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Assessment of the Efficacy of a Recently Proposed Alternative Presentation of the Second Law of Thermodynamics

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Abstract: The traditional presentation of the second law of thermodynamics uses imaginary heat engines undergoing specific imaginary processes to derive the Clausius Inequality, which, in turn is used to derive entropy and exergy relations. The specific and abstract nature of this derivation is an impediment to conceptual clarity and generalization. An alternative method of deriving the Clausius Inequality and other second-law results was recently proposed by the author. It does not rely on imaginary reversible processes occurring inside heat engines; all results can be derived for any arbitrary control volume with heat and/or work interactions. The efficacy of the alternate derivation has been assessed in this work, by comparing students from two class sections of an undergraduate introductory Thermodynamics course. Both sections received identical instruction for the traditional presentation, but only one section was taught the new derivation during one class lecture period; the derivation was then referred to multiple times during subsequent conceptual discussions. Conceptual understanding of both sections was then compared using a second-law concept inventory, and a few supplemental questions. The experiment was repeated for two years during the fall semesters of 2017 and 2018. The results are inconclusive; however, several positive aspects have been described to encourage other instructors to perform similar experiments.

Introduction: A new method of deriving the Clausius Inequality $ds>dQ/T$ has been recently proposed [1], that could provide a unique perspective to students by directly linking entropy generation to local processes. The derivation does not use imaginary reversible processes or heat engines, relying instead on simple arguments involving heat transfer in real arbitrary processes. All second law results can be derived as limiting mathematical cases from these arguments. Reversibility is defined mathematically for the first time. This allows entropy generation to be understood in terms of spatial gradients within the control volume.

The new derivation is presented in a nutshell below, see [1] for details. It was presented in a single lecture to supplement the traditional presentation, and also assigned for homework. It was referenced during subsequent class discussions.

Consider any arbitrary finite control volume across which work or heat transfer occurs. Split the finite control volume into infinite grid points. Consider an infinitesimally small region in two dimensions, represented by grid point '*x*', surrounded by four similar grid points, A, B, C and D, as shown in figure 1. Heat transfer to and from grid point '*x*' occurs along arbitrarily chosen directions. The infinitesimal amount of heat transfer dQ_A through dQ_D occurs across the interfaces of grid points A through D respectively, over an infinitesimally small time interval.

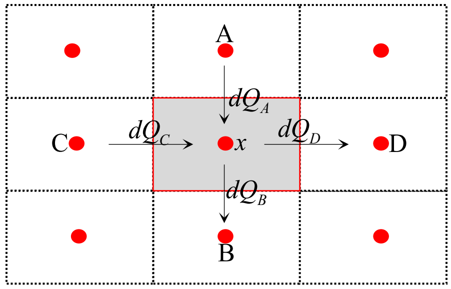


Figure 1. An infinitesimally small region split represented by grid point 'x' surrounded by grid points 'A','B', 'C' and 'D'

The second law states that heat transfer must occur across a negative temperature gradient. This means that$T_A \geq T_x$. Therefore

$$\frac{dQ_A}{T_A} \leq \frac{dQ_A}{T_x}$$

Hence,

$$\frac{dQ_A}{T_x} - \frac{dQ_A}{T_A} \geq 0$$

Similarly:

$$\frac{dQ_B}{T_B} - \frac{dQ_B}{T_x} \geq 0$$
$$\frac{dQ_C}{T_x} - \frac{dQ_C}{T_C} \geq 0$$
$$\frac{dQ_D}{T_D} - \frac{dQ_D}{T_x} \geq 0$$

The equations are written so that the usual sign convention holds; heat transferring into a grid point is treated as positive, while heat transferring away is considered negative. These equation, henceforth called 'interface equations' can be added together, and then rearranged to separate terms with and without denominator T_x. Then:

$$\frac{dQ_A - dQ_B + dQ_C - dQ_D}{T_x} \geq \frac{dQ_A}{T_A} - \frac{dQ_B}{T_B} + \frac{dQ_C}{T_C} - \frac{dQ_D}{T_D} \tag{1}$$
$$\Rightarrow dS \geq \sum_{cs} \frac{dQ}{T}$$

where the summation is over all control surfaces. This is essentially the Clausius inequality at a point. Instead of adding up the interface equations for a single point, an infinite number of interface equations for the entire the finite control volume could be summed up to obtain the result $dS \geq \int_{CS} \frac{dQ}{T}$, where the integration is performed over all control surfaces of the finite control volume, at any instant of time. Integrated over time this result would yield $\Delta S \geq \int_t \int_{CS} \frac{dQ}{T}$, i.e. the Clausius inequality for a finite control volume.

