2023 Annual Conference & Exposition
Baltimore Convention Center, MD | June 25 - 28, 2023
The Harbor of Engineering Education for 130 Years
Paper ID #37036

Oscillators for System ID and Inertia Measurement in Undergraduate Dynamics

Dr. Michael P. Hennessey, University of St. Thomas

Michael P. Hennessey (Mike) joined the full-time faculty at the University of St. Thomas as an Assistant Professor fall semester 2000 and was promoted in 2014 to Professor of Mechanical Engineering (tenured since 2006). He is an expert in kinematics, dynamics, and control of mechanical systems, along with related areas of applied mathematics, such as in automation and transportation. As of summer 2023, he has 54 publications, in journals (11), conferences (41), magazines (1), and patents (1). Mike gained 10 years of industrial and academic research lab experience at 3M, FMC, and the University of Minnesota prior to embarking on an academic career at Rochester Institute of Technology (3 years) and Minnesota State University, Mankato (2 years). Mike holds a Bachelor of Mathematics from the University of Minnesota (with distinction), an MS in Mechanical Engineering from MIT, and a Ph.D. in Mechanical Engineering from the University of Minnesota. He is also a member of ASME, SIAM, and ASEE.

Amir Ahmad Naqwi

Oscillators for System ID and Inertia Measurement in Undergraduate Dynamics

Abstract

Oscillators are a very important topic in undergraduate Dynamics, both in a lab and lecture project setting. This paper shares our experience and lessons learned over many years using two systems: (1) a low-cost translational damped oscillator instrumented with an infrared proximity sensor, along with LabVIEW and the myDAQ from NI in lab for use in a system identification (ID) problem and (2) a cable-based rotational oscillator in lecture as an integrated theory-simulation-design-manufacturing-measurement final project that permits determination of the mass moment of inertia of a symmetric rigid body about a fixed axis. In each case, there is a spectrum of content, from mathematical modeling, to numerics and simulation using MATLAB/Simulink, practical realization in hardware along with either basic or more formal measurement. For the translational oscillator, the methodology for establishing system parameters based on iteration is surprisingly simple, very accurate, and has played especially well with students who lack a strong math background. Regarding the rotational oscillator, representative student work is presented and subsequently analyzed from different points of view, including percent measurement error when compared to a fiducial. Based in part from student feedback (such as through IDEA), we believe that this broad spectrum approach has wide appeal, in particular, use of the translational oscillator, as there is considerable variance in learning styles, areas of emphasis, and abilities within the mechanical engineering student population.

Introduction

Mechanical oscillators, such as the second-order translational "*mbk*" system have been a classic topic in undergraduate mechanical engineering (ME) education for many years. Typically, students are first exposed to them in their introductory ordinary differential equations (ODEs) course and in either a "rigid body dynamics" [1] and/or a "system dynamics" [2] course, all of which are required courses, vs. an elective course, such as Vibrations. Coverage in a lecture setting is common and in some cases simulation software is used, such as MATLAB/Simulink [3]. That said, based on the first author's experience teaching at multiple institutions and from examining the academic literature, oscillators in lab are less common, in part because not every Dynamics course has an affiliated lab. At the University of St. Thomas, within the ME program, there has been a strategic focus on offering students an integrated educational experience in essentially every core ME course, as in representation of theory, simulation, and some kind of lab/hands-on experience [4]. This perspective sets the stage for the paper, which focuses on: (1) a lab experience with a mechanical oscillator of the translational variety and (2) a final integrated project on a mechanical rotational oscillator involving suspension of a symmetric rigid body object by cables. Both of these experiences take place within the Dynamics course (ENGR 322 [4] – [6]).

To contextualize the role of the translational oscillator in an ME lab setting, it is worth studying the recent literature as educators have employed different approaches. Durfee et al. [7] incorporate a scaled "1/4 car" suspension model into a small, inexpensive System Dynamics and Controls take-home lab kit. This physical model is comprised of reconfigurable masses & springs and a rubber band, which is the basis of a 4^{th}-order model. Another, more recent piece of take-home lab equipment with a mechanical translational vibration theme to it is that developed by Tekes et al.

[8] which possesses multiple thin beams that supports creation of different configuration flexible linkages. Analysis support is from SolidWorks finite element analysis (FEA) [9] and MATLAB/Simulink, along with LabVIEW [10] for data acquisition. Bowen et al. [11] have developed a "haptic paddle" with the aid of LabVIEW where students can learn about properties of the classic *mbk* second-order system. Semke [12] describes how to instrument a cantilever beam using LabVIEW and conduct basic vibration analysis, such as determining the natural frequency and damping ratio (assuming use of a second-order model). Lastly, Burchett [13] has developed an instrumented translational dynamic system that forms the basis of a second-order system ID lab experience. Comparison of the experimental results with the simulation results is generally good, but yet off a bit due to the effects of Coulomb friction, which is intended to be part of the learning experience. In our lab students attempt to match the displacement output vs. time profile for a simple laboratory set-up with that from the output of a standard second-order system; in essence, practically solving a system ID problem.

