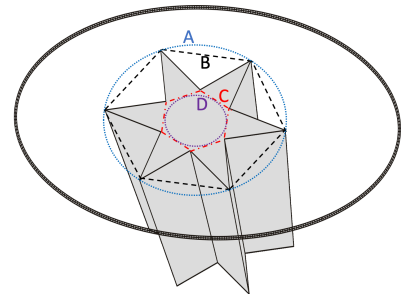


D.



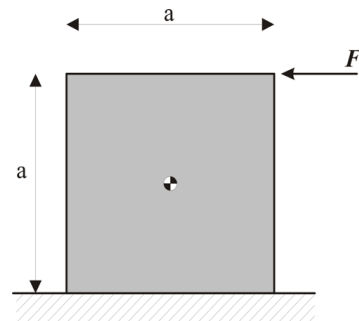
10. A star-shaped base supports a glass tabletop. The footprint is:

- A. Blue dotted big circle A
- B. Black dashed hexagon B
- C. Red small hexagon C
- D. Purple dotted circle D
- E. The star shape.



11. A force F is applied to the block. The weight of the block is W . Determine the necessary condition for the block to tip.

- A. $F \geq \mu W$ and $F \geq 2W$
- B. $F \leq \mu W$ and $F \geq 0.5W$
- C. $F \geq \mu W$ and $F \geq 0.5W$
- D. $F \leq \mu W$ and $F \leq 2W$



12. Angle θ is 30° , and the coefficient of friction is 1. If the dimensions of the block are $a = 1''$ and $b = 1''$, the block will:

- A. remain in equilibrium
- B. tip
- C. slip down
- D. slide up.