The two key points of this derivation are a) The rearrangement of terms at each point based on the denominator and b) Extending equation (1) from an infinitesimal point to a finite control volume, by imagining the summation of an infinite number of interface equations in a similar manner. Note that all terms of interface equations for internal points will contribute to ΔS. Only interface equations written at the boundary of the control volume will contribute to the right-hand-side of the Clausius inequality. Also note that the derivation is unchanged for the three dimensional case; only the number of interface equations to be imaginarily added increases.

All mathematical results of the second law, including Carnot cycle efficiency and exergy relations follow easily from this derivation without any imaginary arguments involving heat engines undergoing reversible processes [1]. The derivation of the Carnot Cycle Efficiency does not even require the concept of entropy. Students can easily understand that the Left-Hand-Side of equation (1) has to be zero at steady state, so, in the limit, $\sum_{cs} \frac{dQ}{T} = 0$. Considering heat interactions between only two control surfaces (hot and cold), the expression for Carnot cycle efficiency is readily obtained. In fact, using this approach, the seemingly abstract concept of entropy can be de-mystified by re-writing equation (1) as:

$$\Delta s = \int_t \int_{CV} \frac{dQ}{T} \geq \int_t \int_{CS} \frac{dQ}{T} \tag{2}$$

There is nothing abstract about adding $\frac{dQ}{T}$ at every point over space and time. It will certainly be appreciated by students who are familiar with computational methods. That entropy is a property, can be demonstrated for ideal gases by writing Δs as $\int_t \int_{CV} \frac{du+pdv}{T}$, which can be expressed as a point function that only depends on end states. Every grid point undergoes identical entropy change per unit mass of ideal gas irrespective of local processes.

Methods: An introductory Thermodynamics course for sophomore mechanical engineering students was chosen for this study. The course is typically offered through double sections with about 20-24 students in each section. To assess the efficacy of the new presentation, one section (experimental or participating group) was taught the derivation during a single 52-min class period while the other section (control or non-participating group) was not. Both groups were exposed to the traditional presentation of the second law: Six 52-minute class periods introducing the second law through reversible heat engine arguments, and nine 52-minute classes covering entropy and entropy generation. Exergy was not covered. The control group spent all 15 class periods learning the traditional presentation, while the experimental group spent 14 class periods doing that, and one lecture period learning the new derivation. Homework related to the traditional presentation was similar for both groups, comprising of problems assigned from the textbook [2]. However, the experimental group had to additionally re-do the derivation for homework, and were directed to refer to it during subsequent class discussion surrounding entropy and entropy generation. They received slightly less instruction (about 10%) on the traditional presentation in lieu of the new derivation over the 15 class periods. Overall instruction was very similar for both groups during the experiment.

A second-law concept inventory developed Jacobs and Caton [3] was chosen as the primary assessment instrument, and administered to both sections. This inventory comprised of 20 multiple choice questions with five possible responses. It was chosen instead of others [4-9] because it was the only inventory with a primary focus on the second law, and specifically developed to address the lack of second-law coverage in other inventories. Still, it did not have questions focused on entropy generation and its relationship with temperature gradients. Hence four supplementary questions were added to the assessment; they are available in the appendix.

The experiment was performed twice during the fall semester of 2017 and again in 2018. Both sections were taught by the author during 2017, while only the participating section was taught by the author in 2018. Only three supplementary questions were used in 2017; a fourth question was added in 2018.

Results and Discussion: Performance on the twenty Jacobs-Caton inventory questions and the supplemental entropy questions are presented in tables 1 and 2, corresponding to fall semester 2017 and fall semester 2018 respectively. The two columns show the mean scores for the experimental and control group respectively. The uncertainty of the population mean was calculated at a 95% confidence level, using a Student's t-distribution.