Regarding the measurement of a rigid body's mass moment of inertia, various techniques have been used for decades, going back to at least the 1950s, such as by Ellett [14] and others [15] – [17]. In all of the cases presented, they use either torsion springs (much like wires as the aspect ratio is quite high) or strings/cables, such as that presented below, but with a slightly different configuration. In almost all cases, small oscillations are considered from which a standard second-order oscillator ODE results and the inertia can be solved for based on the square of the period of oscillation. Usage varies considerably, including determining the mass moment of inertia of large, heavy vehicles as the roll stability depends upon it. Mention is made of the "trifilar" suspension system involving three cables, apparently a popular configuration used [17].

Translational Damped Oscillator System ID (formerly known as Bench Testing of a Proximity Sensor Lab)

Modeling the *mbk* system under study

This lumped-parameter system, shown conceptually in Fig. 1 is approximated as the familiar *mbk* system known to all mechanical engineers, where we are assuming free (vs. forced) oscillations. Here x is measured from the static equilibrium position of the mass. From the free-body-diagram (FBD) in Fig. 1b we can apply Newton's Law Second Law to obtain the governing differential equation (with $kx_s = mg$ cancelling):

$$m\ddot{x} + b\dot{x} + kx = 0 \tag{1}$$

Initial conditions (ICs) are $x(0), \dot{x}(0)$ and a slightly simpler form emphasizing damping and natural frequency is:

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0 \Rightarrow \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0 \tag{2}$$

with ζ being the dimensionless damping ratio ($0 < \zeta < 1$ to make the system oscillatory, or underdamped, and interesting to observe in lab) and ω_n being the natural frequency in units of rad/sec, although it is often referred to in its Hertzian equivalent with units of cycles/sec, or Hz.

These parameters are related to the original parameters as follows:

$$\omega_n = \sqrt{\frac{k}{m}},\ \zeta = \frac{b}{2\sqrt{mk}} \tag{3}$$

The structure of the solution to this ODE is well-known and given by:

$$x(t) = x_m e^{-\zeta\omega_n t}\sin(\omega_d t + \phi) \tag{4}$$

with x_m being the amplitude, $\omega_d = \sqrt{1-\zeta^2}\omega_n$ serving as the damped natural frequency, and (x_m, ϕ) are dependent upon the ICs.

This system is to become the basis of an experimental and computation system ID problem whereby all parameters are determined accurately through measurement of $x(t)$. Given this perspective, it is convenient to assume that $\dot{x}(0) = 0$, but $x(0) \neq 0$. Eqn. (4) then simplifies to:

$$x(t) = x_m e^{-\zeta\omega_n t}\cos\left[\omega_d t - \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)\right] \tag{5}$$

and in effect now there are only 3 parameters: x_m, ω_n, ζ.

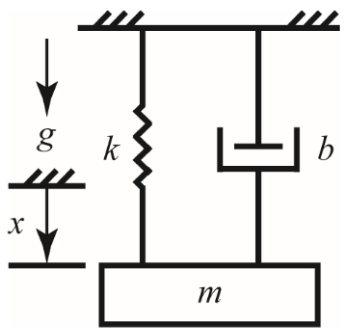


Fig. 1a Translational *mbk* damped oscillator in a gravitational field.

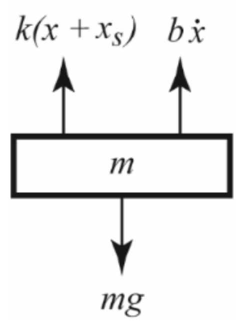


Fig. 1b FBD of mass element introducing the static equilibrium position, x_s.

Essence of lab handout

Objective: To interface and calibrate an infrared proximity sensor, collect data using the myDAQ and LabVIEW for a mass, spring, and damper system (also called a damped oscillator), and through adjustment of modeling parameters in a linear simulation model in MATLAB, achieve a close comparison with experimental data. Critical equipment and set-up activities pertain to:

- Infrared proximity sensor (from SparkFun Electronics [18])
- Wiring and calibration
- Mass, spring, and damper system
- Data acquisition using LabVIEW and myDAQ including wiring

Background Information: Data sheet from SHARP, wiring diagram from SparkFun, sample MATLAB code (M-file), electronic toolkit contents, LabVIEW intro reference book, and notes on mass, spring, and damper solution.

Grading (10 points, verified by instructor): Preparation and submission of a single PDF file to Canvas containing: (1) a quality, tightly cropped picture (such as from your phone's camera) of the myDAQ and proximity sensor, properly wired, (2) a calibration plot harvested from MATLAB relating voltage to distance, (3) your final MATLAB code, (4) a plot harvested from MATLAB that compares theory with experimental results, and (5) explicitly state all 5 relevant parameter values (i.e. m, b, and k, also ω_n, ζ) with proper physical units. In the process you will truncate the data as necessary and numerically iterate to achieve good correspondence (with instructor support).

Here is the suggested step-by-step sequence (may need to repeat for more refined results):

1. Truncate data using a starting array index so that you are starting at a peak (either + or -)
2. Center simulation data with respect to (WRT) experimental data vertically by changing the height h WRT to a reference.
3. Adjust the amplitude x_m to get close to actual data
4. Adjust the natural frequency ω_n by changing its Hertzian frequency
5. Adjust the damping ratio ζ

Please keep track of how many iterations were required (per step number) to achieve good visual correspondence between theory and experimental data. Practically, this means a sufficient number of full oscillations (typically between 4 and 10) where the overlaid plots of both experimental and theoretical/simulation results are very close. This information can be included with your PDF submission. An optional alternative to steps 4 and 5 is to use the "logarithmic decrement" formula (see Appendix, [19]) to establish ζ, and from $\omega_d = \sqrt{1-\zeta^2}\omega_n$ with ω_d effectively measured, ω_n can be determined.

Lab set-up

Figure 2 shows the overall lab set-up and a close-up of the myDAQ with associated wiring to the proximity sensor. The damped oscillator is practical, low-cost, and comprised of a resistance exercise cord and a bocce ball, all supported by a custom frame. From the student's point of view, it is roughly human-sized (making it relatable) and exhibits low damping so that many large oscillations are made possible, making it visually engaging on a lab bench, especially for a small team of students (typically 4).

Experimental procedure details

Key activities are as follows:

- Wiring
- Static calibration of proximity sensor
- Use of LabVIEW to collect dynamic data and import into MATLAB
- Implementation of simple iteration procedure to solve system ID problem

<u>Wiring:</u> For this set-up, the wiring (as shown in Fig. 2b) is straight-forward, given that students have previously learned the basics of LabVIEW and how to acquire analog signals using the myDAQ. That said, wiring errors are not uncommon and inspections are required by the instructor.

<u>Static calibration of proximity sensor:</u> Prior to usage in a dynamics experiment, the proximity sensor needs to be calibrated because as a default, the LabVIEW/myDAQ/proximity sensor set-up measures a voltage directly, but not a distance. The team needs to collect a few data pairs of (voltage, distance) which includes using a yardstick to measure the vertical location. Note that the calibration envelope needs to overlap the oscillation envelope somewhat, but not excessively.

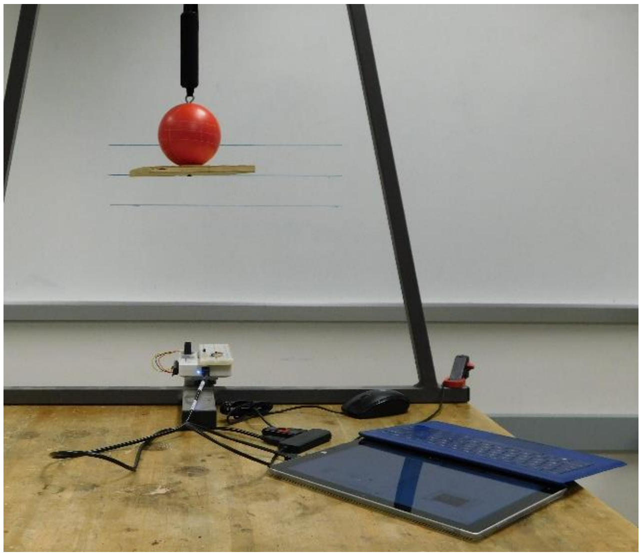


Fig. 2a System level lab set-up of damped oscillator, complete with proximity sensor, wiring, myDAQ, laptop, and support frame.

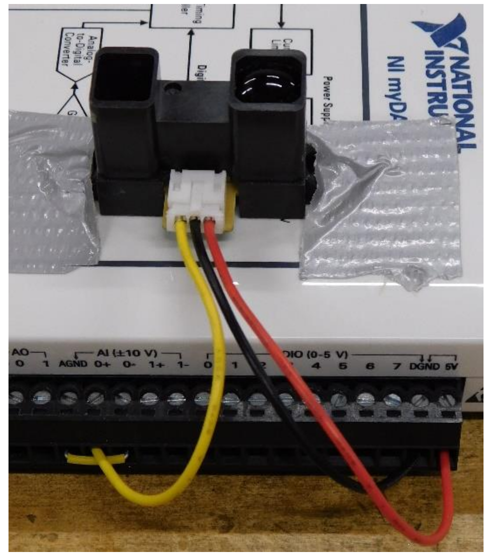


Fig. 2b myDAQ wiring to SparkFun infrared proximity sensor.

This activity works well in a team setting and there is an opportunity for students to showcase and refine their hands-on lab skills since some practical skills are needed. In the end, a simple plot is generated from MATLAB and the data pairs can be used later in the MATLAB dynamics code, supporting spline interpolation in general. On occasion, students will need to redo this activity if strange results occur that differ from that found in the proximity sensor's datasheet [18].

<u>Use of LabVIEW to collect dynamic data and import into MATLAB:</u> Again, this is a team activity, involving some timing, coordination, and data transfer, given use of several VIs from previous labs (conveniently called AI.vi and AISpreadsheet.vi). Both VIs embed the "express VI" DAQ Assistant with the sampling rate set to 20 Hz, as the natural frequency of the system is around 1 Hz and there aren't really any other frequencies represented. Using the Nyquist Sampling Theorem, a practical sampling frequency to use would be 10 × 2 × 1 Hz = 20 Hz (50 ms period), or perhaps slightly larger. The "spreadsheet" VI outputs a data file that can be imported into MATLAB several different ways (e.g. usage of Excel as a via translation environment works well).