Table 1. Population means of Test Scores from fall semester 2017

Test (Max Points)	Mean Score: Experimental Group N = 24, 95% CI	Mean Score: Control Group N =15, 95% CI
Jacobs-Caton (20)	$\mu = 14.25 \pm 0.81$ (13.44 to 15.06)	$\mu = 13.20 \pm 0.63$ (12.57 to 12.83)
Supplemental (3)	$\mu = 0.94 \pm 0.31$ (0.63 to 1.25)	$\mu = 1.00 \pm 0.23$ (0.77 to 1.23)

Table 2. Population means of Test Scores from fall semester 2018

Test (Max Points)	Mean Score: Experimental Group (N = 16, 95%CI)	Mean Score: Control Group (N =25, 95% CI)
Jacobs-Caton (20)	$\mu = 13.29 \pm 1.13$ (12.16 to 14.42)	$\mu = 11 \pm 1.58$ (9.42 to 12.58)
Supplemental (4)	$\mu = 2.20 \pm 0.33$ (1.87 to 2.53)	$\mu = 1.33 \pm 0.45$ (0.88 to 1.78)

The sample means for the Jacob-Caton test are higher for the experimental group than the control group, but this could be because of random chance because the range of population means at 95% confidence overlap. This overlap is also true for the 2017 supplemental problems, while, for 2018, the experimental group performed better than the control group.

Figure 2 shows Student's t-distributions fitted to the data using the MATLAB 'fitdist' function. It can be seen that the experimental group is shifted to the right (higher scores) relative to the control group, but the distributions overlap to a large degree. It was not possible to fit the distributions for the supplemental problems because there were only three (2017) or four (2018) possible scores. The raw data is therefore shown by the histograms of figure 3. The 2018 data (left) is skewed towards higher scores for the experimental group but the 2017 data shows a reverse trend; the only case where the sample mean for the experimental group is lower than the control group.

The study suffered from several weaknesses. Sample sizes were not large enough to use the normal distribution to calculate the uncertainty of population means. The participating and non-participating sections were unequal in size (some sections were undersubscribed due to conflicts between class times and student schedules). For 2018, the participating and non-participating sections had different instructors. Generally speaking, student performance was extremely non-uniform; this exacerbated the small sample size issue. The 2017 scores were higher than the 2018 scores, for the same instructor (participating sections). Counterintuitively, the presentation of the new derivation was probably somewhat better during 2018 because some aspects of the derivation and its presentation were changed since 2017.

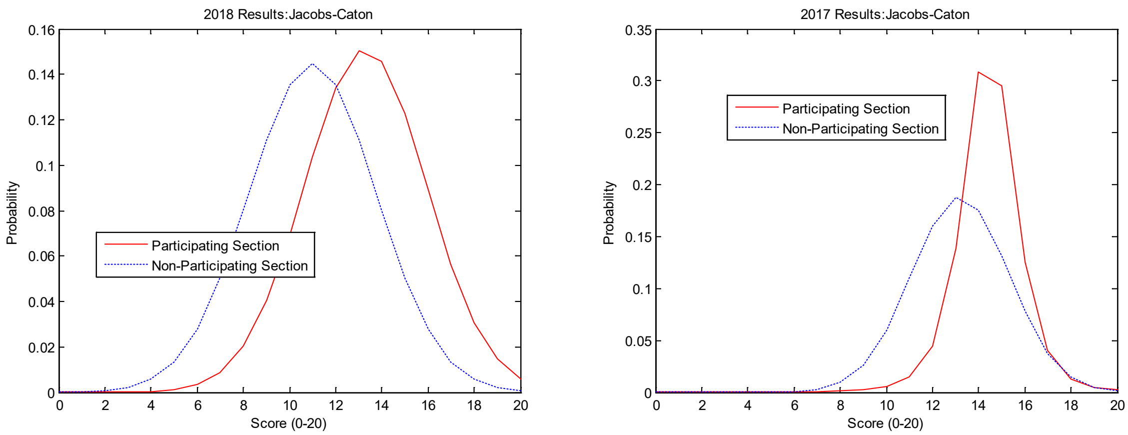


Figure 2. Fitted Student's t-distributions using scores from the Jacobs-Caton inventory for 2018 (left) and 2017 (right). The participating section is the experimental group.

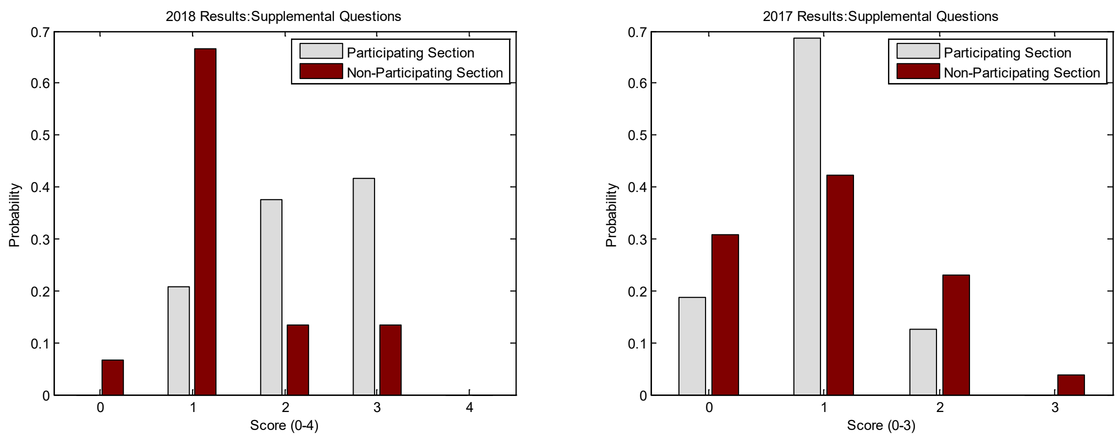


Figure 3. Histograms using scores from the supplemental questions for 2018 (left) and 2017 (right). The participating section is the experimental group.

Despite the inconclusive statistics, it is expected that the new derivation would help answer the supplemental problems better because they were based on the relationship between entropy generation and temperature gradients (see appendix). This is not typically a focus of the traditional presentation but a highlight of the new derivation. Reversibility is defined very specifically; a reversible process is one in which every interface equation (see introduction) must be an equality at every point in space and time. The inequality accounts for irreversibilities and grows with the gradient of$\left(\frac{dQ}{T}\right)$. It is easy to see from the interface equations how irreversibilities and entropy generation would increase due to local temperature gradients. A direct connection between exergy destruction and spatial uniformity of heat/work transfer is made. For processes not involving external heat transfer $\nabla\left(\frac{dQ}{T}\right)$ might be minimized by minimizing dQ everywhere within the control volume, by reducing (solid or fluid) friction. For processes involving heat transfer, the challenge is to transfer the required amount of heat within a finite time duration while minimizing∇T and/or$\nabla\left(\frac{dQ}{T}\right)$. Radiation heat transfer is particularly interesting because it is independent of the local∇T. Theoretically speaking, radiation heat transfer can satisfy the equalities of the interface equations, resulting in reversible heat transfer within finite time duration. Regardless of mode, a heat addition process that is more uniform in space, e.g. by locating multiple heat sources throughout the control volume, would produce lower ∇T and minimize the interface equations. These local arguments are more in line with modern computational methods than eighteenth century reversible processes. Therefore, they might provide a valuable context to engineers and designers who seek to minimize entropy generation.

Conclusions: A new derivation of the Clausius Inequality and subsequently all second law results was taught to one of two sections of an introductory Thermodynamics course offered to mechanical engineering sophomore students. A single 52 minute lecture period was used for the derivation, which was referred to in subsequent class discussions. Both sections were taught the traditional presentation of the second law. The performance of the two sections was then compared using a second law concept inventory and a few supplemental questions over a period of two years. The results were inconclusive because the population means for both sections overlapped for three out of four assessments.

However, it is possible that the new derivation helps understand concepts better through its non-reliance on imaginary devices and processes. It also relates irreversibilities to temperature gradients by precisely defining reversible processes mathematically. This aspect is missing from the traditional presentation of the second law. The study was not perfect and suffered from a number of weaknesses. It might be possible to further improve the proposed presentation and offer it to more students. It is hoped that other instructors incorporate the derivation and perhaps perform similar experiments to assess its efficacy.

REFERENCES

1. Brahma, I. (2018). An alternative derivation of second law results to better relate derivation to practical exergy analysis. *International Journal of Exergy*, *25*(4), 326-338.

2. Cengel, C.A. and Boles, M.A. (2011) Thermodynamics-An Engineering Approach, 7th ed., McGraw-Hill, New York, NY, pp.277–486.