<u>Implementation of simple iteration procedure to solve system ID problem:</u> As alluded to above in the suggested step-by-step sequence, there are 5 simple steps to implement, all of which when viewed as a block set of steps could be repeated several times if need be. The essence of the approach is for the students to make numerical changes in the MATLAB script file and visually review their effect on the resultant plot of experimental and theory/simulation data. No formulas are necessary, unlike that mentioned in [19] and many other sources. To be clear, the method is not an isolated "guess-and-check" approach. Rather, it is a "guess (how much?)-and-check" approach guided by prior knowledge of the analytical solution for the model and an understanding of the associated parameters, including their effect on the solution. That said, a generic, isolated guess-and-check method could eventually work, although it would be irresponsible to promote such an approach, as that would be at the expense of students missing out on the kernel of the lab-based system ID learning experience. This semi-quantitative approach is especially appealing to students who do not possess strong math backgrounds, thereby making experimental study of the

damped oscillator far more accessible to many students. Also, regardless of the student's math background, worthwhile learning takes place while visually and numerically iterating as it forces the student teams to look at the plots and ask themselves questions like "Should the height be adjusted up or down, and by how much?," "Should the amplitude be adjusted up or down and by how much?," "Is the frequency too low or too high?," "Is more or less damping needed?," vs. using a separate formula. Implicit in this activity, which involves a modest amount of iteration per parameter, is reinforcement, repetition, and critical thinking, well-known and appreciated pedagogy concepts. Upon completion of this system ID lab, students will have increased their level of understanding of the various system parameters and their effect on the system.

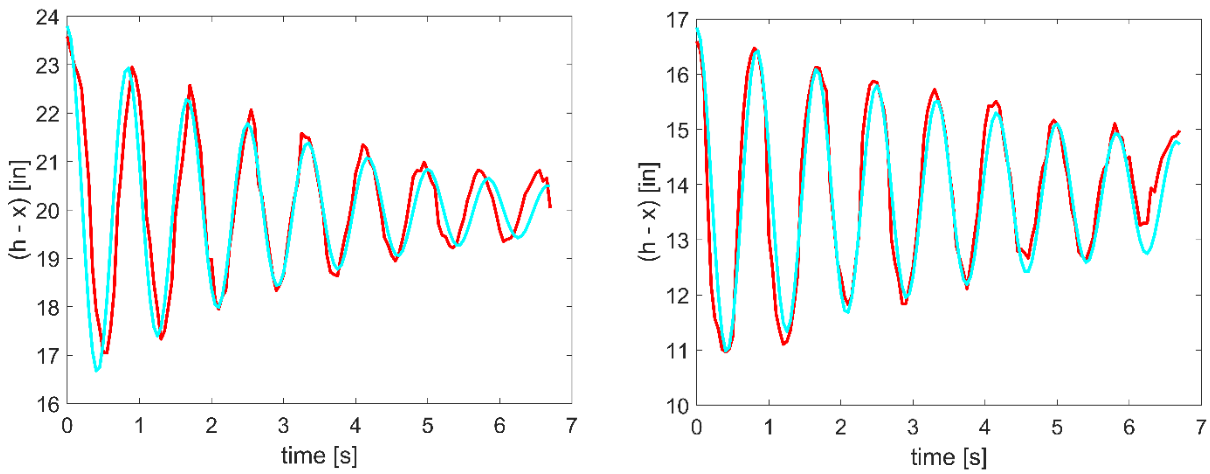
The first step in the process is to identify where the first peak occurs (be it "+" or "-") in the data as characterized by a 1D array index and changing this parameter in the code (just once should be the standard, so no iteration is required). The second step is to effectively vertically center the simulation data WRT the experimental data (guess-and-check). The third step entails adjusting the amplitude of the oscillation so that the first peaks (theory/simulation, experimental) are in close agreement. The fourth step adjusts the natural frequency and this can be determined accurately as typically at least four or so oscillations are plotted. Lastly, the fifth step adjusts the damping ratio. Ideally, after these 5 basic steps the process is complete and all parameters that dictate solution to the system ID problem have been determined (to be fair we assume use of a scale to measure the mass). Assuming reasonably good experimental data as a starting point, this is a realistic expectation, or certainly when one or two more trips through the block-step process are completed to further refine the match of theory/simulation data with experimental data.

Student results

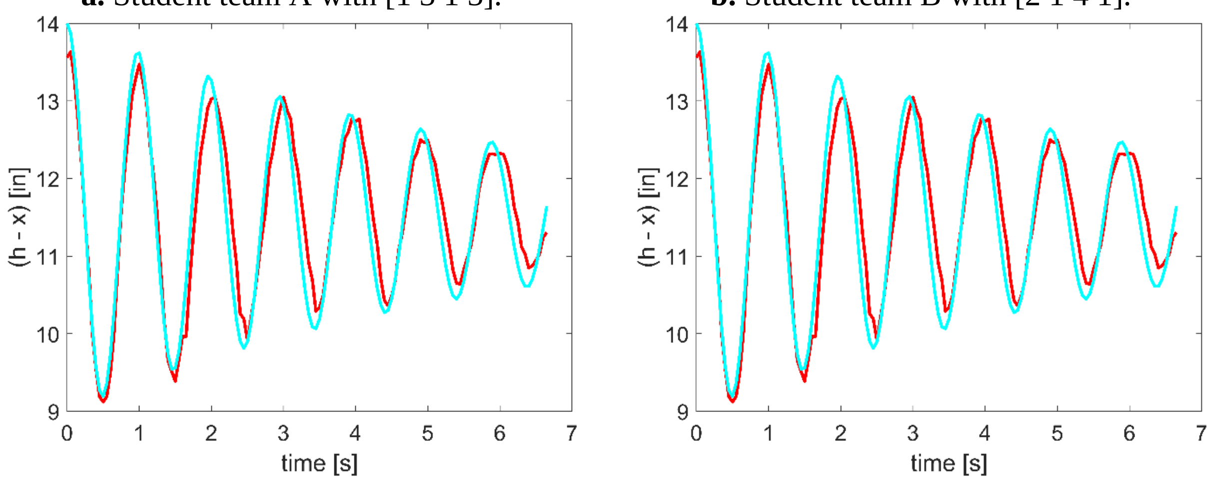
With proper wiring and static calibration of the proximity sensor established, the most significant results are: (1) time-based displacement plots generated from MATLAB that permit comparison of experimental data and theory/simulation data, (2) system ID parameter iteration counts (as in how many different values were tried for a given parameter) for the last 4 steps, and (3) numerical values for all of the 5 parameters. Figure 3 presents sample student work of experimental (red) and theory/simulation (cyan) displacement plots $h - x(t)$ along with a 1×4 "iteration row vector" that keeps track of the total number of iterations per step for steps 2-5.

Critique of student work

Visually, for the narrow linewidth selected (default) in the plots within Fig. 3, in each case the experimental curve and theory/simulation curve are in very close agreement for many oscillation cycles (i.e. about 8). Often the agreement is so close that for a sequence of relatively large time intervals they appear to overlap, which can be impressive. For the student work presented, the iteration row vector average is [3.0 4.3 3.5 4.3]. The average total number of steps, including within multiple block-step iterations is 15.0. Collectively and qualitatively, not many iterations are required for what appears to be very good curve fitting, that implies numerical answers to the system ID problem directly within MATLAB. As a side story, one positive effect that often occurs in this lab is a fun and friendly competition amongst the teams to see which team's result overlaps their experimental data the most. All in the name of oscillator dynamics, where students learn practically about concepts like natural frequency and the damping ratio!



a. Student team A with [1 3 1 3]. **b.** Student team B with [2 1 4 1].



c. Student team C with [3 4 4 6]. **d.** Student team D with [6 9 5 7].

Fig. 3 Experimental (red) and theory/simulation (cyan) displacement plots $h - x(t)$ that supports comparison in addition to iteration row vector.

Assessment of student's "manual" curve fitting using Excel with The Solver

As shown in Figs. 3a-d, qualitatively speaking, the manually obtained curve fits appear to be in good agreement with the experimental data. Subsequently, the curve fitting was optimized using The Solver in Excel [20]. An example optimal fit is shown in Fig. 4 for data set C with small improvements obtained. The objective function used was the RMS error and the underlying algorithm was based on the generalized reduced gradient method with unity weighting. Various measures of student curve fit quality WRT optimal are shown as percentages in Table 1. It is worth noting that the natural frequency ω_n and height h were predicted very accurately by the students (i.e. within about 1%). This is likely due to the multiple periods in the signal and the very symmetric distribution of the data above and below the height. On the other hand, the student's judgement of the damping ratio ζ, while still very good, was not as accurate. If their estimate of the damping ratio was too high, they apparently corrected it by choosing a lower amplitude x_m and vice versa.

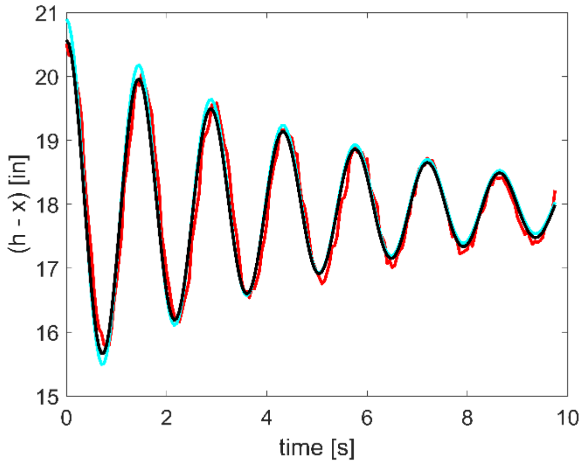


Fig. 4 Experimental (red), manual curve fit (cyan), and optimal (black) displacement plots $h - x(t)$.