3. Jacobs, T. J., & Caton, J. A. (2014). An Inventory to Assess Students' Knowledge of Second Law Concepts. *age*, *24*, 1, Annual ASEE Conference, Indianapolis 2014

4. M. J. Prince, M. A. Vigeant and K. E. K. Nottis, Assessment and repair of critical misconceptions in engineering heat transfer and thermodynamics, in 120th ASEE Annual Conference & Exposition. 2013: Atlanta, Georgia.

5. K. C. Midkiff, T. A. Litzinger and D. L. Evans. Development of engineering thermodynamic concept inventory instruments. in 31st ASEE/IEEE Frontiers in Education Conference. 2001. Reno, NV.

6. S. Yeo and M. Zadnik, Introductory thermal concept evaluation: assessing students' understanding. The Physics Teacher, 2001. 39(November): p. 496 - 504.

7. D. L. Evans, G. L. Gray, S. Krause, J. Martin, K. C. Midkiff, B. M. Notaros, M. Pavelich, D. Rancour, T. Reed-Rhoads, P. Steif, R. Streveler and K. Wage. Progress on concept inventory assessment tools. in 33rd ASEE/IEEE Frontiers in Education Conference. 2003. Boulder, CO.

8. B. M. Olds, R. A. Streveler, R. L. Miller and M. A. Nelson. Preliminary results from the development of a concept inventory in thermal and transport science. in American Society of Engineering Education Annual Conference & Exposition. 2004. Salt Lake City, Utah.

9. R. L. Miller, R. A. Streveler, D. Yang and A. I. S. Roman, Identifying and repairing student misconceptions in thermal and transport science: Concept inventories and schema training studies. Chemical Engineering Education, 2011. 45(3): p. 203 - 210.

APPENDIX A

The supplemental entropy generation questions refer to net entropy generation for the CV enclosing the entire system described, not the surroundings.

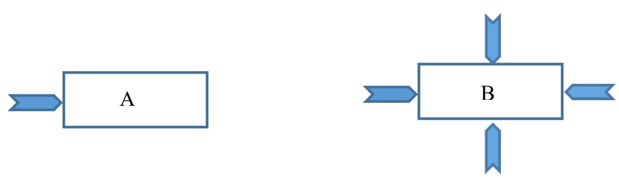
S1. A heat source at 150º C is used to heat saturated liquid at 150º C to saturated vapor. No heat escapes to the surroundings. Theoretically speaking, the calculated the entropy generated for this process would be:

a. $S_{gen} > 0$
b. $S_{gen} < 0$
c. $S_{gen} = 0$
d. $S_{gen} = \Delta S$

S2. Heat source A exists at 1000º C, and heat source B exists at 500º C. Both heat sources are used to transfer 50 kJ of heat to 1 kg of air existing at the same initial state. Therefore the heating process is faster with heat source A. No heat escapes to the surroundings. Then:

a. $S^A_{gen} > S^B_{gen}$
b. $S^A_{gen} < S^B_{gen}$
c. $S^A_{gen} = S^B_{gen}$, both non-zero
d. $S^A_{gen} = S^B_{gen} = 0$

S3. A mass of air A is heated with a single heat source, while the same mass B is heated with four heat sources, as shown in the figure. Both A and B start from the same initial state, and an equal amount of heat is transferred for both cases, resulting in the same final state. Both processes occur over identical time periods, hence the heat source for A is at a higher temperature than heat source B. No heat escapes to the surroundings. Then:



a. $S^A_{gen} > S^B_{gen}$
b. $S^A_{gen} < S^B_{gen}$
c. $S^A_{gen} = S^B_{gen}$, both non-zero
d. $S^A_{gen} = S^B_{gen} = 0$

S4. Jack and Jane weigh the same and travel the same exact distance, and start at the same time. However, Jack runs while Jane walks slowly, arriving much later. Which of the following statements is true from a Thermodynamic point of view?

a. Jack and Jane’s actions are reversible
b. Jack and Jane’s actions are irreversible; however Jack’s actions produce greater irreversibilities.

c. Jack and Jane’s actions are irreversible; however, Jane’s actions produce greater irreversibilities.
d. Jack and Jane’s actions are equally irreversible.