Table 1 Student curve fitting WRT optimal based on Excel Solver and expressed as a percentage

	A	B	C	D
Height	0.42	0.58	0.42	0.17
Amplitude	18.33	5.84	3.03	1.14
Natural frequency	0.03	0.73	1.31	0.24
Damping ratio	8.37	5.84	15.93	4.26

Student feedback

Student feedback generally, such as from IDEA [21] over the last 10 academic years, was very good, both in a quantitative and qualitative sense, with many positive comments being made. While this specific lab wasn't referenced, positive comments were made about the timing of the instrumentation labs in that they followed the software-oriented labs, and the physicality and hands-on nature required by labs such as this one.

Lessons learned and continuous improvement

This lab has been taught every fall and spring semester for the last 10 years, going back to Spring Semester 2013. It has remained largely unchanged from its initial offering, although there have been several small refinements, observations, and some helpful advice to offer based on working closely with the students and the set-ups in lab:

- A plate is now attached to the bottom of the bocce ball for larger infrared proximity sensor signature, improved geometric clarity as to the location of object's bottom, and reduced sensitivity to unavoidable lateral movement. Note that aerodynamic drag, which could affect the governing dynamic model, is still quite low due to the relatively slow speeds involved.
- The initial displacement away from the static equilibrium point can't be too large for two reasons: (1) the resistance exercise cord force/displacement profile becomes noticeably nonlinear and (2) there can be time intervals when the cord is not in tension and basically disconnected which then creates a technically different dynamic system than the standard simple damped oscillator we are intending to study
- The static calibration obtained should be of sufficient quality before advancing to the dynamics experiment or strange displacement vs. time plots will result
- Students need to be reminded to minimize their physical presence within the infrared proximity sensing envelope as this will negatively impact their experimental results
- The instrumentation (i.e. myDAQ, infrared proximity sensor) has been robust and there have been very few failures. When a problem is suspected it is almost always a wiring problem.

Rotational Oscillator used as an Inertia Measurement Instrument
Modeling the system under study

A rotational oscillator about a fixed axis (z) through the center of mass (G) is shown in Fig. 5a in which a symmetric rigid body is suspended by two symmetric massless cables and small oscillations about the equilibrium configuration are considered. Support spacing is characterized by a and b and the nominal vertical separation from the ceiling is given by h. Here $\theta(t)$ is the time-varying angular displacement of the rigid body about the z-axis and $|\theta| \ll 1$ rad to support development of a linear model (i.e. with $\cos\theta \cong 1, \sin\theta \cong \theta$) and we are also ignoring any damping as experimentally it is typically small by design. Observe that this system differs slightly from a commonly used configuration in that in general $a \neq b$ (e.g. see [19]). In the end we seek the governing linear, second-order ODE. To begin the process, from analytic geometry we observe that:

$$\sec\beta = \frac{\sqrt{(b-a)^2+h^2}}{h} \tag{6}$$

where β is the cable's equilibrium deviation angle from vertical. It can be shown that by linearizing the square of the distance between points (A,C), $d^2(A,B)$ and setting it equal to $d^2(A,C)$, $\delta z = 0$, so that only planar-type rotational movements of the body result. A static vertical force balance then yields:

$$T = \|\boldsymbol{T}_1\| = \|\boldsymbol{T}_2\| = \frac{mg}{2}\sec\beta \tag{7}$$

Next, we seek expressions for $\boldsymbol{F}_1, \boldsymbol{F}_2$ given by the product of the tension (the same for each by symmetry) and the appropriate unit vector, so that:

$$\boldsymbol{F}_1 = F\frac{[(b-a\cos\theta)\boldsymbol{i}+a\sin\theta\boldsymbol{j}+h\boldsymbol{k}]}{\sqrt{(b-a\cos\theta)^2+a^2(\sin\theta)^2+h^2}},\ \ \boldsymbol{F}_2 = F\frac{[(-b+a\cos\theta)\boldsymbol{i}-a\sin\theta\boldsymbol{j}+h\boldsymbol{k}]}{\sqrt{(b-a\cos\theta)^2+a^2(\sin\theta)^2+h^2}} \tag{8}$$

The above expression can be linearized to establish that:

$$F = T = \frac{mg}{2}\frac{\sqrt{(b-a)^2+h^2}}{h} \tag{9}$$

with:

$$\boldsymbol{F}_1 \cong F\frac{[(b-a)\boldsymbol{i}+a\theta\boldsymbol{j}+h\boldsymbol{k}]}{\sqrt{(b-a)^2+h^2}},\ \ \boldsymbol{F}_2 \cong F\frac{[(-b+a)\boldsymbol{i}-a\theta\boldsymbol{j}+h\boldsymbol{k}]}{\sqrt{(b-a)^2+h^2}} \tag{10}$$

Finally, from Fig. 5b, taking advantage of symmetry and the rotational analog of Newton's Second Law (i.e. $\sum M_O = J\ddot{\theta}$), it follows that:

$$(-\boldsymbol{r}_1 \times \boldsymbol{F}_1 - \boldsymbol{r}_2 \times \boldsymbol{F}_2)\cdot\ \boldsymbol{k} = J\ddot{\theta} \tag{11}$$

which simplifies to the linear second-order ODE with constant coefficients:

$$\ddot{\theta} + \left(\frac{mgab}{Jh}\right)\theta = 0 \tag{12}$$

Here we recognize this equation as being equivalent to the familiar oscillator ODE:

$$\ddot{\theta} + \omega^2 \theta = 0 \tag{13}$$

where $\omega = 2\pi/T$ is the constant natural frequency and T is the period of oscillation (vs. magnitude from above). For a well-posed problem, thought of as an initial value problem (IVP) we need two ICs: $\theta(0), \dot{\theta}(0)$. From an experimental point of view it is convenient to set $\dot{\theta}(0) = 0$ but $\theta(0) \neq 0$ so as to induce free oscillations. Observe that with several easily pre-measured parameters established (i.e. m, g, a, b, h) along with the measured Hertzian natural frequency $f = 1/T$, the inertia can be determined from the simple calculation:

$$J = \frac{mgabT^2}{4\pi^2 h} \tag{14}$$

thereby forming the basis of an inertia instrument. Lastly, as an aside, we note that with $b = a$, the above expression simplifies to that reported in [19] for the case of nominally vertical cables (i.e. at equilibrium).

Essence of project handout

<u>Objective:</u> Gain and demonstrate expertise in dynamic modeling, simulation, visualization, and experimentation through studying the dynamics of a specific system. In the process, students will:

- Develop the differential equation of motion associated with a rotary inertial element suspended from two cables (i.e. finish off instructor's partial derivation of ODE)
- Conduct simple fabrication and experimental work and calculate the mass moment of inertia in addition to comparing this value with the previously known value, such as from a table or textbook
- Create a hand-drawn simulation model, enter it into Simulink, and simulate angular displacement vs. time
- Create a graphical model of the system using SolidWorks
- Prepare an abbreviated technical report

n

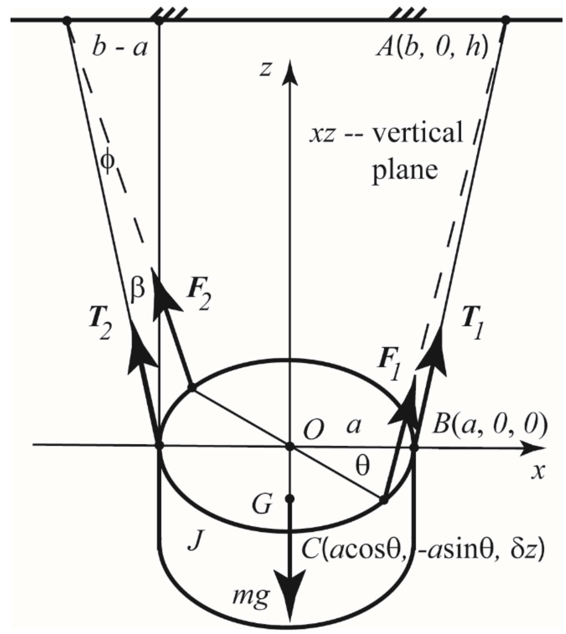


Fig. 5a Rotational oscillator in a gravitational field employing cable (×2) support.

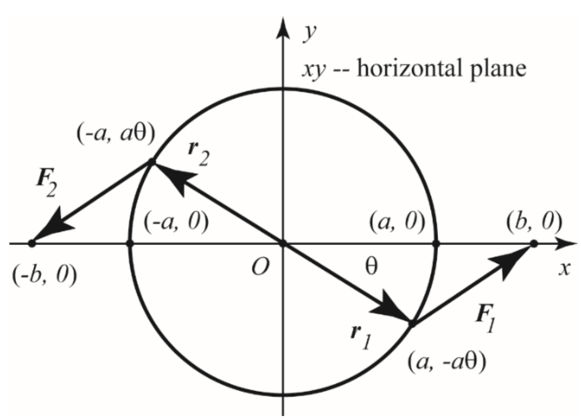


Fig. 5b Projected view onto xy plane of FBD of mass element when $\theta \neq 0$.

Report Contents and Deliverables:

- Title page (5 %)
- Abstract (5 %)
- Derivation of key ODE (20 %)
- Formula for J, the mass moment of inertia (10 %)
- Experimental set-up (20 %)
- Comparison with reference answer (20 %)
- Simulink model (10 %)
- MATLAB plots (5 %)
- Graphics (5 %)

Student results and critique

After a few instances of providing help related to deriving the relevant ODE, essentially all students were able to complete the above list of tasks. From an instructional point of view, the most interesting aspects concerned their experimental set-up and comparison with the reference answer. Figure 6 displays two samples of student work, including a picture of their fabricated "inertia instrument" and the impressive percent error obtained for the first one, which was largely fabricated using 3D printing technology. Typically, the percent error achieved was about 20 %. Some common issues that impacted the accuracy of the results included:

- Choice of cable material, thin and inextensible is preferred
- Choice of object, uniform mass density and a simple symmetric shape is preferred
- Amplitude of oscillations should be small
- Aspect ratio b/a should not be too large or too small, e.g. certainly $0.5 < b/a < 2$ works well and doesn't degenerate into $b = 0$ or make b too large which has the effect of increasing the natural frequency significantly and inducing non-rotational motion
- The ground/cable support points can't move during the oscillation

a. Student A's set-up achieving 0.4 % error when $a = b$.

b. Student B's set-up.

Fig. 6 A sample of student's inertia instruments.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Conclusions

After 10 years of usage, with some very minor improvements along the way, the translational oscillator lab has proven to be an effective lab experience for a wide variety of mechanical engineering students, especially those that appreciate hands-on activities and/or may not possess a strong math background. Over the past 4 years the "rotational oscillator" has shown significant potential as the basis for a final project (in lecture) that integrates a spectrum of useful activities, from theory, simulation, design, basic manufacturing, to measurement, where the inertia of a rigid body can be measured.

Acknowledgements

The authors (and instructors) acknowledge the other recent lab instructors (Drs. K. Acton and T. Secord) along with the Dynamics students and their permission to showcase their results.

References

[1] R. C. Hibbeler, *Engineering Mechanics: Statics & Dynamics*, 15th edition, ISBN 9780134780955, Hoboken, NJ, Pearson, 2022.

[2] C. M. Close, D. H. Frederich, and J. C. Newell, *Modeling and Analysis of Dynamic Systems*, 3rd edition, ISBN 0-471-39442-4, New York, John Wiley & Sons, Inc., 2002.

[3] https://www.mathworks.com/

[4] M. P. Hennessey, "Statics and Dynamics Projects Emphasizing Introductory Design and Manufacturing," *ASEE Annual Conference and Exposition, Pittsburgh, PA, USA, June 22, 2008*, pp. 13-1095.

[5] https://www.stthomas.edu/catalog/current/

[6] M. P. Hennessey, A. Naqwi, and G. O'Brien, "Dynamics Lab booklet," School of Engineering, University of St. Thomas, Fall Semester 2022.

[7] W. Durfee, P. Li, and D. Waletzko, "Take-Home Lab Kits for System Dynamics and Controls Course," *Proceedings of the American Control Conference, Boston, MA, USA, June 3-July 2, 2004*.

[8] A. Tekes, K. Van Der Horn, Z. Marr, and C. Tian, "Dynamics, Vibrations and Control Lab Equipment Design," *Proceedings of the ASME Dynamic Systems and Control Conference, Atlanta, GA, USA, September 30-October 3, 2018*.

[9] https://blogs.solidworks.com/tech/2020/01/introduction-to-solidworks-simulation-finite-element-analysis.html

[10] https://www.ni.com/en-us.html

[11] K. Bowen, M. O'Malley, and W. Marsh, "Haptic Interfaces for a LabVIEW-based System Dynamics Course," *ASEE Annual Conference and Exposition, 2627, June 18-21, 2006*.

[12] W. Semke, "Basic Vibration Analysis in a Laboratory Classroom Using Virtual Instruments," *Proceedings of the 39th IMAC, A Conference and Exposition on Structural Dynamics, V 7, February 8-11, 2021*, pp. 235-244.

[13] B. T. Burchett, "Parametric Time Domain System Identification of a Mass-Spring-Damper System," *ASEE Annual Conference and Exposition, June 12-15, 2005*, pp. 11147-11157.

[14] D. M. Ellett, "How to determine mass moments of inertia for irregular parts," *Product Engineering*, v 28, n 5, p. 184-187, May 1957.

[15] H. R. Angel, "Mass Moment of Inertia," *Machine Design*, v 40, n 12, p. 200, May 23, 1968.

[16] Y. Zheng, M. Lin, and B. Guo, "Instrument for measuring moment of inertia with high precision," *Proceedings of SPIE - The International Society for Optical Engineering, v 7544, 2010, Sixth International Symposium on Precision Engineering Measurements and Instrumentation*, ISSN: 0277786X, ISBN-13: 9780819479402, DOI: 10.1117/12.885551, Article number: 75441G.

[17] G. Previati and F. Ballo, "Trifilar pendulum: Non-small oscillations and calibration," *78th SAWE International Conference on Mass Properties Engineering, May 18, 2019*.

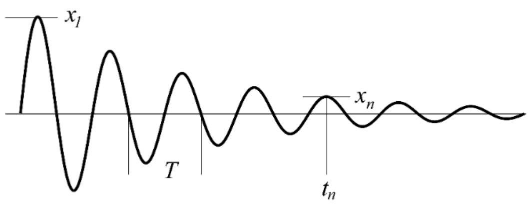
[18] https://www.sparkfun.com/datasheets/Sensors/Infrared/gp2y0a02yk_e.pdf

[19] K. Ogata, *System Dynamics*, 3rd edition, ISBN 0-13-6785745-6, Upper Saddle River, NJ, Prentice-Hall, 1998.

[20] D. Fylstra, L. Lasdon, J. Watson, and A. Warren, "Design and Use of the Microsoft Excel Solver," *Interfaces*, INFORMS, Vol. 28, No. 5, pp. 29-55, Sep. – Oct., 1998.

[21] https://www.ideaedu.org/

Appendix Logarithmic Decrement Damping Ratio Formula [19]



$$\zeta = \frac{\frac{1}{n-1}\left(\ln\frac{x_1}{x_n}\right)}{\sqrt{4\pi^2 + \left[\frac{1}{n-1}\left(\ln\frac{x_1}{x_n}\right)\right]^2}}$$

Fig A. Conceptual experimental data plot from second-order system and damping ratio formula.